S P E C I A L T Y   S E S S I O N   2

PROBLEMS OF NONLINEAR SOIL MECHANICS

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Vice-Chairman Dr.Yu.K.Zaretsky

Ladies and Gentlemen, dear Colleagues,

Allow me to open Specialty Session No.2. The program of this Session is known to you from Bulletin B. Our aim is an exchange of opinions on the development of the nonlinear problems of plasticity, creep and the consolidation of soils. I would like more comments to be made on the methods of calculating the stressed-strained state of a soil base in all ranges of loads, up to the limiting ones.

To our great regret the Chairman of this Session, Professor H.B.Poorooshasb could not come to the Conference, but he has sent us his opening speech. We will begin by listening to it.

I want to remind you, dear Colleagues, of the time schedule and to ask you to keep strictly to it. Five reports will be made. The time alloted ends at 3.40 p.m.;then there will be an intermission until 3.55 p.m.;after which discussion will begin,and last until 5.15 p.m. Fourteen persons have registered for the discussion,and,I assume,there will others,who will wish to speak on the problmes being discussed.If each speaker will take not more than 5 minutes,shall be able to finish in time.

Now we shall ask Mr.Barvashov to read Prof. Poorooshasb's speach,in which a rivew is tiven of the written reports submitted to our Specialty Session.

Prof.H.B.Poorooshasb (Iran), Chairman.

Gentlemen,

Papers were accepted in three areas. Those dealing with the deformation theories, those dealing with the application of such theories to the particular case of foundations (e.g. consolidation,settlement and stress distribution within the soil body) and those dealing with other problems of non linear soil mechanics.

I shall present a brief account of papers submitted in the first two areas. Then ask, from the audiance,two questions which I consider of importance and which I hope will be answered in this session.

The paper by Dr.Fekete discusses the settlement of foundation beams and slabs resting on an elastic half space. It considers the cases where the magnitude and distribution of loads and properties of foundation and supporting media are such that separation of the two media takes place (i.e.,the settlement of ground is larger than that of foundation) and/or when local plastic failure occurs. An iteration process is proposed by means of which the solution to various types of elastically supported foundation beams and slabs is obtained.

Professor Suklje's contribution discusses the possibility and need of using isotaches (i.e.,representing the consolidation test results as contours of e=constant in the e-σ space). He derives equations representing such depencies in the forms $e=e(e,\sigma)$ assuming various known equations relating e, σ and t, the time parameter. Any of these equations when used in conjunction with equations of continuity and compatability may be utilized to solve consolidation problems,given the appropriate boundary conditions.

Chang and Nair announce the development of a single general finite element computer program which incorporates non-linear material properties and is suitable for incremental analysis of plane problems in soil mechanics and rock mechanics.

Ohta and Hata, assuming the clay layer to be non linear elastic plastic material having work hardening or work softening properties and an associated flow rale discuss the mode of its consolidation when subjected to specified boundary conditions.

Professor Hisao Aboshi reports on a series of consolidation tests on a number of geometrically similar but of different size samples and concludes, amongst other things that the C_v value increases somewhat with sample thickness and hence the predictions of consolidation of actual clay layer from the standard oedometer test may lead to underestimation of its settlement rate.

Professor Mikasa presents a new formulation for one dimentional consolidation of highly compressible clays. He derives a linear equation (the Fourier's) in terms of compression strain (not U) assuming the variation of k and m_v being proportional

He further demonstrates that if the linearization assumption employed in derivation of Terzaghi equations are not made, a highly non-linear differential equation obtains. He recommends the usage of this equation when dealing with solution of consolidation problems involving soft clays.

In a very interesting contribution, Murray compares the measured and calculated relations between settlement and time at two road embankment sites. He concludes that the major source of error in these cases was the difference between the measured field and laboratory coefficients of consolidation and the use of a non-linear theory had relatively small influence on the settlement predictions.

Professor Viggiani presents some solutions of the Davis-Raymond consolidation equations which include the depth effects. Two cases are treated (i) uniform surcharge over a large area on the soil surface and (ii) lowering the water table thus producing a triangular excess pore water pressure distribution. The results of the analysis show some deviation from the classical theory.

In his paper Professor Gorbunov-Possadov assumes the soil supporting a strip footing to consist of elastic perfectly plastic material, the yield limit being defined according to Mohr-Conolomb criteria. The method of attack is quite novel and the results of analysis for a strip load acting on such a half space is presented showing the boundary between the elastic and plastic zone.

Professor Ter-Martirosyan considers the problems of a layer of clay subjected to a monolithically increasing time dependent extended load acting on its upper surface. The general equation governing the process of consolidation is presented from which solutions to prescribed boundary conditions can be obtained.

Professor Vinokurov in collaboration with Drs.Kuzmitzky and L.G.Shulika present a solution for foundation resting on the surface of an anisotropic elastic half space. Anisotropy is defined by the ratio, E_x/E_z the ratio of elastic modulii. The results of analysis for four cases of E_x/E_z=1/1 (the isotropic case) 1/05, 1/3 and 2/1 are presented where some variation is observed between the isotropic and anisotropic cases.

Professor V.G.Fedorovsky and his collaborators discuss the stability of methods of solutions of non-linear analysis as applied to foundations. They propose a variant of a combined incremental-iterational method of solutions which posses increased stability.

Professors Konovalov and Salnikov report on a number of experiments carried out on soft grounds. The tests consisted of (a) one dimentional consolidation of a peat layer and (b) two dimentional (radial symmetry) consolidation of a sand layer supported by peat. They demonstrate, for example, the effect of rigidity of the top layer on reducing the amount of settlement.

In his paper, Professor Dinis Da Gama discusses the rationality of using linearly elastic theory in solving said mechanical problems. His analysis shows that the linear approach is on the unsafe side since the bilinear theory leads to less stable conditions associated with layer displacements in the two cases cited.

An interesting observation is reported by Professor Alexiev who discusses the nature of collapse of a silty loess specimen subjected to wetting.

Dr.Feda generally discusses the various constitutive relations and suggests that to him, based on the existing evidence these should be constructed phenomenologically with a necessary and plausible interpretation with respect to the structure of the material.

Dr.Aguime Ramirez, introduces the factor "plastic component of noids ratio" and by assuming this to be a function of stress obtains a yield critéria involving the effective stress and a strain hardening parameter, itself a function of the irrecoverable volumetric strain.

Professors LeLievre and Matyas and Dr.Wang report on an extensive experimental programme completed at University of Waterloo to examine the uniqueness of the state domain and the mode of yielding of cohesive soils.

They conclude, amongst other things, that different yield criteria are required to define the onset of volumetric and distortional strain yielding.

Mr.P.W.Mitchell using the bottom cylinder compression machine to examine the behaviour of a sand sample concludes that the plastic potential is a ballet shaped surface of revolution about the hydrostatic axis in the stress space.

Drs.Ivastchenko and Zakharov start from the equation

$$de^p_{ij} = G\left[\partial f/\partial \sigma_{ij}\right]df$$

where de^p_{ij} is the plastic strain increment, G the strain hardening function, σ_{ij} the stress and F" ... the function of loading test that is the equation of loading locus separating an elastic domain from a plastic one in the space of stresses..."This I take to mean the yield function. They present some evidence which is considered to confirm the validity of the above equation.

I wish to pore two questions, just to get the discussion going. The first one what is infact the nature of the yield function for soils. I believe, for example, that in the case of sand the yield surface when traced in the stress space should contain the space diagonal. Perhaps Dr.Ivastchenko or Zakharov and Mr.Mitchell would like to comment on this.

With regard to the question of consolidation no doubt certain types of soils (e.g. collapsible clays) the classical Terzaghi theory fails to predict, even remotely, the field behaviour. But in the majority of cases, I believe, the discrepancy is due to inadequate information regarding the various involved factors rather than the inadequacy of the simple theory. Would Drs. Murray or Mikasa care to answer.

Vice-Chairman Dr.Zaretsky Yu.K.
Thank you very much Mr.Barvashov.

Mr.Chairman, Ladies and Gentlemen, Colleagues!

Preparations for the VIII International Conference on Soil Mechanics and Foundation Engineering presented an opportunity for assessing the state-of-the-art in the theory of base and foundation design. There have been great successes in this field. One aspect is the increasing transition from purely analytical solutions to ones utilizing to the maximum extent the possibilities of electronic computers. The popular method of design calculations which replaces differential equations by finite differences has been sumpplemented by new methods that have proved highly efficient when an electronic computer is used. These include methods based on finite elements and equivalents, variational-difference methods and, in particular, the very effective method of local variations.

The lack of a generally recognized design model for soil, one that properly represents its mechanical properties, leads to considerable difficulties in foundation engineering research.

The advantages of applying the orderly and comprehensively developed theory of elasticity to soils has been lessened by the obviously excessive values obtained for settlements, deflections and bending moments of structures erected on an elastic base. This theory exaggerates the distributive capacity of the soil. Various models of the base, such as the Winkler model, compressed layer model and others, have been proposed to take these circumstances into consideration, to obtain a decreased value of the distributive capacity of the soil, and to take into account the substantially more rapid decrease in the deformability of soils with the depth.

The real mechanical properties of soil bases are characterized by a nonlinear resistance of the soil to loads, by different behaviour of the soil under tensile and compressive stresses, and by soil deformation which is influenced by all the factors that are revealed in soil tests under conditions of a nonuniaxial stressed state. The problem of taking the real mechanical properties of soil bases into account in structure foundation design is by no means settled only by an attempt to obtain a nonlinear settlement-load relationship. Taking account of the real mechanical properties in this way will have a more essential influence on the diagrams of the reaction pressures and the values of the bending moments in the foundations of structures.

The application of nonlinear laws of deformation in problems concerning the interaction of bases and structures requires the use of numerical methods of integrating the principal equations. The great number of parameters in the physical laws of deformation, the variations in the boundary conditions and the various geometric relationships make the volume of calculations very large. Besides, the question has not yet been settled concerning the convergence to a precise solution of the iteration processes employed. Further work in this line should consist in accumulating the results of numerical solutions to reveal the basic relationships, mainly of a qualitative nature.

Another important trend in dealing with questions of the interaction between structures and clay bases is to take time effects into account in every possible way. The problem here is to apply in design either the laws of clayey soil creep, if the external loads increase slowly and the soil has a low degree of water saturation, or one of the modifications of the theory of consolidation, if the clayey soil is water saturated and the loads on the structure increase with sufficient rapidity. This trend in investigations is being intensively developed; a large number of works have been devoted to the prediction of the course of settlement of clayey soils with time, taking their consolidation and creep into consideration. To date various modifications that take some of the additional factors into account have been proposed for the theories of consolidation and creep. Of especial importance in investigating problems associated with the interaction of elastic structures and soil bases are works dealing with the problem of three-dimensional consolidation and creep of a layer of soil acted on by an external load. A detailed review of the investigations in this field is given in the general report of Professor M.I.Gorbunov-Possadov and Professor S.S.Davidov, and there is no need for me to repeat it here. I wish only to note that it is of considerable interest, in my opinion, to clarify the influence of the creep and consolidation of a soil base on the diagrams of contact pressures under structures of finite rigidity, the transformations of the diagrams with time and their dependence on the load. The commonly held opinion that "time heals all things"; that rheological processes and soil consolidation smooth over concentrations of contact stresses and level off bending moments acting on a structure, may turn out to be unjustified for "all possible cases". These problems await a prompt solution in the near future.

In conclusion I would like to illustrate a possible "practical outcome" of the results obtained in a numerical solution of a certain nonlinear problem in soil mechanics.

Let us assume that in a complex stressed state the shear deformation of the soil is expressed by an equation of state of the kind shown in Fig.1. Then, in a system of

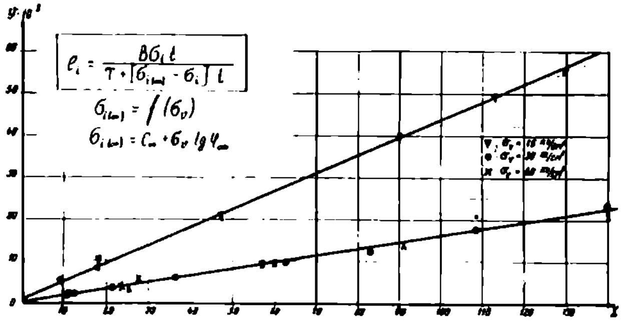

Fig.1. Intensity of shear deformation e_i vs time at various values of σ_v and σ_i plotted in the coordinates: $Y = t\sigma_i/e_i$ and $X=(\sigma_i(\infty)-\sigma_i)t$
Callovian sandy loam θ =-10°C

specially selected coordinates, all the experimentally obtained creep curves should constitute a single straight line. One such experimental relationship, plotted in the above-mentioned coordinates, is shown in Fig.1.

It is maintained that the solution of the boundary value problem, characterizing the relationship between the displacement of a structure and the external load, can be approximately represented in a similar form. As a matter of fact, the displacement of the internal cavity of a soil cylinder acted on by an external load can be represented in this form which follows from an exact solution (see Fig.2). This figure also shows experimental points obtained in the physical simulation of the problem and plotted in the same coordinates.

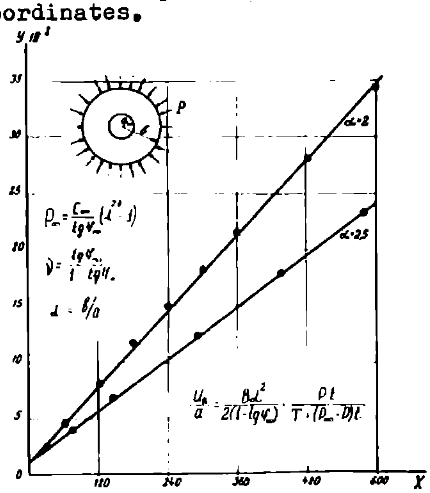

Fig.2

Ua vs time plotted in coordinates:

$$Y = \frac{pt}{Ua/a} \quad \text{and} \quad X = (p_\infty - p)t$$

Ice-soil cylinder, p=50 kg/cm^2

In the report of Malyshev, Zaretsky, Shirokov and Cheremnykh at the present conference, the results of the numerical solution obtained by the method of finite differences for the problem of embedding a rigid round test plate into a half-space, can also be

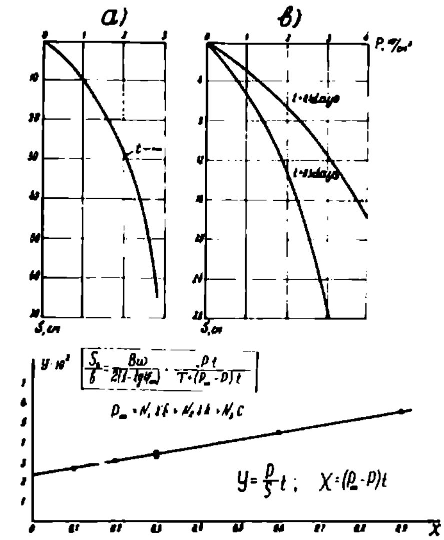

Fig.3. Plate Settlements vs applied load and time
(a) stabilized state
(b) for the time t= 0,1 day and t=0.3 day

approximated well in the form of the relationship given in Fig.3. It should be noted that the proposed integral relationship has been repeatedly confirmed by the data of observations in the field.

This illustrative example shows the possibility of and the necessity for analyzing the results of the mathematical simulation of complex problems of nonlinear soil mechanics.

Thank You!

Now I invite Mr. Wroth from England to make his contribution. Mr. Wroth, will you, please.

Mr. C.P. Wroth (England)

As part of the continuing programme of research in soil mechanics at the University of Cambridge, Roscoe (1970), work has been in progress on developing computational methods which allow solutions to be obtained to boundary value problems using non-linear models of soil behaviour. This contribution outlines some of the recent developments.

Simpson (1973) has carried out finite element computations which have been based on the family of soil models developed at Cambridge. The approach adopted by Simpson has been to keep the computational side of the work as simple as possible by concentrating on two-dimensional problems and employing constant strain triangular elements. In contrast, however, a great amount of effort has been put into the development of mathematical models which provide an adequate description of soil behaviour; these models need to be complex in order to obtain satisfactory solutions to real boundary value problems for real soils.

In some circumstances an elastic model may be adequate, but in others it will not, and a complex elastic/plastic model will be necessary to arrive at an acceptable prediction. An example of the former would be an excavation in an overconsolidated clay for which the stress changes and deformations experienced by the soil are small enough for the behaviour to be considered quasi-elastic. Account may need to be taken of the marked increase of Young's modulus with mean effective stress-that is with increase of depth-and of anisotropy along the lines suggested by Wroth (1972) and Atkinson (1973). But these features of elastic behaviour can readily be incorporated in a finite element computation.

In contrast, the problem of the construction of an embankment or structure on soft normally consolidated clay can only be satisfactorily solved if an elastic/plastic model is used. This not only means that incrementally a bilinear response is obtained from the soil (with the type of response depending on the state of the element in question and whether it is being loaded or unloaded) but also that proper account is taken of the rotation of the principal stress directions. Wroth and Simpson (1972) have used such a model with soil parameters taken from a routine site investigation, in an attempt to match the field data of a trial embankment reported by Wilkes (1972). The model is such that for every increment each element of soil experiences both an elastic and a plastic strain-increment; the principal axes of the elastic component coincide with those of the associated stress-increment whereas the axes of the plastic component coincide with those of stress for the beginning of the increment. Use of a piecewise linear approximation to a non-linear but elastic stress-strain curve, such as the model adopted by Clough and Duncan (1971), would lead to a very different displacement field computed for the ground under the embankment.

Another area of great difficulty is the modelling of the strain-softening behaviour of soils. Most stress-strain curves for real soils display a peak with a subsequent loss of strength as the soil approaches a critical or residual state. The strains will vary considerably along any incipient rupture surface in a soil mass; some elements will have been strained beyond their peak strength and will have 'failed' (although they are being held in equilibrium by neighbouring unfailed elements) while other elements will not have been strained sufficiently to mobilise their peak strength. Overall failure of the soil mass will occur in a progressive manner.

One method of overcoming these difficulties has been developed by Simpson whereby the computer generates extra elements by subdivision of the mesh at various stages of the computation. The choice of elements to be divided is governed by the amount of either strain or displacement that they have experienced. In this way, extra, small elements become concentrated around an incipient rupture surface, and progressive failure of the real situation is directly modelled in the computation without previous quesswork on the part of the operator. Simpson and Wroth (1972) show that for the case of a rigid retaining wall being rotated about its top into a sand bed in the passive mode, the peak in the load-rotation curve observed experimentally for the wall can be reproduced in the computation by using this mesh-forming technique. Without mesh-forming a peak is not obtained.

In parallel with finite element computations an alternative approach using finite difference calculations based on the method of characteristics has been under development at Cambridge and Madrid. This method links together associated fields of stresses and velocities (or strain-increments) so that the stress-strain properties of the soil are satisfied throughout the soil mass. The important development has been the modification of the basic differential equations to take account of varying stress ratio throughout the stress field and varying amount of dilatation throughout the strain-increment field. Some solutions to particular problems of retaining walls have been reported by Serrano (1972) and James, Smith and Bransby (1972) and the general method described by Wroth (1972)

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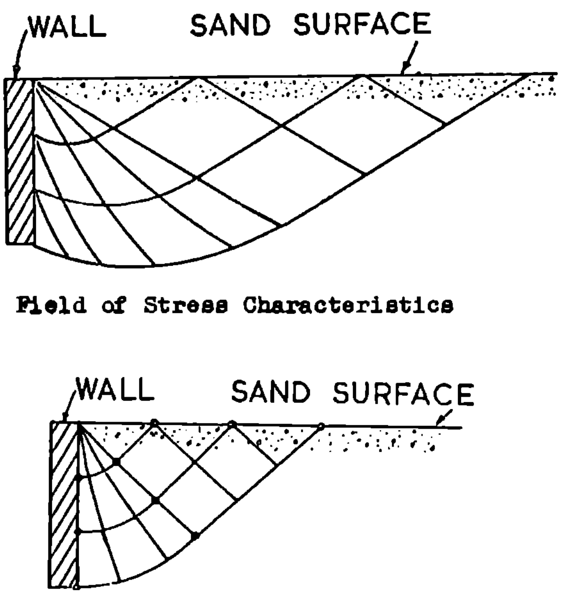
Vice-Chairman Dr.Yu.K.Zaretsky (USSR)

Thank you very much Mr.Wroth. The next will be Dr. Bransby (England).

