The Probabilistic Approach to Soil Mechanics Design

L'Approche Probabilistique dans les Études de Mécanique du Sol

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INTRODUCTION

E. Schultze

The first time statistics in soil mechanics and foundation engineering in context with the problem of safety were discussed, was at the Specialty Session:
Safety factors in Soil Mechanics of the 7th International Conference on Soil Mechanics and Foundation Engineering in Mexico (1969), with Professor G.G. Meyerhof as reporter (s. Proceedings of the Conference, Volume III, p. 479).

Incited by the success of this Specialty Session, Professor P. Lumb organized the 1st International Conference on Statistics and Probability in Civil Engineering in Hong Kong 1971 (s. Proceedings of the Conference, published by Hong Kong University Press 1972), which was followed by the 2nd International Conference of Applications of Statistics and Probability in Soil and Structural Engineering in Aachen 1975, which was organized by myself (s. Proceedings of the Conference, 3 Volumes, publised in English by the Deutsche Gesellschaft für Erd- und Grundbau, Kronprinzenstr. 35a, Essen, West Germany, 1975). During this Conference, the following topics for today were proposed.

1) Partial and Total Safety Factors
   (reporter: Professor Dr.-Eng. Meyerhof)

2) Statistical Estimation and Extrapolation from Observations
   (reporter: Professor Dr.-Eng. Biarez)

3) Optimization in Soil Engineering
   (reporter: Professor Dr.-Eng. Vanmarcke)

The scope of the two last-named Conferences also included, beside soil mechanics, structural engineering. A 3rd International Conference is planned for 1979 in Sydney and it is being organized by Professor O.G. Ingles, University of New South Wales, Australia. The great interest, and the amount of interesting problems involved, was the cause to choose statistics as a topic for a Specialty Session of this Conference, although this time, only soil mechanics and foundation engineering are going to be treated.

In this context, I would like to draw your attention to the figure on the cover of the Proceedings of the 2nd ICASP in Aachen, which shows two intersecting distributions for S (stress) and R (resistance), and which can be taken as a symbol of safety in construction engineering and in soil mechanics and foundation engineering (Fig. 1).

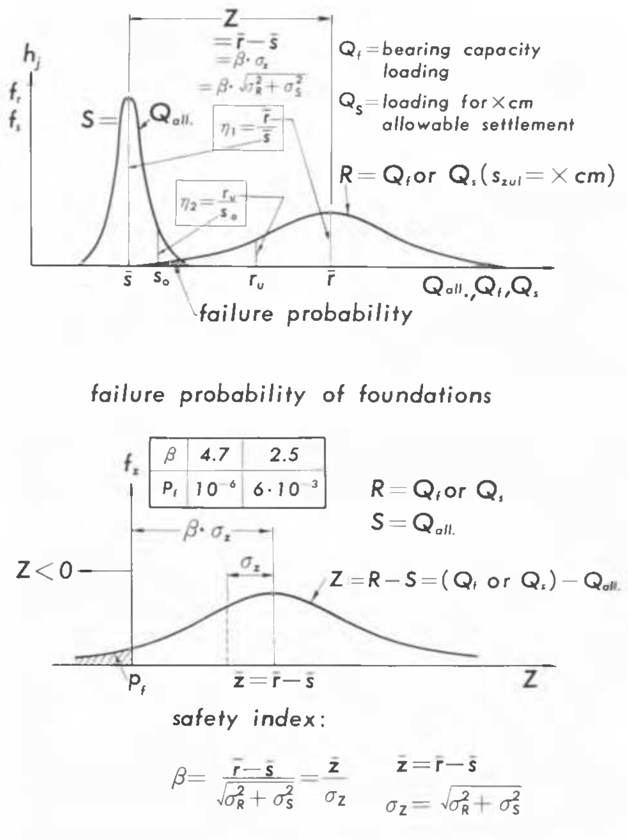

Fig. 1 Failure probability p_f, safety index β and safety factor η

On the basis of this conception, the Institute for Building Technology in Berlin prepared a "Guide for the determination of safety in civil engineering" for construction engineering, the application of which to foundation engineering, especially with regard to the bearing capacity of shallow foundations, is being examined as a research project by myself at the moment. Some conclusions derived from partial results may be of interest for this Session.

The evaluation of a data collection concerning soil examinations which has been accumulated for many years, gave, first of all, the frequency distribution of the more important soil parameters for the different soil classes, which were then divided according to geological regions.

For a further evaluation, the shear parameters and the compression moduli were used. In the first case, the scatter was used to calculate the scatter of the bearing capacity factors - to date for sand with c = 0, and also N_γ and N_q. According to DIN 4017, Blatt 1 the corresponding equations are:

$$N_q = e^{\pi \tan \phi} \cdot \tan^2 (45° + \phi/2) \qquad \text{(Prandtl)} \qquad (1a)$$

$$N_\gamma = (N_q - 1) \cdot \tan \phi \qquad \text{(Muhs/Weiß)} \qquad (1b)$$

As these equations are very sensitive to a variation of ϕ, the variation coefficient $v = \bar{x}/s$ for ϕ benig 0.1, the variation coefficient increased to 0.4 - 0.5 for N. The variation coefficient of the bearing capacity of a foundation, for which the measurements a (length), b (width), t (foundation depth) naturally had to be known, increased correspondingly.

