Similitude Studies on Offshore Structures at Ng=1 Scale

Similitude de Constructions Maritimes sur Modèle Rèduit

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SYNOPSIS Model tests at gravity scale 1 have been performed to predict prototype behaviour and to improve analytical methods. Special attention has been paid to quasi-static and cyclic loading conditions.
The described technique has succesfully been applied to predict deformation and stability behaviour of the Oosterschelde closure caissons.

INTRODUCTION

The technique of physical modelling to predict prototype behaviour is particularly useful, if the mechanical behaviour of the material is incompletely understood or, if a prediction by mathematical modelling runs up against computing difficulties due to the complexity of the problem.

Similarity requirements between model and prototype have to be established from an analysis of the stress-strain and strength behaviour of the soil, which in fact is also the basis for the mathematical model used in an analytical prediction method.

The imperfect understanding of the mechanical behaviour turns out to be the main weakness of the physical modelling technique, although not as much as for analytical procedures, as a variety of mathematical models may yield identical similarity requirements.

SIMILITUDE ANALYSIS

In earth pressure problems boundary displacements and stability are governed by the deformation respectively the strength characteristics of the soil skeleton, and consequently by the effective stresses only. As in general, however, only the total stresses can be specified along the boundaries, a consolidation relation is required to provide the link towards the effective stress distribution within the foundation soil.

The pre-failure stress-strain behaviour of a cohesionless soil is considered to be elastic, strain hardening plastic. Apart from the consolidation phenomenon time rates of stress and strain are irrelevant. All pertinent soil parameters are time independent, although some of them are strongly dependent on the stress history of the soil.

To illustrate the basic features of the stress-strain behaviour of a cohesionless soil, consider an element at current state (s, t, v), where s = effective mean normal stress, t = deviator stress and v = specific volume, represented by point P in the stress space of figure 1.

Suppose this stress state to correspond with the maximum stress ratio $\eta = t/s$, which the element has ever experienced before. At this initial condition there exists a curve Oab, to be considered as a local yield curve, which divides the stress space into a non-yielding zone III and stable-yielding zones I and II.

A stress increment $(\delta s, \delta t)_I$, being applied from a point P on the yield curve section Oa, which moves the stress state of the element into zone I, will cause predominantly permanent or plastic strains to occur, both volumetric ($\delta\varepsilon$) and shear ($\delta\gamma$), for which holds:

$$\frac{\delta\varepsilon^p}{\delta\gamma^p} = \sin\nu_m = f(\eta) \qquad (1)$$

Equation (1) represents a stress-dilatancy relation, in which ν_m = mobilized angle of dilation
η = the current stress ratio t/s

The strain increment produced by the stress increment $(\delta s, \delta t)_I$ will also have a recoverable or elastic component, mainly volumetric due to the change of effective stress s, for which holds:

$$\delta\varepsilon^r = \frac{\kappa}{v}\frac{\delta s}{s} \qquad (2)$$

where κ = a constant
v = the specific volume, being the volume occupied by unit volume of solids

Upon removal of the stress increment $(\delta s, \delta t)_I$ let the state of the element be transferred from P towards point Q on the yield curve section ab, along a path fully within the non-yielding zone III. This process will involve elastic strains only, volumetric according to equation (2) and negligible shear strains.

Subsequently the element is subjected to a stress increment $(\delta s, \delta t)_{II}$, which moves the stress state into zone II, and then unloaded again. This stress increment will cause an incremental volumetric strain, partly permanent and partly recoverable, expressed by a relation

$$\delta\varepsilon = \delta\varepsilon^r + \delta\varepsilon^p = \frac{\lambda}{v}\frac{\delta s}{s} \qquad (3)$$

from which the elastic part, conform to equation (2), vanishes at unloading, leaving a permanent plastic strain:

$$\delta\varepsilon^p = \frac{\lambda - \kappa}{v}\frac{\delta s}{s} \qquad (4)$$

where λ and κ are constants.

In the meantime by the stress increments $(\delta s, \delta t)_I$ and $(\delta s, \delta t)_{II}$ the non-yielding zone III has expanded towards the limiting curve Oa'b'.