Dr.P.L.Bransby (England)

I wish to describe briefly the method of associated fields,a calculation method which is an alternative to the finite element method and which has some advantages.The method of associated fields is based on the finite-difference calculation of Sokolovski,but the Sokolovski calculation is extended to take account of variations with strain of the mobilised angle of internal friction.The corresponding displacement field is developed simultaneously with the stress field so that a solution can be obtained for stresses,displacements and strains.

The field of stress characteristics for a retaining wall is illustrated in Fig.1,together with the associated field of displacement characteristics. The complete solution, consisting of the linked stress and displacement fields,satisfies (i) equilibrium (ii) kinematics and (iii) the stress-strain law for the soil (which may be non-linear and can incorporate realistic volume changes of the soil). The method is described in detail by Serrano (1972) and James,Smith and Bransby (1972).



(a) Field of Stress Characteristics

(b) Field of Displacement Characteristics

Fig.1. Typical associated fields of stress and displacement characteristics for the passive earth pressure problem

Two of the advantages of the method over the finite element method are as follows:

(1) the method lends itself to simple and realistic hand calculations of stresses and displacements for certain simple problems of practical interest,e.g.the displacements and stresses near a retaining wall or a sheet pile wall that can reasonably be taken as smooth.

(2) the method is intrinsically well suited to deal with stress or displacement fields with a singularity (or a strong discontinuity) of stress or displacement, such as occurs near the top of a retaining wall.

The method can easily be extended to deal with inertia forces,seepage forces etc.

These features of the method of associated fields suggest that it has considerable potential both in research and practice,especially as it builds on much accumulated experience of plasticity in metals and in soils.

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Vice-Chairman Dr.Yu.K.Zaretsky (USSR)

Thank you Mr. Bransby. Now I pass the word to Mr. Goldin (USSR)

A.L.Goldin (USSR)

To evaluate the stress-strain state of earth-structures and foundations a model is most commonly used of a linear deformable elastic medium. However recently it has become evident that soil mechanics problems must be solved with due regard for a non-linear relationship between stresses and strains. In the USSR the problem is investigated by refining both analytical methods (M.V.Malyshev, Yu.K.Zaretsky, V.I.Solomin, V.N.Shirokov, A.L.Kryzhanovsky,etc.) and numerical methods (L.A.Rosin, L.M.Rasskasov, etc.).

According to the author among numerical methods the finite element method seems most advantageous. The use of the finite element method enables stress-strain computations for non-homogeneous domain of a complex geometry to be performed.

The paper deals with physically non-linear elastic media. Assuming moderate non-linearity and using G.Kauderer model the following relationship be applied for a plane strain case

$$\left.\begin{aligned} \sigma_x &= 3Ke_0 + 2G\gamma(\psi_0)(e_x - e_0), \\ \sigma_y &= 3Ke_0 + 2G\gamma(\psi_0)(e_y - e_0), \\ \tau_{xy} &= G\gamma(\psi_0)\gamma_{xy} \end{aligned}\right\} \qquad (1)$$

where there is a correlation between three-dimensional deformations and average stresses while the intensity of deformations due to shear is nonlinearly related to the tangential stress intensity.

K,G- confining pressure and shear moduli

Assuming $\gamma(\gamma_0)=1-g_1\gamma_0^2$ and presenting stresses, strains and displacements as series with respect to a very small parameter

$$\lambda=\frac{g_1 K}{G^2(3K+G)};\quad \sigma_x=\sigma_x^{(0)}+\lambda\sigma_x^{(1)}+\ldots,\quad e_x=e_x^{(0)}+\lambda e_x^{(1)}+\ldots,\quad u=u^{(0)}+\lambda u^{(1)}+\ldots,$$

we get for a zero- and first-order approximation

$$\left.\begin{aligned}
\sigma_x^{(0)}&=\frac{(1-\nu)E}{(1+\nu)(1-2\nu)}e_x^{(0)}+\frac{\nu E}{(1+\nu)(1-2\nu)}e_y^{(0)},\\
\sigma_y^{(0)}&=\frac{(1-\nu)E}{(1+\nu)(1-2\nu)}e_y^{(0)}+\frac{\nu E}{(1+\nu)(1-2\nu)}e_x^{(0)},\\
\tau_{xy}^{(0)}&=\frac{E}{2(1+\nu)}\gamma_{xy}^{(0)}
\end{aligned}\right\}\quad(2)$$

$$\left.\begin{aligned}
\sigma_x^{(1)}&=\frac{(1-\nu)E}{(1+\nu)(1-2\nu)}e_x^{(1)}+\frac{\nu E}{(1+\nu)(1-2\nu)}e_y^{(1)}-\frac{E^3}{3(1+\nu)^4}\Psi^{(0)}(2e_x^{(0)}-e_y^{(0)}),\\
\sigma_y^{(1)}&=\frac{(1-\nu)E}{(1+\nu)(1-2\nu)}e_y^{(1)}+\frac{\nu E}{(1+\nu)(1-2\nu)}e_x^{(1)}-\frac{E^3}{3(1+\nu)^4}\Psi^{(0)}(2e_y^{(0)}-e_x^{(0)}),\\
\tau_{xy}^{(1)}&=\frac{E}{2(1+\nu)}\gamma_{xy}^{(1)}-\frac{E^3}{2(1+\nu)^4}\Psi^{(0)}\gamma_{xy}^{(0)}
\end{aligned}\right\}\quad(2a)$$

where ν, E - Poisson's ratio and elasticity modulus.

$$\Psi^{(0)}=e_x^{(0)2}+e_y^{(0)2}-e_x^{(0)}\cdot e_y^{(0)}+\frac{3}{4}\gamma_{xy}^{(0)}$$

In the tensor form we have

$$(\sigma^{(0)})=[D](e^{(0)})\quad(3)\qquad\qquad(\sigma^{(1)})=[D](e^{(1)})-(T^{(0)})\quad(3a)$$

where

$$[D]=\frac{E}{(1+\nu)(1-2\nu)}\begin{bmatrix}1-\nu & \nu & 0\\ \nu & 1-\nu & 0\\ 0 & 0 & \frac{1-2\nu}{2}\end{bmatrix}\quad(4)\qquad (T^{(0)})=\frac{E^3}{6(1+\nu)^4}\Psi^{(0)}\begin{pmatrix}2(2e_x^{(0)}-e_y^{(0)})\\ 2(2e_y^{(0)}-e_x^{(0)})\\ 3\gamma_{xy}^{(0)}\end{pmatrix}\quad(4a)$$

The finite element method may be effectively used for solving problems with the stress-strain relations of the type (3)-(3a). The existing finite element programs, in particular the program developed by the Mathematical Laboratory of the B.E.Vedeneev All-Union Research Institute of Hydraulic Engineering, allow to evaluate stress and strain components of the zero-order approximation (a linear problem of the elasticity theory). To obtain the basic equation for the 1st-order approximation components the Lagrange principle for minimizing the total energy of the system may be applied.

By describing the variation of the work done by the internal forces over the corresponding strains for the total element assembly of a certain domain S the following equation combining the nodal forces and the nodal displacement of the 1st-order approximation can be derived:

$$[K](q^{(1)})-(\Omega^{(0)})=(\mathcal{F}^{(1)})\qquad(5)$$

where $[K]$ - stiffness matrix of the system

$(q^{(1)})$ - nodal displacement of the 1st-order approximation

$(\mathcal{F}^{(1)})$ - nodal forces of the 1st-order approximation

$$(\Omega^{(0)})=\int_S[B]^T(T^{(0)})\,ds$$

The following matrix $[B]$ can be obtained from the relationship

$$(e^{(1)})=\begin{pmatrix}\frac{\partial u^{(1)}}{\partial x}\\ \frac{\partial v^{(1)}}{\partial y}\\ \frac{\partial u^{(1)}}{\partial y}+\frac{\partial v^{(1)}}{\partial x}\end{pmatrix}=[B](q^{(1)})$$

Using a set of equilibrium equations in the finite element formulation and presenting the nodal force vector (F) and the generalized nodal force vector (f) corresponding to the forces prescribed on the contour of the domain as a series with respect to a small parameter, two sets of equations can be obtained

$$[E_1]\{(\mathcal{F}^{(0)})-(f^{(0)})\}=0 \quad \text{and} \quad [E_1]\{(\mathcal{F}^{(1)})-(f^{(1)})\}=0 \qquad (6)$$

where $[E_1]$ - an identity matrix with zero elements all "offdiagonal" positions and unity element on each of the diagonal positions.

The correlation between the vector of the displacement of the element points and the nodal displacement vector can be presented as an equation

$$(u)=[U](q)$$

where $[U]$ is the approximation matrix. Using the expression for the work of the nodal forces along the possible displacements we get a set of 1st-order approximate equations

$$[E_1]\{[K](q^{(1)})-(\Omega^{(0)})\}=0 \qquad (7)$$

Thus, the vector components $(q^{(1)})$ can be found from Eqs (7) at given vectors of fictitious nodal forces $(\Omega^{(0)})$ After evaluating $(q^{(1)})$ the moments $(\sigma^{(1)})$, $(e^{(1)})$ and $(u^{(1)})$, are calculated, and then the (σ), (e) and (u) components considering $(\sigma^{(0)})$, $(e^{(0)})$ and $(u^{(0)})$.

Vice-Chairman Dr.Yu.K.Zaretsky. (USSR)

Thank you Mr. Goldin. I want to invite Prof. Šuklje to make his report. Prof.Šuklje, will you, please.

L.Šuklje, (Yugoslavia)

In my written contribution to the problems of non-linear consolidation of saturated soils I have discussed analytical expressions for experimentally obtained volumetric rheological relationships for soils and their consideration in the differential equation of consolidation.

According to Florin (1961), this equation accounting for one-dimensional consolidation of partly saturated soils (including saturated soils as a special case) has the form:

$$\frac{\partial e}{\partial t}=\frac{1+e}{\gamma_w}\cdot\frac{\partial}{\partial z}\left(k\,\frac{\partial \mu}{\partial z}\right)-\frac{e_a+He_w}{\mu+\mu^\circ}\cdot\frac{\partial \mu}{\partial t}$$

when, besides the internationally agreed symbols e, t, γ_w, k, z the following notation has been used: H for Henry's coefficient of the solubility of air in water, u^o for the pore pressure including the atmospheric pressure but excluding the excess-pore pressure, and the suffixes a and w for air and water. Equation (1) has been deduced with the simplifying assumption that the air-pressure equals the water-pressure.

The following connections between the dependent variables have been considered:

$$\sigma'=\sigma-\mu$$

$$e=e_a+e_w \qquad (4)$$

$$e_a=-e(z,o)\,\mathcal{S}_{ro}H+\frac{(z,0)+\mu^\circ}{\mu(z,t)+\mu^\circ}\left[e_a(z,o)+e(z,o)\,\mathcal{S}_{ro}H\right]$$

(cf.Šuklje 1969), where S_{ro} denotes the initial saturation degree. The total stress is assumed to be known:

$$\sigma=\sigma(z,t)=\sigma_o(z)+2(t) \qquad (5)$$

The numerical solution of the equation (1) can be given for an arbitrary rheological relationship

$$\frac{\partial e}{\partial t}=f(e,\sigma') \qquad (6)$$

In order to elucidate the effects of: the initial saturation degree, the initial porosity (previous secondary consolidation), the load interval and the length of the seepage paths, J.Kozak has programmed, in collaboration with the Writer, the above defined problem for the following form of the rheological relationships:

$$f(e,\sigma')=e_o\exp\frac{A+B\ln\frac{\sigma'}{\sigma_o}-e}{C+D\ln\frac{\sigma'}{\sigma_o}} \qquad (7)$$

where e_o, A, B, C, D, σ_o are the constants corresponding to the experimentally obtained isotache set. Furthermore, the simplifying assumption k=const had been taken and the following boundary conditions considered:

$$\mu(o,t)=\mu(h,t)=o \qquad (8)$$

$$\sigma(o)=\sigma_{oz},\ \frac{\partial\sigma_o(z)}{\partial z}=\left\{\left[\gamma_s+\gamma_w e(z,o)\,\mathcal{S}_{ro}\right]:\left[1+e(z,o)\right]\right\}-\gamma_w \qquad (9)$$

Figs 1 and 2 represent some results of the numerical calculations made according to the prepared programme, by considering the isotache set (parameters in equation 7) corresponding to a lacustrine clay (see Šuklje-Simončič 1972, Fig.2) and to the boundary values shown in Figure 1.

The comparison of pore-pressure versus time plots (Fig.2) proves that the influence of the initial porosity and of the load interval onto the pore-pressure development is equally or even more important than the influence of the degree of saturation.

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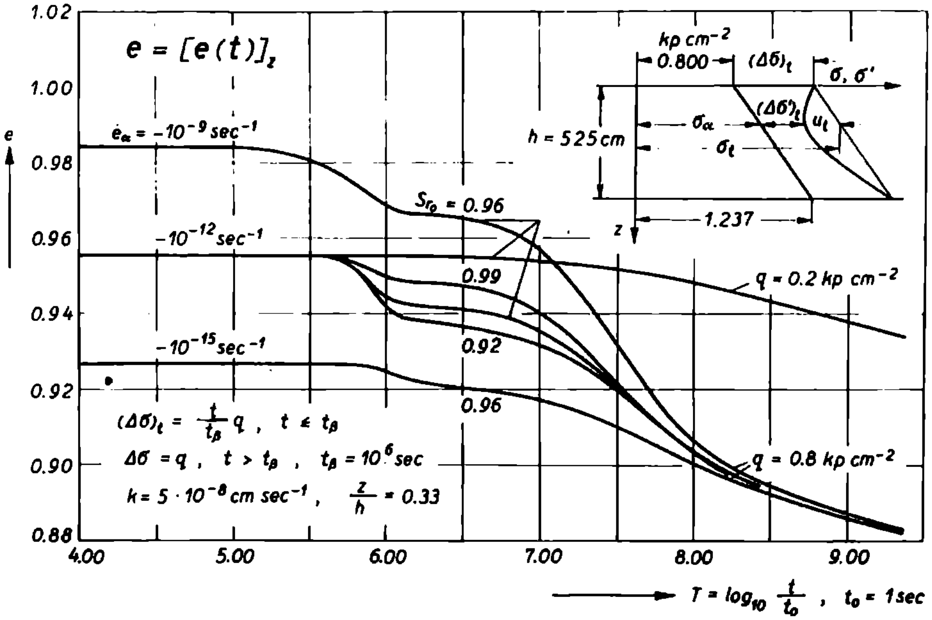
e = [e(t)]z
e
$\dot{e}_\alpha = -10^{-9}$ sec^{-1}
-10^{-12} sec^{-1}
-10^{-15} sec^{-1}
S_{r_0} = 0.96
0.99
0.92
0.96
q = 0.2 kp cm^{-2}
q = 0.8 kp cm^{-2}
kp cm^{-2}
0.800
$(\Delta\sigma)_t$
σ, σ'
$(\Delta\sigma')_t$
u_t
σ_α
σ_t
h = 525 cm
1.237
z
$(\Delta\sigma)_t = \frac{t}{t_\beta} q$, $t \leq t_\beta$
$\Delta\sigma = q$, $t > t_\beta$, $t_\beta = 10^6$ sec
$k = 5 \cdot 10^{-8}$ cm sec^{-1}, $\frac{z}{h}$ = 0.33
$T = \log_{10} \frac{t}{t_0}$, t_0 = 1 sec

Fig.1

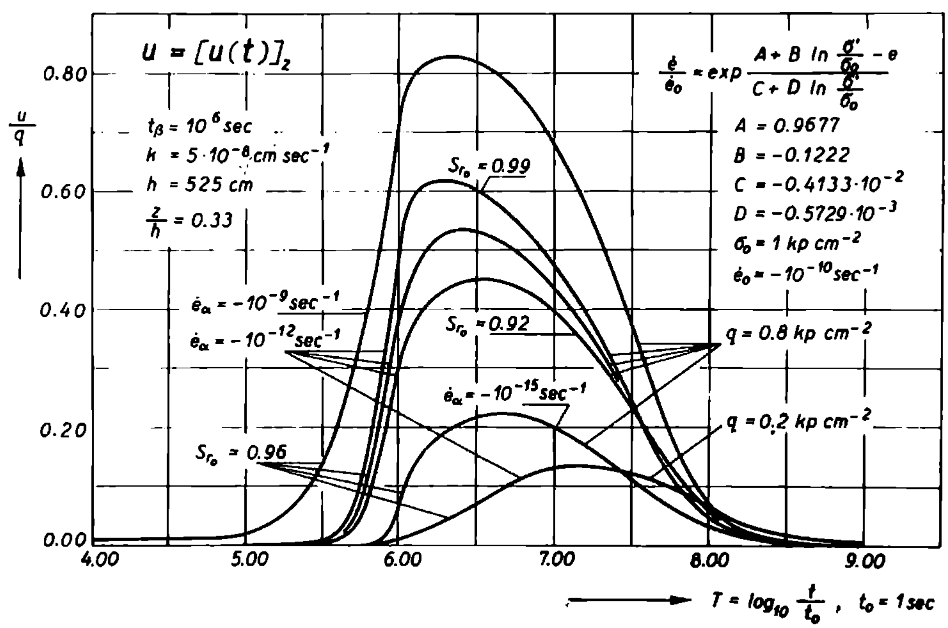
u = [u(t)]z
u/q
$t_\beta = 10^6$ sec
$k = 5 \cdot 10^{-8}$ cm sec^{-1}
h = 525 cm
$\frac{z}{h}$ = 0.33
$\frac{\dot{e}}{\dot{e}_0} = \exp \frac{A + B \ln \frac{\sigma'}{\sigma_0} - e}{C + D \ln \frac{\sigma'}{\sigma_0}}$
A = 0.9677
B = −0.1222
C = $-0.4133 \cdot 10^{-2}$
D = $-0.5729 \cdot 10^{-3}$
σ_0 = 1 kp cm^{-2}
$\dot{e}_0 = -10^{-10}$ sec^{-1}
S_{r_0} = 0.99
S_{r_0} = 0.92
S_{r_0} = 0.96
$\dot{e}_\alpha = -10^{-9}$ sec^{-1}
$\dot{e}_\alpha = -10^{-12}$ sec^{-1}
$\dot{e}_\alpha = -10^{-15}$ sec^{-1}
q = 0.8 kp cm^{-2}
q = 0.2 kp cm^{-2}
$T = \log_{10} \frac{t}{t_0}$, t_0 = 1 sec

Fig.2

Vice-Chairman Dr. Yu. K. Zaretsky (USSR)

Thank you very much Prof. Šuklje. The next will be Mr. Kisiel from Poland.

I. Kisiel, (Poland)

The Coulomb's condition of the limit state of soils and all its posterior modifications are applicable only to the soils with limited clay content. The internal friction becomes negligible in the case of very cohesive soils; this fact appears evidently in the Skempton's "Ø=O- method".

The reason of the mentioned fact are the rheological properties of clay fraction being quite different from properties of granular fractions: gravel, sand or silt. The clay particles show an oriented shape /platelets or needles/, whereas the grains of all other mentioned fractions have a compact shape. The clay particles have very small dimensions and very large specific surface. The individual particle of clay contains very small amount of material /10^6 times smaller as the silt particle/. In the state in-situ the clay fraction is always fully saturated and has therefore the properties of a viscous suspension. At the same time all other fractions are characterized by existence of dry friction between the grains. Therefore the clay fraction makes not only a binding material between silty and sandy grains, but also some kind of grease between them, which facilitates their shearing displacement with respect to each other.
displacement with respect to each other.

By amount of clay fraction exceeding 2% the cohesion in the soil occurs. By increasing of clay content the cohesion also increases; if the clay content exceeds 20% the influence of friction becomes negligible and this is connected with a considerable alternation of rheological behaviour of soil.

It is well known, that the minimal porosity of sand and the minimal porosity of the densiest arrangement of equal spheres are the same: n_{min}=25,9%. If one puts between the big spheres in such arrangement the smaller spheres, which will represent the silt grains and will contact with the big spheres not disturbing the arrangement of the latest /fig.1/, then

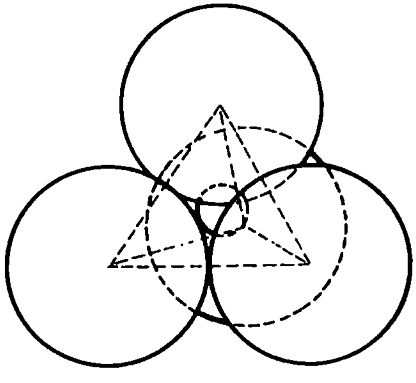

Fig. 1

the porosity of such arrangement is equal to n_{min}=20,8%, what is near to the value of n=20%, mentioned above. If all pores of such arrangement will be filled by clay-water mixture, the properties of the arrangement will not change; in particular there will be no change in the friction between big spheres, although the cohesion will take place. If however the content of clay fraction exceeds 20%, then all the grains do not contact with each other. The influence of dry friction will diminuish rapidly with increase of clay content. When the clay content exceeds the maximal porosity of grain skeleton, the grains become a part of filler of the clay-water mixture, what will manifest in increasing viscosity in comparison with the viscosity of pure clay-water mixture.