To simplify matters, only one influence factor was normally distributed, the others were assumed to be constant. Otherwise, the Monte Carlo Method would have had to be been applied.

For the second condition for the permissible load, those loads had to be calculated, which produce 1 resp. 3 resp.5 cm settlement. As normally only the distribution of the soil parameters v and w in the equation

$$E_s = v \, (\sigma/p_a)^w \qquad (2)$$

are known from soil examinations, the difficulty is to decide how to combine one with the other, and how large the unknown stress σ has to be assumed, which can only be decided by iteration.

To obtain the permissible values of S from these R distributions, the corresponding variation coefficient v_s has to be known. This value was fixed to 0.05 for permanent loads and to 0.30 for variable loads. The position of the R- and S-distributions has to be calculated for a failure probability for $p_f = 10^{-6}$ for the limit value of the bearing capacity according to the "Guide" by which the safety index $\beta = 4.7$ can be computed according to the equation (Fig. 1):

$$p_f = \frac{1}{2\pi} \int_{-\infty}^{\beta} e^{\frac{-t^2}{2}} \, dt \qquad (3)$$

t = integration variable

Furthermore

$$\beta = \frac{\bar{r} - \bar{s}}{\sqrt{\sigma_R^2 + \sigma_S^2}} = \frac{\bar{z}}{\sigma_z} = \frac{\bar{r} \cdot (1 - 1/\eta)}{\sqrt{\sigma_R^2 + \sigma_S^2}}$$

$$\bar{z} = \bar{r} - \bar{s} \; ; \quad \sigma_z = \sqrt{\sigma_R^2 + \sigma_S^2}$$

By this means, the distance $\bar{z}$ is fixed for a given distribution R for the distribution S, whereby the failure probability p_f has not been exceeded.

Related to the mean values, the safety factor is:

$$\eta = \bar{r} / \bar{s}$$

In this case, the difference between the variation coefficient for permanent load $v_s = 0.05$ and $v_R = 0.5$ is very large (Fig.2). A fairly good approximation is achieved, if the distribution S is substituted by a vertical line, i.e. the mean value (Fig.3). Then the failure probability turns into the fractile. The above mentioned equations can now be simplified, as $\sigma_s = 0$, $v_s = 0$:

$$\beta = \frac{\bar{r} - \bar{s}}{\sigma_R} = \frac{\bar{r} - \bar{s}}{v_R \cdot r} = \frac{1 - 1/\eta}{v_R}$$

$$\eta = \frac{1}{1 - \beta \cdot v_R}$$

If $\beta \cdot v_R > 1$, a negative safety is obtained, therefore the following must be true:

$$\beta < \frac{1}{v_r}$$

In this case, for $v_R = 0.5$,

$$\beta < 2$$

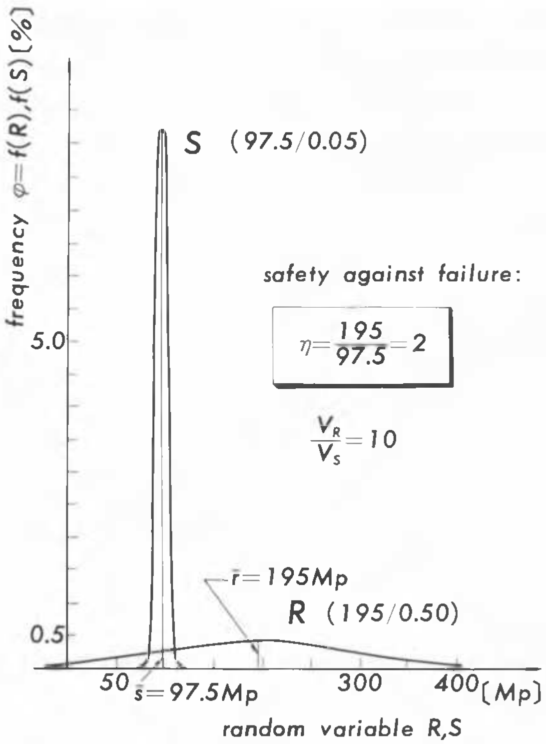

Fig. 2 Limit case of failure probability for a large scatter of R compared to S (v_R/v_S = 10)

