Dynamical Behaviour of Piles in Nonlinear Stratified Soil

Comportement Dynamique des Pieux dans un Sol Stratifié Non Linéaire

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SYNOPSIS A suitable form of the general differential equation governing the dynamical behaviour of a pile is used in connection with a non-linear or hysteretic subgrade reaction, in order to compute the bending moments, deflections, natural frecuencies, etc. of piles in homogeneous or stratified ground on a bedrock subjected to seismical vibration of harmonic type. The influence of the end conditions of the pile, the relative percentage of soft or hard layers, and the local plastic yielding of soil along the pile are also investigated.

INTRODUCTION.

Several mathematical models have been dealt with the dynamical behaviour of piles under seismical loading or alternative horizontal forces. Those considering some equivalent cantilever beam, (GRAY (1.964), PRAKASH & SHARMA (1.968)), etc., are at present outpassed by those based upon integration of MINDLIND'S equations (ZABALLOS & LORENTE (1.976)). This integration however cannot be performed when the ground is non elastic or stratified, a very common case.

For this kind of problems very useful tools are the Winkler model, in spite of its limitations, and some more sofisticated elasto-plastic models as used by TUCKER (1.964), PRAKASH (1.973), AGARWAL (1.973), etc.

In this paper a model of this type is described, along with its aplication to the analysis of the behaviour of single piles under seismical loading.

THE MATHEMATICAL MODEL

The differential equation governing the deflections y(z) of a pile under dynamical loading in a soil represented by its horizontal subgrade reaction k(z), is:

$$\frac{\partial}{\partial z^2} EI(z) \frac{\partial^2 y(z,t)}{\partial z^2} + k(z)\, y(z,t) + m(z) \frac{\partial^2 y(z,t)}{\partial t^2} + C(z) \frac{\partial y(z,t)}{\partial t} = p(z) \qquad (1)$$

where:

EI (z) = flexural stiffness of the pile
C (z) = damping factor
m (z) = pile mass per unit length

Eq.(1) is very complicate for application to stratified soil or when k=k(z, y) (elasto-plastic soils) or when some type of loading p (z) exists along the pile shaft. To our knowledge these problems have not yet dealt with in the literature.

In order to perform an easier integration of eq. (1) a very practical way is to consider that, in a general sense, under harmonic type loading of period T, the pile deflections will vary harmonically too. Thus

$$y = y(z)\, e^{\frac{2\pi t}{T} i} = y(z,t)$$

and equation (1) becomes the complex one:

$$\frac{\partial}{\partial z^2} EI(z) \frac{\partial^2 y(z,t)}{\partial z^2} + y(z,t) \cdot \left[k(z) - \left(\frac{2\pi}{T}\right)^2 m(z) + i\, \frac{2\pi}{T}\, C(z) \right] = p(z) \qquad (2)$$

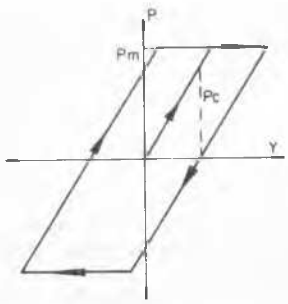
This equation has been integrated by means of polynomical series, under the loading boundary conditions stated below.

<u>Discussion of the parameters</u>

The determination of the dynamical coefficient of horizontal subgrade reaction is beyond the scope of this paper (c.f. ZEEVAERT, 1.972). It has been assumed in the elastic range a constant value along the pile for soft, cohesive soils, and linearly increasing value for granular soils.

The damping effect C(z) of the ground due to viscous reactions and energy dissipation --

through hysteretic stress-softening, is very difficult to introduce in the model. The actual behaviour has been here assumed to follow the simplified bilinear loop of fig.1. - The shape of the loop varies along the pile according to the effective geostatic stresses and the strength properties of each layer.



The viscous component is in fact disregarded but it can be shown that its importance is only noticeable in the vicinity of the natural frecuencies where, in any case, the forces - in the pile are extremely high, even taking damping into account.