One should keep in mind, that the foregoing reasoning is of greatly approximate character: the grains of sand or silt have no spherical shape. Nevertheless the coincidation of calculated and observed results is of course very interesting. Recently some experiments of the rheological properties of soils were performed in Poland. It was stated, that by exceeding of 20% content of clay fraction the properties of soils were changed in a fundamental manner. The mentioned results will be published soon.

It follows from the foregoings, that to understand correctly the properties of the clayey soils one should know at first the properties of clay-water mixture. The knowledge of mentioned properties would be an object of the research of the new branch of applied mechanics, which is proposed to call a "clay mechanics".

The fundamental assumption to be made in the clay mechanics is the assumption on the geometry of clay structure. A model of structure of a normally consolidated undisturbed clay was proposed by Tan Tjong-Kie, 1954, 1959 and was called "the cardhouse structure" (fig.2). Since the claywater mixture is a

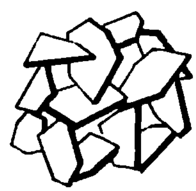

Fig. 2

disperged medium, to investigate its properties one should apply the principles of the statistical physics; in particular for such medium the following laws are valid:

a/ If the clay sample under consideration contains n particles, then the probability, that n_m particles contact immediately with each other /so-called "mineral-to-mineral" contact/ is equal to:

$$\frac{n_m}{n} = \left[1+\exp\left(-\frac{\Delta E}{RT}\right)\right]^{-1}, \qquad /1/$$

b/ the rate of deformation $\dot{u}$ of sample is equal to:

$$\dot{u} = 2\frac{n_1 r_1 kT}{h}\exp\left(-\frac{\Delta F}{RT}\right)\sin h\left(\frac{\alpha q}{kT}\right) \qquad /2/$$

In both mentioned formulas from which the first was proposed by Resendiz,1965,and the second by Murayama and Shibata,1958,1964 the denotations are:

$\Delta E = \Delta F + \alpha q$ the experimental energy of activation /Mitchell et al.,1968/

ΔF is free energy of activation,

α is the parameter,depending on r_1, see below,

q is an external load acting on individual particle,

n_1 is the number of particles moving in the direction of acting load q,

r_1 is the average distance between the particles in contact,

k is the Boltzmann's constant,

h is the Planck's constant

R is the gas constant,

T is the absolute temperature.

Using the mentioned laws one could develop the formulas for the elasticity and viscosity moduli of clay. Obviously,one couldn't calculate directly the values of mentioned moduli by use of formulas,because they contain the statistical values n_m,n_1 and r_1. Nevertheless the formulas give valuable indications, which manner of dependencies and which physical factors influence the values of rheological parameters and how the experiment should be performed to obtain these parameters of the macrovolume of clay.

Murayama and Shibata have proposed the statistical method to obtain the rheological parameters of soil. Mitchell and co-workers, 1964,1968, gave valuable extensions of the method. Resendiz gave a method to evaluate the ultimate strength of clays. The mentioned researchers have found,that the long-term strength of clays could be obtained by use of the dependency between rate of strain $\dot{\gamma}$ on maximal deviatoric stress τ_{max} /fig.3/.

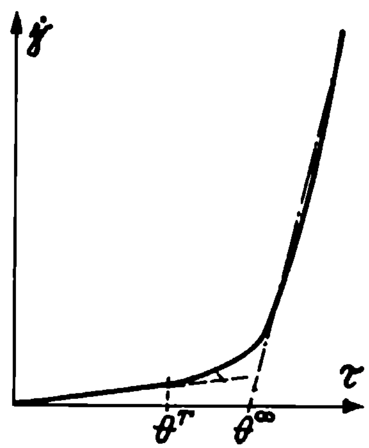

Fig.3

By exceeding the long-term strength the character of mentioned dependency varies considerably. The similar results were obtained by Dmitruk and Suchnicka 1964 /published 1966/ during investigation of clays of opencast brown-coal mines. Mitchell et al.,1968,gave the method of direct determination of physical parameters occuring in the formulas /1/ and /2/.

Murayama and Shibata in their papers do not take into account the possibility of the phenomenon of reversible creep,which is observed of course in all experiments by small loading of sample. Resendiz doesn't make any proposal of the rheological model of clay. Therefore the more detailed description of clay behaviour is indispensable. There is possible to describe the clay behaviour as follows:

1. In the first phase of small loading of sample there exist both the instantaneous and the delayed deformations being fullyreversible. The nonlinearity in this phase is negligible. As a rheological model the standard Zener's model with constant elasticity and viscosity moduli could be used /fig.4 /.

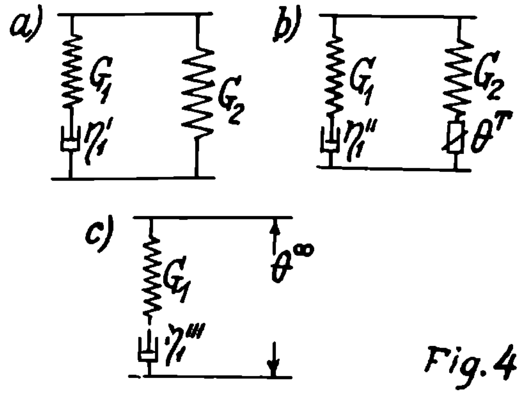

Fig.4

2. When the load exceeds some value which could be called "Tan's limit" /Tan,1954 observed it at first/ and denoted by , the irreversible deformation becomes observable. This deformation increases with time and increasing load up to the value of long-term strength of clay,denoted by θ^{∞}. In this phase the modulus of viscosity changes by comparison with the previous phase and becomes nonlinear. Nevertheless both the first and the second phases could be described at least for practical purposes by use of rheological model M/V, developed 1958 and independently by Folque, 1961 /fig.4b/. One could assume the constant rheological parameters.

The both described phases are of maximal practical importance.

3. If the load exceeds the value of θ^{∞} the viscous flow begins. This flow is distinguished by strong nonlinearity. To represent the rheological behaviour of clay in this phase a modified nonlinear Maxwell-model of fluid could be used /fig.4c/.

Introducing the degree of orientation of clay particles g_r which values are: $g_r=1$ by parallelly oriented particles and $g_r=0$ by randomly oriented particles,one could introduce g_r instead of n_1 into the formula /2/. By loading of clay the value of g_r changes with time and intensity of load. The particles will re-arrange more parallel to each other and perpendicularly to the direction of load. Then the formula /2/ becomes:

$$u=2\,\frac{2g_r+1}{3}\,\frac{r_1 kT}{h}\,\exp\left(-\frac{\Delta F}{RT}\right)\sin h\left(\frac{\alpha q}{kT}\right) \qquad /3/$$

Because of $g_r=g_r(q,t)$, the formula /3/ gives the possibility to examine the change of viscosity modulus of clay with time.

More detailed investigation of this problem will be published in "Studia Geotechnica" in 1974.

I am of meaning that the problem of fundamentals of clay mechanics is of significant theoretical and practical importance and I am pleased with the possibility to submit this point of view during the present Conference.

Vice-Chairman Dr.Yu.K.Zaretsky (USSR)

Thank you Mr.Kisiel for your discussion. The next will be Mr. Drescher (Poland)

A.Drescher (Poland)

Reffering to the subject of the present session let me make some remarks concerning the description of the mechanical behaviour of granular media,which do not exhibit the time-scale effects i.e.rheological effects. In the every-day engineering practice the distinction between the range of small deformations and of advanced flow of a material is usually made, and the theory of elasticity or plasticity is applaied respectively. In order to describe the material in both ranges by one constitutive relation it seems therefore natural to combine these theories. This concept, known for several years,may lead, however,to various constitutive relations depending on the assumed simplifications regarding real behaviour of a material. In our approach, which I would briefly discuss, the following two experimental facts are taken as the basis for the theory: 1/ the current behaviour is loading or deformation paths dependent, 2/ the material response is different for loading and unloading process,what can be observed even for very small cycles. Both facts lead to the following incremental constitutive law

$$d\varepsilon_{ij}=A_{ijkl}\,d\sigma_{kl}+d\lambda\,(\partial f/\partial\sigma_{ij}) \qquad (1)$$

$d\lambda > 0$ for $f=0,\ df=0$

$d\lambda\ \ 0$ $f<0$ or $f=0,\ df<0$

It is now assumed that any material function appearing in /1/ may depends on density variation $\bar{\rho}$

$$\bar{\rho}=\bar{\rho}'+\bar{\rho}''=\frac{\rho-\rho_0}{\rho_0}=d\varepsilon_{kk} \qquad (2)$$

or its reversible and irreversible portions $\bar{\rho}',\bar{\rho}''$. For isotropic A_{ijkl} moduli matrix the bulk modulus K and shear modulus G can be introduced

$$K=K(\bar{\rho}',\bar{\rho}'') \qquad (3)$$
$$G=G\ (\bar{\rho}',\bar{\rho}'')$$

The yield condition f is assumed to be a closed surface in the stress space,depending on $\bar{\rho}''$

$$f=f(\sigma_{ij},\bar{\rho}'') \qquad (4)$$

Thus the concept of density-hardening originated by Drucker,Gibson and Henkel, and Roscoe et al. is assumed. Leaving aside the discussion of the yield criterion and flow rule let us confine our attention on reversible strain increment.

There can be distinguished two extremal cases for A_{ijkl} matrix or moduli K and G: 1/ K and G depend on reversible density variation $\bar{\rho}'$, ii/ K and G depend on total density variation $\bar{\rho}=\bar{\rho}'+\bar{\rho}''$. For i/ no coupling of reversible and irreversible deformation exists, while for ii/ coupling is present. To demonstrate the influence of coupling consider purely hydrostatic stress state, Fig.1. When no coupling is assumed the unloading curves do not depend on irreversible deformation,and their local inclination depends on stress level only. In other words each of the unloading curves is obtained by translation of the virgin elastic curve along the ε_{kk} axis. If the ii/ hypothesis is assumed the unloading curves depend on irreversible deformation, and are obtained by translation of the virgin elastic curve along the σ_{kk}.axis. The above shifting rules hold true for proportional stress ors strain paths.

In order to verify the concept of coupling the experiments in uniaxial strain state were carried out. Fig.2 presents the loading and unloading curves for sand, plastic spheroids and wheat grains .It is seen that the unloading curves do not obey the horizontal shifting rule valid for case i/ and thus may indicate the existence of coupling. The detail discussion of the proposed constitutive relation and of the test results is made in Ref. /1/.

/1/ T.Hueckel and A.Drescher- Description of non-linear and inelastic behaviour of granular media. Int.J.Solids Struct. /to be published/.

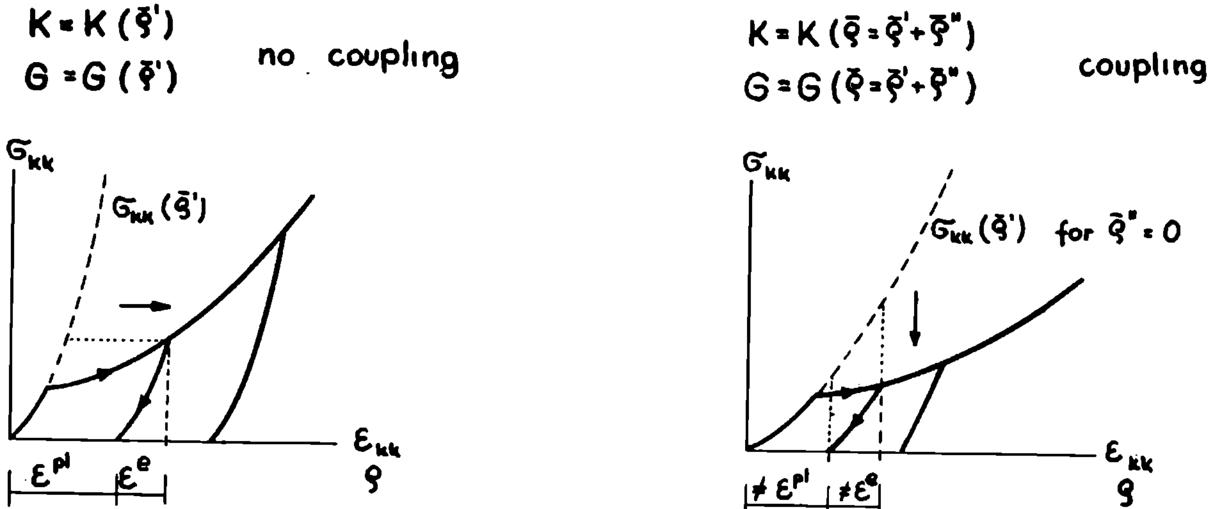
K = K (ρ̄')
G = G (ρ̄')
no coupling
K = K (ρ̄ = ρ̄' + ρ̄")
G = G (ρ̄ = ρ̄' + ρ̄")
coupling
σkk
σkk (ρ̄')
εkk
ρ
εpl
εe
σkk (ρ̄') for ρ̄" = 0
≠ εpl
≠ εe

Fig. 1

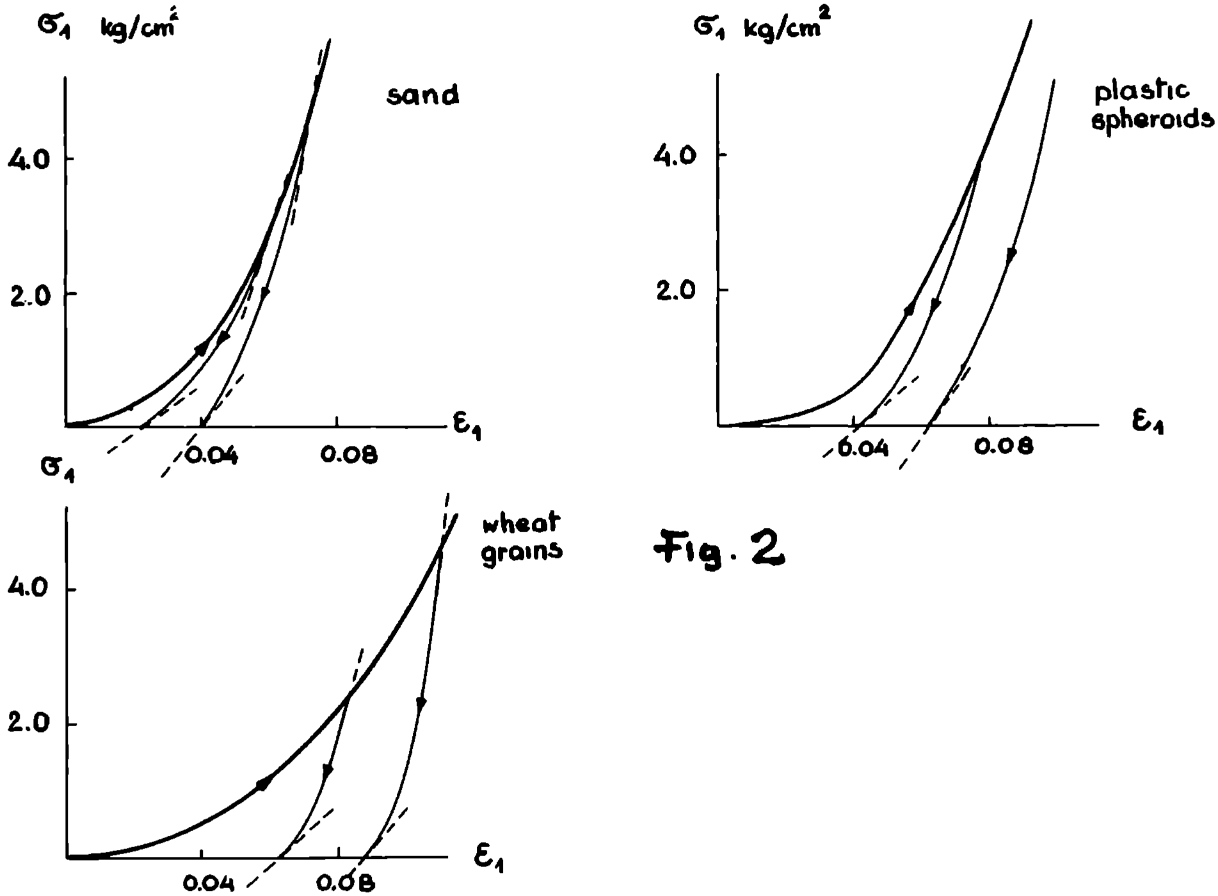
σ1 kg/cm²
sand
4.0
2.0
ε1
0.04
0.08
plastic spheroids
σ1
wheat grains

Fig. 2

Vice-Chairman Dr.Yu.K.Zaretsky (USSR)

Thank you very much Mr.Drescher. Now I pass the word to Mr.Feda (Czechoslovakia)

J.Feda (Czechoslovakia)

Plastic strain increments of soils may be evaluated by means of plastic potential functions. Those proposed are not identical and when deciding which of them should be used for constitutive equations it is important to know what are their differences and why they differ.

Fig.1 presents the form of some plastic potentials in the

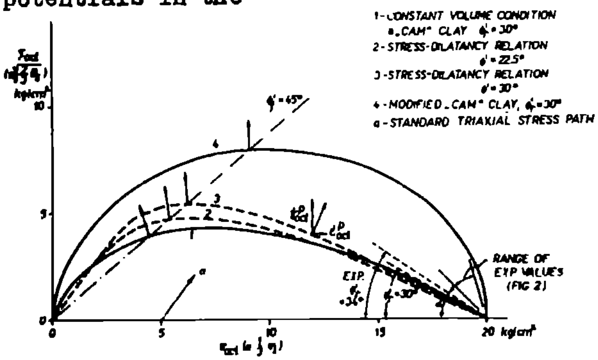

Fig.1. Plastic potential curves according to different theories

(τ_{oct}, σ_{oct}) plane for axially symmetrical (triaxial) state of stress+. Curve 1 was deduced from the incremental relation of the Bishop's energy equation (Bishop,1950) and coincides practically with the plastic potential for "Cam" clay (Roscoe and Burland,1968) Its equation reads

$$\frac{\tau_{oct}}{\sigma_{oct}} = \mathrm{tg}\,\phi'_r \,\lg \frac{\sigma^o_{oct}}{\sigma_{oct}} \qquad /1/$$

(σ^o_{oct} - hardening parameter, ϕ'_r- residual or constant volume angle of internal friction) For "Cam" clay tg ϕ'_r should be replaced by $M\sqrt{2/3}$ where

$$M = \frac{6 \sin \phi'_r}{3 - \sin \phi'_r} \qquad /2/$$

Curves 2 and 3 follow from the stress-dilatancy relation (Rowe, 1962; Barden and Khayatt 1966)

$$\frac{\sigma_a}{\sigma_r} = \left(\frac{\sigma^o_r}{\sigma_r}\right)^{\frac{2K-1}{2K}} \qquad /3/$$

(σ_a, σ_r - axial and cell pressure, σ^o_r-hardening parameter)

$$K = \mathrm{tg}^2 \left(45^o + \frac{\phi'}{2}\right) \qquad /4/$$

(ϕ'=22.5° for curve 2 and 30° for curve 3).

+ II and I on Fig.1 are the second basic invariant of the stress deviator and the first invariant of the stress tensor.

Curve 4 represents modified "Cam"clay (Roscoe and Burland, 1968),i.e. the relation

$$\frac{\sigma_{oct}}{\sigma^o_{oct}} = \frac{M^2}{M^2 + \frac{9\,\tau^2_{oct}}{4\,\sigma^2_{oct}}} \qquad /5/$$

It is rather interesting to see the curves 1 and 2 (or 3) to differ not too much although the way of their deducing are fundamentally different.

One simple possibility to test the reliability of the respective plastic potential curve is to consider the plastic strain increments at the peak strength. For dense sand e.g. with the peak angle of internal friction ϕ'=45° and a rather high dilatancy rate curve 1 seems the most suitable, curve 4 cannot be accepted.

