Development of the Mechanics of Granular Materials in Japan

Développement de la Mécanique des Matériaux Granulaires au Japon

T. MOGAMI Professor Emeritus, Tokyo University

1. Introduction

The chapter concerning deformation of soil is a very important one in the text book of soil mechanics. However, at the present so many pages are left in blank. Recently studies about this problem from several view points were performed by scholars. Among them the work due to late professor Roscoe and others and the work due to Rowe were most influential to researchers in our country. The former was primarily a theory of deformation of clay but they thought it applicable to sand. The latter concerned mainly with deformation of granular materials.
The conception of granular material is an abstraction of the real soils. The properties of this abstracted material would differ a little from the real soils.
At the dawn of our soil mechanics, the great Coulomb began to write down the first page from the chapter of earth pressure considering soils as granular materials. It would be wise to follow our forefather in promoting the studies of this important problem.
At almost the same time, Professor Murayama and the present writer began to study the mechanics of granular materials such as dry sand from the microscopic view point. Fortunately we could have excellent followers and considerable development of this field could be achieved in these several years.
About four years ago, a special committee to study the mechanics of granular materials was established in our Society of Soil Mechanics and Foundation Engineering. This committee was helpful in collecting and exchanging knowledge and opinions proposed by members. The report of this committee was prepared already, but not yet published.
On this occasion of the International Conference, the writer wants to report the development in our country of the field basing on the materials presented to the said committee. However, topics are divergenced widely and the writer cannot introduce, even understand all contributions of members. The writer can only make his subjective report about the development of the mechanics of granular materials in these several years.
The present aim is to introduce the recent development, but we have to reffer to some older contributions at home and abroad. Strictly speaking, the dynamics of the granular materials, for example, liquifaction of sand by vibration have to be included in this report, but the writer does not touch this problem.

2. Observations of the deformation of granular materials

The deformation of the granular materials as a whole can be made clear by experiments on models, for example, the movement of sand particles behind a wall when the wall is shifted is observed by preparing suitable model box containing sand with side glass walls through which we can see or take photographs of the movement. In this case coloured stripes on the sand surface are helpful. Many authors applied this method in the past to know the behaviour of sand, late professor Takabeya showed clear and beautiful photographs in many cases of the movement. [F. Takabeya, 1931]
Anyway this method is a qualitative and educational help to understand the existence of slip surface, for example. In addition, the observation is limited to the phenomena appeared on the surface of the mass.
Models using a pile of metal balls or metal discs sometimes of photoelastically sensitive material are also helpful.
Matsuoka conducted shear tests on model of granular material composed of alminium rods having uniform and different diameters. He investigated the change in the distribution pattern of the inclinations of contact planes of rods in the course of the increasing shear strain. [Matsuoka, 1974]
Three kinds of cylinders having different diameters are piled randomly in a two-dimensional bi-axial compression apparatus and in a two-dimensional simple shear apparatus, the material of cylinder was photo-elastically sensitive. [Oda and Konishi, 1974]
During the course of shear tests with these apparatuses, change in granular fabric and forces between cylinders are observed. Observed forces are transferred into stress by calculation. With increase of major principal stress in bi-axial compression, the normals of the contact planes have tendency to concentrate to the direction of this principal stress, whereas in simple shear tests similar phenomena are observed when shear stress increased. It can be said that the predominant direction of the normal rotates gradually with that of principal stress during shear

deformation and both directions coincide with each other.
Degree of concentration of the normals is parallel to the stress ratio σ_1/σ_3 or τ/σ_N. Rotational angle ψ of major principal stress is related to stress ratio as

$\tau/\sigma_N = \kappa \tan\psi$, where κ is a constant.

They observed also that the number of cylinders which are sliding one against another is **only** a few and other cylinders are flocked into groups like rigid bodies.

3. Distribution of particles in a granular material

It would be a matter of common sense that the distribution of particles in a granular material is an important factor to determine the mechanical character there-of. Years ago, Smith, Foote and Busang [Smith, Foote and Busang, 1929] studied this problem by pouring equal spheres into a box. They found that they could not have the densest packing and their conclusion was that spheres are not arranged in a simple array but densely packed groups of spheres and loosely packed ones are mixed randomly. The present writer found the same thing when he arranged steel balls in a rectangular frame placed on a horizontal plate as randomly as possible. Balls were observed to construct denser and looser groups, this pattern was like islands separated by straits which correspond to looser parts. [Mogami, Imai, 1969].
Fukumoto [Fukumoto] observed voids in a granular matter put in a box and fixed by rasin, his conclusion was similar to the above. The minimum void ratio, coordination number in a granular material which is composed of glass beads of large and small sizes were observed by Oda [Oda, 1975], and he found that the structure of particles in the box is not uniform.
Above cited experiments on model granular materials composed of regular shaped particles such as spheres and Fukumoto's laborious experiments lead us to the conclusion that in a granular material there exist groups having denser and looser packing of particles, and the distribution of such groups is irregular. It seems that there is some resembrance between the above said distribution of particles in a granular material and the molecular arrangement in polycrystalline metal, that is, recrystallization of metal crystals from melted metal begins in many places and the crystals grow into various directions depending on conditions, there exist amorphous metal between regularly grown crystals.

4. Triaxial tests on sands and gravel

Triaxial tests are now recognized as routine tests but the writer thinks that it is worth while to introduce tests performed in our country recently.

i) Large scale tests

In 1970 Fujita Corporation constructed a super size triaxial compression apparatus. [Fujita Corporation 1975] The size of the sample is 120 cm in diameter and its height is 240 cm. The maximum side pressure is 30 kg/cm^2, and the axial force is 680 t. After careful preparatory experiments, they are now carrying on tests on coarse materials. Up to now, they performed triaxial compression on over 100 kinds of coarse materials.
Because our final aim of study of granular materials is to get reliable informations for the safety of actual large scaled earth structures and almost all theories which we have at the present are confirmed by small sized experiments in laboratories so that the construction of such a large scale apparatus must be an encouraging event. With the results obtained by this apparatus, it was found that almost all general propositions which are concluded basing on small sized experiments are also valid for large specimens.
Ishii et al [Ishii, Fujiwara, Iwahashi, Sakami, 1974] performed triaxial tests by this large scale apparatus and also by small scaled apparatus, size of samples for small apparatus is 5 cm in diameter and 10 cm in height. They observed that the stress strain curve for dense sand obtained by large scale apparatus is approximated by the similar curve obtained by small scaled tester on loose sand.
However, when the samples are very uniformly prepared, appreciable difference in shape of stress strain curves for similar tests by large and small scaled apparatuses could not be observed. Hence they conclude that for large scale actual earth structure, the influence of non-uniformity of void ratio, especially the influence of loose part to stress strain characteristics would be appreciable.
Imai gave a comment that the scale of the dense or loose part compared with the loading area would be the problem. [Imai, unpublished.]
According to a report of the Fujita Corporation, as far as it is confirmed by the test results up to now, large scale apparatus gives smaller angle of internal friction, larger axial strain before failure compared with the results by smaller apparatus, and in case of large apparatus volumetric strain at failure is in general contraction.