For hysteretic behaviour eq. (2) becomes

$$\frac{\partial^2}{\partial z^2} EI(z) \frac{\partial^2 y(z,t)}{\partial z^2} + \qquad (3)$$

$$+ \left[\alpha k(z) - \left(\frac{2\pi}{T}\right)^2 m(z)\right] y(z,t) + p(y) = 0$$

where p(y) is a reaction force corresponding to the deflections of the pile, with P_m being the value of the constant plastic reaction (fig.1) and $\alpha = 1$ or 0 depending on the branch of the loop. No allowance is made for liquefaction or other dynamical effects.

APPLICATIONS

The mathematical model has been applied to - the problem defined in Fig.2. It is assumed that the pile supports a fraction (M=200 t.) of the total weight of a building and that - the pile head is connected to a rigid pile - cap with only possibility of horizontal displacement, without rotations, as it is usually thecase.

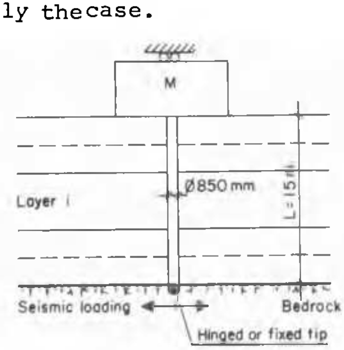


Fig.2. Problem analysed.

The building and the pile are subjected to - inertial forces due to the transmission of - the bedrock vibrations.

According to the records of actual earthquakes maximal displacements of bedrock are typically in the range of 1 cm, with periods - varying from 0,125 to 4 seconds; these ones have been also used for computations.

The following cases have been studied:

a) <u>Influence of the subgrade reaction</u>

As it could be expected, the variation of -- the subgrade reaction along the pile influences the moments an deflections of the pile. Considering an homogeneous layer of cohesive soil (k=340 t/m2) and another one of granular soil with k linearly increasing with depth (k=45,33zt/m2), the results of the computations are shown in fig.3.

The effect of the subgrade reactions is the shifting and separation of the two natural - frecuencies, with smaller bending moments -- and deflections for cohesive soil.

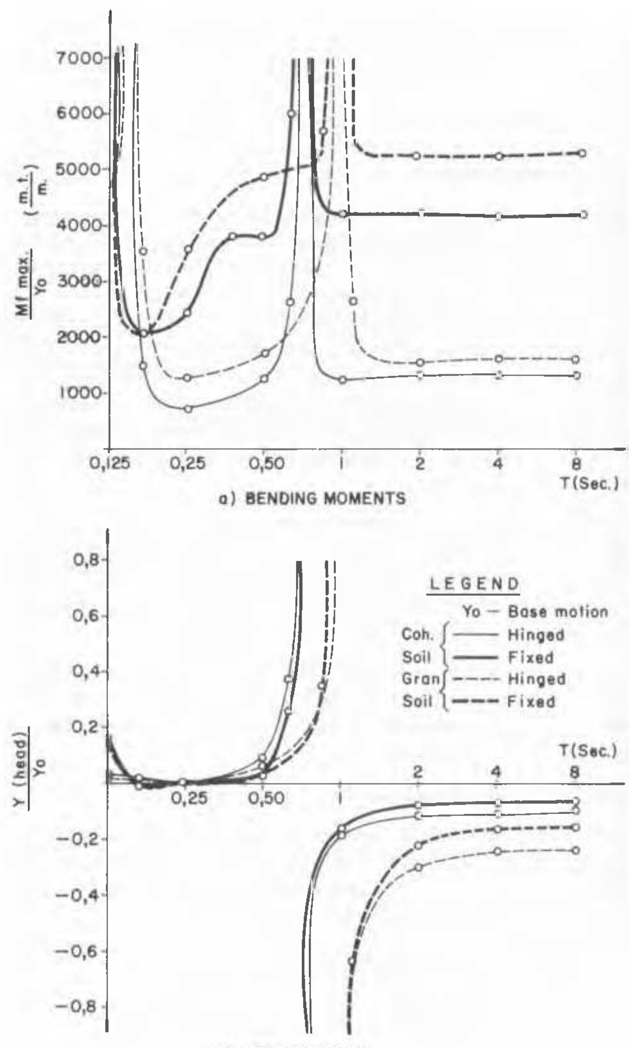


Fig.3.- Influence of the subgrade reaction variation and the pile point condition.