Let two elements of soil m and p at current stress states $P_o(m)$ and $P_o(p)$ in figure 2, having experienced past maximum stress ratio η and maximum compressive stresses s(m) and s(p) according to points P(m) and P(p), be subjected to similar stress paths $P_0P_1(m)$ and $P_0P_1(p)$, as described by:

$$\frac{s}{s_o}\sqrt{\{(\frac{t}{s})^2 + 1\}}_{(m)} = G(\eta) = \frac{s}{s_o}\sqrt{\{(\frac{t}{s})^2 + 1\}}_{(p)} \qquad (5)$$

There is considerable experimental evidence that for such similar paths at the above initial conditions holds the stress-strain relation:

$$\frac{t}{s} = \eta = \sin\phi_m = g(\gamma) \qquad (6)$$

with ϕ_m = mobilized angle of internal friction.

Equation (6) represents a strain hardening relation.

In an actual earth pressure problem the boundary displacements are a function of the strains within the foundation soil and the geometrical scale only:

$$\delta = L \cdot F(\varepsilon, \gamma(x, y, z, t)) \qquad (7)$$

where ε and γ = total strains
L = specific length of model or prototype
δ_V = displacement components: δ_V, δ_H, δ_ω

For a true model F in the model equals F in the prototype, which is satisfied if all strains are identical at similar locations and similar time in model and prototype.
Then the prediction equation is:

$$\delta = \delta_m \cdot n_L \qquad (8)$$

with $n_L = \frac{L}{L_m}$ = geometrical scale
and the primary similarity requirements become:

$$n_\varepsilon = 1 \qquad (9a)$$
$$n_\gamma = 1 \qquad (9b)$$

According to equation (6) the condition (9b) requires

$$n_t = n_s \qquad (10)$$

Condition (10) will be satisfied if corresponding elements of model and prototype are subjected to similar stress paths according to relation (5) and have experienced identical stress history.

As sand cannot be densified easily by compressive stresses only, its current specific volume is very much a function of the past maximum stress ratio t/s. Therefore, if the same sand is used in model and prototype tests and if the same specific volume arises from an equal initial isotropic stress condition, there is a fair guarantee for identical stress histories.

As elastic shear strains can be neglected, integration of equation (1) after substitution of (6) now yields:

$$n_{\varepsilon^p} = n_\gamma = 1 \qquad (11)$$

for plastic volume strains occurring from stress increments of type I.

By substituting $\delta\varepsilon = \frac{\delta v}{v}$ in equations (2) and (4) and considering that κ and λ are constants, it is observed that similar increments of stress δs in model and prototype cause equal changes of volume δv. Then by integration over the entire stress path it may be shown that for all elastic volume strains and for plastic volume strains of type II holds

$$n_{\varepsilon^r} = n_{\varepsilon^p} = 1 \qquad (12)$$

if the initial specific volume is the same in model and prototype:

$$n_{v_o} = 1 \qquad (13)$$

Condition (13) is satisfied to a close degree of approximation whenever corresponding soil elements in model and prototype have experienced identical stress histories as specified previously and otherwise differ only by their current initial values s_o.

For an earth pressure problem on a rigid, strain hardening plastic, cohesionless soil in drained loading, as shown by figure 3, the stress distribution within the plastically deforming zone may be expressed by

$$s = \bar{\gamma}\, r\, F_1 + q\, F_2 \qquad (14a)$$

where q = surcharge pressure
r = radius vector to soil element
$\bar{\gamma}$ = buoyant unit weight = $\frac{\rho_s - \rho_l}{v}\, g$
$F_1 = F_1$ (ϕ_m, boundary conditions)
$F_2 = F_2$ (ϕ_m, boundary conditions)
and a similar relation for the deviator stress t (14b)

Equation (14) serves to specify the boundary loading requirements to ensure similar stress paths for geometrical similar points.
Sufficient conditions for equation (10) to be satisfied at geometrical similar points are geometrical similar boundaries and identical values of the functions F.
According to equation (14) this, in turn, requires:

$$n_s = n_t = n_q = n_{\Delta\rho}\, n_g\, n_L\, n_v^{-1} \qquad (15)$$

where: $\Delta\rho = \rho_s - \rho_l \qquad (16)$

Equation (15) expresses both the boundary loading conditions and similarity of stress paths.
For equation (15) to be a sufficient condition for similarity, relation (14) must remain essentially unaffected by elastic and plastic strain increments according to equations (2) and (4).