ii) Tatsuoka's experiments

Systematic studies of sands by tri-axial apparatus were conducted by Tatsuoka, some interesting conclusions were deduced from these researches. His experiments were: [Tatsuoka, 1972]

Sand: two kinds of river sand
Density of samples: three kinds of density, those are, void ratio was 0.80, 0.68 and 0.54 respectively.
Tri-axial apparatus: Bishop-Henkel type
Tests performed:

a) Tri-axial compression
1) σ_r is constant, σ_a increasing, drained, strain control tests, axial strain rate is 0.24 %/min.
2) $p=\frac{1}{3}(\sigma_a+2\sigma_r)$ is constant, σ_a increasing, drained tests, stress control
3) σ_a is constant, σ_r decreasing, drained, stress control tests
4) σ_r is constant, σ_a increasing, undrained, back pressure is 1.5-3.0 kg/cm^2
5) q/p' is constant p' varies, drained, stress control tests

b) Tri-axial extension

1) σ_a is constant, σ_r increasing, drained, stress control tests
2) $p=\frac{1}{3}(\sigma_a+2\sigma_r)$ is constant, σ_a increasing, drained, stress control tests
3) σ_r is constant, σ_a decreasing, drained, strain control tests, axial strain rate 0.24 %/min.
4) σ_r is constant, σ_a decreasing, undrained, strain control tests
5) q/p' is constant, p' varies, drained, stress control tests,

where
σ_a: axial stress
σ_r: side pressure
ε_a: axial strain
ε_r: radial strain
p: mean pressure and $p=\frac{1}{3}(\sigma_a+2\sigma_r)$
q: shear stress and $q=\sigma_a-\sigma_r$
v: volumetric strain and $v=\varepsilon_a+2\varepsilon_r$
γ: shear strain and $\gamma=\varepsilon_a-\varepsilon_r$

Some findings from the experiments were:
α) Basic line
The relationship between the pressure and the void ratio when a sand sample is compressed isotropically is represented by a curve on the p,e plane. (Figure 1 and the curve AC is designated shortly as ICC). Samples which are

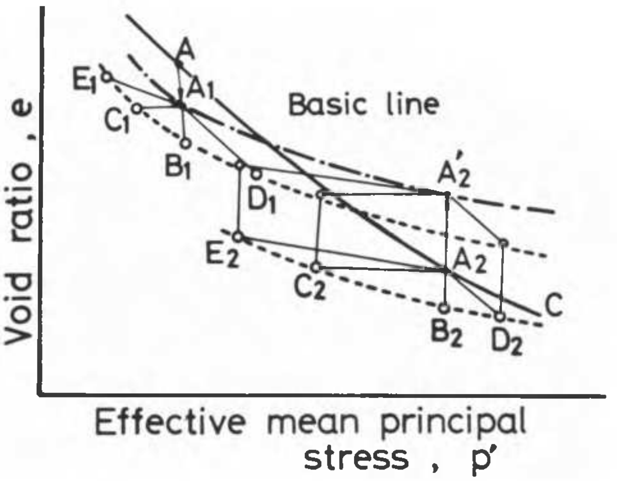

Fig. 1 Basic Line

brought to a point on this curve ICC are sheared through various stress paths. If these stress paths are on one surface, a relationship between pressure, shearing stress and void ratio can be found independently of stress paths. This surface corresponds to the state boundary surface due to Roscoe. Tatsuoka found that this anticipation is negative.
However, points on the ICC curve are shifted a little parallel to the axis e which is allowed by the reason that the mechanical response of sand does not show distinguishable change by such a small difference of void ratio. If all stress paths are rewritten as if the initial point of shear would be the shifted point on the p,e plane, he could show that all shifted stress paths can be considered to be on one surface.
He named this surface "state surface" and the curve on the plane p,e which is constructed by shifted points the "basic line". The state surface thus defined cannot be determined uniquely for a definite kind of sand. It depends on initial density of sand and also on the kind of testing, that is, the surface for compression test is different from that for extension test.
β) Equi γ curve
On a q/p',p' plane, the relationship between q/p' and p' is represented by a curve for each test, and if on such curves points which correspond to a definite shear strain, it can be observed that these points lie on one curve. In another word, when samples are sheared from the state o,p', the shear strain is determined for the combination of values of q and p' independently of stress path.

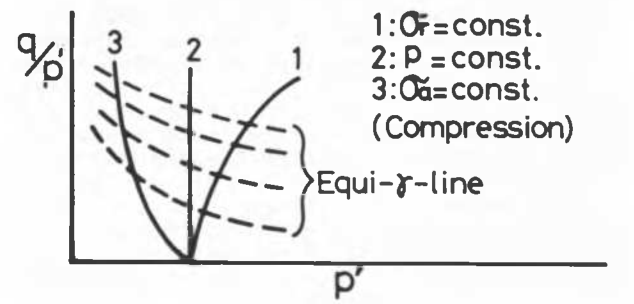

Fig. 2 Equi-γLine

Curves on a (q/p', p') plane which is constituted by points corresponding to equal shear strain are named "equi γ curves".
γ) Tatsuoka showed the relationship between q/p' and $dv^p/d\gamma^p$ on the figure 3. It shows that the value q/p' is determined independently of the stress path only when the value of $dv^p/d\gamma^p$ is small, that is, only when the state of sample reaches the peak. It was shown that the strain increment ratio dv/dγ is