Another simple possibility is to decide if there is a corner on the σ_{oct}-axis (i.e. $\alpha < \pi/2$, curve 1,2,3) or not ($\alpha = \pi/2$, curve 4). The value of tgα (Fig.1) may be expressed by the relation

$$\mathrm{tg}\,\alpha = \frac{\dot{\varepsilon}_{oct}}{\dot{\gamma}_{oct}} = \frac{\dfrac{\dot{\varepsilon}^p_v}{\dot{\varepsilon}^p_1}}{\left(3 - \dfrac{\dot{\varepsilon}^p_v}{\dot{\varepsilon}^p_1}\right) 2} \qquad /6/$$

(dotted symbols increments, exponent p-plastic component). For isotropic samples $\dot{\varepsilon}^p_1 = \dot{\varepsilon}^p_3 = \dot{\varepsilon}^p_3 = \dot{\varepsilon}^p_v$ /3, i.e. $\dot{\varepsilon}^p_v / \dot{\varepsilon}^p_1 = 3$ and from eq.(6) tg$\alpha = \infty$, $\alpha = \pi/2$. Presence of a corner indicates therefore an anisotropic sample with $\dot{\varepsilon}^p_v / \dot{\varepsilon}^p_1 < 3$ ($\dot{\varepsilon}^p_3 < \dot{\varepsilon}^p_1$)

Fig.2 shows the angle α from triaxial tests of sand evaluated from the boginning of the stress-strain diagram (when τ_{oct} is quite small on the stress path a of Fig.1). An empirical correction of measured values of $\varepsilon_v / \varepsilon_1$ was applied so that for loose samples $\alpha \to \pi/2$, as observed with the experiments of Pooroshasb et al.(1966). The results are therefore of qualitative value only indicating that for loose samples the corner is gradually avoided.

On Fig.2 the effect of the stress level may be seen: α increases at higher cell pressure what means that small shear stress in these cases produces elastic distortion only which is physically acceptable.

The anisotropy of samples with $\alpha < \pi/2$ results therefore not only from the sample nature (original or inhereted anisotropy,e.g for a sand sample from the mode of sample preparation) but from the stress path (in a broad sense) too (induced or deformation anisotropy). Higher cell pressure partially reduces the original anisotropy (if the mova-

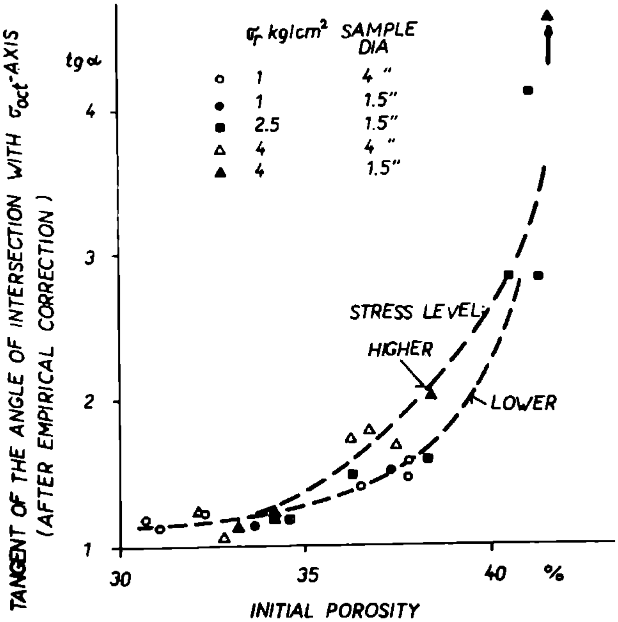

Fig.2. Measurement of the angle α (Fig.1) of Zbraslav sand (with empirical correction applied), standard triaxial compression test

bility of sandy grains is sufficient, e.g. at lower densities). This may explain the data of Fig.2. Dominant hydrostatic pressure promotes structural isotropy (see also Smith's, 1972, results).

One may therefore conclude that the plurality of plastic potentials agrees with the physical nature of soils and the respective form depends on the inhereted and induced anisotropies of samples.

Although there are relatively small differences between plastic potentials of sands and clays the views on their yield surfaces differ substantially. On contrary to clays the yield surface of sands is often considered to be "opened", i.e. it does not contain points on the σ_{oct}-axis (Poorooshasb et al., 1966; Roscoe, 1970; Barden, 1971). For the same soil it is difficult to imagine a closed plastic potential and an open yield surfaces. Under hydrostatic pressure strain hardening must be principally admitted since even for ideal random packings of spheres, i.e. ideal granular materials, the structure is found to be highly irregular (see e.g. Bernal and Mason, 1960), so that isotropic loading will induce anisotropic internal stresses. Further on the mode of a sand sample preparation determines a sort of its loading history and for lower stress ratios an almost elastic response may be expected (at least for dense samples). Both these points cover the results of Schlosser's (1965) tests reproduced on Fig.3. The yield surface for sand and glass spheres was measured to be closed.

It seems to me that the proper form of the

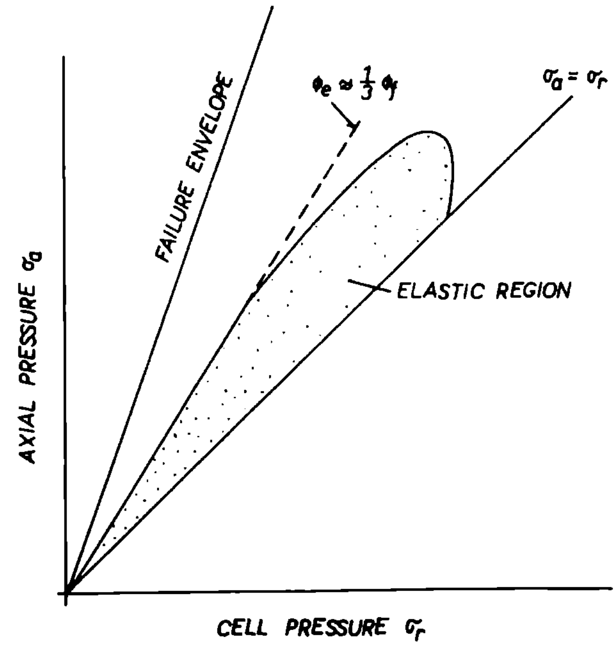

Fig.3. Yield surface of sand according to Schlosser (1965)

yield surface of sands necessitates further exploration. This will decide if the respective flow rules are associated or non-associated.

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Vice-Chairman Dr.Yu.K.Zaretsky (USSR)

Thank you very much Mr.Feda for your contribution. Mr. Ohta (Japan) will you,please,

Hideki Ohta (Japan)

I would like to talk about the non-linearity both in the current analytical method,that is to say, Terzaghi's theory and the incremental theory of three dimensional consolidation which has been developed by myself. The incremental theory is based on the incremental stress-strain relations for anisotropically preconsolidated clay derived from the stress-void ratio relations of clays under two stress conditions, that is, constant effective mean principal stress and proportional loading. A brief example of the application of the incremental consolidation theory is given in my paper entitled "Immediate and consolidation deformations of soft clay stressed by uniform strip load" presented on the Main Session No.2 of this conference.

In my country, Japan, we have a discussion on the effectiveness of sand drain method. A lot of papers on the field observations were presented. Gentlemen / attending this meeting would be interested in these papers, but they were written in Japanese unfortunately. About the half of the authors of these papers insisted the effectiveness of drainage by sand piles and another half were in opposition. The estimation of the rates of dissipation of pore pressure and consequent settlement of the ground surface is indeed very difficult in the practical sense. In most of the designs of consolidation, the settlement of the ground surface is linearly related to the average degree of consolidation. If we use the e-log p relation in order to estimate the settlement, we shall find the result is differ from the linear estimation to the amount of a few decades of percentage. To say further, the use of the average degree of consolidation results a considerable error in the estimation of settlement in the case of normally consolidated clay. Of course,we always find it difficult to decide the soil constants prior to the design works.

In the case of two or three dimensional consolidation, the modified use of Terzaghi's theory must lead to the results far from the observation. Consolidation settlement due to the expulsion of pore water is hardly separated from the immediate settlement which seems to take place within ten days after the application of load. The effect of dilatancy is commonly neglected although it is considerable. The incremental theory of consolidation can describe the three demensional phenomena at least more adequately than the modified use of Terzaghi's theory. It is vitally important to estimate the distribution of the total stress throughout the clay layer in both cases. The incremental stress-strain relations on which the incremental theory of three dimensional consolidation is based are essentially coincident with those derived from the Cambridge original energy theory. Using the incremental stress-strain relations in the estimation of total stress distribution is rather laborious because of their complexity in mathematical forms,especially because of their log functions. These log functions in the incremental stress-strain relations can be substituted by hyperbolic functions with sufficient accuracy. By this substitution, the undrained stress-strain relation for normally consolidated clay derived from the incremental stress-strain relations is reduced to the hyperbolic stress-strain relation proposed by Kondner. Kondner's hyperbolic relation is useful to estimate the total stress distribution and the results can be easily compared with that given by the theory of elasticity

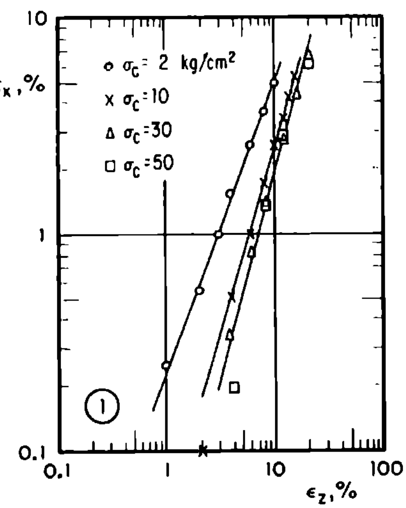
Vice-Chairman Dr.Yu.K.Zaretsky (USSR)

Thank you Mr.Ohta for your discussion. The next will be Mr.Marsal (Mexico)

Raul J.Marsal (Mexico)

The stress-strain analysis of earth structures requires that the modulus of deformation (M) and the Poisson's ratio (ν) be known for different stress levels and strain conditions. Frequently, Kondner's equation is used to evaluate M and some law of variation of ν is adopted. A brief account of experimental data obtained in triaxial compression tests with rockfills and sand-gravels is presented, in order to improve our understanding of the behavior of Poisson's ratio.

Empirical Correlations. If radial (ε_x) and axial (ε_z) strains measured in triaxial compression tests are plotted in logarithmic paper (Fig 1), points define linear relationships which are valid for the whole range of stresses, except beyond failure for specimens that show a peak stress. A similar result is



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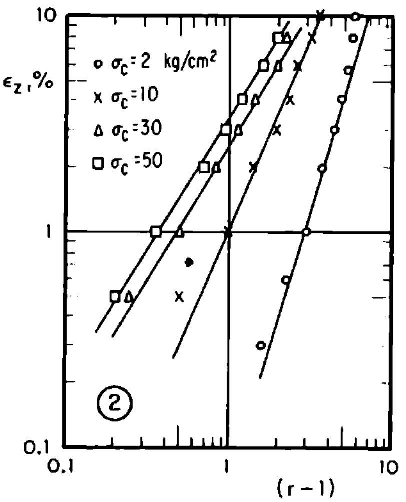
obtained when logarithm of the deviator stress $(\sigma_1-\sigma_3)$ is drawn in terms of log ε_z. To deal with dimensionless variables only, the deviator stress is substituted by the difference (r-1), r being the principal stress ratio σ_1/σ_3, as shown in Fig 2. The linearity of (r-1) vs ε_z fails near the origin and in the vicinity of failure. Therefore,



$$\varepsilon_x = \alpha\, \varepsilon_z^{\beta} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

$$\varepsilon_z = a(r-1)^{b} \quad \ldots\ldots\ldots\ldots\ldots \quad (2)$$

From these empirical facts, the following equations are derived:

$$\nu = \left|\frac{\Delta\varepsilon_x}{\Delta\varepsilon_z}\right| = \beta\left|\frac{\varepsilon_x}{\varepsilon_z}\right| \quad \ldots\ldots\ldots\ldots \quad (3)$$

and,

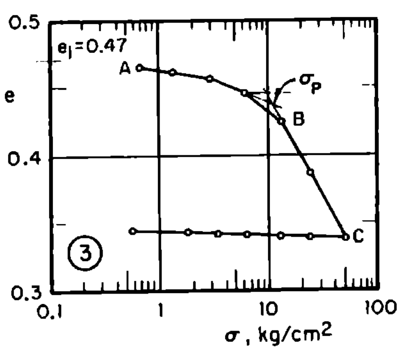
$$\nu = n(r-1)^{c} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (4)$$

where,

$$n = \alpha\ \beta\ a^{\beta-1}$$

$$c = b(\beta-1)$$

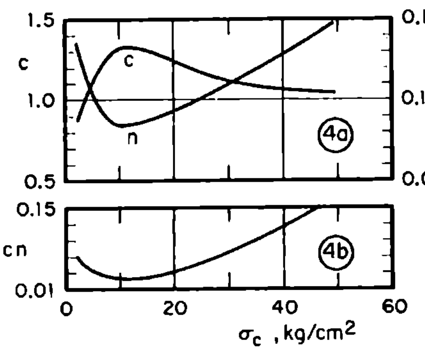
Preload Stress. Similarly to undisturbed clays tested in one-dimensional compression, the void ratio vs log pressure curves of most granular materials show a change in behavior related to stresses induced by a previous load, e.g., compaction (Fig 3). The pressure



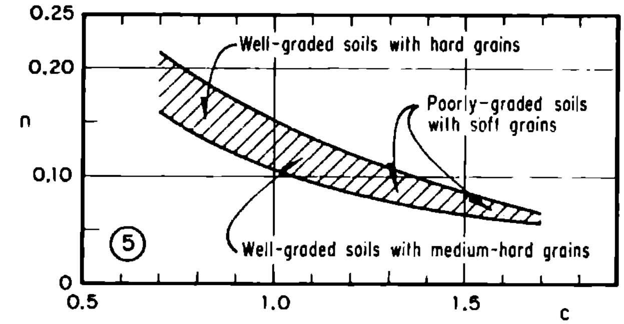
at which the above change occurs will be called the preload stress (σ_p) and the method for its determination will be similar to that proposed by Casagrande for the preconsolidation pressure (p_c) of a natural clayey soil. This concept is necessary for the analysis of parameters β, n and c.

Parameter β. The results of triaxial compression tests on rockfills disclose that the parameter β ranges between 1 and 2, for confining pressures σ_c in the interval 1-50 kg/cm². Materials that undergo substantial grain breakage during testing have β-values varying from 1.5 to 2.0, when σ_c increases within the abovementioned interval. The parameter β tends to unity for $\sigma_c < \sigma_p$. Sound, well-graded sand-gravels in dense state show that β is independent of σ_c and equal to 1.5, approximately. Note that β = 1 implies that ν = constant, or vice-versa, as occurs in a constant volume triaxial test.

Coefficients n and c. The values of n and c for a series of triaxial compression tests are interrelated, as shown in Fig 4a. For confining pressures (σ_c) smaller than the preload stress (σ_p), the coefficient n decreases with the ratio σ_p/σ_c; then n attains a minimum value for $\sigma_c = \sigma_p$, and from this stress onwards increases steadily with σ_c. The behavior of the exponent c is the opposite. However, the product cn is not constant (Fig 4b).



With the help of the empirical correlations between ε_x, ε_z and (r-1) discussed before, values of n and c for different granular materials were computed. Upon these results, it was found that points fall within a rather narrow band shown in Fig 5. The analysis of individual values does not disclose a clear trend for the points of each material to concentrate on a certain region of the band or to follow a definite path along it, when the confining pressure varies. After a careful comparison of above data, it was concluded that the spreading of (n,c) points in the band mainly reflects changes in gradation of the specimens due to particle breakage.



For practical purposes, it is tentatively suggested to compute Poisson's ratio selecting the coefficients n and c, upon the following guidelines:

1) Well-graded materials with hard grains, have values located in the left side of the band. 2) Poorly-graded soils with soft grains, fall in the right end of the band, for low confining pressures; but (n,c) values tend to the middle region when σ_c increases. 3) Well-graded soils with medium-hard grains, concentrate on the central portion of the band.

Vice-Chairman Dr.Yu.K.Zaretsky (USSR)

Thank you very much Mr.Marsal. Mr. Moroto, will you, please.

Moroto Nobuohika (Japan)

The (e,p.q) state space has been used for soils. I think that state of granular material will be more reasonably represented by an entropy space. The concept of entropy was first introduced by Mogami. Mogami's entropy was obtained from microscopic consideration. While macroscopic entropy of granular material has not be seen yet. Then, I propose the macroscopic entropy as

$$S = \int \frac{\Delta W}{p} = \frac{-e}{1+e} + \int \eta \, d\gamma \qquad (1)$$

where W is increment of plastic work done
p is mean pressure
γ is deviator strain
η is stress ratio (q/p)

Where the shearing strain is getting large,

$$S = \frac{-e}{1+e} + \eta \gamma \qquad (2)$$

I represented state of the material in terms of S,p, γ. The entropy gives a yield function. The form of pS would be an approximate plastic potential.

The entropy given by Eq.(2) may be related to Mogami's entropy.

Chairman
Vice-Chairman Dr.Yu.K.Zaretsky (USSR)

Thank you Mr.Moroto for your discussion. The next will be Mr. Wiener from GDR.

K.H.Wiener. (GDR)

From the results of numerous recent and also older experimental investigations it is known which effects of deformation occur on sand samples when these are subjected to different test conditions and especially to different stress-conditions. It was tried to mathematically formulate a stress-strain-law which,when applied to the usual test conditions, leads to theoretical dependeces between states of stress and states of deformation which are near experimental results. This non-linear stress-strain-relation has been set up in an infinitesimal form in order to allow its general application.

The explanation of the change of volume basis upon the following considerations. It is supposed that this change is caused by the superposition of three effects:

1. An increase of the average of principal stress is followed by deformations of the grains and, due to these, by secondary grain displacements. This part is reversible; an increase in stresses leads to a decrease in volume.

2. An increase of the average of principal stress gives rise to interlocking of the grains,because part of these have an instable position, especially in the case of low densities. This part is not reversible; an increase in stress being followed again by a decrease in volume.

3. An increase in the stress deviator gives rise to a change in volume,due to the kinematic conditions of the grain skeleton similar to a system of mutually supporting spheres. This change is taken as an increase.

The third effect of volume change,which is commonly called dilatancy,leads simultaneously to corresponding shear strains. Referring to this deformations the directions of stress increments and the directions of strain increments are assumed to be identical. The relation is formulated in such a way that with the occurance of a stress state satisfying a spatial failure condition the increments of shear strains gets infinite.

The stress-strain-relation contains five material constants,which can be determined, for example,by means of suitable triaxial tests. Anisotropy was not taken into consideration. The constants depend on the material properties of the sand and on the density at begin of the stressing.

The application of the theoretical relation on the conditions of triaxial tests brought about results which greatly reflect the experimental behaviour. The same is true for oedometric tests,for which the known effects of initial loading, unloading,reloading and of repeated loading and unloading could be theoretical simulated. The law which was found yields an explanation for the difference of the angles of internal friction received by triaxial or plane deformation conditions. Furthermore, indications on the value of moduli of deformation as function of the spatial principal-stress-relations and on non-linear behaviour of pressure at rest could be derived which agrees to the results of experimental research.

Vice-Chairman Dr.Yu.K.Zaretsky (USSR)

Thank you Mr.Wiener for your discussion. The next will be Prof. Vyalov (USSR)

S.S.Vyalov (USSR)

At the present time properties of soils are described by three separate models that are not related to one another: Hooke's elastic body model which describes soil behaviour in the pre-limiting state; Coulomb's body model which describes the limiting state and the Kelvin-Terzaghi model, which describes percolation consolidation. This approach gives sufficiently satisfactory results for engineering practice,but,at the same time, it has essential restrictions. Indeed, by substituting a broken line,for the actial settlement vs load curve, we make several very essential assumptions (Fig.1). In the first

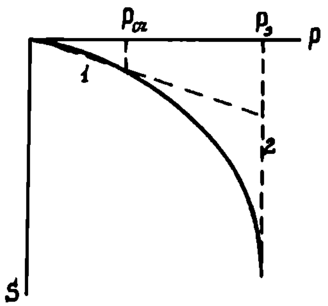


Fig.1.

place, in the pre-limiting state we consider only a small part of the curve,restricting it by the proportionality limit, when in fact the soil is subject to a much larger range of loads prior to failure. In the second place, in dealing with the limiting state we omit the deformations and consider the stresses only. In the third place, such an important soil property as its internal friction is taken into account only in considering a limiting state and is altogetner disregarded while considering the pre-limiting stage,although it is evident that not only strength, but the deformations as well depend on the magnitude of the internal friction of the soil. In other words, at small loads we regard the soil as a body having elastic properties; at large loads we regard it as a binding medium having altogether different characteristics. Thus,depending on the load,we deal with two different kinds of bodies.