The effect of the pile condition is remarkable as concerns the bending moments (fig.3-a), also with smaller values for cohesive soil, but the incidence on the displacements and resonant frecuencies is negligeable.

It is interesting to note the occurrence for high frecuencies of bending moments below those approaching the static condition ($T \to \infty$).

b) <u>Stratified ground</u>

Equation (2) becomes specially usefull in the very common case of several layers of soil underlain by bedrock. Integration is carried out by establishing the transfer matrices between the extremities of the pile across the boundaries of consecutive layers.

For the sake of simplicity only three layers have been considered , as alternance of soft, cohesive soil (C) (k=330 t/m2) and granular soil (G) (k=150.z t/m2). Any possible combination has been investigated, from 3 equal cohesive layers to 3 granular ones.

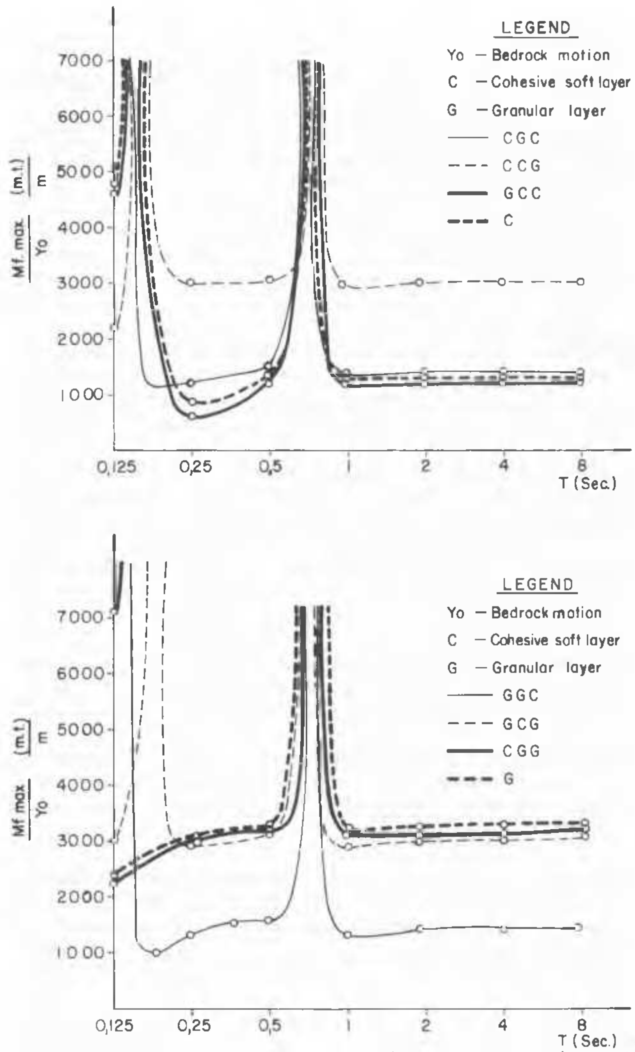


Fig.4.- Maximal bending moments in a pile embedded in three layer ground (hinged tip).

The maximal bending moments (not always at the same position)referred to the amplitude of the bedrock motion appear in fig.4, the letters C and G denoting the corresponding layer alternance from the surface. The pile tip is hinged.

As it can be seen, outside the natural frecuencies the maximal bending moment depends on the stiffnes of the deepest 5 m., on the bedrock, i.e. the deformability of the ground in the vicinity of the dynamical source.

The first natural frecuency is quite stable for any combination of layers, whereas the second one shows a certain influence of the intermediate soil layer.

Outside the natural frecuencies the bending moments lay under the expected static values as in the homogeneous case.

From this analysis it is clear that the non resonant design bending moment can be estimated for stratified ground by reducing the actual soil sequence to an equivalent homogeneous case, as

$$M_{str.} = \Phi(\lambda,\xi)\cdot M_{hom}$$

λ being a factor depending on the percentage of hard of soft layers (whichever had been taken as reference for the homogeneous case)and

$$\xi = \frac{\text{depth of c.of g. of hard/soft layers}}{\text{embedded lengt of pile}}$$

Fig. 5 shows the function $\Phi(\lambda,\xi)$ for the particular case computed. Differences must be expected for other values of the relative subgrade reactions of the homogeneous cases.

