Vertical Vibrations of Embedded Footing

Vibrations Verticales d'un Mur de Fondation Encastrée

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SYNOPSIS It has been recognised that displacement amplitude of embedded footing subjected to vibrations decreases with embedment and is due to the increase in static stiffness of the media. This increase in stiffness of the medium is estimated using Mindlin's theory and is compared with the more rigorous solution obtained by finite element method. A simple modified procedure is suggested to estimate the damping ratio for any embedment ratio and thus to predict the resonant amplitude and resonant frequency. The predicted values are compared with the experimental values found in the literature. The agreement between the predicted and the experimental values is good. The advantage of the method is that it is simple and needs no computer and also takes into account partially the shape of footing. This then may replace the concept of equivalent radius in the case of square and rectangular footings.

INTRODUCTION

The response of embedded footing subjected to vertical, sliding and rocking oscillations is a topic of research in recent years. Rigorous solution is not yet available. Several approximate solutions are available(Novak and Beredugo,1972;Anandakrishnan and Krishnaswamy,1973). Finite element technique has been used by Lysmer and Kuhlemeyer(1969)and by Kaldjian(1969) to obtain solutions. Finite element method needs a computer to obtain solution. Simple approximate solutions are therefore needed for routine jobs. Therefore an attempt is made herein to present an approximate solutions of embedded footing subjected to vertical vibration. It is known that displacement amplitude decreases with embedment and is due to the increase in static stiffness of the medium(Richart et al.,1970). The increase in static stiffness can easily be estimated using the Mindlin theory(1936). Penzien(1970) has employed it to estimate the static stiffness in the study of soil-pile interaction problems. The Mindlin equation for displacement is integrated over a circular and a rectangular area and the spring constant for these loaded areas are evaluated at various depths. These values are compared with the solution of Kaldjian(1969) obtained by using the finite element method and with the approximate solution of Novak and Beredugo(1972). The present solution is found to agree with the Kaldjian solution fairly well and the difference is of the order of 10% only. This is a good agreement when compared to the simplicity of the solution presented. The results are used to predict the response of embedded footing and the predicted values are compared with the experimental values of other investigators. The agreement between the predicted value and the experimental value is good

ANALYSIS

a) Circular footing: If a point vertical load acts within the soil mass at a depth 'c', then according to the Mindlin theory(1936), the vertical displacement due to this is given by

$$w = \frac{P}{16\pi G(1-\mu)}\left[\frac{3-4\mu}{R_1} + \frac{8(1-\mu)^2-(3-4\mu)}{R_2} + \frac{(z-c)^2}{R_1^3} + \frac{(3-4\mu)(z+c)^2-2cz}{R_2^3} + \frac{6cz(z+c)^2}{R_2^5}\right] \quad ..1$$

where $R_1 = (x^2 + y^2 + \overline{z-c}^2)^{\frac{1}{2}}$

$R_2 = (x^2 + y^2 + \overline{z+c}^2)^{\frac{1}{2}}$

μ = Poisson's ratio; G = Shear modulus

Integrating the above equation over a circular area of radius R and letting z = c = H(depth of embedment), one can get displacement of a circular loaded area located at depth c, as

$$w_c = \frac{pR}{8G(1-\mu)}\Big[(3-4\mu) + (5-12\mu+8\mu^2)\cdot\left\{(1+4m^2)^{\frac{1}{2}} - 2m\right\} + (10-16\mu)\left\{m/2 - m^2/(1+4m^2)^{\frac{1}{2}}\right\} + m - 8m^4/(1+4m^2)^{1\frac{1}{2}}\Big] \quad ..2$$

where m = H/R (c = H); p = load intensity

To take into account the rigidity of the footing, Eq.2 is multiplied by a factor $\pi/4$. This factor is the ratio of the surface displacement of a rigid circle on the surface to the displacement at the centre of a corresponding uniformly loaded flexible circle. It is assumed that this ratio apply approximately to the case of circle founded below the surface(Poulos and Davis, 1968). The Eq. 2, therefore reduces to,

$$w_c = \frac{\pi pR}{32G(1-\mu)}\,C' = \frac{P}{32GR(1-\mu)}\,C' \quad ..3$$

where C' refers to terms in bracket in Eq. 2

and P is the total load. The spring constant is then defined as

$$P/w_c = GR/C = k_{zH} \qquad ..4$$

where $1/C = 32(1-\mu)/C'$ and k_{zH} is the spring constant at surface only. In Fig. 1 is shown the variation of the stiffness increase factor k_{zH}/k_z against H/R for $\mu = 0$ to $\frac{1}{2}$