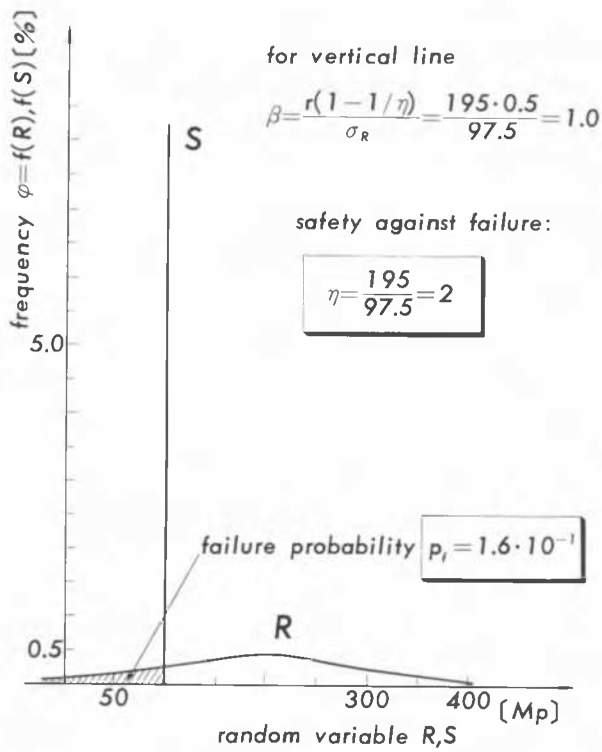

Fig. 3 Approximation of the failure probability in the case of Fig.2 by a fractile

For the often demanded safety factor for the bearing capacity η = 2, a value of β = 1 is obtained for v_R = 0.5 and also a failure probability $p_f = 1.6 \cdot 10^{-1}$.

These results are very different from those obtained for variable loads and for construction engineering, where the difference between v_R and v_S is much smaller (Fig.1)

That is why the necessary failure probability which is about 10^{-6} (see above) for construction engineering, is much larger in foundation engineering.

According to the Recommendations of the Committee for Waterfront Structures, p. 13 (EAU 1970) published by Wilhelm Ernst & Sohn, a fractile, in this case corresponding to the failure probability, of 10^{-1} is regarded as sufficient.

PARTIAL AND TOTAL SAFETY FACTORS

G.G. Meyerhof

In two earlier papers (Meyerhof 1970 and 1976) the author reviewed the safety factors in soil mechanics and foundation engineering. It was shown that estimates of the performance of earth structures and foundations should include adequate safety factors against both the ultimate limit state (mainly instability against sliding, bearing capacity, overturning, uplift, seepage and erosion) and the serviceability limit state (mainly total and differential movements, cracking and vibration). The inevitable uncertainties in problems of design and construction may be either objective (such as loads, soil resistance and deformation) or subjective (such as analysis, judgement, experience and human errors).

The magnitude of partial and total safety factors is governed by the reliability of information (mainly loads and load effects, resistance, deformation, design and construction), the economy of construction and maintenance, the probability and seriousness of failure during service life. The safety margin is influenced by the loads and load

Table 1. Partial Variability and Partial Safety Factors

Variability Coefficient	Loads	Soil Properties	Analysis Construction	Safety Factor (90% Reliability)
Very Low <0.1	Dead Loads Stat. Water Pressure	Unit Weight	Earthworks Earth Retg. Structs.	<1.1
Low 0.1 - 0.2	Pore Pressure	Index Prop. (Sand) Friction	Foundations	1.1 - 1.3
Medium 0.2 - 0.3	Live Loads Environ. Loads	Index Prop. (Clay) Cohesion		1.3 - 1.6
High 0.3 - 0.4		Compress., Consol. Penetr. Resistance		>1.6
Very High >0.4		Permeability		

Table 2. Values of Minimum Partial Safety Factors

Category	Item	Safety Factor
Loads	Dead Loads	0.9 - 1.2
	Live Loads	1.0 - 1.5
	Static Water Press.	1.0 - 1.2
	Environmental Loads	1.2 - 1.4
Soil	Cohesion (c)	1.5 - 2
Strength	Friction (tan ϕ)	1.2 - 1.3
	Cohesion plus Friction	1.3 - 1.5

Table 4. Values of Minimum Total Safety Factors

Failure Type	Item	Safety Factor
Shearing	Earthworks	1.3 - 1.5
	Earth Retg. Structs.	1.5 - 2
	Offshore Foundations	1.5 - 2
	Foundations on Land	2 - 3
Seepage	Uplift, Heave	1.5 - 2.5
	Piping, Exit Gradient	3 - 5

Table 3. Total Variability and Total Safety Factors

Variability Coefficient	Stability	Settlement	Stability Safety Factor (99% Reliability)	Settlement Factor (90% Reliability)
Low 0.1 - 0.2	Slopes (Sand)		1.3 - 1.9	
Medium 0.2 - 0.3	Earth Retg. Structs. Slopes & Foundts. (Clay)	Foundations (Clay)	1.9 - 3.3	1.3 - 1.6
High 0.3 - 0.4	Foundations (Sand)	Foundations (Sand)	>3.3	>1.6
Very High >0.4	Piles (Dynamic Analyses)			

effects for dead, live and environmental (water, wind and earthquake) loads, the soil resistance and deformation (including effects of sampling and disturbance, specimen size, rate and variation of loading, anisotropy, plane strain, local and progressive failure, pore pressures and drainage), analysis (method, accuracy, assumed mechanism, simplified soil profile and weak zones) and construction (geometry, quality and control of materials and workmanship, maintenance during service).