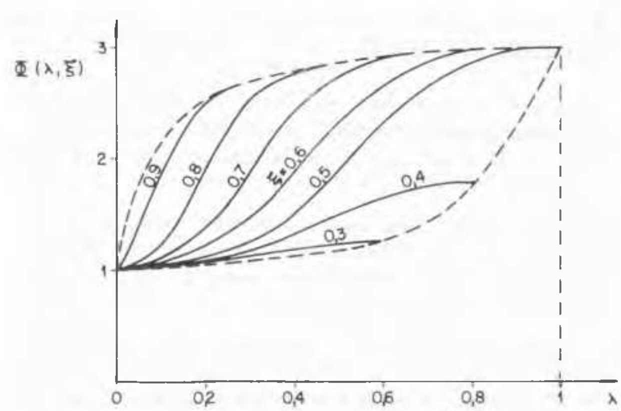


Fig.5.- The function $\Phi(\lambda,\xi)$

c) <u>Nonlinear behaviour</u>

In actual cases the bending moments at natural frecuencies may attain limited values as a result of viscous damping, plastic yielding of soil and stress redistribution along the - pile. These values should be used in the de-- sign when vibration of the piling at natural frecuencies is unavoidable.

For the same case of fig.3, hinged pile tip, and granular soil with hysteretic response of the tipe of fig. 6, the behaviour in the vicinity of the first natural frecuency (T ≃ 1 sec)

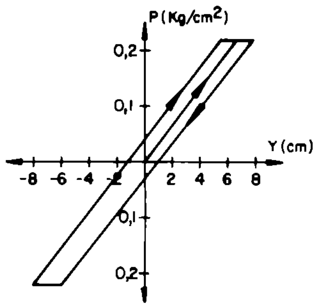


Fig. 6

has been studied. Results appear in fig.7.

It has been assumed that the building on top of the pile follows very closely the bedrock frecuency. This hypotesis must be checked in any actual case, according to the embeddment of the basements, heigh of the building, etc.

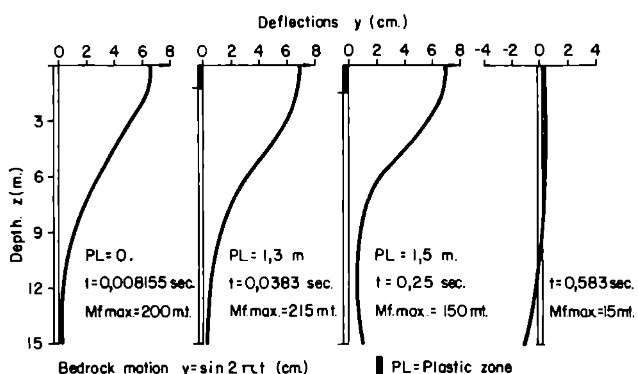


Fig.7.- Pile deflections in homogeneous elastoplastic soil (k=45,33z t/m2) in - the vicinity of the first natural frecuency (T ≃ 1 sec).

It can be seen that with a short length (≃ 1,50 m.) of plastified soil an energic dam--- ping occurs, with bending moments 12 to 15 times those of the static case.

The limit deflection of about 6 cm. is introduced in the elasto-plastic law of soil behaviour according to its triaxial shear strength.

CONCLUSIONS

The differential equation for dynamical loa-- ding on piles can be adapted to stratified -- elasto-plastic ground on bedrock.

The inertia of the superestructure must be introduced when studying the dynamical behaviour of a pile foundation in order to determine - natural frecuencies in the range of seismical vibrations.

With fixed pile point the maximal bending moments (outside the natural frecuencies) are 2 to 3 times greater than those with hinged point.

It is possible to reduce the problem of stratified soil to the homogeneous case by weighting the relative layer thicknesses and its position along the pile shaft. The strength of the soil near the bedrock is decisive for the pile bending.

Plastic yielding of the soil in about 2 pile diameters is highly efficient in reducing to limited values the bending moments at natu-- ral frecuencies.

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