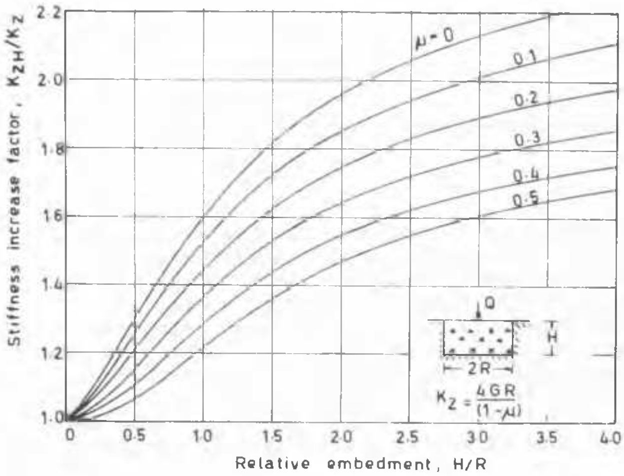

Fig. 1 Stiffness increase factor k_{zH}/k_z Vs. relative embedment, H/R - for circular footing.

b) Rectangular footing: The settlement below the corner of a rectangle of side a x b is obtained by integrating the Eq. 1 with respect to dx and dy and replacing P by p dx dy. The integration yields,

$$w_{cor} = \frac{pa}{16\pi G(1-\mu)}\Bigg[(3-4\mu)\left\{m \ln \frac{S_1+1}{m} + \ln(S_1+m)\right\} + (5-12\mu+8\mu^2)\Bigg\{m \ln \frac{S_2+1}{(m^2+4n^2)^{\frac{1}{2}}} + \ln \frac{S_2+m}{S_3} - 2n \tan^{-1}(m/2n) + 2n \cos^{-1} \frac{(4n^2/\overline{S_2+1}+1)}{S_3}\Bigg\} + \frac{2n^2 m(1+m^2+8n^2)}{S_2 S_3^2 (m^2+4n^2)} + n \sin^{-1} \frac{m}{S_3(m^2+4n^2)^{\frac{1}{2}}} + (5-8\mu)\, n \cos^{-1} \frac{2nS_2}{S_3(m^2+4n^2)^{\frac{1}{2}}}\Bigg] \qquad ..5$$

where $m = b/a$; $n = z/a$; $S_1 = (1+m^2)^{\frac{1}{2}}$; $S_2 = (1+m^2+4n^2)^{\frac{1}{2}}$ and $S_3 = (1+4n^2)^{\frac{1}{2}}$

since $p = P/ab$,

$$w_{cor} = PC'/16\pi Gb(1-\mu) \qquad ..6$$

where C' is the terms in bracket in Eq.5. In order to take into account the rigidity of the footing, the same procedure as used in circular footing is to be adopted. But unlike circular footing, there is no rigorous solution for settlement of a rigid rectangle resting on the surface. The solution is available for flexible footing only. To get the solution for rigid footing, it is assumed that the average settlement of flexible footing is taken as very nearly equal to the settlement of a rigid rectangular footing. This is true for a circular footing (Timoshenko and Goodier, 1951). The average settlement due to uniform pressure distribution over rectangles of various ratios of the sides is given by Timoshenko and Goodier(1951). So the average settlement divided by the settlement at centre of flexible footing should give the correction factor to be used in Eq. 6. The following table gives the correction factors thus calculated.

Table No. I Correction factors for Rectangles of various sizes

m = b/a	1	1.5	2	3	5	10
Correction factor	0.848	0.848	0.849	0.854	0.871	1.04

The spring constant is then calculated as

$$k_{zH} = P/w^*_{cen} = Gb/C \qquad ..7$$

where $1/C = 16\pi(1-\mu)/C'$; $k_{zH}/Gb = 1/C \qquad ..8$

In Figs. 2 & 3, k_{zH}/k_z is plotted again t the embedment ratio H/a for a square and rectangular footings.

