Dynamic Behaviour of Soil Supported Foundations

Le Comportement Dynamique des Fondations sur le Sol

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SYNOPSIS This paper discusses the results of experimental and theoretical investigations of the foundations resting on soils. The experimental work describes the dynamic behaviour of model foundations consisting of blocks and pile foundations and enumerates the parameters which influence the natural frequency and amplitude of motion.

The theoretical investigations are based on the mathematical models of soil-supported foundations generated in the computer. Vertical, horizontal and rocking modes of vibration of these foundations have been analysed and simple to use Design Charts have been prepared for blocks having different possible shapes of bases and plinths.

1. INTRODUCTION

The problem of dynamic behaviour of foundations supported on soils is of interest in the design of machine foundations and resistance of buildings to earthquakes or blasts. The need to develop effective and economical design for foundations subjected to out-of-balance forces has been on the increase in recent years. The design considerations include the dynamic behaviour of the foundation soil system, particularly its natural frequency and the amplitude of motion.

Extensive work has been done on determining the behaviour of block foundations resting on soil. Of particular mention are Richart (1962) and Whitman & Richart (1967) who have analyzed different modes of vibration of a soil-supported block, their approach being, mainly, to take a lumped mass-spring-dashpot system. Thus the mass of the foundation, stiffness of the support and the damping of the system essentially define the parameters required for dynamic analysis of foundations.

In this paper, an attempt has been made to supplement some of the work done already by experimental observations. Behaviour of pile foundations under dynamic loading has been studied by model foundations. Model solid blocks with plinths have also been vibrated and the effect of embedment of the base investigated.

Some theoretical aspects of the design of foundations subjected to out-of-balance forces are discussed. Design Charts, based on a number of stepped-block configurations generated in the Computer, are presented for easier calculations of natural frequency in the vertical, horizontal and rocking modes of vibration.

2. EXPERIMENTAL INVESTIGATIONS

Experimental investigations were based on the behaviour of model footings resting on a sand bed and subjected to steady state vibration. There were a number of model foundations of various sizes, shapes and weights yielding different test systems.

2.1 SAND TANK & LOADING FRAME

A sand tank was fabricated using metal sheets with inside dimensions of 1.35 m x 1.35 m x 1.2 m high. The walls of the tank were adequately reinforced to ensure adequate resistance to the lateral earth pressure.

The soil used was fine to medium RAVI Sand (75 % retained between B.S. Sieves No.7 and No.100) and was compacted in the tank in eight layers of 15 cm thickness each using a vibratory compactor. Density tests performed during and after the compaction process indicated a density of 105 pcf. The shear strength parameters of the sand at this density were cohesion = 90 psf and angle of internal friction=32°.

The loading frame was designed so that its resonant frequency did not lie within the predicted testing range and the structure was made as rigid as possible. The loading frame consisted of vertical columns and a cross beam to which the vibrator was attached. The height of the vibrator in relation to the footing was adjustable as required.

2.2 MODEL FOUNDATIONS

The model foundations consisted of three different types :

i) Steel plates of different sizes.

ii) Steel piles of different diameters and lengths with the steel plates in (i) serving as pile caps.

iii) Concrete blocks (Base and plinth Combinations).

Block Diagram of the experimental setup is shown in Fig:1.

3. EXPERIMENTAL RESULTS

This section describes the results obtained by performing dynamic tests on pile foundations and stepped blocks.

3.1 PILE FOUNDATIONS

The changes in natural frequency and resonant amplitude of a model plate-type foundation were studied by adding different piles to the foundation. Variations in the diameter and length of the piles provided a number of test results, a summary of which is given in Table I. This table shows that for all cases of addition of piles, an increase in the natural frequency of the foundation has been observed. In other words, the natural frequency of the system appears to be dominated more by the 'additional' stiffness provided by the piles rather than the additional 'mass' of both the piles and the 'participating' soil mass around them. However, noticeable reduction in the amplitudes of resonant motion were observed for different cases and a reduction of more than 50 % was recorded by adding four 45 cm long piles of 1.25 cm diameter to the plate. This reduction is mainly due to the following two factors :

i) Increase in the mass ratio.
ii) Increase in the damping.

Increase in the mass ratio of the system is due to the increase in the mass of the foundation by additional mass of the piles. Secondly, the effective mass of the system increases due to embedment effect of the piles which can be explained by considering the effects of skin friction along the piles. During vibration the particles of soil around and beneath the piles also undergo periodic motion causing a greater effective mass of the soil.