The factors of safety (ratio of resistance of structure to applied load effects for freedom of danger, loss or unacceptable risk) are usually based on characteristic loads including uncertainties of the analysis, and characteristic resistance or deformation including uncertainties of construction. Typical values of the partial coefficient of variation of loads, soil properties, geotechnical analysis and construction are given in Table 1 and they have been used to estimate the corresponding partial safety factors for a 90% reliability. The range of these partial safety factors is supported by customary values of minimum partial safety factors on loads and soil strength shown in Table 2. Similarly, typical values of the total coefficient of variation of the stability and settlement of earth structures and foundations are given in Table 3 and they have been used to estimate total stability safety factors for a 99% reliability and total settlement factors for a 90% reliability. The range of these total stability safety factors is in reasonable agreement with conventional values of the corresponding minimum total safety factors in stability estimates shown in Table 4.

The upper values of the above mentioned partial and total safety factors apply to normal loads, service or operating, while the lower values can be used for maximum loads and worst environmental conditions and also with performance observations, large field tests, analyses of similar failures, at the end of the service life and for temporary works. The conventional total safety factors are associated with nominal life-time failure probilities of the order of 10^{-2} and 10^{-4} for earthworks and foundations, respectively, and these values appear to be acceptable in practice (Meyerhof, 1970 and 1976).

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STATISTICAL ESTIMATION AND EXTRAPOLATION FROM OBSERVATIONS

J. Biarez and J.L. Favre

The authors are truly pleased to see one of the Session of this Conference being devoted to the application of Statistics and Probability to Soil Mechanics. Prof. Meyerhof, Prof. Schultze, Prof, Lumb and several colleagues made it possible especially after Hong-Kong and Aachen.

Well known relationships between parameters have been used by engineers in Soil Mechanics for a long time (Terzaghi, Casagrande, Skempton). Along the same lines careful studies of correlations have also been developped here and there. But recently the general theories of Probability and Statistics have been applied which are able to identify mathematically the population of each variable, their distribution in space, the relationship among them. These concepts give a probabilistic knowledge of strains and stresses and their positions with respect to given acceptable or maximum bounds.

Soil Mechanics people know well that the soil layers are usually heterogeneous and measurements are often widely scattered and their statistical analyses appear sometimes to be hopeless for the mathematician or for the engineer. As for the finite element method, the application of statistics must be coupled with an increased quality of measurements and sampling methods. The authors sincerely wish that a Session of the next Congress be devoted to a discussion of these techniques. They feel that some improvement is needed in order that the finite element method and the statistical analysis be generalised on sound bases.

Whatever be the quality of mathematical tools, we think it useful to recall that the parameters used in Soil Mechanics are related through the concepts of the Mechanics of Continuous Media. Through this theory, we are able to classify variables and to correlate them inside the same class or between given classes but not anyhow. Let us consider the example of the determination of the bearing capacity of a foundation. This bearing capacity belongs to the class of solutions and thus must depend on three classes:

mechanical properties (c, ϕ)
exterior body forces (γ)
and boundary conditions (B, D, L)

The cohesion and the friction angle give with a reasonable accuracy the law of perfect plasticity. On the other hand, if settlemets are needed the Young's modulus and the Poisson's ratio are too crude parameters to represent properly the non linearity of stress-strain relationships. Sometimes, the geometric conditions are of a secondary nature: e.g. R_p is related to c and ϕ, and if the depth is large enough, the role of B and D will be small for a penetrometer or a pile.

Numerous parameters are considered in Soil

Mechanics. Even if it is not a priori obvious, most of them have a meaning related to the concepts of Mechanics of Continuous Media. It is important to consider the fact that this theory may consider the whole mass of soil as continuous. In the same way, we may consider the grain itself as a continuous body with given mechanical properties Eg, Dg, Cg, ϕg which are generally not essential. In the latter point of view, the mechanical properties at the grain interfaces must be introduced like $\bar{c}$ and $\bar{\phi}$, and the boundary conditions of the assemblage which are often complex. These boundary conditions are approximately written when the volume of grain in a unit volume is computed with e, n and γd. The geometrical anisotropy may be represented if necessary by the statistical orientation of contact planes. The geometry of the squeleton is also represented by the granulometry also in a first order of approximation.

To get the constitutive equation of the continuous medium, homogeneous stresses and strains must be applied on the sample. In this way, measurement can be made at the sample contours only like in the "triaxal" apparatus. Through the foregoing analysis, we may deduce the mechanical properties of the continuous medium from those of the discrete simplified medium: c $\longrightarrow$ $\bar{c}$ and γd. (See the table)

The grains interfaces properties may be related to the properties which are independent of the density of the continuous medium. They are obtained in several cases:

1. Mechanical properties after large strains annealing the past history of the material: c, ϕ perfect plasticity, C_c (see Table Ronde 1972).