In a partially drained problem only the total stresses can be controlled along the boundaries. From the general stress field equations [1] it can be obtained that the only additional requirement to be satisfied is:

$$n_\sigma = n_s \qquad (17)$$

where σ = fluid pressure

For condition (17) to be accomplished model and prototype have to be subjected to similar time rates of loading, being prescribed by the consolidation equation, which for a general elastic plastic soil reads:

$$(K + {}^4/_3\, G)\nabla^2\varepsilon^r = \frac{\mu}{k}\frac{\partial\varepsilon}{\partial T} \qquad (18)$$

where $(K + {}^4/_3\, G)$ equals D = constrained modulus of recompression
ε^r, ε = elastic resp. total incremental volume strains
μ = dynamic viscosity of the fluid
k = intrinsic permeability of soil skeleton

As ε^r and ε are to be identical in model and prototype, the time scale resulting from equation (18) is:

$$n_T = n_\mu\, n_L^2\, n_k^{-1}\, n_D^{-1} \qquad (19)$$

MODEL AND PROTOTYPE TESTS

A major project of the Delta-works in Holland is the closure of the Oosterschelde estuary. One of the proposed solutions was to build a permeable caisson dam, which would both protect the land against inundation by reduction of the tidal movements behind the dam and preserve the natural salt water environment in the enclosed bassin.

To study the feasibility of such a dam a near to prototype size caisson has been subjected to a storm loading condition on a similar foundation soil at about 8.5 metres water depth. The stability and deformation behaviour of this test caisson have been predicted by several analytical and experimental methods, one of those being model tests at gravity scale 1.

The prototype caisson had dimensions:

width : 15 m
length: 27.7 m
height: 12 m

and a weight of 1350 tf in 8.5 m water depth.
The storm loading program applied consisted of 6 stages of each 300 horizontal load cycles, as specified in the next table.

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[1]) Stress field equations for plain strain deformation under partially drained condition of a rigid, strain hardening plastic, cohesionless soil:

$$ds \pm 2s\tan\phi\, d\theta \pm s\left(\frac{\partial\phi}{\partial x}dz - \frac{\partial\phi}{\partial z}dx\right)$$
$$= \left(\bar{\gamma} - \frac{\partial\sigma}{\partial z}\right)(dz \pm \tan\phi\, dx)$$
$$- \frac{\partial\sigma}{\partial x}(dx \mp \tan\phi\, dz)$$

stage number	F_H(static) tf	F_H(cyclic) tf	number of cycles
0	20	25	300
1	40	50	300
2	80	100	300
3	120	150	300
4	160	200	300
5	200	250	until steady state

$F_H = F_H(\text{static}) \pm F_H(\text{cyclic})$

F_H applied at 6.75 m above caisson base

cycle period 3 seconds

The foundation soil consisted of recent deposits of medium to fine sand with d_{50} = 150 - 160 μm and average porosity n = 41 - 42% to a depth of at least 25 m.

Model tests were carried out on geometrical scale n_L = 25, using the same sand and molasses as pore fluid.
The stress scale was chosen according to equation (15), with $n_g = n_v = 1$ and $n_{\Delta\rho} = 1.3$.

The time scale was selected according to equation (19), with $n_\mu = 2 \cdot 10^{-3}$, $n_k = 1$ and $n_D = 5$.

Figures 4 and 5 show the horizontal and vertical displacements of the caisson centre as measured in the prototype (M) and predicted by the model tests (P).