Evidently,it would be expedient to work out a generalized soil model which would take into account: (1) the nonlinear relationship between stresses and strains,(2) the influence of internal friction on the process of deformation, as well as on the strength,(3) the influence of the shear stress on the volume strain (dilatation), (4) the correlation between the strength and strain characteristics, and (5) the development of the process in time.

In the general form the equation which takes into account the above properties will be:

for the pre-limiting state

$$\varepsilon_i = f(\sigma_i, \sigma_o, \mu, t); \varepsilon_o = \varphi(\sigma_o, \sigma_i, \mu, t) \quad (1)$$

for the limiting state

$$\varepsilon_i = const; \sigma_s = \varphi(\sigma_i, \mu, t) \qquad (2)$$

Here σ_i = intensity of tangential stresses

ε_i = intensity of shear strains

$\sigma_o = \frac{1}{3}(\sigma_1 + \sigma_2 + \sigma_3)$ mean normal stress

$\varepsilon_o = \frac{1}{3}(\varepsilon_1 + \varepsilon_2 + \varepsilon_3)$ mean strain

$\mu = \frac{2\sigma_2 - \sigma_1 - \sigma_3}{\sigma_1 - \sigma_3}$ Lode parameter

t-time

Equations (1) and (2) represent the fact that both the shear strain ε_i and the volume strain ε_o depend on the stress deviator σ_i and on the hydrostatic pressure σ_o. The introduction of the Lode parameter μ describes the dependence of the deformation process on the kind of stressed state. The introduction of the time factor describes the rheological properties of the soil. The transition to the limiting state results from the condition that this state is due to the fact that the shear strain reaches its limiting value $\varepsilon_i = \varepsilon_s$. These equations are shown as curves in Fig.2: the first curve represents

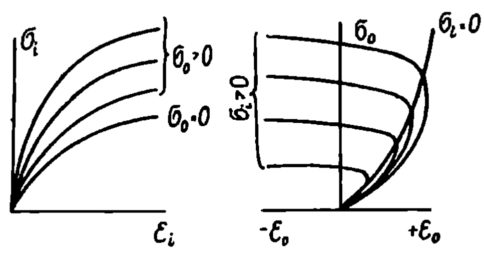


Fig.2

shear strains at various hydrostatic pressures, the second curve represents volume strains at various shear stresses. Equations (1) and (2) may have various specific forms. Experimental data are best represented by power (3) and hyperbolic (4) laws of deformation and by the power (5) and linear (6) strength conditions that follow from these laws:

$$\varepsilon_i = \frac{1}{(H+\sigma_o)^n} \left(\frac{\sigma_i}{A} \right)^{m_t \alpha} \qquad (3)$$

or

$$\varepsilon_i = \frac{1}{H + V_o} \frac{1}{A\left[T+t(1-\delta^{\sigma_i/\sigma_s})\right]} \qquad (4)$$

$$\sigma_s = B(H+\sigma_o)^{\lambda} \qquad (5)$$

or

$$\sigma_s = (H+\sigma_o)\,tg\,\psi \qquad (6)$$

Here H and ψ =Mises-Botkin parameters corresponding,

H= resistance to hydrostatic compression,

ψ = angle of friction on an octahedral area,

$n, m, \alpha, A, T, \delta, B, \lambda$ = parameters

A very important question is raised: to what degree does the nonlinear character of the strain affect the behaviour of the soil in bases of structures? This question can be answered by means of test data obtained by A.L.Mindich and the author.

Tests were carried out by pressing a strip loading plate into a layer of clayey soil lying on a rigid base. The deformative and strength properties of this soil were investigated in a triaxial compression apparatus and are described by equations (4) and (6).

Test results revealed (Fig.3) that all the strain characteristics of the soil were represented in the nature of the settlement vs load curve. As a matter of fact,the shape of this curve is a good approximation of the hyperbolic relationship which is similar to the one obtained from the triaxial tests. The following can be pointed out: the relationship between the hydrostatic pressure σ_o and the volume strain E_o for the given soil could,according to the triaxial compression tests,be assumed linear. It would be natural to assume that the settlement vs load relationship obtained in the tests in pressing a loading plate into the soil,is determined by a certain combination of the hyperbolic law (4) for shear strain and the linear law $\sigma_o = KE_o$ for volume strain. However, the shear strain had a greater effect on the character of the settlement vs load relationship which is discribed by a hyperbolic function such as (4)

$$S = B\,\frac{1-\nu}{E_o}\,\frac{P}{1-P/Ps}, \qquad (7)$$

where parameter $B = \frac{\alpha b}{1+B/h}$ takes into account

the influence of the thickness h of the layer being compressed and the width b of the plate.

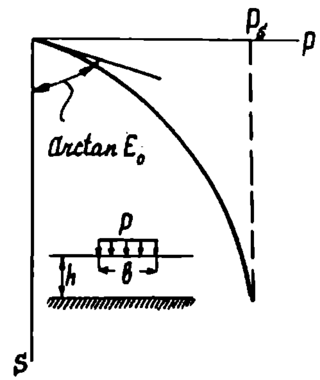


Fig.3

It should be pointed out that the magnitude of the limiting load ps is included in the settlement equation. This magnitude, in its turn, is a function of the strength characteristics H and ψ , and the thickness of the layer being compressed: $Ps = \phi(H, \psi, B/h)$

Thus,the settlement vs load curve can be described in the whole range of stresses, including the limiting state,by taking into account the nonlinear stress-strain relationship and by introducing the strength into this relationship. This enables the real properties of the soil to be more fully utilized.

Vice-Chairman Dr.Yu.K.Zaretsky (USSR)

Thank you Prof.Vjalov. Now I pass the word to Mr. Vinokurov (USSR)

Vinokurov E.F. (USSR)

The research of stressed and strained state of non-homogeneous anisotropic bases had been performed. The theory of the work is based on the book "Iteration Method of Bases and Foundations Estimation by Computers" written by the author. The "Minsk-22" computer program for the solution of axially symmetric and plane problems is composed.

The estimated diagrams of soil basis are given in Fig.I, the character of the common strain modulus changes by the depth of the basis being various: 1) and 2) $-E_x=E_y=500 kg/cm^2$ and $E_x=E_y=200\ kg/cm^2$ is isotropic medium; 3) strain moduli were changed within $E_x=200+500\ kg/cm^2$ and $E_y=200+500\ kg/cm^2$; 4) modulus E_y was increased with the depth, E_x is constant; 5) and 6) the both moduli of common strain were changed by a step-like law. Shear moduli were computed using Barden's formula and the normal formula of elasticity theory. Poisson's coefficient was assumed to be 0.3.

The results of estimations are presented in Fig.2. Compressive stress sheets for estimated diagrams I and 2 coincide with compressive stresses computed theoretically as for isotropic basis. The step-like changes of the strain modulus with the depth involve uneven changes in compressive stress sheets. In case when the upper layer is more compressive than the lower one, the compressive stresses in the second layer increase greatly (diagram 5) as compared to estimated diagram N 1. An inversely, when the second layer is more compressive than the first one, the compressive stress sheet decreases (diagram 6).

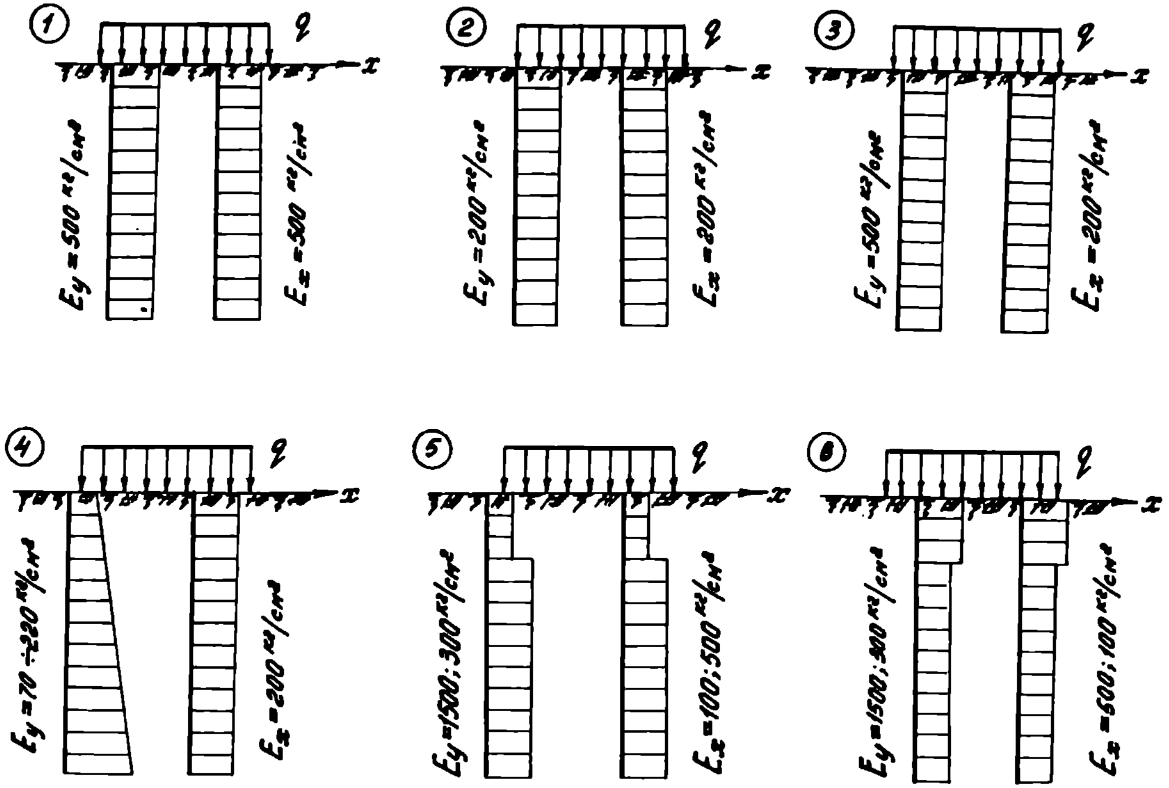


Fig.2. Compressive stress sheets and the epures of vertical translation along the loading axis for:

1 and 2-strain moduli being constant;
3 - strain moduli being constant, but varying in values.

In the presence of a soft intercalation, the compressive stresses sheet has jumps at the boundaries of the soft layer. The epures of the vertical translation along the loading axis are given in the right-hand side. The curves based on diagrams N 1 and N 2 characterize the shaded part with the results for diagram N 3.

Conclusion: The theory of linearly strained isotropic medium can be used as approximate in the determination of stresses in anisotropic basis for constant values of E_x and E_y only, as well as for particular range E_x/E_y relation changes. This theory cannot be used in determining the components of translation.

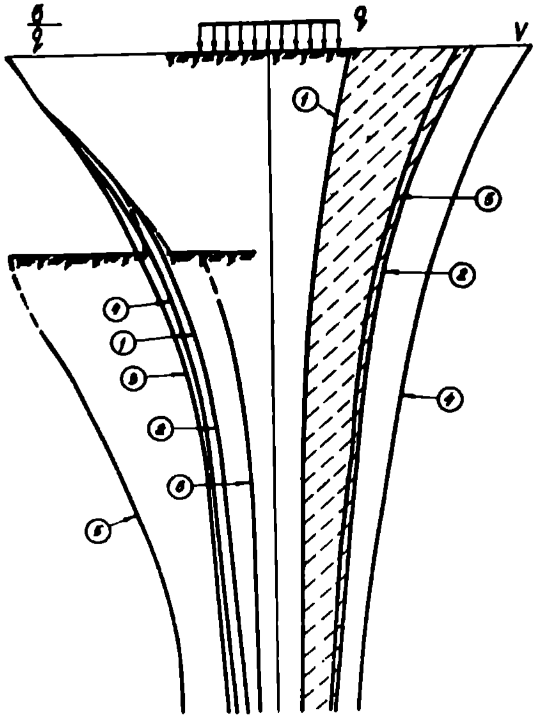


Fig.1. Diagram of changes in strain moduli E_x and E_y by the depth of soil basis

Vice-Chairman Dr.Yu.K.Zaretsky (USSR)

Thank you very much Mr. Vinokurov
The next will be Mr. Yasuhara (Japan)

K.Yasuhara (Japan)

1) It may be said difficult to estimate the secondary compression of peaty soil as well as silty soil at the field. However, we suppose to be able to estimate it by applying the third expression in Table 1[1)] to coefficient of consolidation c_v. The expression was proposed by us taking a large difference of permeability between primary and secondary compressions in consolidation. When we use this expression, c_v value shows approximately constant independently of the height of specimen.

2) We have found a fact that the consolidation of soft subgrade of road is progressed under repeated loading of traffic, and they made clear in laboratory that repeated loading brings about a large secondary compression, by using an oedometer not only in silty soil but also in muck. However, its effect in muck was smaller than in silty soil. Also, we have found another fact that precompression is increased with increasing of over-consolidated ratio in both soils.

3) It can be said important to investigate the difference between the secondary compression rate in muck under the one-dimensional consolidation of laterally confining, that is, K_0- consolidation, and the shearing creep rate in it in triaxial consolidation to which lateral deformation is allowed. Fig.1[1)] shows the relation between effective stress ratio and logarithmic creep rate. As seen from the figure, the two significant points are indicated as follows : i) The value of shearing creep rate is different from the one of secondary compression rate, ii) When we compare the shearing creep rate after isotropic consolidation with the one after unisotropic consolidation, the latter is larger than the former. This difference may be significant at the field because its stress condition is similar to the latter.

Table 1
Treatment of Secondary Compression on Oedometer Test

1) Terzaghi

$$C_v = \frac{k}{m_v \gamma_w}$$

C_v: coef.of consolidation in total process

m_v: coef.of compressibility in total process

2) Japanese Society of SM & FM (Prof.Mikasa)

$$C_v = \frac{k}{m_{v1} \gamma_w} \cdot \frac{m_{v1}}{m_v} = \frac{k}{m_{v1} \gamma_w} \cdot r = C'_v \cdot r$$

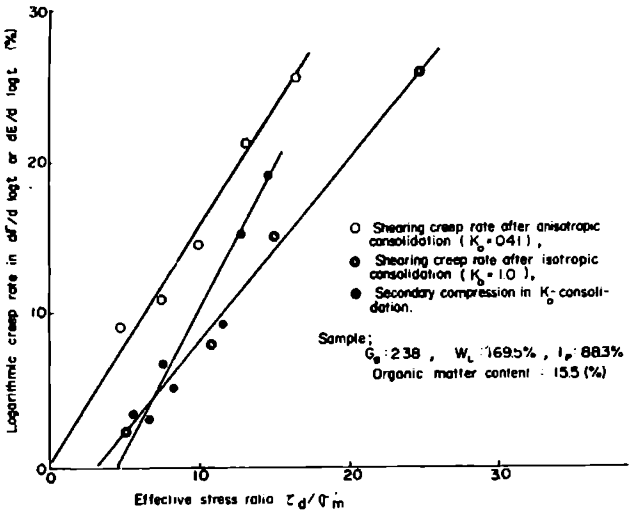
where, m_{v1}: coef. of compressibility in primary compression,
C'_v : coef.of consolidation in primary compression,
r : primary compression ratio

3) The writers

$$C_v = \frac{k_1}{m_{v1} \gamma_w} \cdot \frac{m_{v1}}{m_v} \cdot \frac{k}{k_1} = C''_v \cdot r \cdot \frac{\alpha + 1}{\alpha}$$

$$\alpha = \frac{k_1}{k_2}$$

where, k_1: coef. of permeability in primary compression process,
k_2: coef. of permeability in secondary compression process,
C''_v: coef. of consolidation in secondary compression



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Vice-Chairman Dr.Yu.K.Zaretsky (USSR)
Thank you Mr. Yasuhara. Now. Mr. Thamm from FRG, will you, please.

B.R.Thamm (FRG)

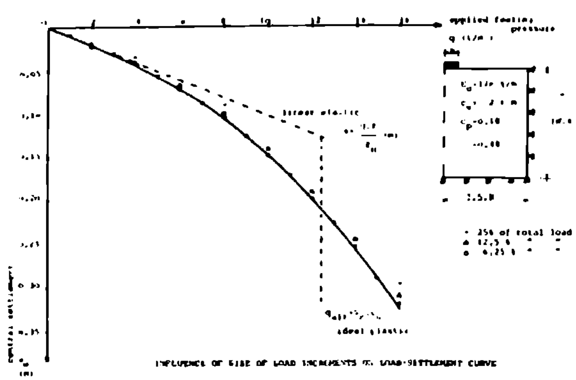
Mr.Chairman,ladies and gentlemen,
My report is restricted on the calculation of initial settlements and initial excess pore pressures developed during undrained loading of a normally consolidated non-sensitive clay.

The stress parameters selected for representing the state of stress are in terms of octahedral stresses, where $\Delta\sigma_{oct}$ is the change in the mean normal stress and $\Delta\tau_{oct}$ is the change in shear stress. The term $\Delta\tau_{oct}$ HENKEL uses in his pore pressure equation is supposed to be mathematically wrong since 1967, but this equation unfortunately is used still in 1973 by many authors.

For practical purposes up to a stress level of 0.5 (where the stress level is defined by the current value of $(\sigma_1 - \sigma_3)$ over $(\sigma_1 - \sigma_3)$ at failure or better a factor of safety of 2.0) $\Delta u = \Delta\sigma_{oct}$ and we have an almost linear elastic solution of the problem.

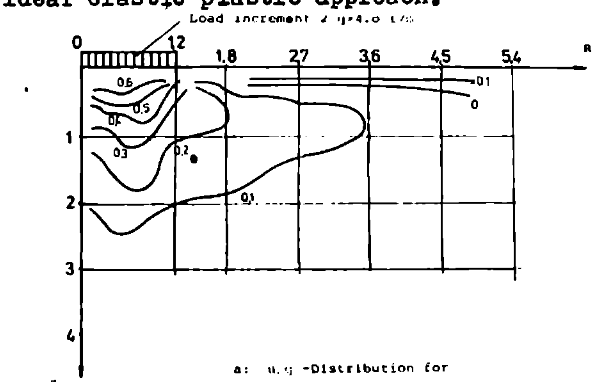
For stress levels between 0.5 and 1.0 however, we use a non-linear elastic approach similar to DUNCAN and CHANG in combination with a non-linear pore pressure response, where the pore pressure parameter α is a function of the current stress level. The halfspace investigated by a finite element program consists then of a non-homogeneous, non-linear elastic, isotropic, saturated and porous media.

The first slide shows the initial central

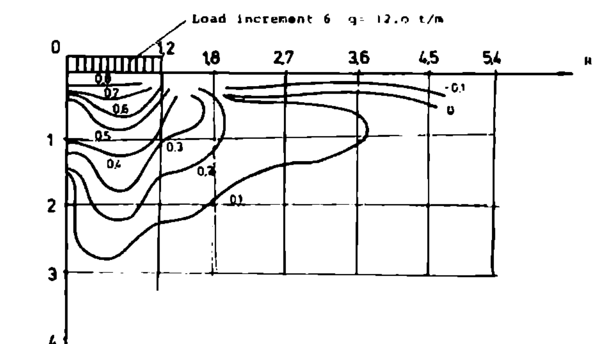


INFLUENCE OF SIZE OF LOAD INCREMENTS ON LOAD-SETTLEMENT CURVE

settlement of a circular footing over the applied footing pressure and compares the results with an ideal elastic-plastic approach. You may note, that the finite element model with above assumptions is somewhat stiffer beyond 0.25m settlement than the ideal elastic plastic approach.



a: u/q -Distribution for load increment 2



b: u/q -Distribution for load increment 6
NORMALIZED EXCESS PORE PRESSURE DISTRIBUTION

Here, the discretization technique has to be developed and investigated more clearly.

In the second slide the excess pore pressures Δu are normalized by the applied footing pressures for different load increments. Load increment 2 being in the almost linear elastic range and load increment 6 being in the plastic range.

It is felt that one of the most important developments in soil mechanics today is the increased use and study of field measurements and their relation to soil properties.

I hope that the developed program may furnish the engineer to study field measurements and their relation to results of laboratory tests. Thank you!

Vice-Chairman Dr.Yu.K.Zaretsky (USSR)

Thank you Mr. Thamm for your report. Mr. Solomin (USSR) will be the next.

V.I.Solomin (USSR)

Here are given some results obtained recently at the Chair of Structural Mechanics of the Cheliabinsk Polytechnical Institute under supervision and with participation of the Research Institute of Foundations and Underground Structures in Moscow.