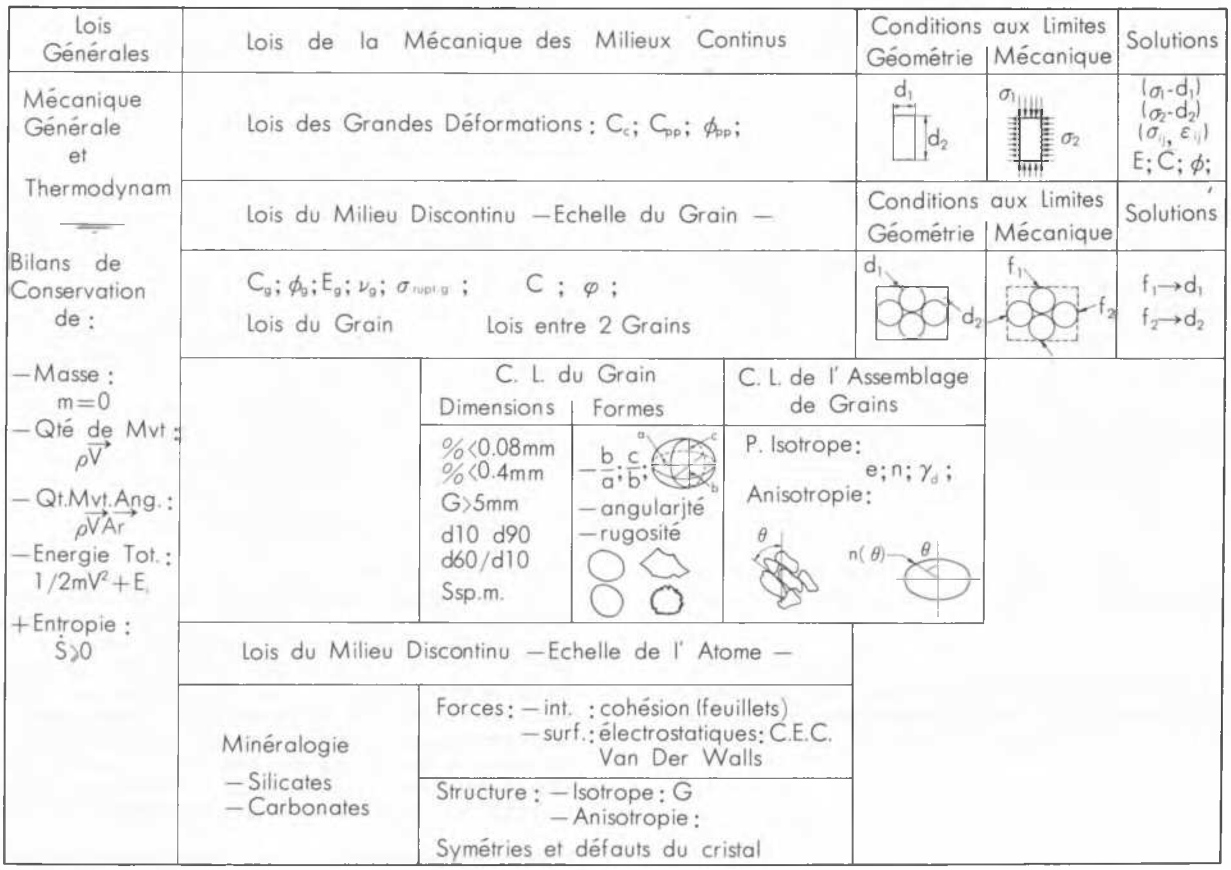
2. Properties corresponding to normalized boundary conditions on an initially loose sample like γdopt, Wopt, Wp, Wl, Ip.

3. Variable grouping like (E / σc), (Cu / σc), (E'/ σc) for clays. Thus C_c is not related to γd and thus cannot depend on Rp.

The numerical value of the density or of the water content if the soil is saturated must be supplied. Another way is to consider normally consolidated soils at small depths because the density depends in the consolidation stress σc.

Statistical estimations and observations extrapolation are applied to the three calsses of data that we defined above. They are also applied to the solutions of the corresponding equations of the continuous medium as well as of the discrete medium.

As a conclusion the authors believe that the



concepts of the mechanics of continuous media are a good basis for the classification of the data and observations.

In the area of observation and its first treatment statistical analysis is an essential tool. Probability theory is rather applied to solutions depending on the statistical analysis using models (forecasting and optimization theory).

Papers submitted for discussion

The communications which will be submitted for the discussion are the following:

A. *Papers related to coefficients of constitutive equations either of the continuous or the discrete medium*

The problem of the accuracy of measurements connected with the operator or the testing procedure has not been touched upon. This question appears to be very important however with the development of new techniques for in-situ testing particularly. It is also crucial if bank of data are to be collected from different sources. The studies often compare one technique with another or one organization with another without condering any Analysis Variance.

Concerning the display of data, most of the communications give histograms, means and variances. This practice should be soon generalized given the importance of linear estimations.

Recordon and Despond give a study of coefficient of variation of the parameters of a clay layer to identify its homogeneity. Furthermore they show a same classification of the parameters with respect to their coefficients of variations between two different soils.