Addition of piles also means transmission of more energy away from the foundation by the propagation of siesmic waves. This produces greater radiation damping and hence results in lower amplitudes of motion as has been observed.

TABLE I - EFFECT OF PILES ON THE FREQUENCY & AMPLITUDE OF THE VIBRATING FOOTING AT RESONANCE.

Size of the Footing Plate = 30 x 30 cm
Thickness = 4.75 mm
Frequency at Resonance of the Footing without piles = 85 H_z
Amplitude at Resonance = 47.5×10^{-4} mm
Wt. of the Plate = 3.155 kg

Dia of Piles cm.	Length of Piles cm.	No. of Piles.	Total mass of the foundation system kg	Frequency at Resonance H_z		Amplitude at Resonance mm x 10^{-4}	
				Piles at corners.	Piles at the Centre of sides	Piles at Corners.	Piles at the Centre of sides.
1.25	15	4	3.836	89	89	42.5	45.0
1.25	30	4	4.408	90	90	32.5	33.75
1.25	45	4	5.000	92	92	20.0	22.5
1.785	30	4	5.909	94	92	22.5	25.0
2.5	30	4	8.182	97	97	15.0	16.25
3.75	30	4	13.182	108	108	11.25	15.0
1.25	15	8	4.460	90		22.5	

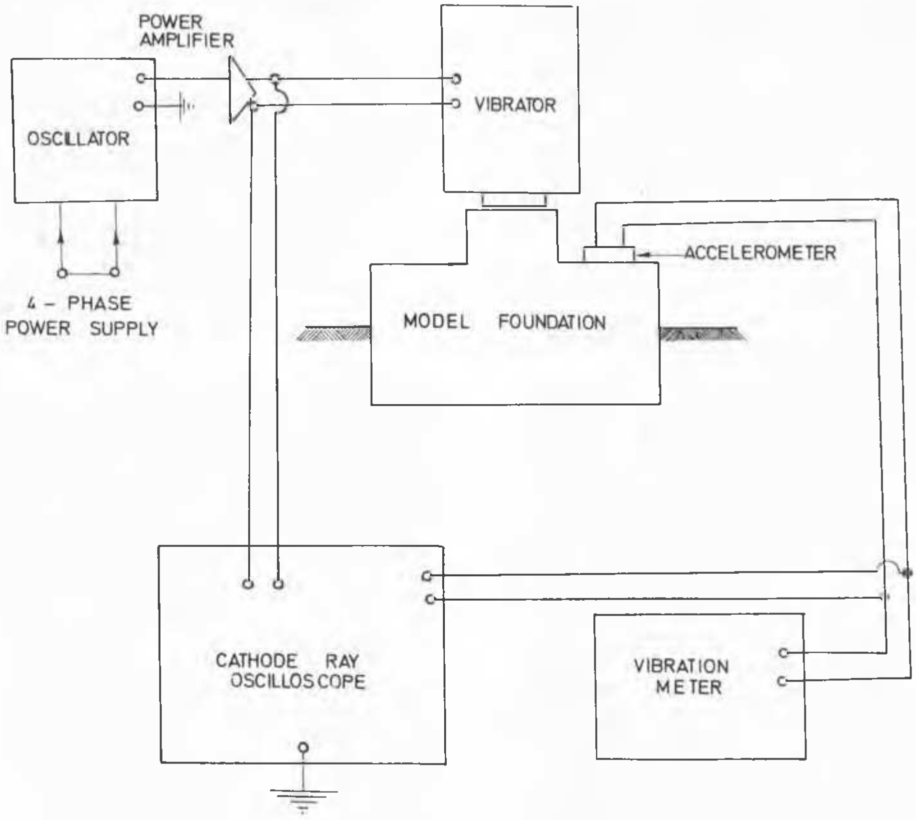

Fig. 1 Block Diagram Showing Instrumentation

Static pile loading tests were carried out on the three 1.25 cm dia piles (15, 30 and 45 cm long) and the stiffnesses of these piles were 4.0×10^2, 5.5×10^2 and 6.75×10^2 kg/cm, respectively. Natural frequencies were calculated taking into account the pile stiffnesses with and without the masses of the piles. Comparison of these results are given in Table II. It is clear from this table that the consideration of just the pile stiffnesses (as given by formulae in Irish & Walker (1969)) yields higher values of natural frequencies and it seems advisable to incorporate the mass of the piles as well.