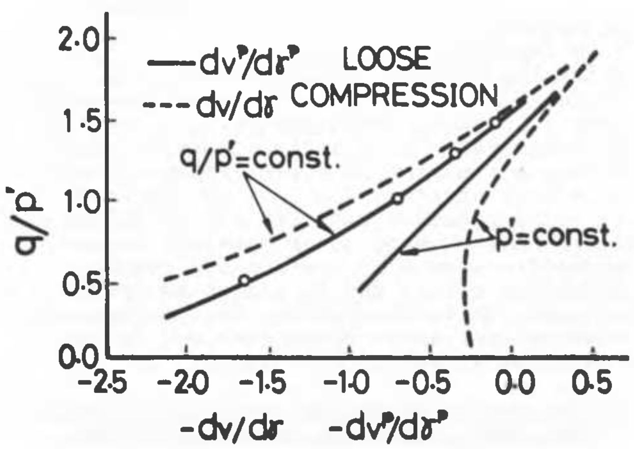

Fig. 3 Stress Path Independent Only When the Peak Approaches

stress path dependent unless the value of q/p' is close to the peak value.

iii) Oda's experiment

Oda [Oda, 1972] performed very laborious experiments to study the influence of initial fabrics to the mechanical properties of sand. After Brown he defined the term "fabric" as the spatial arrangement of solid particles and associated voids. Samples of sand were produced by pouring sand in a cylinder in several kinds of preparing ways. Before and at some stages of loading, polyester resin was poured into the sand and the structure of the sample was fixed and cut in slices and the distribution of particles, distribution of number of contact point and distribution of direction of contact planes were investigated with a petrographic microscope. He assumed that if we pick out all contact surfaces in a unit volume of sand and they are gathered and sticked into one surface, so we could have an ellipsoidal surface. The ratio of the principal planes of this surface can be calculated with the help of the distribution function of directions of contact planes obtained by experiments. Such ratio is designated as S_Z/S_X. This ratio represents the fabric characteristics of the sample and the change of this ratio during loading was investigated.

It was observed that this ratio is linearly connected with stress ratio and $dv/d\varepsilon_1$. Hence, by elimination of the ratio, the linear relationship between the stress ratio and $dv/d\varepsilon_1$ can be deduced.

The continuous reconstruction of the initial fabric with increase of axial strain occurs principally in preferred direction of sliding along the unstable contacts among neighbouring grains and partially rolling of each grain to make preferred re-orientation of long axes of grains.

In his experiments neither failure plane nor zone could be seen macroscopically in any specimens throughout the process of progressive deformation (axial strain less than 15 %). There was no evidence that long axes of particles tend to arrange parallel to the potential failure plane. These findings are in accord with Rowe's view that slip bands are the delayed results of failure and not the cause.

iv) Moroto's experiment

Performing tri-axial compression tests, Moroto [Moroto, 1976] could have conclusions that, 1) plastic shear strain is a function of stress ratio 2) plastic volumetric strain is also a function of stress ratio 3) total volumetric strain is the sum of that due to all round pressure and that due to dilatation connected with shear strain 4) plastic volumetric strain is the sum of plastic volumetric strain due to all round pressure and that due to dilatation, the latter is a function of plastic shear strain. In the above, the stress ratio means q/p'.

5. Imagination about the behaviour of particles when a granular mass is deformed

Basing on experiences in experiments, opinions about the behaviour of particles in granular mass during deformation as a whole are proposed by authors. Such opinion is, in one hand, is useful for physical understanding of the problem and in another, it would be a help to construct one's theory which extends one's narrower knowledge to wider unexperienced problems. However, our observation of the movement of particles is limited to the surface of a sample and even in case of the very laborious experiments by Oda, which was explained before, observations of the surfaces of slices cut from sample discontinuously at stages of loading can only be done. Hence, strictly speaking, we can only have imagination about the movement of particles. However, the complete observation of the behaviour is not needed for us, because only statistical properties of movement of particles have concern with the deformation of sample as a whole. Thus, some imaginations about the movement of particles are presented in this chapter.

As stated before, studies about the distribution of particles in a sample show that particles are located in the form of groups, some are denser and others are looser.

Particles belonging to both kinds of group constitute a kind of structure, one is loose or weak structure, another is dense or strong structure. Weak structure collapses when it is loaded, whereas the strong one can support load acted from without.

However, particles which constitute, for example, a weak structure could change into stronger one by the movement of particles after collapse and the reverse could be possible for stronger structure. From old times, it has been recognized that there exist many small arches in a sand mass. These arches support forces so that particles constituting arches are pressed stronger than others. Hence, it can be said that in a sand mass there exist non-uniformity of locations of particles, that is, geometrical non-uniformity and non-uniformity of forces acting between particles, that is, mechanical non-uniformity.

When a sand sample is compressed, some numbers of weak structure are collapsed which result in the contraction of sample as a whole.

In this stage of loading, number of weakest groups which collapse by loading increases and the geometrical non-uniformity decreases, whereas stronger groups behave like rigid bodies and any changes would not be observed. When this stage comes to end, very small slips in a somewhat stronger groups would begin, because without such movements of particles any movement cannot occur in any group of particles. Such small slip would induce the increase of volume in groups of this kind and the total volume of sample increases. Slips which occur in this stage would be small and the geometrical and mechanical non-uniformity of sample would decrease, but complete uniformity would not be achieved.

As such change in configuration of particles spreads, the property of sample as a whole approaches to be elastic, so that the apparent elastic region in the stress-strain curve of the sample can be observed.

On the other hand, in some of weaker groups among the stronger ones thus constructed, small slips but larger than those occurred in earlier stage of loading would be generated. Such slips are limited in some small regions and do not go through the whole sample so that even once such slip occurs in some group of particles it would cease to slip, because unfavourable arrangement of particles in this region is lost by this slip. Losing of unfavourable arrangement of particles is equivalent to the losing of resistance to movement of particles, this circumstance produces the cause of occurrence of stick-slip motion. Small sudden changes, in another word, jerky change in the stress-strain curve are observed in this stage as many authors already experienced. The scale of slip line in this stage is small but enough large to be observed, some author including the present writer, observed the appearance and vanishing of small slip lines in places of sample. [Yamaguchi, 1975]

The number of such small slippage increases with progress of loading and in final stage, these slip lines could be united into one slip line or the sample could be deformed by slippage along many slip lines in places.

It is not useless to emphasize that the movements of particles occur always in some limited number of groups and other groups of particles behave like rigid bodies.

6. Theories of deformation of granular material

In order the estimation of deformation by calculation to be possible, some theory to describe the deformation character of the granular material is needed. Theories have been proposed by authors and they are introduced in this chapter.

i) Plastic theory

It is well known that a theory was constructed along similar way to the mathematical theory of plasticity but taking the special characteristics of soils into account. This is due to late professor Roscoe and others and independently by Oota and this is the theory especially for clay.

However, Roscoe and others suggest that the theory would be applicable to sand if some modifications are added.

The theory is based on several fundamental conceptions.