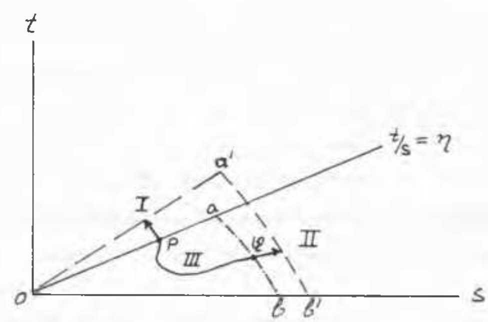


Fig. 1

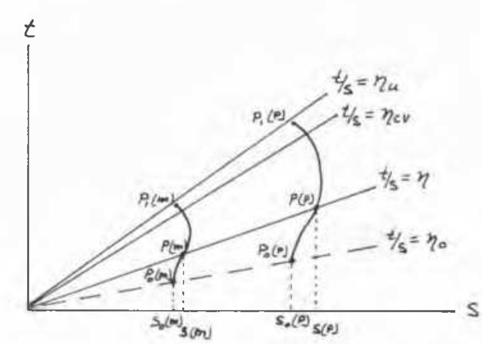
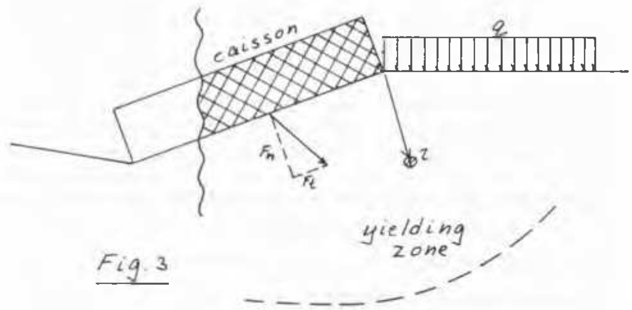
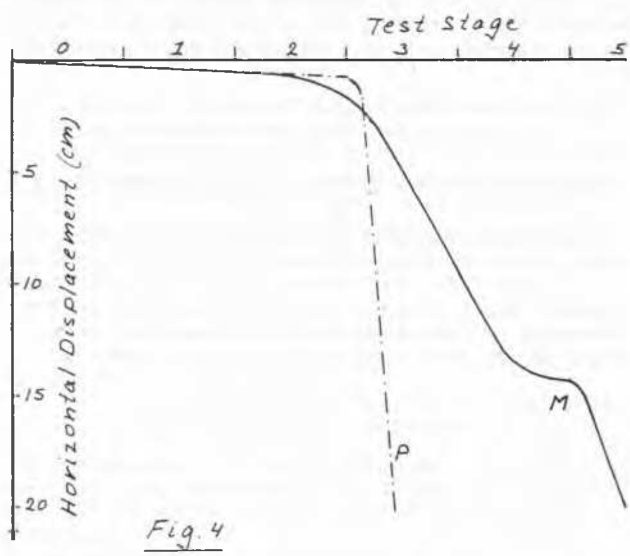
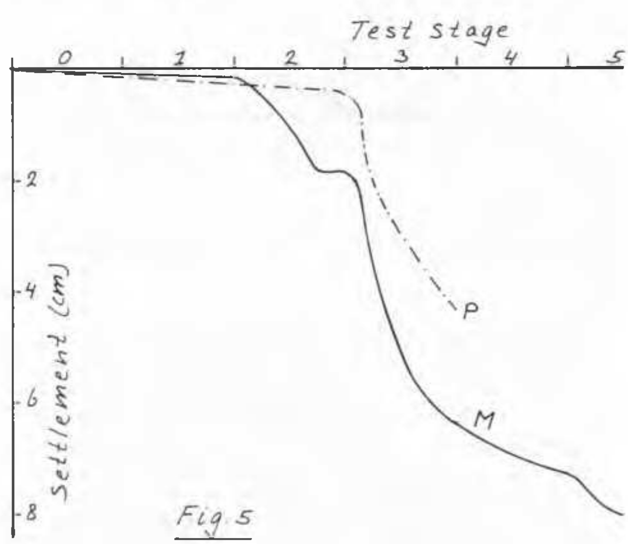


Fig. 2







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