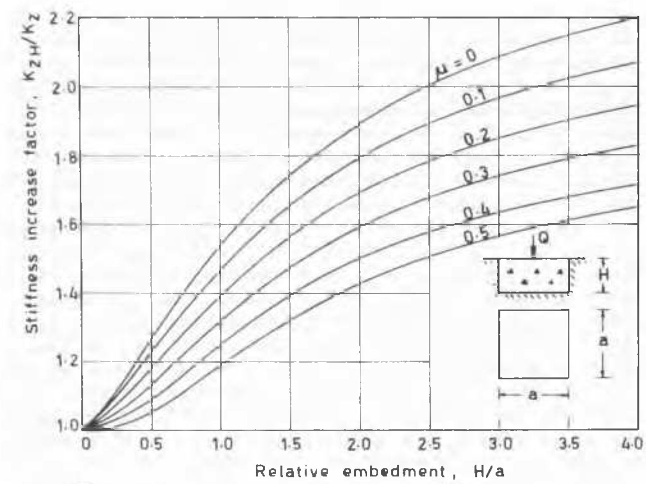

Fig. 2 Stiffness increase factor k_{zH}/k_z Vs relative embedment, H/a - for Square footing

DISCUSSION

i) Variation of Spring Constant with Depth: In Fig. 4, the ratio k_{zH}/k_z is plotted against the ratio H/R and is compared with the more rigorous finite element solution obtained by Kaldjian(1969) and also with the approximate solution of Novak and Beredugo(1972). The variation between the authors's value and Kaldjian value is seen to be less than 10% except in the initial range of the ratio H/R where the difference is of the order of 15%. But it must be pointed out that all available theories overestimates the decrease in amplitudes for small embedment ratio(Novak and Beredugo, 1972). Therefore this difference in this range is expected. The authors's solution when compared with Novak and Beredugo's solution shows reasonably good agreement in the initial

$^*w_{cor} = \frac{1}{2} w_{cen}$

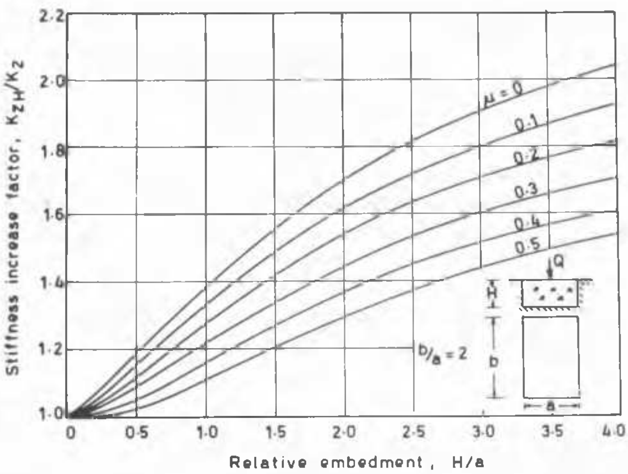

Fig. 3 Stiffness increase factor k_{zH}/k_z Vs relative embedment, H/a - for rectangular footing

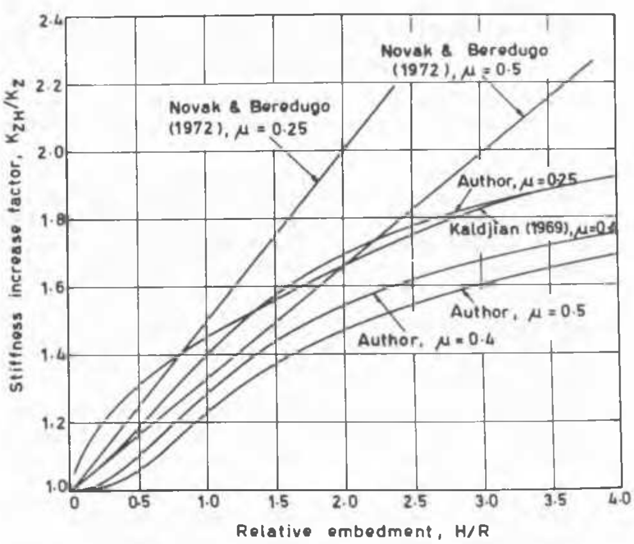

Fig. 4 Comparison with other theories

ranges of the ratio H/R and when this ratio is greater than two, the difference becomes large. At H/R = 2, the difference is about 12%. In all the cases, the authors value is less than that obtained by other solutions.