1) Yield function

When the state of stress of a material is represented by a point in the stress space, the border of elastic and plastic regions is represented by a surface in this space called yield surface and the equation of this surface is designated as yield function. As far as the stress point remains in one side of this surface which indludes the origin the material is elastic.

2) Hardening rule

Almost all kinds of metal show hardening property when deformed. When the hardening law is wanted to be represented, some parameter which varies during loading process is inserted in the yield function. Hence we consider that the yield surface changes its location as the stress point moves along the stress path. The manner of this movement gives the hardening law. Of course this should be determined by experiments.

3) Flow rule

The rule which gives the relationship between the increment of the plastic strain components and the stress components is the flow rule. If the ratio of each plastic strain component depends only on the stress components and not on the increments of stress component, that is, the increment of plastic strain is independent of stress path, we can introduce a function called the plastic potential. Thus

$$d\varepsilon^p_{ij}=\frac{\partial\psi}{\partial\sigma_{ij}}d\lambda ,$$

where $d\lambda$ is a factor which determines the magnitude of the plastic strain components. The plastic potential proposed by late professor Roscoe et al is

$$\psi=\frac{q}{p'}+M\ln p'$$

and this is same as their yield function.

In 1966, Poorooshasb et al proposed a theory of deformation of sand, in this theory they denied the equality of plastic potential and the yield function and their yield function was

$$f=\frac{q}{p'} ,$$

but Poorooshasb improved the theory in 1971, the improved yield function is

$$f=\frac{q}{p'}+M\ln p'$$

The flow rule adopted by Roscoe et al is expressed as

$$p'dv^p+qd\varepsilon^p=Mp'|d\varepsilon^p| ,$$

where

$$p'=\frac{1}{3}(\sigma_1'+2\sigma_3') , \quad q=\sigma_1'-\sigma_3'$$

$$dv^p=d\varepsilon_1^p+2d\varepsilon_3^p , \quad d\varepsilon^p=\frac{2}{3}(d\varepsilon_1^p-d\varepsilon_3^p)$$

in case of tri-axial compression.

In this case the plastic potential function is

$$\psi=\frac{q}{p'}+M\ln p'$$

From the flow rule we can get

$$\frac{q}{p'}=M-\frac{dv^p}{d\varepsilon^p}$$

in case of tri-axial compression.

Tatsuoka gives some comments

i) Poorooshasb determines his plastic potential from the directions of $d\varepsilon^p_{ij}$ obtained by anisotropic consolidation tests the stress ratio being constant, however, because $d\varepsilon^p_{ij}$ is stress path dependent when the stress is still small, the plastic potential can be recognized to exist only near the peak of stress.

ii) By experiments to obtain the plastic strain increments for given stress increments along several stress paths, it was shown that the plastic strain increments seem to be determined independent of the stress path when the stress path is that produce only a few amount of plastic volumetric strain. Hence, it is anticipated that the existence of the plastic potential has to be discussed excepting the volumetric strain which is produced by isotropic compression. [Tatsuoka, Ishihara, 1974]

ii) <u>Stress ratio and strain increment ratio type theories</u>

Theory of this type was proposed originally by Rowe, and called stress-dilatancy theory. This theory was presented as that of deformation of sand. Coincidence of the results of the theory with those obtained by experiment is reported to be quite satisfactorily by Rowe himself and others. [Barden, L., Khayatt, A.T. (1966), Barden, L., Ismail, H., Tong, P. (1969)].
However, careful investigation of figures in the original papers, it can be said that the coincidence between theory and experiment is good near the peak of stress strain relationship, but it is not always good in the earlier stage of the relationship. It would be due to the fact that the existence of the plastic potential for sand is questionable in this stage. Several theories of this type were proposed by young researchers in our country which will be introduced in this chapter.

i) Oda's theory [Oda, 1972, 1974]
After finding that the parameter S_z/S_x which represents the fabric characteristics is linearly connected with stress ratio and $dv/d\varepsilon_1$, the stress ratio is shown to be linearly related to $dv/d\varepsilon_1$ by elimination. This is

$$\frac{\sigma_1}{\sigma_3} = C_1\frac{dv}{d\varepsilon_1} + C_2$$

He obtained the equation of this type for simple shear, but in this case he considered that the direction of major principal stress is not always coincide with that of the major principal strain rate.

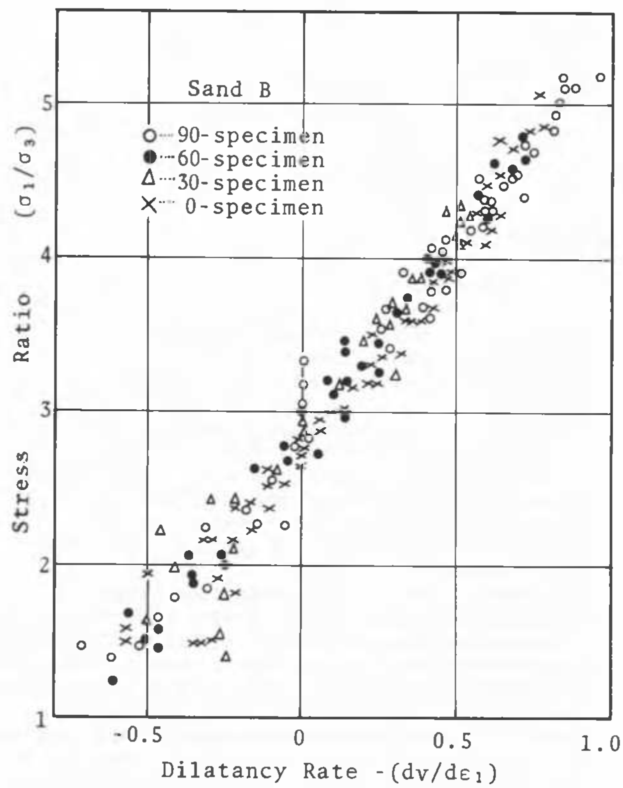

Fig. 4 Stress Ratio-Strain Increment Ratio Relation by Oda

ii) Matsuoka's theory [Matsuoka, 1974, 1976]
By model tests on alminium rods, the change in the distribution of the inclinations of contact planes is investigated. Finding this change pattern he obtained a stress-ratio-strain increment ratio relationship. His relationship is

$$\frac{\tau}{\sigma_N} = \lambda\left(-\frac{d\varepsilon_N}{d\gamma}\right) + \mu \text{ ; valid on the}$$

mobilized plane,
where
- σ_N: effective normal stress
- τ : Shear stress
- ε_N: normal strain in perpendicular direction to mobilized plane, compression being positive
- γ : shear strain