TABLE II

Pile Length cm.	Natural Frequency H_z		
	Experimental.	Theoretical	
		Without Pile mass.	With Pile mass.
15	89	110.0	99.5
30	90	127.0	108.0
45	92	144.0	114.5

3.2 STEPPED BLOCK FOUNDATIONS

Three block-type model foundations, made of concrete, were subjected to vertical motion to determine the effect of partial and full embedment. Sizes and shapes of these foundations are as under :

Block Foundation.	Base size	Plinth Size
No.1	22.5 x 22.5 x 7.5 cm	--
No.2	22.5 x 22.5 x 7.5 cm	12.5 x 12.5 x 7.5 cm
No.3	22.5 x 22.5 x 15 cm	12.5 x 12.5 x 7.5 cm

The plinths in Blocks 2 and 3 were located at the Centre of the base.

Results of the dynamic tests of these blocks are given in Table III.

TABLE III

Block Foundation No.	Natural Frequency H_z			Resonant Amplitude mm x 10^{-4}		
	Surface	Half Base Embedment	Full Base Embedment	Surface	Half Base Embedment	Full Base Embedment.
1	96	98	100	35.0	30.0	17.5
2	95	--	100	32.5	--	22.5
3	95	97	102	32.5	30.0	15.0

Two observations can be made from the test results indicated in Table above.

Firstly, the effect of ambedment of base, be it embedded partially or fully, results in a slight increase in the natural frequency of the system. Secondly, there is a reduction in the amplitude of motion at resonance. This behaviour can possibly be explained by the fact that the effect of embedment is essentially to increase the resistance to motion of the foundation. Thus the effective spring constant is increased. This is substantiated by FALDJIAN's work (1969) who showed that the spring constant increases with an increase in the depth of embedment. At the same time embedment of a foundation block produces side friction between the walls of the foundation and the soil causing an increase in the effective constant area of the foundation thereby increasing the dissipation of energy in the form of interfacial slip. Increase in the contact area also means an increase in the participating soil mass and so these factors help to lower the amplitude of motion. These observations are supported by Barkan (1962) and Whitman and Richart (1967).

Thus, the above leads to the conclusion that the frictional force due to the embedment influences both the spring stiffness and the damping of the system.

4. THEORETICAL INVESTIGATIONS

This section describes some theoretical work done on the subject of foundation vibrations and may prove helpful to the engineers engaged in the design of foundations subjected to dynamic loads.

4.1 SPRING CONSTANT

Mirza (1971) demonstrated that an increasing value of spring stiffness increases the natural frequency of a foundation but this effect is different on different modes of vibration. The natural frequency registers a maximum increase in the fundamental mode of vibration but there is a progressive reduction of this effect such that the higher modes (5th and above) are hardly influenced by an increase in the stiffness of the supporting medium. Hence, in machine foundation design where lumped parameter approach is preferred and where only one or two degrees of freedom are considered in a particular plane of motion, the spring constant is a critical factor.

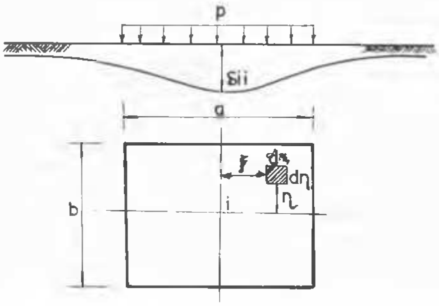

Fig. 2 Vertical Displacement due to unformly loaded Rectangular area on isotropic half space

The commonly adopted formula for evaluating the spring constant of soils is given in Equation (1). This was suggested by Timoschenko and Goodier (1951) and is a constant of proportionately between static load and displacement of a circular plate as determined from the theory of elasticity.

$$K = \frac{4 G r_o}{1 - \upsilon} \qquad \ldots \quad (1)$$

Cheung and Zienkiwicz (1965) have used Equation (2) in evaluating load-deflection behaviour of soil while carrying out the static analysis of plates and tanks on elastic foundations. Equation (2) is called Bonssinesq equation and gives the deflection at the centre of a uniformly loaded rectangular area a x b (Fig. 2).

$$\delta_{ii} = 2\int_{\xi=0}^{\xi=\frac{a}{2}} 2\int_{\eta=0}^{\eta=\frac{b}{2}} \frac{P_i(1-\upsilon^2)}{a\, b\, \pi\, E} \cdot \frac{d\xi \; d\eta}{\sqrt{\xi^2 + \eta^2}} \quad .. \ (2)$$

Values of the spring constant were calculated for various b/a ratios by equations (1) and (2) and their results are compared in Fig. 3. Bonssinesq Equation has been made a reference and the value obtained by equation (1) are shown as a percentage difference. This figure shows that for some cases of rectangular base areas of foundations, equation (1) differs from equation (2) by more than 20 %.