Concerning the models of distributions, D. Athanasiou-Grivas and E. Harr use for ϕ and C normal, log normal and beta p.d.f. with a coefficient of variation of 0.15 as recommended by Lumb.

Using the Monte Carlo method, these authors compute for three average values of the friction angle, the three bearing capacity factor with average, coefficient on variation, probability of fit using W test and a log normal law. Their results are reported in very interesting tables. The probabilities of fit are low in the interval 0.4 - 0.5 and they increase with the friction angle. It would be interesting to know the influence of these probabilities on those of rupture that the authors give in function of the safety factor.

In the area of probabilistic models for parameters, M. Matsuo and A. Asaoka give a very interesting study of the true value of the undrained shear strengh. Two influencing factors are considered separetely: the stress independent disturbance measured by the O.C.R of a disturbed sample as defined by Ladd and Lambe and the dependency of the true value of the strength on the consolidation process. The authors make the hypothesis that the ratio su/po is equal to a constant for a given material. In the case of a layer of a natural marin clay the probabilistic characterization of the true value is only function of the depth. The model has been tested on 1708 measures divided into four groups according to four degree of disturbance. The criterion of the test is a non-biased estimation of the Kullback Leibler Information Criterion. The basic assumptions of the models have been carefully checked as: (1) coefficient of variation is assumed to be constant; (2) the autocorrelation is equal to zero. The difficulty related to the separation of the error terms in the classical convolution models is thus here overcome.

B. *In the area of relations between parameters of constitutive equations*

Many communications are often dealing with the rheological properties of soils which is an essential aspect. Four papers are of this nature:

The report of Biarez and Favre intend a global study of some of these parameters of constitutive equations and compactness. Marine sediments are analysed to judge of the influence of carbonates. Their work is based on a particular factor analysis using the χ^2 metric. This technique respects the distributional equivalence and the symmetric role of observation and characters which are interpreted simultaneously on the same factorial axes. Each variable is splitted in several modes and a 0.1 table is used. In this way, the analysis is not restricted to the linearrity of the model. The results are rather of a qualitative nature and the smallness of the sampling does not give a good stability to the model. Three classes of material and two different influences of carbonates are shown. In particular, they are opposite on W_L and C_u.

These methods seem to be a good application of factor analysis using the concepts of mechanics.

Max Kalin gives an interesting discussion of independent variables to find the best regression. The sample may grow from 258 to 718 when some missing data are computed using regressions between parameters of geometry and composition. The regressors of the Darcy permeability K are identified through the study of the table of the coefficients of partial correlations and using rheological consideration giving F. The two samples (n = 285 and n = 718) give approximately the same regression coefficients. The author notes surprisingly enough that K increases with the degree of saturation.

A. El Nihmr and V. Rizkallah give some results on mixed polynomial functions $y = \Sigma bz + \varepsilon$

where z are monomes of degree N in two variables. They obtained good regressions for E with respect to the water content and the normal stress and for c' and ϕ' with respect to the density and two criteria of grain size distributions. (% < 2μ, % < 20μ)

B. Cambou gives an example of the technique consisting of going from the discrete continuous medium. The author introduces the deterministic mode of M. Auvinet of the building of spheres randomly distributed in a grain size distribution and randomly thrown down. A statistical study of all the geometric parameters of the discrete medium is done and their correlations are examined to identify through summation the global parameters of the continuous medium. It should be noted the limitations of the model (grain shape) and its possible developments (mechanical model) to predict constitutive parameters.

These studies using deterministic or probabilistic model of the discrete medium could be a way to investigate constitutive equations.

C. "*Boundary conditions*" *and* "*Solutions*" *observations*

The use of random space processes brings a new examination of boundary conditions: contours of "homogeneous" layers, poorly controlled geometries of structures between soil and structure or along kinematical discontinuities.

The paper by A. Thomas is dealing with the characterization of homogeneous volumes and their automatic mapping (to found the function p = f(x,y)). This very interesting work takes into account the data and the methods of the geologist, the correlations by multivarite analyses between the geological and mechanical criteria. The writer also gives a probalistic solution of p = f(x,y) by a French method called interpolation with "μ.quadratic average", which requires less assumptions than the method of autoregressive filters with covariograms. The author presents two applications: a mapping of risks and an optimization of sampling.

E.H. Vanmarcke gives a simple one homogeneous variable model for the autocorrelation function and the variance function. The essential fact is the correct choice of the parameter of decrease. He comes to simplified formulas for characteristics of the random process and to an easy estimation of the scale of fluctuations in function of the equidistant sampling interval. The monodimensional model of a great comodity can be generalized to 2 or 3 dimensions and can be complicated by considering the variable as a sum of several independent contributions. The author gives 3 examples of computing for the natural water content w and initial void ratio e of the San Francisco "bay mud", for results of SPT (with previous elimination of the dependence of mean and standard deviation with the depth), for index of compressibility, following the horizontal level, under the foundation of a nuclear power plant complex. At last the author gives the principle of computing the correlation between spatial means in function of the process characteristics. Due to space limitation it is difficult to discuss shortly a work which has been engaged for 7 years now.