The problem of action of a circular rigid plate or a rigid strip on a sand soil was solved by using symmetrical loads. For this purpose physical equations were applied, they were obtained in the three-axes-pressure instruments and establish the relationship between the invariants of stressed and deformed states. By calculating there were revealed some phenomena earlier observed only in the experiments, they are: the transformation of the diagram of reaction pressures from a saddle-shaped to a parabolic one, when loads increase; the more rapid, than in a linearly-deforming half-space, attenuation of vertical displacements, deformations and stresses to the depth; the formation of a rigid core and some others. The problem was calculated in the displacements by an electronic computer using the method of finite differences.

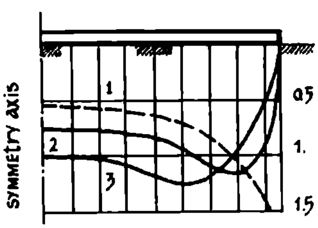


Fig.1

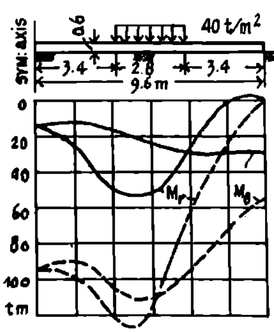


Fig.2.

Fig.1 shows the diagram of reaction pressures under the circular plate, they are the ratio between actual and average pressures, I is the linear solution for a half-space, 2, 3 are the non-linear solutions with average pressures of I and 2 kp/cm^2 respectively.

The results described as well as the data of other investigations indicate that the equations of continuous medium and the methods of deformational theory of plasticity are acceptable for sand soils.

Another problem takes into account the non-linearity of deformation of reinforced concrete foundation structures. In most investigations devoted to this question the limited states of strength are considered in various forms. It is necessary to develop the methods of calculation of internal strains and displacements occuring before plasticity moments appear, since the requirements of crack-resistance and the restriction of mutual settlements are more strict than those of strength and usually determine the amount and diameter of reinforcement. To solve this problem there were used physical equations suggested by the Research Institute of Concrete and Reinforced Concrete in Moscow. These equations give the opportunity to apply the methods of deformational theory. The investigations done by an electronic computer indicate good co-relation between the results of the calculations and those of the experiments.

The data obtained for beams and circular axial-symmetrically loaded plates show that even in those cases when stresses in the reinforcement do not reach a yield limit and the pressures on soils do not exceed normative ones, the differences between the results of linear and non-linear solutions may be significant. It is explained not only by the decrease of the rigidity of a foundation structure bur also by the change of the reaction pressures.

Fig.2 shows the diagrams of bending moments occuring in the circular plate resting on an elastic half-space and loaded with a ring-shaped load. The dotted lines are the linear solution, the solid lines are the non-linear solution.

The application of the non-linear physical equations of deformation of soil and foundation structures allows to reveal significant reserves of strength of footings and foundations.

Vice-Chairman Dr.Zaretsky Yu.K.(USSR)

Thank you Mr. Solomin. Mr. Uriel will you please make your discussion

A.O.Uriel (Spain)

It is a well known fact that if we carry out drained cylindrical triaxial tests on a given sand, for different density conditions, we get stress-strain curves of the shape represented in Fig.1. The dotted lines correspond to the common definition of axial strain,

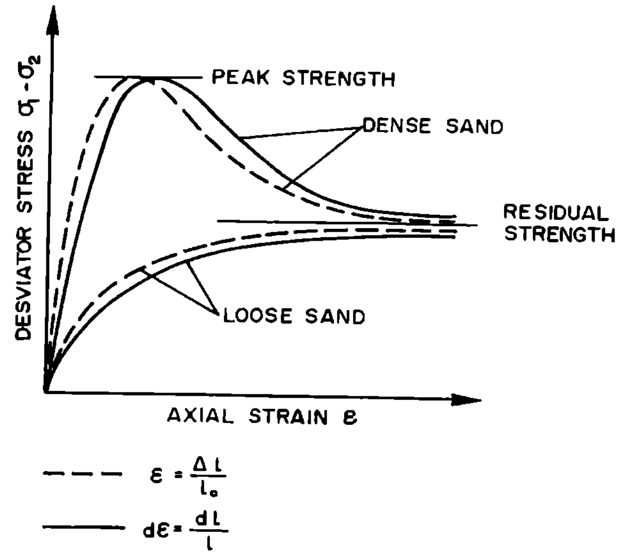


FIG. 1

in other words, with relation to the initial length of the sample. The continuous lines correspond to the definition of strain used in this discussion, related to the length of the sample at each instant of the test. Dense sand exhibits peak and residual strength, whereas loose sand has only residual strength. There should be a certain critical initial density which separates the two kinds of behaviour.

The first derivative of the stress-strain curve with respect to the axial strain gives the variation of the modulus of deformation in the direction of the major principal stress with the axial strain (Fig.2). The

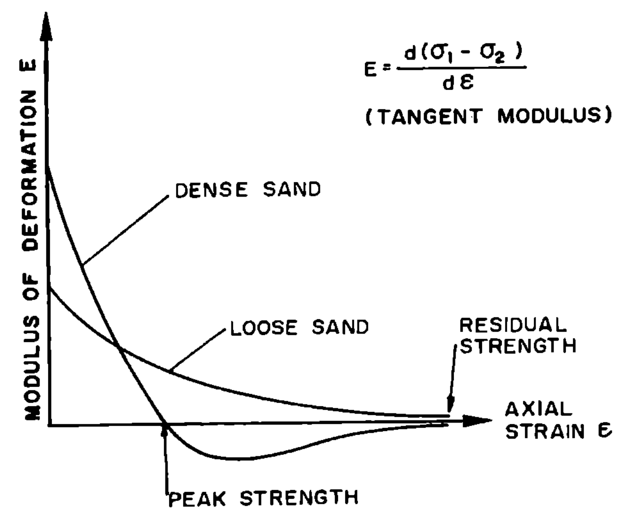


FIG. 2

tangent modulus for the peak strength, in the case of dense sand, is equal to zero and it is negative from this point onwards. The tangent modulus for the residual strength is again equal to zero.

Consider the problem of the vibration of a mass suspended from a spring, with a damping force proportional to the displacement velocity, for the particular case in which the damping parameter is equal to the square root of the spring constant (Fig.3). If we plot elongation against time, we get the curves of the figure, depending on the magni-

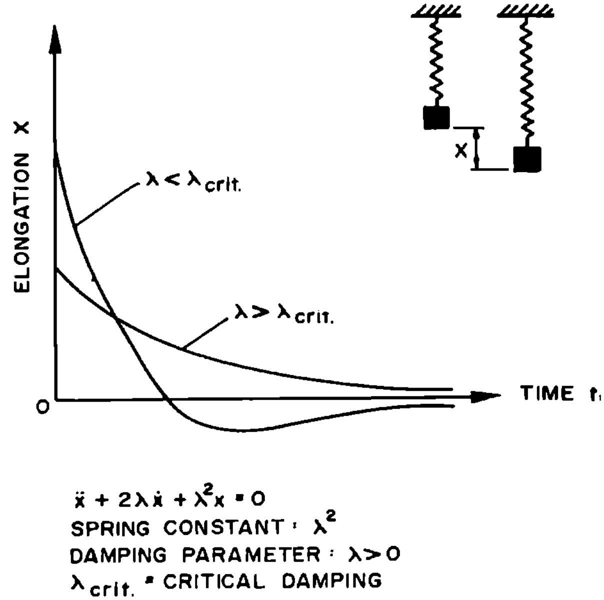


FIG. 3

tude of the damping parameter with relation to a certain value called critical damping. There is a large similarity between these curves and those of the previous figure. So, the role of the initial sand density in the stress-deformation process is equivalent to the damping in the problem of the vibrations.

The canonical form of the differential equation of the damped vibrations has been written in the figure. By analogy, we can suppose that the stress-strain relationship of the sand under the cylindrical triaxial conditions is governed by the same differential equation, in terms of the tangent modulus:

$$\frac{d^2E}{d\varepsilon^2} + 2\lambda\frac{dE}{d\varepsilon} + \lambda^2 E = 0 \quad (\lambda > 0) \qquad (1)$$

where λ is an intrinsic constant of the material, likely related to the shape and size distribution of the grains.

Let $\omega = -\dot{E}_0/E_0$, where E_0 and $\dot{E}_0$ are, respectively, the initial tangent modulus and the initial value of its first derivative.

Should the parameter λ of the differential equation be less or greater than ω, there will not be peak strength.

Let r_o denote the ratio between the initial density of the sample (after introducing the cell pressure σ_3) and the critical density, which determines the existence of peak strength. Then, r_o is greater than one for the dense samples and less than one for the loose samples. Therefore, $\omega/\lambda = \rho_o (\rho > 0)$ is a certain function of the ratio r_o, such that $\rho_o - 1$ has the same sign as $r_o - 1$.

Assuming that the residual strength is a material property, which does not depend on the initial conditions of the test (cell pressure and initial density), and taking into account the condition σ_1 equals σ_3 for ε equals zero, we can integrate the differential equation (1), obtaining the following expressions for the tangent modulus and the deviator stress:

$$E = \frac{\lambda \sigma_3}{(2-\rho_o)} \frac{2 \sin \phi_r}{1-\sin \phi_r} e^{-\lambda \varepsilon} \left[1 + \lambda(1-\rho_o)\right] \qquad (2)$$

$$\sigma_1 - \sigma_3 = \left[1 - e^{-\lambda \varepsilon} - \lambda \frac{1-\rho_o}{2-\rho_o} \varepsilon e^{-\lambda \varepsilon}\right] \frac{2 \sin \phi_r}{1-\sin \phi_r} \sigma_3 \qquad (3)$$

The initial tangent modulus is:

$$E_o = \frac{\lambda \sigma_3}{(2-\rho_o)} \frac{2 \sin \phi_r}{1-\sin \phi_r} \qquad (4)$$

Equation (4) shows that E_o is proportional to the cell pressure σ_3 and increases with the increasing initial density, as expected. Since E_o has to be positive, $\rho_o \leq 2$.

The expression for the deviator stress corresponding to the peak strength is:

$$(\sigma_1 - \sigma_3)_p = \left(1 + \frac{\rho_o - 1}{2-\rho_o} e^{-\frac{1}{\rho_o - 1}}\right) \frac{2 \sin \phi_r}{1-\sin \phi_r} \qquad (5)$$

and it follows from equation (5) that:

$$\frac{\sin \phi_p}{1-\sin \phi_p} = \left(1 + \frac{\rho_o - 1}{2-\rho_o} e^{-\frac{1}{\rho_o - 1}}\right) \frac{\sin \phi_r}{1-\sin \phi_r} \qquad (6)$$

It may be seen that the peak strength only depends on the residual strength and the initial density of the sample, and does not depend on σ_3, since the Mohr-Coulomb failure criterion has been assumed.

The necessary strain for the peak strength is:

$$\varepsilon_p = \frac{1}{\lambda(\rho_o - 1)} \qquad (7)$$

which decreases with the increasing initial density and does not depend neither on σ_3 nor on ϕ_r.

The suggested semiempirical approach to the constitutive equation of a granular material is not as simple as it would be desirable, but reality is more complex. Compared with other expressions simpler than this one, such as the well known Kondner s hyperbolic stress-strain relationship, is has the advantage of being able to account for the existence of peak and residual strength in the densest conditions, and it may be useful in analysing certain problems involving large deformations, for instance the stability of residual slopes.

Finally, it is interesting to say that the volume change during the test may be predicted in a similar way and, even, that this method can be extended to any stress path, by using a more general linear second order differential equation with the appropriate coefficients, which depend on the incremental tendency of the stresses and are invariable as long as the ratio of the stress increments remains unchanged.

Vice-Chairman Dr.Ju.K.Zaretsky (USSR)

Thank you very much Mr.Uriel.

At the beginning of this Session we heard Professor Poorooshasb's report. He suggested two problems for discussion.

Let us formulate them again.

1) What is the nature and the form of the yield surface of soils? Does a yield surface traced in a stressed space contain a space diagonal?

2) The discrepancy between prediction by means of the classical Terzaghi theory and prediction by means of experimental data is usually explained in the majority of cases by the inaccuracy of the initial information regarding the soil factors involved rather than by the inadequacy of the simple classical theory.

I now ask all those interested to take the floor and express their opinions on these two problems. Mr. Mustafayev, will you, please.

Mustafayev A.A. (USSR)

The solution of non-linear problems of soil mechanics of strained bodies as applied to foundations on soil is known to involve considerable mathematical difficulties, thus justifying the development of engineering methods of their solution using the simplified foundation models.

In this communication, based on the principally new method of equivalent transformations, the method is reported for calculation of foundation setting (according to the second threshold state) with consideration of the real non-linear regularity of soil medium strain.

As a basis of the problem formulated serves a model of a medium consisting of soil with a hypothetic property convenient for calculation and differing from the real one by the fact that the regularities of linearly-deformable uniform isotropic body are applicable to it in the whole range of stressed state, to each stressed state of the nonlinearly deformable foundation soil in question (Curve 1) corresponding univocally quite definite hypothetic state (Curve 2), characterized by linear strain module E and Poisson ratio ν (Fig.1). The real soil is

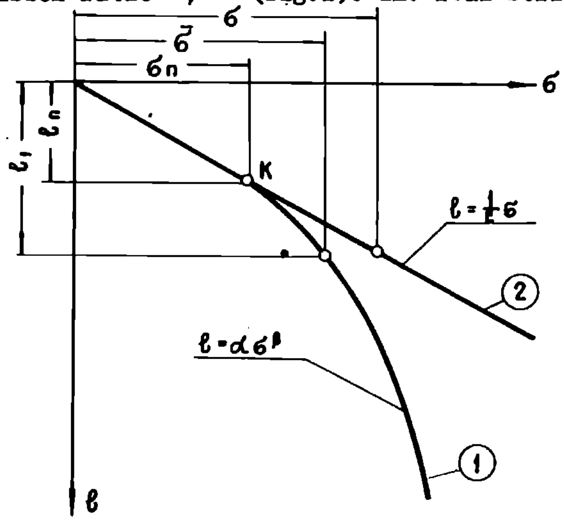


Fig.1. The calculated diagram of the "Equivalent transmissions" method

declined from its hypothetical state starting from the limit of proportionality (point K) because of formation of structurally irreversible strains in it.

The conditions relating the stress-strain state of the two foundation soil models under consideration, may be expressed both according to strain and stress depending on the problem in mind. The problem is solved using a phenomenological method which allows construction of a strain-stress curve for the foundation soils concerned based on experimental data, and relating it with the linear behaviour of a hypothetical medium. The condition of soil strain adequacy in its two stressed conditions under study leads to a formula for distribution of packing stress along the structure axis in the nonlinearly strained soil foundation:

$$\sigma_z = \left(\frac{1}{\alpha E}\right)^{1/\beta} \sigma(z)^{1/\beta} = \bar{K}(E, \alpha, \beta)\,\sigma(z)^{1/\beta} \quad (1)$$

Here α and β are the parameters of nonlinear strain of the foundation soil, determined by rectification of the experimental $e - \sigma$ curve, approximated by the $e = \alpha\sigma^{\beta}$ function on the logarithmic coordinate grid.

The σ (z) function determines the vertical stress distribution in the foundation in keeping with the solutions of the theory of uniform linearly strained body.

In case of unidimensional stressed state of the foundation the experimentally found strain-stress relationship for soils is a compression curve. If in this case the foundation contraction due to compression, is been as a two-dimensional stressed state the intensity of the load applied may be presented in the form

$$\sigma(z) = \sigma = \frac{\sigma_x + \sigma_y}{1 + \xi}$$

For a three-dimensional stressed state one has:

$$\sigma(z) = \sigma = \frac{\sigma_x + \sigma_y + \sigma_z}{1 + 2\xi}$$

where $\xi = \frac{\nu}{1+\nu}$ is a coefficient of soil lateral pressure.

The values of design parameters α, β and E for the case of foundation loading concerned may be determined from the $e - \sigma$ curve constructed based on the data of field tests which were made using an experimental load of a standard round (F=600 cm^2) or quadratic (F= 5000 cm^2) die.

To study the regularities of nonlinear soil strain under unidimensional and three-dimensional stressed experimental study was made in laboratory conditions.

Evaluation of data obtained in numerous experiments with cohesive soils of undisturbed structure made on the stabilometer show that the $e - \sigma_1$ curves are similar to each other over the whole range of variation of horizontal main stress value ($\sigma_2 = \sigma_3$) i.e. these curves are obtainable from one curve, for example, for the case of unobstructed free lateral expansion ($\sigma_2 = \sigma_3 = 0$) – by multiplying its ordinates by some value being a function of horizontal main stress. Therefore the $e - \sigma$ relationship for the spacial axially symmetrical loading of foundation soil may be presented as a product of two functions of which one $\varphi(\sigma_1)$ is a function of only vertical main stress, while the other $\psi(\sigma_2 = \sigma_3)$ of only horizontal one:

$$e = \varphi(\sigma_1)\,\psi(\sigma_2 = \sigma_3)$$

For the adopted power approximation of the $e - \sigma_1$ function graph, (2) is presented as:

$$e = \alpha(\sigma_2 = \sigma_3)\,\sigma_1^{\beta_0}$$

where β_0 is a parameter of experimental parabolic $e - \sigma_1$ curve constructed using the data of soil testing under the conditions of unobstructed free lateral expansion.

As the $e - \sigma_1$ curve obtained based on the compression testing over the whole variation range $\sigma_2 = \sigma_3$ crosses all the curves of the $e - \sigma_1$ family for axially symmetric loading of the same kind of soil (Fig.2), it is possible to determine the α parameter from the data of two simplest tests-compression and crashing at $\sigma_2 = \sigma_3 = 0$

The setting of a separate footing on a nonlinearly strained soil foundation may be calculated, on an analogy with the formula accepted, using the expression:

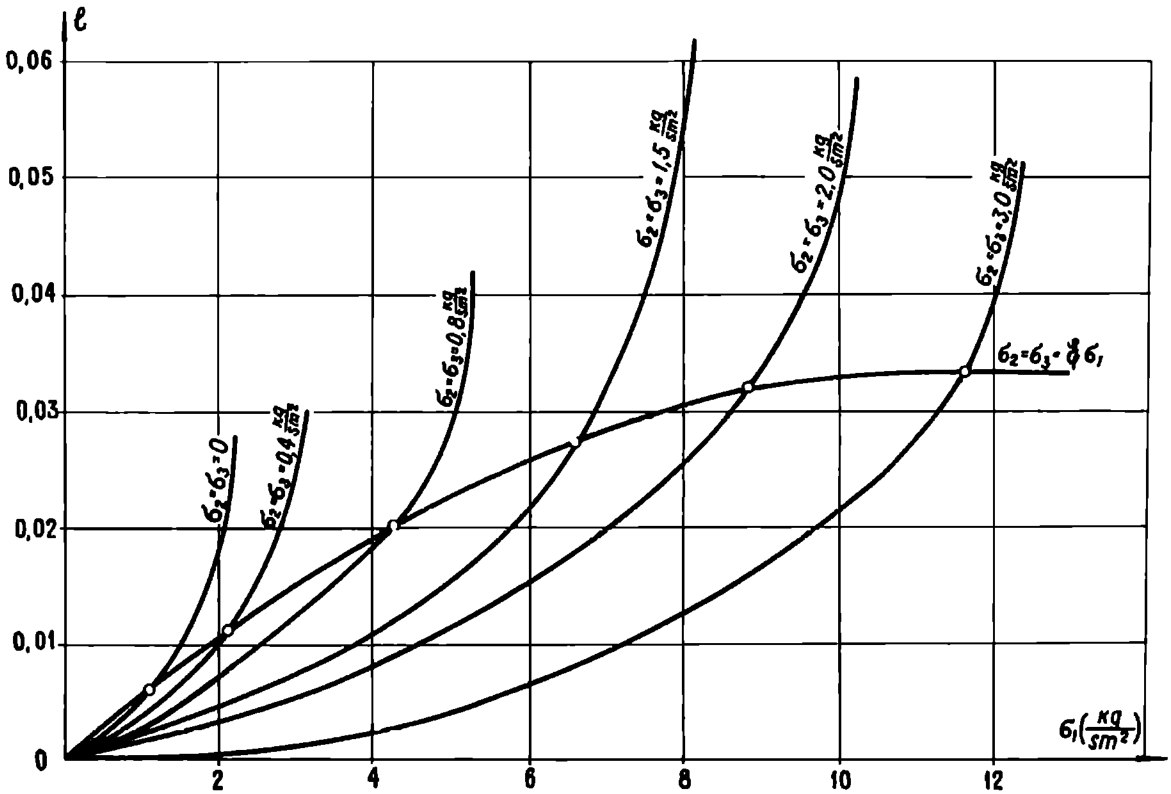


Fig.2. The experimental curves of dependence of the relative deformations on stress for the cases of one-linear and three-linear condition of bound soils of undisturbed structure

$$S = \sum_{i=1}^{n} h_i d_{k_o} \ln \frac{\bar{\sigma}_i(z)}{\sigma_o} \qquad (3)$$

where n is a number of horizontal layers constituting the compressed foundation; h_i is a thickness of the i-th soil layer; σ_o is a natural pressure in the middle part of the given layer; $\bar{\sigma}_i(z)$- vertical normal pressure in the middle part of the given layer, determined from formula (1); d_{k_o} - relative compression coefficient, determined for the i-th layer as an angular coefficient of semi-logarithmic graph of compression curve.