The author suggests a very simple empirical relationship for calculating the spring stiffness which is given in equation (3).

$$K = E\sqrt{A} \quad \ldots \quad (3)$$

This relationship is easy to use and for a number of geometrical base configurations agrees to within 10 % of equation (2). It is suggested that initial guidance to the values of spring stiffness may be obtained by equation (3) and a more detailed and rigorous approach be followed to finalize the value of the spring constant for a particular foundation-soil system.

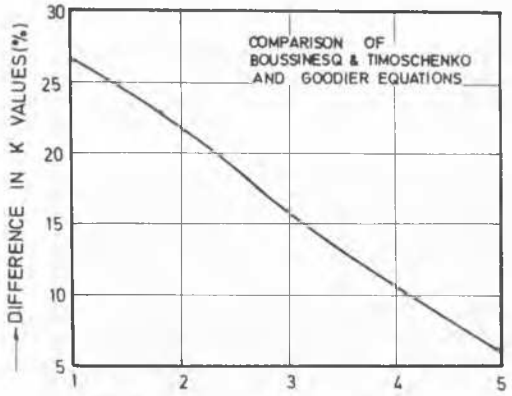

Fig.3

4.2 DESIGN CHARTS

The design of foundations subjected to out-of-balance forces is a trial and error procedure. Initial dimensions are governed by such factors as the dimensions of the machinery or the structure to be supported, the space available for the foundation and the normal load-carrying capacity of the supporting medium.

In order to facilitate the trial designs and to avoid excessive number of trials, design charts for the vertical, horizontal and rocking motion for a stepped block (Fig. 4) have been prepared. These charts are shown in Figs. 5.1 to 5.3 and give frequency parameters which can then be used to calculate the natural frequency by the following equations.

$$f_{nz} = \frac{1}{2\pi}\sqrt{\frac{K_z \cdot g}{\gamma L}} \cdot \lambda_z \quad .. \quad (4)$$

$$f_{nx} = \frac{1}{2\pi}\sqrt{\frac{K_x \cdot g}{\gamma L}} \cdot \lambda_x \quad .. \quad (5)$$

$$(f_{n\emptyset})_{xz} = \frac{1}{2\pi}\sqrt{\frac{K_\emptyset \cdot g}{\gamma L}} \cdot \lambda_{\emptyset x} \quad .. \quad (6)$$

$$(f_{n\emptyset})_{yz} = \frac{1}{2\pi}\sqrt{\frac{K_\emptyset \cdot g}{\gamma L}} \cdot \lambda_{\emptyset y} \quad .. \quad (7)$$

Thus for a given operating frequency of the machine, for the known soil and foundation block material properties and by fixing the length of the foundation, a suitable frequency parameter can be calculated from the equations given above. (The natural frequency of the block should preferably be 30 to 50 % above or below the operating frequency). The design charts in Figs.5.1 to 5.3 can then be used to get the other dimensions of the foundation block by selecting suitable length-breadth and breadth-depth ratios.

It is to be noted that these charts can also be used to find out the frequency parameters of foundation blocks whose dimensions are already fixed (providing these dimensions fall within the ranges of α and β given) and then the natural frequency can be calculated from equations (4) to (7).

CONCLUSIONS

The experimental work described in the paper relates to pile foundations and blocks with plinths. It is shown that addition of piles to plate type foundation results in an increase in the natural frequency of the system and a reduction in the amplitude of motion. Inclusion of mass of the piles to the mass of the foundation produces better agreement between the theory and the experiment. The effect of embedment of a block foundation results in a slight increase in the natural frequency of the system and a reduction in the amplitude of motion. Thus the embedment of a block increases the spring stiffness as well as the dissipation of energy.

In the theoretical work discussed in the paper, the choice of a suitable spring constant is discussed and a simple empirical relationship is suggested. Design, Charts, for various shapes and sizes of 'stepped' block foundations are presented to facilitate the calculation of natural frequency for vertical, horizontal and rocking modes of vibration.