ii) Damping Coefficients: The damping coefficients is the most eluding parameter to predict. Novak and Beredugo(1972) have given an expression for the damping coefficients in terms of embedment ratio and two stiffness parameter as

$$C_H = (C_1 + SH/R)\ R^2 \sqrt{\rho G} \qquad ..9$$

At surface, ie., H/R = 0, $C = C_1\ R^2 \sqrt{\rho G}$ ..10

where $C_1 = 3.4/(1-\mu)$

C_H = coefficient of damping for any depth of embedment, H

ρ = mass density

G = shear modulus of soil

The critical damping c_c in the case of surface footing is given by

$$c_c = 2\sqrt{k_z\ m} \qquad ..11$$

where k_z is the spring constant of rigid circular footing at surface and is equal to $4GR/(1-\mu)$. The critical damping for any depth of embedment can be written as

$$(c_c)_H = 2\sqrt{k_{zH} m} = \sqrt{k_{zH}/k_z} \cdot c_c \qquad ..12$$

Dividing Eq. 10 by 11, the expression for damping factor at surface is obtained as

$$D = \frac{3.4R^2\sqrt{\rho G}}{2(1-\mu)\sqrt{k_z\ m}} \qquad ..13$$

Dividing Eq. 9 by 12, the damping factor D_H for any embedment is obtained as

$$D_H = \frac{(C_1 + SH/R)\ R^2\sqrt{\rho\ G}}{\sqrt{k_{zH}/k_z}\ .\ 2\sqrt{k_z\ m}} \qquad ..14$$

Finally dividing Eq. 14 by 13

$$D_H/D = \left[1 + \frac{1-\mu}{3.4} \cdot SH/R\right] \Big/ \sqrt{k_{zH}/k_z} \qquad ..15$$

The value of parameter S is determined from a set of experimental results(Anandakrishnan and Krishnaswamy) and an average value of 2.9 is obtained. This value of S is very much different from the constant value suggested by Novak and Beredugo (1972). Further, it is to be pointed out that this value is arrived at after analysing the results of both circular and rectangular footing and in the latter case the concept of equivalent radius is retained. Probably this may not be correct and the parameter S may depend on the shape of footing. Once D_H is determined for different embedment ratio, the resonant amplitude and frequency are calculated easily.

iii) Comparison with Experiments: The predicted values of resonant amplitudes and frequency have been compared with the experimental values of Anandakrishan and Krishnaswamy(1973) and the agreement between the two is very good. Tables showing the comparison of the predicted values and the experimental values are not presented herein due to lack of space but can be found elsewhere(Ramamurthy,1976). The predicted values of resonant amplitude ratio, R_A (defined as the ratio of resonant amplitude for an embedded footing to that of a footing at the surface) and the resonant frequency ratio, R_f are compared with the experimental values of Novak and Beredugo (1972) and is presented in Figs. 5 & 6. It is clear from these figures that the agreement between the predicted value and the experimental values is very close and in any case do not overestimate the decrease in resonant amplitude or increase in resonant frequency. In Table II, the predicted value are compared with experimental values of Fry(1963). The predicted amplitude agree closely with the experimental values whereas the resonant frequency overestimated by a factor of 2. However the resonant frequency ratio shows good agreement between the predicted value and the experimental value.