Another kind of random boundary conditions is dynamic sollicitations like earthquake and wave forces which can be handled either as processes or considering extreme values whose p.d.f. parameters may be estimated.

M. Faccioli uses Bayesian estimation. The author has collected a lot of seismic data on circumpacific belt and has identified four types of sites. For each site he gives with linear regression an attenuation law of maximum acceleration or velocity which have a log-normal distribution (conditional to the magnitude and the focal distance). The latter is really the a priori Bayesian distrubution. Then the author gives the data of dynamical properties of the soil to get with the help of a constitutive law a "numerical observation" on the site which is augmented by an error term to obtain the observed value. Using other hypotheses on relationships between parameters of the site, the bed-rock and the attenuation law the author computes the a posteriory probability of the velocity (or the acceleration) given an estimation of the latter in the bed-rock with respect to the error term. An essential discussion is in connection with the distribution of the error term for which surface records are strongly needed.

At last M. Ingles and Noble and M. Ingles, Hasofer, Rutten and Chen consider two problems of pavement control: The first paper after having defined a Test Utility Concept examines the performance of 332 pavements showing the overdesign and the over testing. A Bayesian analysis of an example is a good illustration of these 2 phenomenous. This probabilistic approach seems very much profitable. The second paper deals with the causes of failure of a pavement followed since 37 years. A particular care is brought to evaluate traffic data and failure data (3 types of cracking and 8 types of subsidence). The statistical analysis shows the failure causes, i. e. injection of cement grout under slabs (mudjacking) and excessive wheel loads.

E. Alonso, J. Casanovas, J. Murcia and J. Santos are dealing with a problem of sequential decision (acceptance, rejection, new information) applied to the warranty against damages on buildings founded on piles. A lot of statistics of prior and actual damages is used for the choice of the maximum settlement which is the decision criteria. This maximum settlement is computed by the authors using the maximum settlement given by elasticity theory. The p.d.f. of the maximum settlement can be estimed from the parameters of a log-normal distribution of E the modulus of Elasticity. A Bayesian estimation is used for the parameters. Thus with models of cost of foundation and reconnaissance, the authors compute the global cost and the risk. An important discussion is based on real cases and on the different implications of this strategy.

Finally we thank V.F.B Demello and P. Bilz for the comments and informations they gave in their paper on the probabilistic approach in Soil Mechanics.

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RISK AND DECISION ANALYSIS IN SOIL ENGINEERING

E.H. Vanmarcke

There is no doubt about the pervasive nature of uncertainty in geotechnical engineering. Much of what we do as professionals is designed to combat and minimize this uncertainty: exploration and testing, application of better analytical procedures and models of soil behavior, and gathering data about loading and environmental conditions. But no amount of effort can ever completely eliminate the risk. Even after extensive and expensive laboratory and field investigation, there always remains some uncertainty about true field conditions. Design level loads are never entirely known and neither is the soil behavior under design level loads or environmental conditions. Therefore, the geotechnical engineer must make decisions, that is, must choose among alternative solutions, in design, exploration, and surveillance, under uncertainty.

In the interest of safety, to compensate for uncertainties, the geotechnical engineer prudently makes conservative assumptions about the soil parameter values, the applied load, and the allowable deformation, and frequently about all of these. However, as the uncertainties remain largely unquantified, the degree of conservatism achieved may greatly vary from project to project, and depends on such factors as the engineer's attitude towards risk and his confidence in the results of a particular set of analyses. Moreover, the confidence is not necessarily justified. A recent study on embankment deformation prediction (1) suggests that geotechnical engineers tend to be overconfident in predictions they themselves make. Failure to make sufficient allowance for real uncertainty jeopardizes safety, while overreaction to uncertainty leads to "wasteful over-conservatism and less satisfactory solutions than if reasonable risks were accepted"(2).

Decision analysis provides a framework for organizing factual information about costs and risks involved in decision situations. Consider the following specific situations in which geotechnical engineers must make decisions under risk:

Site Selection: Choose among alternative locations, or whether to stay or to relocate.

Exploration: Choose amount or type of exploration, spacing of borings.

Design (e.g., earth embankment): Choose embankment height, section geometry, material properties.

Site Improvement: Choose whether or not to compact, excavate or preload.

Construction: Choose method of construction, rate of construction.

Surveillance or Inspection: Choose level of inspection effort: how frequently, how extensively?

These decisions are frequently interrelated and, to some extent, hierarchical. For example, a hazardous condition(e.g., solution activity) about which one speculates at first, becomes better defined as a project progresses through exploration and construction, and it may be kept in check by a surveillance program during operation. The above-mentioned decision situations have the following features in common: in each case, the engineer must make a choice among alternatives which imply different levels of cost and risk. Usually, the least expensive alternative (or "solution") implies the greatest risk, while the solution involving least risk costs most. This trade-off between risk and cost is an essential feature of decision making in geotechnical engineering.