NOTATION

G	Shear modulus
r_o	Equivalent Radius
K, K_x, K_z, $K_\emptyset$	Spring stiffnesses
δ_{ii}	Deflection at the Centre
a, b	length and breadth of the loaded area
ν	Poisson's ratio
A	Base area
E	Modulus of elasticity
γ	Unit weight foundation block material
L	Length of the block

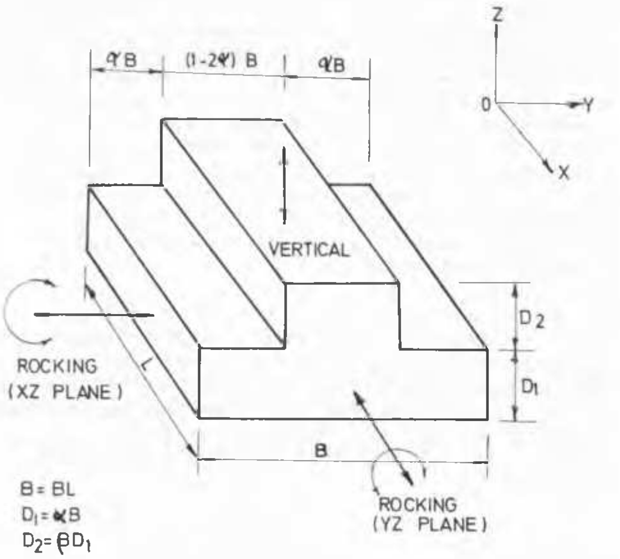

Fig. 4 Stepped block configuration for Natural Frequency Calcalations.

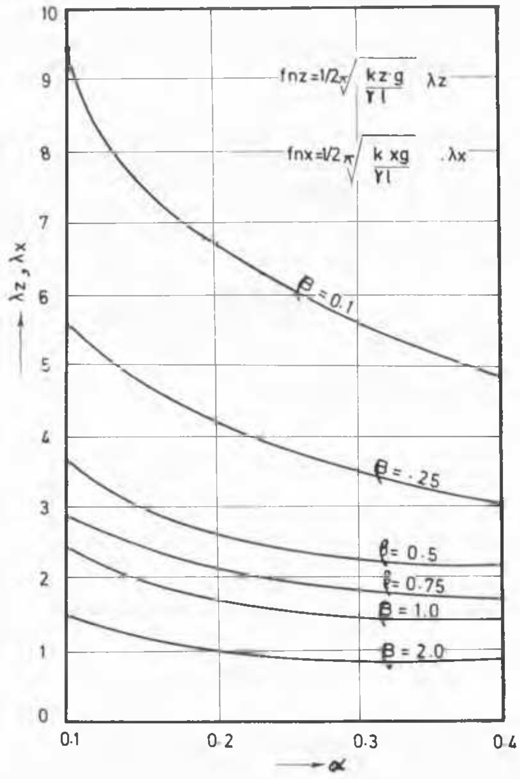

Fig. 5.1 Frequency Parameter for Vertical and Horizontal motion.

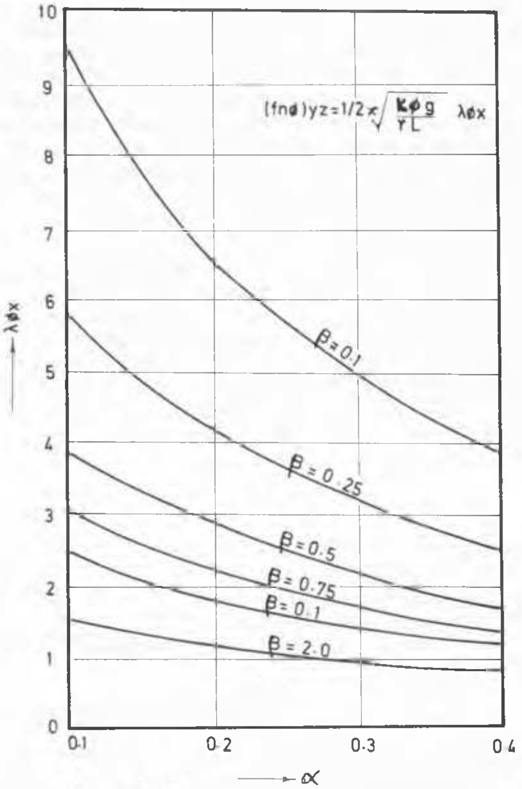

Fig. 5.2 Frequency Parameter for rocking motion in YZ Plane.