CONCLUSION

A simple method to predict the response of an embedded footing is presented. The Mindlin theory is used to estimate the spring constant at various depth and is compared with the more rigorous solution of Kaldjian and also with the approximate solution of Novak and Beredugo. A modified

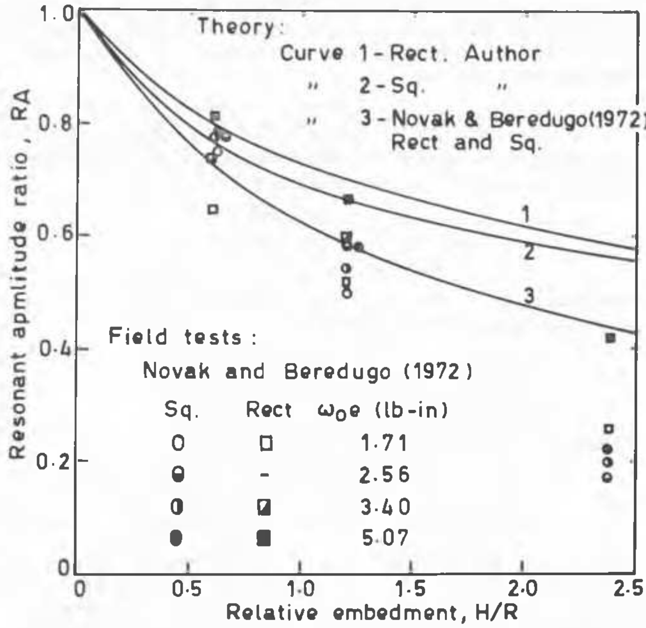

Fig. 5 Comparison of theoretical and experimental resonant amplitudevariations with embedment

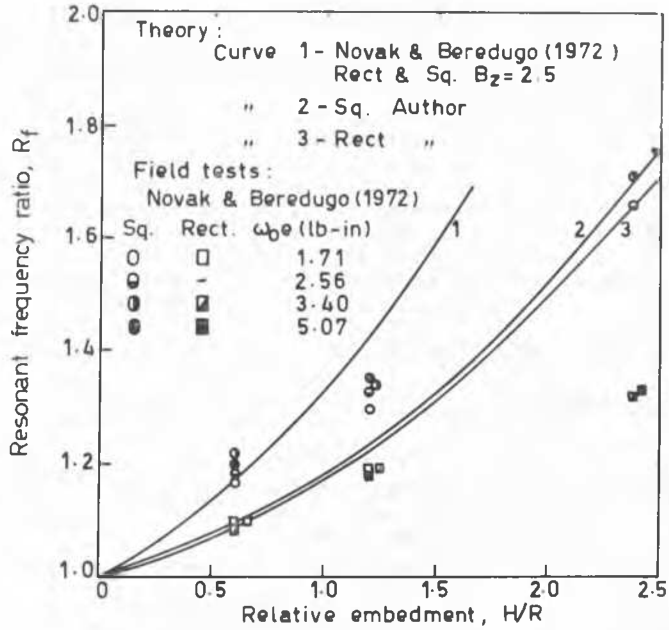

Fig. 6 Comparison of theoretical and experimental resonant frequency variations with embedment

method is suggested to calculate the damping coefficients at various depth. The predicted values of amplitude and frequency is compared with the experimental values and is found to agree well. The advantage of the method is that an attempt is made to take into account the shape of the footing.

Table II- Comparison of Predicted and Experimental Values of Resonant Amplitude and Resonant Frequency
Base Description(Fry,1963); Shape = circular; Dia = 7.30'; Ht. = 2.08'; Wt. = 30971 lbs; Mass ratio = 6.4

m_e in $\frac{lbs}{sec^2}$	H/R	Resonant Amplitde in inch		Resonant frequency in rad/sec	
		Predicted	Experimental	Predicted	Experimental
1.468	0.0	.02411	.0235	166.9	94.0
	0.57	.02093	.0210	217.6	100.2
1.106	0.0	.01816	.0178	166.9	94.0
	0.57	.01577	.0150	217.6	100.2
0.735	0.0	.01207	.0120	166.9	94.0
	0.57	.01048	.0100	217.6	100.5
0.368	0.0	.00604	.0062	166.9	100.0
	0.57	.00524	.0056	217.6	100.8

Note: The values of shear modulus G and Poisson's ratio μ, have been taken from Richart et al (1970) as G = 5340 lbs/in^2 and μ = 0.355

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