He emphasizes that this relationship should be applied to a plane composed of particles most heavily mobilized. Such consideration leads him to the conception of the mobilized plane (potential sliding surface), that is, the plane on which the value of τ/σ_N is maximum. To deal with the behaviour of assemblage of particle in three dimensions, he considered that there would be three compounded mobilized planes and he assumes that his stress-ratio-strain increment relationship can be applied to each of these compounded mobilized planes and also assumes the validity of the law of superposition.
In this way, he could derive the relationship between principal strains and ratios of

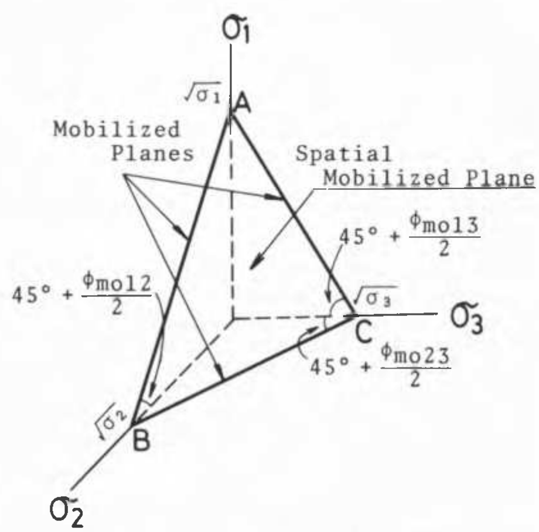

Fig. 5 Spatial Mobilized Plane

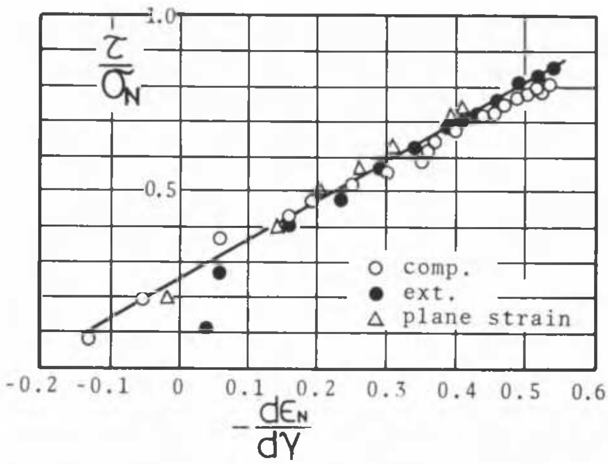

Fig. 6 Stress Ratio-Strain Increment Ratio Relation by Matsuoka

principal stresses. In addition, he defined the space mobilized plane as a plane which cuts the three principal stress axes at $\sqrt{\sigma_1}$, $\sqrt{\sigma_2}$ and $\sqrt{\sigma_3}$ respectively. In this case

$$\frac{\tau}{\sigma_N} = \sqrt{\frac{J_1 J_2 - 9J_3}{9J_3}}, \quad d\varepsilon_N = \frac{J_3}{J_2}\left(\frac{d\varepsilon_1}{\sigma_1} + \frac{d\varepsilon_2}{\sigma_2} + \frac{d\varepsilon_3}{\sigma_3}\right)$$

and $$\frac{d\gamma}{2} = \frac{J_3}{J_2}\left[\frac{(d\varepsilon_1 - d\varepsilon_2)^2}{\sigma_1\sigma_2} + \frac{(d\varepsilon_2 - d\varepsilon_3)^2}{\sigma_2\sigma_3} + \frac{(d\varepsilon_3 - d\varepsilon_1)^2}{\sigma_3\sigma_1}\right]^{\frac{1}{2}},$$ where J_i is the ith invariant of principal stresses.

He showed that his stress ratio-strain increment ratio relationship is also valid on this space mobilized plane. He could also show that the failure criterion that the value of τ/σ_N reaches to some constant on the space mobilized plane is simply written as

$$J_1 J_2 / J_3 = \text{constant}$$

and when this condition is plotted on an octahedral plane, the plotted curve is very similar to the curve for Mohr-Coulomb criterion.

iii) Tatsuoka's modification

Tatsuoka studied to have the stress ratio-strain increment ratio equation for three dimensional granular material with initial anisotropy, his line of thought follows the Matsuoka's but he extended. He started from the stress-dilatancy equation due to Rowe for simplicity's sake and he assumes the superposition law and the existence of plastic potential functions, he obtained the stress-ratio-strain increment ratio equations for,

tri-axial compression: $$\frac{\sigma_1}{\sigma_3} = -K\frac{d\varepsilon_2 + d\varepsilon_3}{d\varepsilon_1}, \quad (\sigma_1 > \sigma_2 = \sigma_3)$$

tri-axial extension: $$\frac{\sigma_1}{\sigma_3} = -K\frac{d\varepsilon_3}{d\varepsilon_1 + d\varepsilon_2}, \quad (\sigma_1 = \sigma_2 > \sigma_3)$$

general stress state: $$\frac{\sigma_1}{\sigma_3}\,\frac{\frac{\sigma_2}{\sigma_3} + \frac{1}{K}}{\frac{\sigma_2}{\sigma_3} + 1} = -K\frac{d\varepsilon_3 + \frac{1}{K}(\sigma_2/\sigma_3)d\varepsilon_2}{d\varepsilon_1},$$
$(\sigma_1 > \sigma_2 > \sigma_3)$

plane strain: $$\frac{\sigma_1}{\sigma_3}\,\frac{\frac{\sigma_2}{\sigma_3} + \frac{1}{K}}{\frac{\sigma_2}{\sigma_3} + 1} = -K\frac{d\varepsilon_3}{d\varepsilon_1}, \quad (d\varepsilon_2 = 0),$$

where $$\frac{\sigma_2}{\sigma_3} = K^{-\frac{1}{K+1}}\left(\frac{\sigma_1}{\sigma_3}\right)^{\frac{K}{K+1}} a^{\frac{1}{K+1}}$$

a: a parameter representing the anisotropy between σ_1 and σ_3 or σ_2 directions

In these latter two cases, anisotropy is assumed not to exist between σ_2 and σ_3 directions. [Tatsuoka, 1976]

iv) Moroto's theory [Moroto, 1976]

Moroto considered that the plastic work increment is written as

$$dW_s^p = p\,dv_d^p + \frac{2}{3}q\,d\gamma^p,$$

where dv_s^p is the volumetric strain increment due to volume change by dilatancy effect of shear strain. Both strain increments can be obtained as functions of stress ratio q/p', this was confirmed by his experiments.