The proposed method of calculation of structure foundation strain presents no special difficulties, being very simple and always giving reliable results.

Vice-Chairman Dr. Yu.K.Zaretsky (USSR)

Thank you very much Mr. Mustafayev. The next will be Mr. Gudehus G. (FRG)

Gudehus G., (F R G)

The questions concerning elastic range raised by Prof. Poorooshasb can be partly answered for sand under cuboidal deformations in our apparatus. Strictly speaking sand has no elastic range at all: due to the non-linear behaviour of the individual grains non-dissipative continuations of strain histories are impossible. However, there are ranges in stress space that can be reached with rather small dissipation (below 20% of the total work, say). The boundary of such a range ("yield surface") is not very marked. It is state-dependent and generally non-isotropic. As far as we have made tests it contains the diagonal in stress space. For further details see our contribution in Proc. Symp. on the Role of Plasticity in Soil Mechanics, Cambridge, 1973.

Vice-Chairman Dr. Ju.K.Zaretsky (USSR)

Thank you Mr. Gudehus. Then I want to invite Mr. Rochette (U.K.)

P.A. Rochette (U.K.)

Summary. In addition to the very elaborated mathematical formulations of a non-linear response, it is proposed that there is room for progress from approaches using simpler tensorial models and realistic parametric representations (including the structural properties). For accurate results,the complexity of the model appears less efficient than the degree of representativeness of the test procedure and boundary conditions. This is illustrated on a simple usual model and new basic non-linear properties are proposed.

1. Principle of solution for non-linear materials. When the response of the soil or rock is non-linear, i.e.varies with the strain, all the points of the soil mass give a different answer,and the principle of superposition does not apply. It is necessary to integrate compatible stresses and strains all through the mass in order to satisfy the laws of motion and the mixed boundary conditions. The steps are the following:

a) Choice of a stress-strain relationship framework. A preliminary study of the material should enable the choice of one or several approximate basic behaviours for which solutions for stress and strain distributions in a mass exist; for instance the material can be tentatively considered as elastic, viscoplastic,etc.

b) Testing programme to measure the constitutive parameters of the material. Only when the behaviour framework has been chosen,the parameters can be separate in special tests, and measured. A testing programme of measuring typical soil behaviours (shear,triaxial, consolidation etc) would produce the overall effect of unknown parameters in unknown proportions, as long as the empirical data are plotted without the guidance of a framework model. The relationships empirically fitted do not represent intrinsic properties; they depend on the test procedures. Unless a finite strain and stress increment can be divided into zones of different mechanisms (the same zones for all the variables chosen, in order that each zone represents a unique particular mechanism), the parametric method is arbitary and would give nonreproductible results. An approximate, although coherent and basic approach for parametric representation will be described in a following paper.

4. Account for material anisotropy and structural properties. In the case of saturated soils and rocks,a complex structure can be represented at least approximately using constitutive properties which are directional. For instance C.R.Calladine, 1971,distinguishes different ratios of elastic and plastic deformations according to the direction around a given location.

5. Representativeness of the model for the real loading conditions and changes of behaviour with time. Practically the simples usual stress-strain relationship assumes a straight line in the diagram "e-log p" (or preferably volumetric strain "v"-log p). However the following comments show that sucha line is highly variable with the material and the engineering problem: a) Interpretation of the test curve.

- The diagram consists of a network of lines (1) and (s) which each corresponds to a different age (or history) of the material (see points a, a', a" in figure 1a)
- The rebound line(s) is not necessarily horizontal (L.Bjerrum, 1967) nor nearly horizontal (L.Bjerrum, 1973),as shown on a typical figure 1a. There is a locus (q1) for the critical pressures. A slight change of history (change of line a" into a"f in figure 1c) could affect considerably the value of the critical pressure.
- Figure 1b illustrates how different histo-

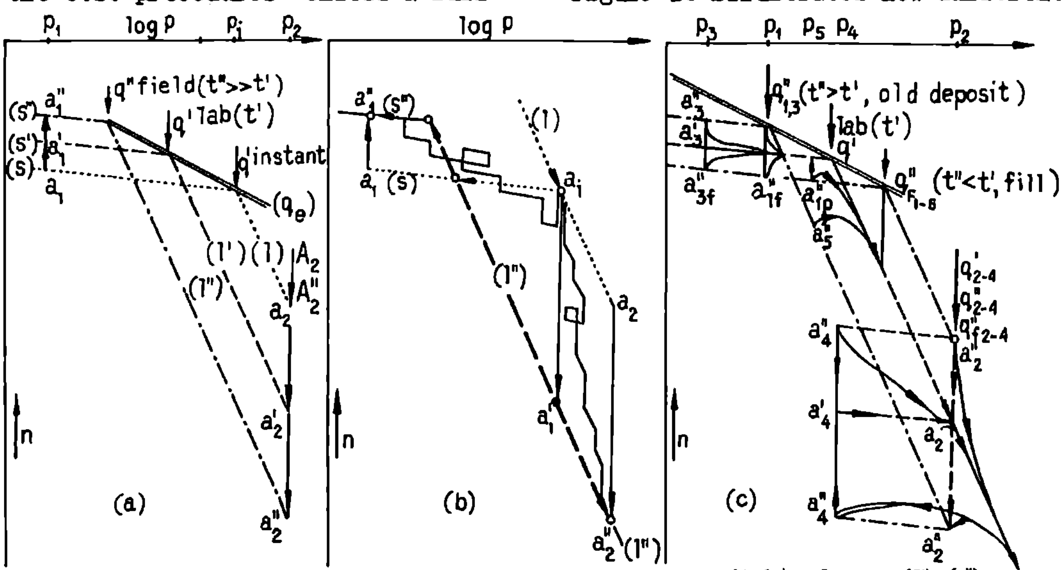


FIG 1 Compression test: (a) critical pressure q; (b) initial field values on(1"),(s")curves (c)laboratory load curves.

ries of erosion and deposition from the same state a_i produce states which are all situated in the locus (s") or (1").

- A laboratory test carried out at a certain rate corresponds to a line (s') and (1'), that is to a certain time rate t'. Different curves will be obtained on figure 1c depending whether the history of the material in the field has a rate slower than the test (curve 1"), or faster than the test (curve $1"_f$). The different curves exhibit large variations for the critical pressure.

b) Basic properties from the constitutive diagram

- Figure 2a shows how the material may exhibit a consolidation or a heave, depending on the previous history.

- The intersections of the critical pressure locus (q_1) and (Q_s) with the extreme pressures determine the existence and the values of the maximum and minimum porosity of the material for its given history. The line (l_g) and boundary conditions; and even a correct integration throughout the mass would lead to non-representative results, unless if the sample and the tests exceptionally happened to be a true scaled-model of the mass and engineering problem.

c) Behaviour of the mass of the material under given or changing boundary conditions

Since non-linear behaviours lead to relatively complex formulations, the benefit of this more accurate and realistic method would be lost if excessive simplifications are used in the determination of the mass behaviour. The classical numerical methods should be

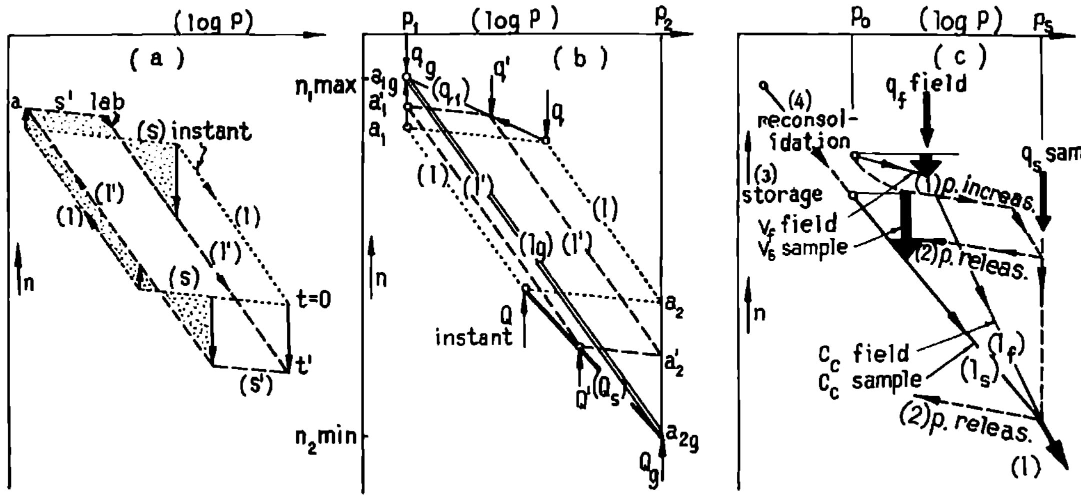


FIG2 Computation of: (a) time effects; (b) critical pressures q and extreme densities (c) sampling effect.

considered, in particular the finite element techniques for non-linear media. The numerical methods are summarised and selected for instance in J.T.Oden, 1972, chapter 17.

a) Proposed solution for usual soils and rocks. The simplest model assumes that $\dot{T}$ the stress increment tensor (co-rotational stress rate), is a linear function of the strain increment (D is the stretching), through the linear operator $H(T)$. This constitutive equation of hypo-elasticity (C.Truesdell, 1955) $\dot{T}=H(T)\{D\}$, applies in any case of anisotropy and non-colinearity of $\dot{T}$, T and D, providing that the model is representative of the material, in which case $H(T)$ is found an isotropic function of T. Then the stress increment can be represented by a form of the second order in T(T, Tokuoka, 1971, equation 4.7), and the function $H(T)$ can be determined by experiments (the parameters are defined in T.Tokuoka, 1971, equations 4.10 to 4.12).

The representation of the equation for the computation of $\dot{T}$ in the above tensorial form enables a direct calculation of $\dot{T}$ when T and D are given, without any assumption on the normality conditions nor on colinearity between the principal directions.

e) Simplified relationship. The following constitutive equation is useful for common soil and rock engineering problems:

$$\dot{T}=a\ \{tr\ D\}\ 1 + b\ \{tr\ D\}\ T + c\ D + d(DT + TD)$$

a,b,c,d are functions of the principal stress invariants.

For a preliminary solution d can be neglected when the history of the soil is not too complex (e.g. not too overconsolidated).

2. Rigorous solution for non-linear soils and rocks. The use of a "material element" which is not oversimplified (infinitely small, anisotropic, etc) is necessary to take account of such phenomena as yield, hysteresis, generalised plasticity, imperfect memory. The use of "interinsic stresses and deformations" enable constitutive equations which are frame independant automatically. This approach is described in W.Noll, 1972.

3. Parametric representation of a non-linear behaviour. A parametric representation is often used to simplify the computation techniques using finite elements. The accuracy of the results depend on the convergence (difficult to appreciate), and practically on the physical reality of the simplifying representations. Represents a closed cycle (with identical rebound) as would happen after a very long time, at a geological scale.
-Figure 2c shows the considerable variation of the critical pressure and amount of volume change, resulting from the history of an "undisturbed" sampling.

c) Basic properties for rebound and reload conditions.
- The rebound (s) may vary between the elastic and plastic cases (see figure 3a).
- The reload curves produce cycles as in figure 3b
- A load and reload curve exhibits a three slope configuration whenever the pressure step is large enough (see figure 3c)

d) Computation of the deformation.
- It is accepted that the curve e-log p is close to a straight line (fig.4)

Now it is proposed that the curve e-log t is approximately a straight line (fig.4b)
-The deformation involves three components as as illustrated by figure 4c: the history component, the pressure term, and the time effect.

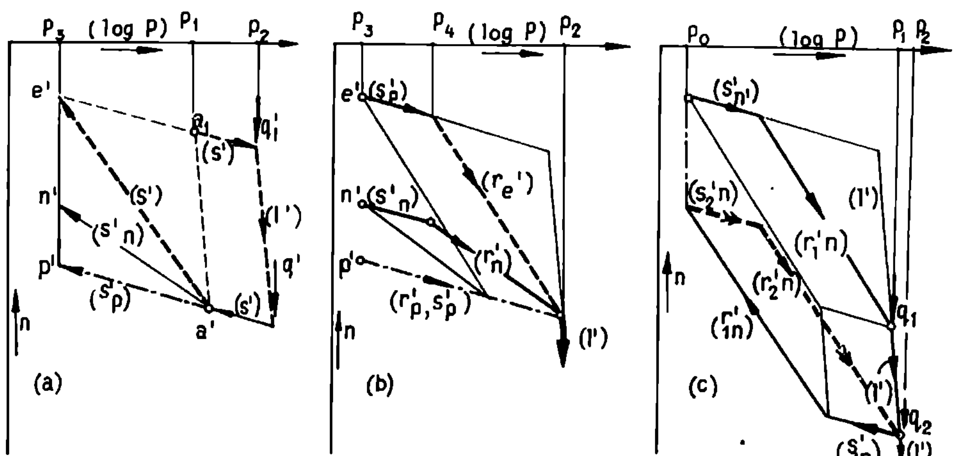


FIG 3 Mechanisms for(a) rebound; (b) reload; (c) typical curve with 3 slopes($p_o \ll p_1$)

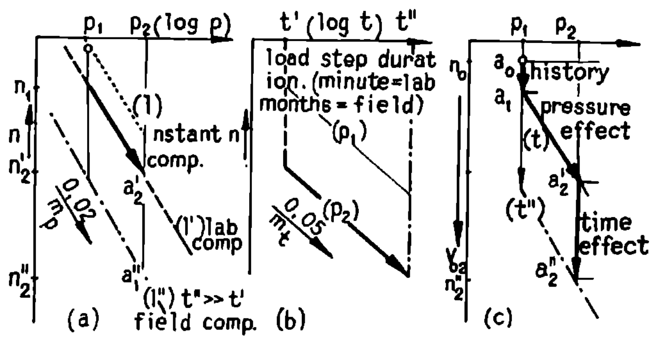


FIG 4 (a): compression and (b): consolidation mechanisms
(c): compression $V_{o2} = n_o - n'_1 + m_p \log\left(\frac{p_2}{p_1}\right) + m_t \log\left(\frac{t''}{t'}\right)$

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Vice-Chairman Dr.Ju.K.Zaretsky (USSR)

Thank you Mr. Rochette. Mr.Ivashchenko, will you please, make your contribution?

I.N.Ivashchenko (USSR)

Prof.H. Poorooshasb has touched on important aspects associated with application of flow theories for soils.

Our communication has dealt upon some specific features of the loading locus: convexity, smoothness and asymmetry relative to the space diagonal. Sand and clay soils test have shown that drastic change of the loading direction causes not only variation of the shape and dimensions of the loading locus; but also the displacement of the whole locus in the space of stresses. And the envelope of all possible positions of the loading locus is the limit surface,characterising the strength of soil. It has been established that limit surface is not a symmetric function of three invariants,but it depends on the principal stress ratio. The position of limit surface is not significantly influenced by the stress path including the paths of unproportionate stresses.

At all possible displacements of the loading locus,space diagonal does not go beyond its limits. At the same time the characteristic feature of soils is their plastic strains due to hydrostatic pressure. As a consequence of this loading locus is closed and convex in the direction of space diagonal and touches it from within.

The authors express their thanks to Prof. Poorooshasb and Dr.Yu.Zaretsky for their attention and comments to our communication.

Vice-Chairman Dr.Ju.K.Zaretsky (USSR)

Yu.K.Zaretsky (USSR)

Concluding Remarks

Dear Colleagues,

Our Session has come to an end. We have heard five planned reports and nineteen persons took part in the discussion. The aim of our Specialty Session was to discuss problems in soil mechanics that deal with the nonlinear problems of plasticity,creep and consolidation, and the application of theory to design. We had no intention of covering the entire scope of existing problems. Therefore,three problems were proposed for discussion:

1. Computation algorithms for a nonlinearly deforming soil base.
2. Analysis of the computation results.
3. Consolidation of nonlinearly compressible water-saturated soils.

The discussion revealed that a great number of investigators are interested in these problems and we had the opportunity of becoming acquainted with new solutions in this field. However, we thought that there would be a more lively and interesting discussion on problems concerned with the working out of computation algorithms and new research in the application of numerical methods for solving problems in nonlinear soil mechanics. Only Mr.Wroth (Great Britain) and A.L.Goldin (USSR) spoke constructively on this problem. A.L.Goldin's and A.P.Troitsky's report dealt with the application of the finite-elements method to a physically nonlinear elastic medium. The possibility of efficiently applying the finite-elements method was shown for cases when the stresses and strains are assumed to be expanded in a series with respect to the small parameter.

In the discussion V.I.Solomin (USSR) reported on the solutions of several urgent problems in soil mechanics making use of the finite-differences method. Problems were solved for physically nonlinear media having substantial nonlinearity. E.F.Vinokurov (USSR) reported on the results of calculations on nonhomogeneous anisotropic bases in the elastic zone, using the same finite-differences method. B.R.Thamm illustrated the application of the finite-elements method for determining the initial stressed-strained state of a base and the initial excess pore pressure, which is related to the stress deviator and the spherical stress tensor. However, it should be pointed out that many mathematical aspects of utilizing numerical methods for problems in the calculation of physically nonlinear media were not brought up in the discussion. Such problems as the convergence of the iterative process to a precise solution, as the existence and uniqueness of solutions were not mentioned at all. To our regret, the local variations method, worked out in the USSR, was not mentioned in the discussion, as well as other variational-differences methods.

At the same time, many reports dealt with the formulation of the physically nonlinear relationships applicable to soil systems. Although this problem was not proposed for this Session, I fully agree with the speakers, since in the final analysis this is the most essential problem. Here we can note many interesting reports presented by A.Drescher (Poland), J.Feda (Czechoslovakia), Prof.Kisiel (Poland), R.Marsal (Mesico), A.O.Uriel (Spain).

Apparently, sufficient material has accumulated at the present time to discuss this problem separately. To our regret, the problem of equations of state for soil systems is still far from being solved.

I want to thank Professor L.Šuklje (Yugoslavia) and Professor M.Mikasa (Japan) for their very interesting reports on the problem of nonlinear consolidation and Professors T.Yamanouchi and K.Yasuhara (Japan) for their remarks on secondary consolidation. Professor S.S.Vyalov (USSR) presented laboratory test data on pressing a loading plate into a layer of clayey soil lying on a rigid base. He approximates these test results by a proposed relationship. In his report Professor A.Mustafaev (USSR) sets forth an engineering method for determining settlements of a base, whose soils are subject to nonlinear laws of deformation.

It is impossible to deal in detail with all the remarks made in our discussion. It will be expedient only to point out that the discussion concentrates our attention in the future on the following: (a) to expediently intensify the investigation of actual mechanical properties of soils with the aim of obtaining generalized equations of state; (b) to continue working out computation algorithms on the basis of the finite-elements, the finite-differences and the variational-differences methods, that have justified our expectation in practice; (c) it is essential to continue the investigation of problems dealing with the consolidation of highly compressible soils and to work out methods for calculating their settlements and the interaction of structures with a base, made up of water-saturated soil.

In conclusion it should be emphasized that the interest exhibited by specialists to the subject-matter of Specialty Session No.2 can be explained by the fact that it becomes more and more evident that it is essential in designing bases and foundations of structures to take into account the actual mechanical properties of soils.

It is my opinion that this Session was useful, fruitful and timely.

Allow me to close our Session. Thank you.