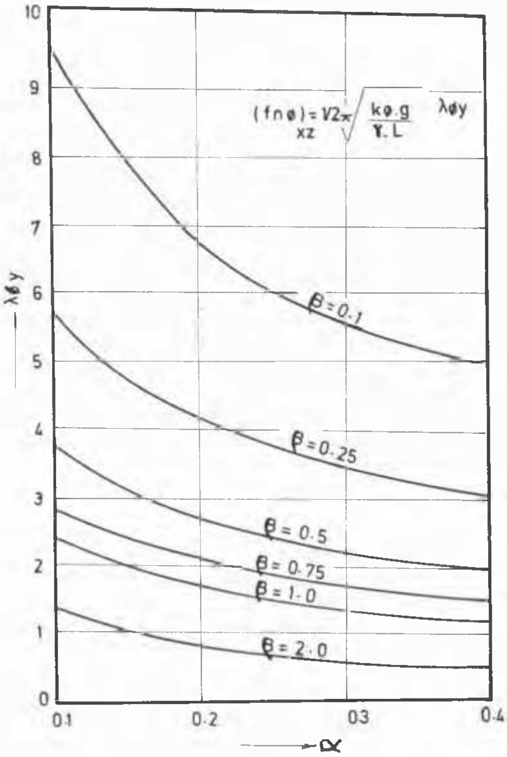

Fig. 5.3 Frequency Parameter for rocking motion in XZ Plane.

f_n Natural frequency

$\lambda_x, \lambda_z, \lambda_\phi$ Frequency Parameters

REFERENCES

Akhtar, M. (1976), "Dynamic Behaviour of Soil-Supported Foundation", M.Sc. Thesis, Department of Civil Engineering, University of Engineering & Technology, Lahore.

Barkan, D.D. (1962), "Dynamics of Bases and Foundation", translated from the Russian by L. Drashevska, McGraw-Hill, Book Co., New Rork.

Chae, Y.S. (1969), "Vibration of Non-Circular Foundations", Journal of the Soil Mechanics and Foundations Division, ASCE, Vol.95, No. SM6, Proc. Paper 6918, pp. 1411-1430.

Cheung, Y.K. and Zienkiewicz O.C. (1965), "Plates and Tanks on Elastic Foundation - An Application of Finite Element Method", Int. Journal Solid Structures, 1965, Vol.1, pp. 451-461.

Irish, K. and Walker, W.P. (1969), "Foundations for Reciprocating Machines", Concrete Publications Limited, 60 Buckingham Gate, London.

Kaldjian, M.J. (1969), Discussion of "Design Procedures for Dynamically loaded Foundations", Journal of Soil Mechanics and Foundations Division, ASCE, Vol.95, No. SM1, pp. 364-366.

Lee, S.S. (1971), "Effect of Embedment on Response of Vibrating Footings", A.I.T., Bengkok, Thesis No. 402.

Lysmer, J, and Richart F.E. (1966), "Dynamic Response of Footings to Vertical Loading", Journal of the Soil Mechanics and Foundations Division, ASCE, Vol. 92, No. SM1, Proc. Paper 4592, Jan., pp. 65-91.

Mirza, W.H. (1971), "Determination of Free Vibration Characteristics of Foundations Resting on Elastic Supports", Proc. of the Symposium on Soil-Structure Interaction, Birmingham University, U.K., July 1971, pp. 174-182.

Richart F.E. (1962), "Foundation Vibrations", Transanction, ASCE, Vol. 127, Part I, pp. 863-898.

Richart F.E. and Whitman R.V. (1967), "Comparison of Footing Vibration Tests with Theory", Journal of the Soil Mechanics and Foundations Division, ASCE, Vol. 92, No. SM6, Proc. Paper 5568, Nov., pp. 143-168.

Timoschenko, S.P. and Goodier, J.N. (1951), "Theory of Elasticity", McGraw-Hill Book Co., Inc., New York.

Whitman, R.V. (1966), "Analysis of Foundation Vibrations", Vibration in Civil Engineering, Butterworth, London.

Whitman, R.V. and Richart, F.E. (1967), "Design Procedure for Dynamically Loaded Foundations", Journal of the Soil Mechanics and Foundations Division, ASCE, Vol. 93, No. SM6, pp. 169-193.