Hence we have

$$\gamma^p = G(\eta), \quad v_d^p = D(\eta), \quad \text{where } \eta = q/p'$$

Thus it follows,

$$dW_s^p = pX(\eta)d\eta \qquad (1),$$

where

$$X(\eta) = D'(\eta) + G'(\eta)$$

He points out the equation (1) is not a perfect differential so that the plastic work done depends on stress path. If we introduce a quantity S_s such that

$$dS_s = \frac{dW_s^p}{p} = X(\eta)d\eta,$$

this quantity S_s is a stress path independent quantity. When several samples are compressed uniformly to the same state and then they are sheared along different stress paths until they reach equally the state of failure, the total plastic work done or total energy dissipated in each sample is different with each other depending on each stress path, but the quantity S_s for all samples which attains the final state by different stress path are equal. In addition, Moroto showed that his state variable S_s is closely related to the writer's entropy which will be explained later. Plastic strain is a function of η, so that we can write

$$dS_s = Md\varepsilon^p, \quad (d\varepsilon^p = \frac{2}{3}d\gamma^p)$$

From the definition of S_s we have,

$$dS_s = \frac{dW_s^p}{p} = dv_d^p + \frac{q}{p}d\varepsilon^p = Md\varepsilon^p$$

This equation has the same form as Roscoe's flow rule.

v) Other contributions

Satake [Satake, 1975] writes several papers in which, for example, he presents a geometrical meaning of yield criterions proposed by authors. His contributions suggest that there exists geometrical coordination or harmony in the mechanics of granular material. However, his mathematics is a little difficult.

Tokue [Tokue, 1975, 1976] with models of granular material consisting of spheres in two or three dimensional space, considers equilibrium of forces between particles and condition of compatibility of deformation with assumptions. It is a noticeable thing that the equation of the ratios of increments of strain can explain the results of real tri-axial shear tests on sand performed by Shimobe, Miyamori, whereas equation of each component of strain cannot be confirmed to be true.

Kobayashi (1975) derived a stress ratio-strain increment ratio relationship from the consideration of equilibrium of force and compatibility of deformation for two particles in tetrahedral-closed packed hexagonal array.

vi) The writer's theory [Mogami, 1965, 1969]

A theory of another type basing on the distribution of voids in granular material was proposed by the writer.

The configuration of particles in a granular material is very complicated but the void ratio has been thought to be a measure of the mechanical properties of the material. Even if the void ratio be constant, the distribution of void ratios can be different. When N particles are put into a box of volume V, the void ratio of the material is constant regardless of the arrangement of particles. When nine numbers giving coordinates of three arbitrary points in a particle not lying on one plane are given, the position and inclination of the particle is determined. Hence to indicate the arrangement of N particles in the box, 9N numbers are needed. These nine numbers are designated as a whole. If one number is attached to each particle, some quantity which indicates a state of the mass is defined as a function of N quantities $a_1, a_2, \ldots\ldots, a_N$.

Different set of N quantities corresponds to different state, so that we can use the notation I_i for ith state of the mass. We cannot give arbitrary numbers $a_1, a_2, \ldots, a_N$, we have, for instance, a restriction that two particles cannot occupy the same place at the same time.

When such restrictions are satisfied, sets of numbers such as $a_1, a_2, \ldots, a_N$ are called the mathematically possible configurations and are designated as Z. Particles are located on the surface of the earth so that they cannot be in the state of floating. Hence, even if, a state can exist mathematically, it cannot appear actually. Then the number of states which can exist physically is less than Z and they are designated Z'. Moreover, as known from observations by authors, the possible states are still more limited, the more our knowledge about the law of change of the configuration increases, the more will the number of possible state decreases. Hence the mechanically possible states will be designated as Z''.

It can be assumed that a probability is assigned to each state of configuration, so that we could say,

for mathematically possible states,

$$P(I_1)\neq 0,\ P(I_2)\neq 0,\ \text{-----},\ P(I_Z)=0$$

for physically possible states,

$$P(I_1)\neq 0,\ P(I_2)\neq 0,\ \text{-----},\ P(I_{Z'})=0$$

$$P(I_{Z'+1})=0,\ P(I_{Z'+2})=0,\ \text{-----},\ P(I_Z)=0$$

for mechanically possible states,

$$P(I_1)\neq 0,\ P(I_2)\neq 0,\ \text{-----},\ P(I_{Z''})\neq 0$$

$$P(I_{Z''+1})=P(I_{Z''+2})=\text{-----}=P(I_{Z'})=\text{-----}=P(I_Z)=0$$

And moreover every state is equivalent as far as the mechanical properties are concerned, because some deformation law of the sample can be found even if the configuration of particles in the sample is not exactly identical. Hence we have the reason to assume

$$P(I_1)=P(I_2)=\text{------}=P(I_{Z''})$$

In the theory of information, the degree of uncertainty or the entropy is defined by the equation,

$$S=-K_1\Sigma P(I_i)\ln P(I_i)\ ,$$

where K_1 is a constant and logarithm means natural logarithm.

If the probability of the state is equal, the above equation has the form,

$$S=-K\ln W\ ,$$

where $P(I_1)=P(I_2)=\text{------}=P(I_{Z''})=W$

Probability of mathematically possible state can be calculated with the help of combinatory analysis. Even though the entropy for the mechanically possible state of assembly cannot be obtained, but when the mechanical process is stationary the entropy for the mathematically possible state can be used in place of the entropy for the mechanically possible state, because we need only to know the increment of the entropy in developing the theory. The entropy in information theory can be that in thermodynamics, so that we can proceed the thermodynamical consideration with above obtained entropy. In case of the deformation of a granular material such as sand, the internal energy stored in the mass in form of elastic energy is far less than the energy dissipated in the material, hence the energy supplied to the material is approximately proportional to the entropy production in the material.

By such consideration the writer could derive a relationship between the void ratio and the angle of internal friction, that is

$$\sin\phi=\frac{k}{1+e}\ ,$$

where k is a constant.