The remainder of this short paper attempts to outline the steps in a general approach to the analysis of decisions in soil engineering; when one is faced with multiplicity of natural hazards (e.g., environmental, geological). Special emphasis is given to the problem of dam safety [3]. First, one must consider all events of concern and their associated consequences, referred to here as the hazard potential. For example, in dam design and surveillance, the event of concern is a catastrophic failure which results in a sudden discharge of the reservoir contents. The major consequences are loss of human life and property damage in the downstream area.

In analyzing the benefits of measures aimed at reducing geotechnical risks, it is useful to express costs as well as risks in relative or marginal terms, referenced to an existing structure or to an available preliminary design. In other words, one of the alternatives serves as a reference for evaluating the incremental cost and the fractional changes in risk generated by the other alternatives. In decisions about repair, inspection, or additional exploration, the reference action may be to "do nothing." In design decisions, the reference solution may be the design obtained on the basis of "common practice." If the reference action is taken, no added cost is incurred ($\Delta c = 0$) and some particular level of risk (p) is incurred. The risk p denotes the probability of "failure" within a specified period of time (for example, one year or the intended life of the structure).

The next step is to consider all alternatives which differ from the reference alternative. These are measures or strategies for providing added protection against "failure": changes in design, site improvements, repair measures, surveillance programs, reservoir lowering, etc.. The benefits of added protection take the form of a reduction in the probability of failure, and hence in the probable losses (economic and social) resulting from failure. Assume that the risks with or without the added protection are denoted by p' and p, respectively. The relationship between p and p' can be expressed as follows:

$$p' = p(1-r) \tag{1}$$

in which r = a measure of the effectiveness of the added protection. It usually ranges from 0 to 1 in value: $r = 0$ indicates a totally ineffective measure (implying $p' = p$, i.e., there is no change in the annual risk); $r = 1$ indicates full (or 100%) effectiveness, implying that $p' = 0$, i.e., the risk has been eliminated. A measure which is 90% effective ($r = 0.9$) reduces the annual risk by a factor of 10. A negative value of r means that the measure under consideration is expected to increase the risk of failure.

Concentrating now on the economic component of the benefits, let C denote the component of the hazard potential which can be expressed in monetary terms. The average monetary losses with and without the added protection are Cp' and Cp, respectively, and the difference between these losses is the average economic benefit resulting from the measure under consideration:

$$b = Cp - Cp' = Cpr \tag{2}$$

Provided r is not negative, b will range between zero and the maximum value Cp, depending upon the effectiveness of the measure.

In the foregoing treatment, no attention has been paid to the actual causes or modes of failure which contribute to the probability of failure. For example, in the case of a dam, catastrophic failure may be caused by overtopping, internal erosion, ground motion during earthquakes, etc. Specific strategies for protection are often designed to limit the hazards posed by a particular cause or mode of failure: spillway capacity is upgraded to counter the threat posed by floods, piezometers are installed to monitor seepage, etc. It is therefore useful to express the risk p as a summation of probabilities each of which relates to a specific cause or mode of failure:

$$p = \sum_j p_j \tag{3}$$

where p_j = the risk due to failure mode or hazard j. Implementation of a design change or protective strategy results in the reduced risk p' which can be similarly decomposed:

$$p' = \sum_j p_j' = \sum_j p_j(1-r_j) = p\left[1 - \sum_j \left(\frac{p_j}{p}\right) r_j\right] \tag{4}$$

in which p_j' = the risk due to failure mode j following implementation of the strategy under consideration, and r_j = the measure of effectiveness of the strategy in reducing the risk in mode j. The relationship $p_j' = p_j(1-r_j)$ is entirely analogous to Eq.1. r_j may be interpreted in the same way as r (the "overall" effectiveness measure) except that the former is referenced to a specific causative hazard; the quantity r_j measures the fractional risk reduction in mode j. Comparing the equation above to the relationship $p' = p(1-r)$, the following simple relationship between the values r and r_j, and the relative risks p_j/p is obtained:

$$r = \sum_j \left(\frac{p_j}{p}\right) r_j \tag{5}$$

This equation expresses the overall effectiveness r as a weighted combination of the r_j values. The weighting factors (which sum to one) are the relative risks p_j/p. Each relative risk value is expressed as a percentage of failures likely to be caused by a particular hazard, and can be estimated from information about the causes of past failures supplemented by professional judgment and perhaps by analytical work. For example, in the case of an existing dam, the relative risks will depend primarily on dam type, age, and site geology [3].

Sometimes, consideration of "r" alone governs a decision. For instance, suppose that an engineer must decide whether or not to raise the crest of an existing earth dam. There are two competing hazards: overtopping (the risk of which will be reduced by raising the crest) and instability (the risk of which will be increased). The overall effectiveness will depend only on the relative risks and the effectiveness indices r_1 and r_2. A negative value of r would guarantee the outcome of the decision (regardless of the added cost).

In conclusion, many geotechnical engineering decisions can be usefully examined in a decision analysis framework. Such an approach attempts to put technical and socic economic issues into proper focus by organizing factual information about risks, costs and losses, both monetary and non-monerary.

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