Experiments show that the above formula is valid. Also the writer with Yoshikoshi could find out by experiments that the constant k contained in the formula is likely to be linealy connected with logarithm of the uniformity coefficient of the material. [Mogami, Yoshikoshi, 1969, 1971]

In this occasion, the writer expresses his appreciation to Gudehus' kind discussion which was helpful to improve the prototype of the writer's theory. [Gudehus, 1968]

7. Practical problems

The committee of our Society introduced in the introductory chapter had, as a matter of course, field engineers as members of whom interest was naturally related to their practical problems. Up to the present the studies of the mechanics of granular materials have not yet been sufficiently developed so that informations obtained by the committee were not enough to satisfy their requirements. However, their participation made the activity of the committee to be sound. Our final aim of research concerns with problems of actual ground so that field investigations are wanted to be performed more in future.

i) Practical formula

As shown before the writer presented a formula to estimate the angle of internal friction. He thinks that it would be helpful more or less for practice, even if its accuracy is not so high, because almost all soil mechanical calculations are based on this angle and the reliable sampling method has not yet been developed. Moroto suggested that the k-e_{min} diagram can be used for practical classification of sand and gravel. [Moroto, 1975]

ii) Initial anisotropy

Oda studied the influence of the difference in initial fabric to the mechanical properties of sand sample in tri-axial compression and he found that the influence of initial anisotropy is sometimes not negligible. Oda also reported that in actual sand layers, appreciable anisotropy in arrangement of particles is recognized. [Oda, Kazama, Enomoto, 1976] Tanimoto [Tanimoto, Noda, Fudo, Noguchi, 1974] reported that the wave velocity along a shore line and that along the direction perpendicular to it is different. The velocity of P-wave along the shore line is around 10 % larger than that in perpendicular direction, whereas S-wave showed the reverse character. Hence it can be concluded that the resisting power of the sand layer in perpendicular direction would be larger. This was confirmed by a simple direct shear test which showed 10 % difference in the angle of internal friction. Yanagi also gave an example which show the importance of anisotropy of the original layer for its mechanical properties. [Yanagi, 1971]
Investigations due to Oda et al and Mikasa [Mikasa, Takata, Kinoshita, 1970] also revealed that the bearing capacity of the ground is also depended on the initial structure of the layer of either natural or artificial.
Above cited all studies indicate that the initial structure of the ground is very important in connection with the soil mechanical properties of ground. The relationship between actual soil properties in the ground and those of samples obtained by tests in laboratory is wanted to be studied more.

iii) Age effect

Nishigaki et al [Nishigaki, Okajima, Yoshida, 1974] performed mechanical tests on sand samples from alluvial and diluvial layers. Samples are taken by hand-trimming method. It was shown that ϕ_d was not equal to ϕ_{cu} and the former was larger. They found a stress level σ_c, under this level, in consolidated undrained tests, samples showed similar characters in failure envelope, dilatancy characteristics to overconsolidated clay. Over this stress level, samples had similar character to ordinary clay. The stress level for alluvial sand is far larger than the overburden pressure and it seems to change with weathering and strength of particle. After sampling of undisturbed sands from various horizons in southern Kanto, Tohno conducted unconfined compression tests on them. [Tohno, 1975, 1976] Also investigations by a scanning electron microscope were performed. The relation between the geological age and the density of sample is very clear. Observations with electron microscope show the existence of finer particles between sand grains for all samples. Sand grains found in hydraulic fill have few contacts between them, whereas sand grains deposited in late pleistocene are in tight contact. Sand grains of middle pleistocene deposit were in contact at least at two points. It is a matter of common sense that the unconfined compression strength of saturated sand is zero but samples from quaternary deposit had the strength, which probably is due to the cementing effect of finer particles between sand grains.

iv) Finer fraction

Since the actual sandy ground contains always some amount of finer particles under 74 μ, the influence of this part to sand cannot be ignored.
Several studies about this problem were presented [Kurata, Fujishita, 1962; Nishi, Ishii, 1971; Kitago, Nara, 1974] which show that when the clay content increased beyond some limit the sand-clay mixture loses sandy characters and approaches to clay. This clay content is around 30-40 %, but it seems that it depends slightly on its water content. Some researches are made on the influence on dynamical properties of sand and clay mixture but we do not touch these problems now.

v) N value of standard penetration test

In our country, N value of the standard penetration test is prevailingly utilized when the mechanical characteristics is investigated. Recently the influences of several factors upon the N value are investigated. Originally N value is offered as a number which shows the relative density of ground. From studies in laboratory, Gibbs and Holtz found that the overburden pressure has influence on the relation between the value N and the relative density D_r, and after the time of this study results of many researches about the problem were presented. Rodenhauser [Schmertman, J.H., 1975] found that the N value is proportional to the value $\sigma_0^{2/3}$ which is expressed by the formula,

$$\sigma_0 = \frac{1}{3}(1+2K_0)\sigma_{v'}\ ,\quad \sigma_{v'}=\text{overburden pressure}$$

The value K_0 would probably be large for very overconsolidated diluvial sand, artificially compacted sandy ground etc. so that the studies about the influence of the value K_0 are wanted. The N value and density of the artificially made ground which was reclamed with sand were observed by Saito [Saito, A., 1977]. The N value after improvement of ground was found to be very high but the relative density was not so large, which was contrary to his expectation. He considers that high value of K_0 after the improvement resulted the increase of the value N.
After examination of Saito's data, Imai gave a comment. The relationship between the relative density and the value of $\sqrt{N/\sigma_{v'}^{2/3}}$ calculated by data of sand before reclamation is plotted(Fig. 7), thus obtained straight line showed good coincidence with the relationship obtained by Gibbs-Holtz for saturated coarse sand in laboratory.
But the relationship for sand after reclamation gives values of N three to four times

larger than those for sand before reclamation.

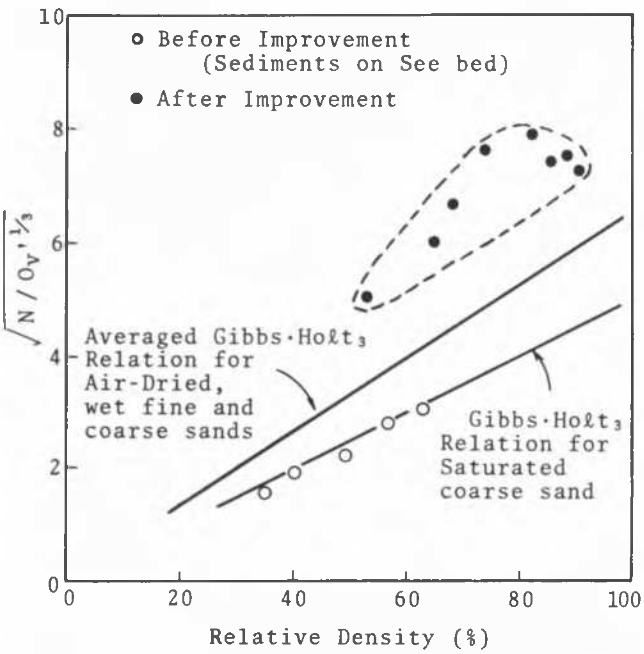

Fig. 7 Change in N-σ_v'-D_r relation by improvement.

This shows the value K_0 increased from 0.43 to 4-6. Actually by LLT tests Saito showed that K_0 value increased from around 1 to 3-6. Miyata et al [Miyata, Takano, Miyamura, 1976] observed values of N, D_r, and K_0 before and after improvement by compaction. They reported that K_0 increased 1.3 to 2.4 times, D_r 1.2 to 1.5 times and N 3 to 5 times. Because the N value is proportional to the square of the relative density D_r so that the change in N value due to the change in relative density is estimated as 1.4 to 2.3 times, this estimated value attains only a half of the actual increase. Hence it must be concluded that the N value after improvement includes the increase due to the horizontal stress. If this effect is neglected, the relative density estimated from N value would be overestimation.

vi) N value and cementation

Fukuoka and Oda investigated N values of consolidated sands from tertiatiary to quarternary. [Oda, N., 1973] The relative density of this sand is 106 % and is hard enough to be block-sampled. These sand samples have cohesion component of shear strength in unconfined compression.
After disturbing this sand and compressed to around 100 % relative density, they made samples and the samples were uni-axially compressed.
They found that the cohesion component vanished, the change in the angle of internal friction could not be observed. The N values of the original ground are 5 to 14 larger than those of the artificially made ground compacted to the state of similar relative density to original. N values of this artificially made sandy ground have good coincidence with those estimated from the relative density by Gibbs-Holtz formula.
The reason of higher N values of original sandy ground would be due to cementation of sand grains. However, considering the facts that 1. when this sand is disturbed and saturated and is brought to unsaturated state, apparent cohesion attains to almost a half of the cohesion of original undisturbed sample, 2. N values of this sand in unsaturated and dry states are almost equal and 3. in general, dilatancy index obtained by triaxial test for over-consolidated sand is larger than that for ordinarily consolidated sand 4. cemented sand is almost always overconsolidated and 5. the measurement of N value is accompanied by the decrease of volume. They presented their opinion that large N value of cemented sand could be due to the deformation characteristics.
Reffering also Hanzawa's [Hanzawa, H., Matsuda, E., 1976] discussion about the relationship between relative density and N value, Imai offers his comment that it would be very interesting if N value of sand layer is depended on the dilatational characteristics which is essential to the granular materials.

8. Remarks

i) Fundamental equation of deformation

The existence of the state surface suggests a functional relation, $f(e, p') = g(q/p', C)$---1, where C is a constant depending on initial condition. On the other hand, the existence of the equi-γ line gives a functional relation $F(q/p', \gamma) = G(p')$---2. By elimination of p' from equations 1 and 2, a functional relation $H(q/p', e, \gamma) = C$ can be deduced.
This relationship would be same as the equation of stress ratio-strain increment ratio type which was obtained by authors (chapter 6) starting from different assumptions, because the equations which give numerical values having good coincidence with those obtained by experiments should essentially be same inspite of the difference in appearence. Theoretical equations in the chapter 6 show better coincidence with experimental results in the peak region than in the initial stage of loading.
This would be due to the fact that $dv^p/d\gamma^p$ is determined for the value of q/p' independently of stress path only when q/p' is close to peak. In this region, q/p' is almost constant so that the flow rule due to Roscoe and others also gives the equation of stress ratio-strain increment-ratio type. In this flow rule, the volumetric strain due to allround pressure is included. However, as Moroto and Tatsuoka pointed out, only the volumetric strain which is dependent on dilatancy due to shear strain should be considered.
In the chapter 5, imaginary opinion about the behaviour of particles in granular material was given. As stated in that chapter, geometrical and mechanical uniformity would be achieved when the peak approaches.
This would be the reason of the above said fact and also this gives the basis for allotting equal probability for all states of

configurations of particles which the writer adopted as his fundamental assumption, though he has not any direct experimental proof for this.

ii) Deformation problem in soil mechanics

Problems of soil mechanics are essentially statically indeterminate, so that the problem can be solved when the deformation characteristics and the boundary conditions of the problem are completely known. It is very hard to know the boundary conditions completely because the conditions depend on many unknown things. Even if the fundamental equation is obtained and confirmed by precise experiment in laboratory, too many conditions necessary to get the solution are always left unknown, so that the solution of the problem is still impossible.

The writer recognizes of course the importance of the fundamental equations and the researchers' efforts to get them are highly appreciated. However, he wants to point out the other side of the problem.

It could be easily understood that the earth pressure cannot remain constant because of the complexity of the material and the surrounding. Hence it can be said that the deformation has also the similar character because the deformation depends on stresses in the ground.

In addition, in case of ideal granular material, the theory of earth pressure shows that the pressure has the value between limits, these are active and passive earth pressures. Hence it would not be strange to think that the deformation of the ideal granular material lies between limits because of the frictional character of the material.

It is the writer's imagination that the existence of above said limits of deformation would correspond to the fact that there exist many state surfaces which are confined in some region for one kind of sand depending on the initial etc..

iii) Practical problems

As introduced in the chapter 7, practical approaches to the mechanics of granular materials are gradually progressing also in our country.

These researches are wanted to be developed more and more in the future.

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Notations

ε_a, ε_1 : axial strain
ε_r : radial strain
γ : shear strain $=\varepsilon_a-\varepsilon_r$
ε : shear strain $=\frac{2}{3}\gamma$
ε_{ij} : strain component in connection with ith and jth axes
v : volumetric strain
σ : stress
σ_i : ith principal stress
σ_N : normal stress
σ_a : axial stress
σ_r : radial stress
τ : shear stress
σ_v' : overburden pressure
p : mean pressure $=\frac{1}{3}(\sigma_a+2\sigma_r)$
q : shear stress $=\sigma_a-\sigma_r$
J_i : ith stress invariant
$J_1=\sigma_1+\sigma_2+\sigma_3$, $J_2=\sigma_1\sigma_2+\sigma_2\sigma_3+\sigma_3\sigma_1$, $J_3=\sigma_1\sigma_2\sigma_3$
q/p : stress ratio
S : entropy
S_s : entropy
D_r : relative density
ϕ : angle of internal friction
e : void ratio

The letter p written as super-script means plastic and the prime as super-script means effective stress.