WRITTEN CONTRIBUTIONS

AN EXPERIMENTAL INVESTIGATION ON THE SIMILITUDE IN THE CONSOLIDATION OF A SOFT CLAY, INCLUDING THE SECONDARY CREEP SETTLEMENT

H. Aboshi (Japan)

It is very important to establish the law of similitude in the consolidation of clay including the secondary or creep settlement, in order to apply consolidation test results in the laboratory to the settlement prediction of actual foundations. There are many theories on the secondary consolidation, however experimental endorsement on these theories usually stands on the oedometer or triaxial test of small size. There seems to be the lack of data which fill the gap between laboratory test results and field observations. In order to make clear the change of settlement characteristics with the thickness of clay layers, including the effect of secondary consolidation, a series of oedometer tests using different specimen sizes were carried out. The ratio of the diameter to the thickness of each specimen is kept constant, to avoid the different effect of friction and stress distribution on the side wall. The oedometers used and the method of loading are tabulated below,

No.	1	2	3	4	5
D cm	6.0	14.1	60.	120.	300.
H cm	2.0	4.8	20.	40.	100.
Load	Loading Lever		Hydraulic Pressure		Direct Loading
Place	Laboratory			In-situ	

Careful and long time preparations have been performed to obtain homogeneous thick clay specimens for the present investigation. A trench of 10 m x 15 m with 1.5 m of depth was cut in the surface sand layer in our University and filled with a sedimentary marine clay from Hiroshima Bay, in perfectly slurry condition. This artificial clay layer was consolidated more than 6 years with a thin sand layer on it, to make this test fill homogeneous and to recover the structural strength among soil grains to a certain degree. Average physical and mechanical properties are as followed,

Clay	Silt	Sand	LL	PL	PI	G	wn	qu
27%	68%	5%	100.2%	58.2%	42.0%	2.65	80.0%	$1.5 t/m^2$

There are many factors which influence the rate of secondary consolidation, the consolidation pressure, preconsolidation, sustained loading, temperature, rate of increase in effective stress, sample thickness, side friction, etc.[1)]

Care has been paid on these factors, to clarify only the effect of sample thickness on the settlement characteristics.

Loading steps were kept perfectly same in each test. After preliminary consolidation, which is necessary to perform the comparison test in a normally-consolidated region, settlements of long duration by one loading step from 0.2 to 0.8 kg/cm^2 for each specimen are measured and compared. To avoid the effect of sustained loading, preliminary stage was limited up to the end of primary or 100% consolidation, and to decrease side friction, silicone oil was used. To keep the temperature during the experiment constant was impossible because larger ones were performed in-situ. However, it was the season of the year that the temperature change was very small, at least during the primary stage.

Settlement curves are shown in the Figure and the main conclusions obtained are as followed,

1. The value of c_v obtained from the consolidation curves somewhat increases with the thickness of the specimens. So the prediction of consolidation of an actual clay layer from the standard oedometer test may underestimate its settlement rate.
2. The final settlement at the 100% primary consolidation increases with the thickness, owing to the secondary consolidation contained in the primary stage.
3. $d\epsilon/d \log t$ or the gradient of the creep strain in each test, gradually decreases to a constant value, so the creep settlement curves of every tests become almost parallel in their final stages.
4. However these creep settlement curves do not coincide in one line on the creep curve of standard oedometer test, as shown in Suklje's Isotaches theory.[2)]
5. This seems to mean that the state of clay structure at the end of hydraulic retardation may be affected by the effective stress history or the duration of primary stage, and on the other hand, the rate of creep settlement may not be affected by it.

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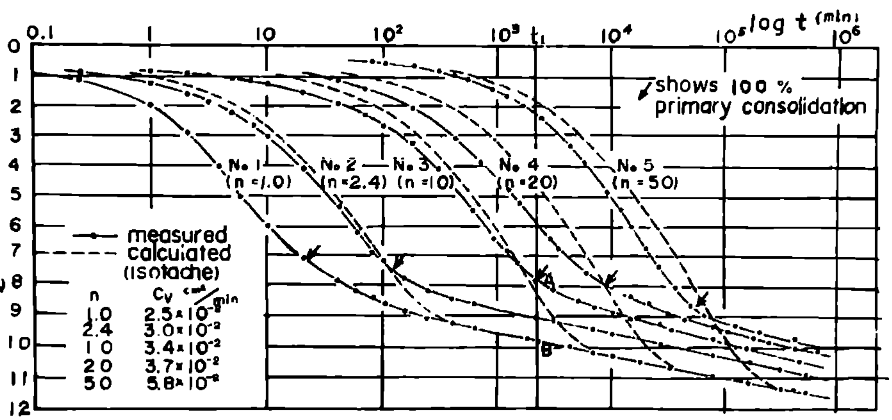


VOLUMETRIC FRACTURE OF SOILS - AN EMPIRICAL APPROACH. A. P. Alexiev (Bulgaria)

A physical theory of soil strength was exposed in brief recently (Alexiev,1970). Its basic concept has been that a"fracture surface" defined by ratios of gaseous, liquid and solid areal contents begins to develop long before the occurrence of soil failure. A fracure governing relation was obtained:

$$\left[\frac{(W_g/W_l)^{\alpha(t)}}{(W_l/W_s)^{\beta(t)}}\right]_{\text{Temperature}} \qquad (1)$$

where W_g, W_l, W_s are soil phases contents on the fracture surface, $-1 \leq \alpha(t) \leq 1, -1 \geq \beta(t) > 1$ are composite time functions determining the fracture surface topography in dependence on solid particle size and shape, solid-liquid and liquid-gas interactions (bonds). On this basis was shown that only four structural states are possible. Changing properties in dependence on these states is illustrated by Smalley (1972) for flowslide systems.

In fact(1) is not uniquely defined so it determines a volumetric fracture zone instead of a surface one, which is well in accord with some recent general views on material fracture(Gilman,1969;McClintock,1971). The coincidance of rupture surfaces with the directions of zero-extension found by James in sand(Roscoe,1970) may be an evidence for the mechanism suggested.

An attempt was made to check experimentally some of these conclusions in the case of a three-phase silty soil(loess) representing the"quasi-stable"structural state for which:

$$W_g/W_l > 1, W_l/W_s < 1; \; 0 < \alpha(t) < 1, \beta(t) < -1 \qquad (2)$$

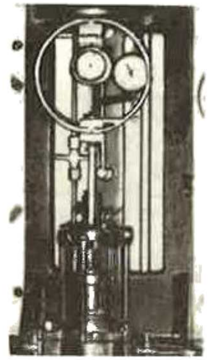
A relative non-sensitivity of soil strength to water content change in this material has been expected, and to be followed by a plasticity flow when the structural state of the soil transforms into "quasi-liquid" one:

$$W_g/W_l < 1, W_l/W_s > 1; -1 < \alpha(t) < 0, \beta(t) > 1 \qquad (3)$$

as well as a return back to"quasi-stable" state when the conditions (2) are restored.

A cylindrical loess specimen posed on a porous disc and surrounded by mercury in a perspex tube has been stressed axially, in plane strain conditions(mercury level maintained constant by means of air pressure change in the triaxial cell shown in Fig.1), until the vertical stress in situ for this soil was attained. It is allowed then pressurized water to enter the cell wetting the specimen, and a gradual transition to"quasi-liquid"but not collapsible behaviour has been observed: a slow and almost constant rate axial stress relaxation has been occurring. This strain behaviour is similar to the second stage of a"natural"loess collapse phenomenon, but here the whole process has occurred by this way.

The common manner to return to(2)is to expel some water from the soil, yet an alternative technics has been applied. The cell pressure was decreased and the specimen allowed to expand under the action of pressurized air entrapped in soil voids. A sponge structure(clearly seen in the upper part of the specimen in Fig.2)has been obtained and a plasticity flow reduction has occurred simultaneously.



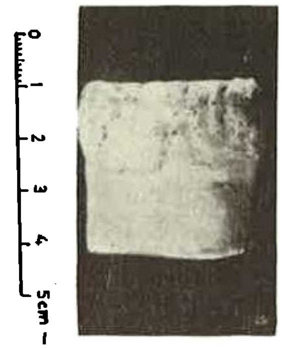


Fig.1 (on the left). General view of apparatus utilized.

Fig.2 (on the right). Specimen after testing.

So it is found again(for previous data see: Alexiev,A.P.,1969,Com.ren.Acad.bul.Sci.T.22, 805 et 806)that a presumed typical brittle-bond soil may restore its"quasi-stable"aspect, after being in"quasi-liquid"state, without reducing of water content. Consequences of the technics applied will be discussed in more detail elsewhere(Alexiev,1973), but not in purpose to obtain strong quantitative conclusions. This is not possible to achive by the techics described nor by any other if a sufficiently rigorous control of thermal conditions is not assured. But a more general and detailed proving is quite feasible having in mind some sophisticated technics existing, in paricular the last models of the Cambridge simple shear(biaxial) apparatus and any true triaxial apparatus.

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CONTINUUM THEORY FOR PLASTIC DEFORMATION OF GRANULAR MEDIA /G.Aguirre-Ramirez/ (USA)

Problems of mechanics of continua associated with the constitutive relationships for plastic deformation of granular media have received considerable attention. Some of the earlier suggestions to use the Mohr-Coulomb failure criteria as a yield criteria (Drucker and Prager 1952) have not lead to acceptable models because predictions are not in agreement with experimental results. However, results of a remarkably intensive experimental and theoretical program have shown that the deformation of certain soils can be described remarkably well by a simple isotropic work-hardening idealization (Roscoe and Burland 1968). The complex rheological behavior of sands was also compared with existing theories and it was shown (Roscoe 1970) that the well known methods of plasticity and physical interpretation of the Mohr-Coulomb limit states have to be modified.

This report deals with a phenomenological theory for the mechanical behavior of granular media which shows stress-strain behavior similar to that of an elastic work-hardening plastic metal. The theory is a mathematical generalization of the critical state theory (Roscoe and Burland 1968, Schoffield and Wroth 1968). In the theory it is conjectured that the irrecoverable deformation of the granular mass is described by the plastic component $\underline{\varepsilon}''$ of the linear strain tensor and the plastic component e'' of void ratio. In soil mechanics the strain tensor $\underline{\varepsilon}$ is related to the void ration e by making use of the observation that for most granular masses of interest the compressibility of the solid skelton is several orders of magnitude smaller than the compressibility of the bulk material.

The theory is wholly dependent on the concept of a critical state for the granular mass. This concept (Rendulic 1936) is equivalent to the assertion that there exists a unique scalar valued function N of the effective stress $\underline{T}$ such that

$$e = N(\underline{T}) \quad (1)$$

i.e., void ration is a function of stress. In the report, instead of (1), the existence of a unique scalar valued function M of the effective stress $\underline{T}$ such that

$$e'' = M(\underline{T}) \quad (2)$$

is postulated, i.e., the plastic component of void ratio is a function of stress. It is then shown how to interpret (2) in order to obtain a yield criterion of the form

$$F(\underline{T}, k) = 0 \quad (3)$$

where F is the yield function and k is the strain hardening parameter which is such that

$$k = k(\theta'') \quad (4)$$

θ'' being the irrecoverable volumetric strain.

The theory is also put in terms of triaxial stress parameters g, p defined by

$$p = \tfrac{1}{3}\,\mathrm{tr}\,\underline{T}\,, \quad g^2 = \tfrac{3}{2}\,\mathrm{tr}\,\underline{\tilde{T}}^2, \quad \underline{\tilde{T}} = \underline{T} - p\underline{1} \quad (5)$$

with $\underline{1}$ the unit tensor. Given a yield function $Q(p, J, k)$ in g, p space the following expression is obtained:

$$\dot{\underline{\varepsilon}}'' = B(p, J, k)\,\underline{\tilde{T}}\,\dot{\theta}'' \quad (6)$$

$$B(p, J, k) = \sqrt{\frac{3}{2J}}\,\partial_g Q(p, g(J), k)\left(\partial_p Q(p, g(J), k)\right)^{-1} \quad (7)$$

for the increment of the plastic component of the deviatoric strain $\underline{\varepsilon}''$. The yield function $Q(\ )$ and yield criterion $Q = 0$ together with a strain hardening function $k(\theta'')$ are used to obtain an expression for θ'',

$$\theta'' = \theta''(J, p) \quad (8)$$

As examples it is shown how to generalize the Cam-Clay model (Schoffield and Wroth 1968) and modified Cam-Clay model (Roscoe and Burland 1968).

Finally as part of the report it is shown how the resulting theory can be adapted for the use of the finite element method in the solution of boundary value problems.

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The design of offshore structures or bridge bents on battered piles involves the analysis of highly non-linear soil-pile interaction system. The proposed method of analysis assumes that the linearly elastic structure is two dimensional and supported by inter-dependent nonlinear spring supports.

At present analytical prediction of the axial pile displacement curve is difficult. Often loading tests have to be performed on full-sized piles at the site to ensure the reliable curves before designing important structures. On the other hand analytical treatment of the lateral pile behavior is regarded appropriate. A finite difference scheme for a beam-column (Fig. 1) can solve for the transverse and rotational reactions of a pile which is connected to the pile cap with arbitrary degree of fixity (Awoshika and Reese, 1971). In solving the lateral behavior transverse subgrade reaction curves which vary along a pile at different levels have to be generated from proper soil criteria.

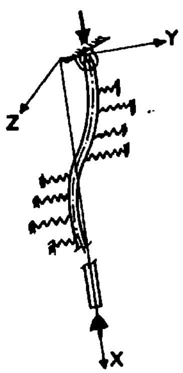

Fig. 1 Laterally Loaded Pile

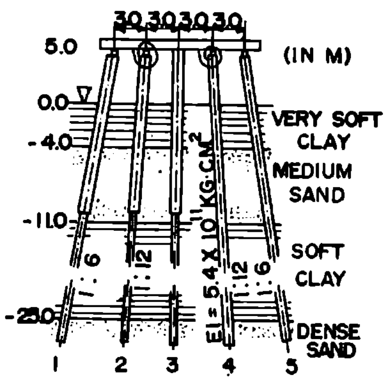

Fig. 2 Numerical Example

The algorithm developed herein consists of a successive displacement correction method or a tangent modulus method to find the equilibrium conditions at all the joints where piles are connected to the structure. A computer program was developed for the iterative solution of the problem.

Given any set of static loadings, a spring supported structure is solved for displacement by the stiffness matrix method.

$$(U) = [S + K]^{-1}(F) \qquad (1)$$

where (U) is the load vector, [S] is the structural stiffness matrix, [K] is the initial tangent modulus matrix of pile reaction calculated by giving arbitrary small displacement to pile top in each direction one at a time, and (F) is the load vector. The unbalanced joint forces are calculated for the initial displacement obtained from Eq. 1.

$$(dF) = (F) - [K](U) - (R) \qquad (2)$$

where (dF) is the vector of the unbalanced joint forces and (R) is the pile reaction vector. The unbalanced forces are induced because of the nonlinearity of the pile reaction and the superposition of different modes of pile deflection to get the resultant pile action. The displacement correction, (dU), necessary to dissolve the unbalanced joint forces is obtained through Eq. 3.

$$(dU) = [S + K]^{-1}(dF) \qquad (3)$$

The displacement is corrected by adding (dU) to (U). The process represented by Eqs. 2 and 3 is repeated until enough convergence is observed for (dU).

Assuming that the axial pile response is independent of the lateral response, the tangent modulus matrix of each pile is expressed by Eq. 4.

$$[k] = \begin{bmatrix} p_x & 0 & 0 \\ 0 & q_y & q_z \\ 0 & m_y & m_z \end{bmatrix} \qquad (4)$$

where p_x is the tangent modulus of axial reaction, and q_y, q_z, m_y, and m_z are the partial derivatives of the transverse and rotational reactions with respect to the y and z axes. These partial derivatives are calculated numerically for an arbitrary small interval.

An example of a grouped pile foundation is illustrated in Fig. 2. A beam with flexural rigidity three times as that of a pile is used for footing. Piles are connected in three different ways, pinned, fixed and rotationally restrained. Curvilinear axial pile top displacement curves are employed. A set of bilinear or curvilinear transverse subgrade reaction curves used for the computation are generated based on the soil criteria (Awoshika and Reese, 1971).

Computation results for the varying horizontal load with a constant vertical load are shown in Fig. 3.

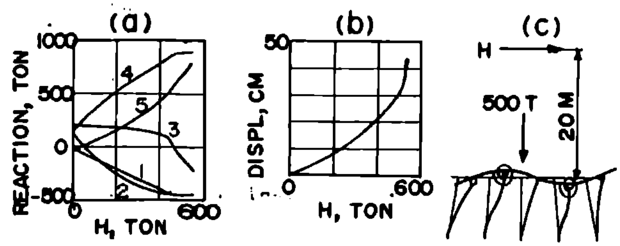

Fig. 3 Computation Results. (a) Distribution of Axial Pile Reaction, (b) Lateral Displacement of Footing, (c) Deflection.

Fast convergence was observed for the entire range of loading. Equilibrim check between the external loads and the total pile reaction was always proved exact.

The method which is presented for the analysis of elastic structures supported by battered piles is rational and efficient for use in designing. This method accounts for the nonlinearity of soil-pile interaction system in a rigorous manner. Since this method can predict the exact behavior of grouped pile foundation for the complete range of loading, it can distinguish the most critical condition of the foundation.

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THE FINITE ELEMENT SOLUTION OF CONSOLIDATION PROBLEMS USING THE LAPLACE TRANSFORM.

I.R.Booker (Australia)

Previous investigations of the finite element solution of consolidation problems Christian and Boehmer 1970, Yokoo et al (1971) have used a "marching" technique to obtain solutions. Such techniques although suited for calculations at small times are not efficient for large time calculations. The author Booker (1973) has devised a method, based on the Laplace transform, which overcomes this difficulty.

For simplicity consider a porous elastic soil occupying a volume V, tractions T act on portion of the surface S_T while the remainder S_D is fixed; portion of the surface Sp is free to drain the remainder being impermeable. If $\underline{\sigma}'$ is the vector of effective stress components (tension positive), $\underline{\varepsilon}$ is the vector of strain components, $\underline{u}$ is the displacement vector, p is the excess pore pressure and θ the volume strain, then Booker (1973) has shown that the correct solution ($\underline{u}$,p) to the problem outlined above is that which satisfies the boundary conditions on S_P, and S_D and minimises:

$$\Phi = \int_V \tfrac{1}{2}\bar{\varepsilon}^T \bar{\sigma} - \bar{p}\,\bar{\theta} + \frac{\gamma_w|\nabla\bar{p}|^2}{2sk}\,dV - \int_{S_T} (\underline{\bar{T}}\cdot\underline{\bar{u}})/s\; dS \quad (1)$$

where k is the permeability of the soil γ_w is the density of water and a bar denotes a Laplace transform, viz:

$$\bar{\underline{u}} = \int_0^\infty e^{-st}\,\underline{u}\,dt.$$

This minimisation problem may be solved by the finite element technique, let $\underline{\delta}$, $\underline{q}$ be the vectors of nodal displacements and pore pressures, then the following approximations may be made:

$$\underline{u} = C\,\underline{\delta}\;;\; p = \underline{a}^T\underline{q}$$

and $\quad \underline{\varepsilon} = B\,\underline{\delta}\;;\nabla p = E\underline{q}\;;\;\theta = \underline{d}^T\underline{\delta}.$

where $\underline{a}$,B,C,$\underline{d}$,E are all known and depend on the particular type of element used. Φ may then be approximated and the minimisation leads to the equations:

$$\begin{bmatrix} K, & -L \\ -L^T, & (k/\gamma_w s)O \end{bmatrix}\begin{bmatrix} \underline{\delta} \\ \underline{q} \end{bmatrix} = \begin{bmatrix} m/s \\ o \end{bmatrix} \qquad (2)$$

where $K = \int(B^T DB)dV$, $L^T = \int \underline{a}\,\underline{d}^T dV$, $O = \int E^T E dV$, $\underline{m} = \int_{S_T} C^T \underline{T} dS$ and D is the matrix of elastic constants.

Equation (2) may be solved by eigenvalue techniques. If the determinant of equations (2) has zeros $s = -\alpha_n$ and if

$$\underline{r}_n^T = [\Delta_n^T,\; Q_n^T]$$

are non-trivial solutions of (2), with $s = -\alpha_n$ and $\underline{m} = 0$, then it is easily shown that:

$$\bar{\underline{r}} = \underline{r}_o/s + \Sigma\, a_n\, \underline{r}_n/(s+\alpha_n) \qquad (3)$$

where $\bar{\underline{r}}^T = (\bar{\underline{\delta}}^T, \bar{\underline{q}}^T)$; $\underline{q}_o = o$ $K\underline{\delta}_o = \underline{m}$

and $\quad a_n = (\underline{Q}_n^T\, L^T\, \underline{r}_o)/(\underline{Q}_n^T\, L^T\, \underline{\Delta}_n)$

Equation (3) is easily inverted to give the expression:

$$\underline{r} = \underline{r}_o + \Sigma\, a_n\, e^{-\alpha_n t}.$$

For numerical evaluations it is sufficient to calculate only a few eigenvalues, this is illustrated in Fig.1, the marching technique together with the author's technique (3 eigenvalues, 10 finite elements) have been used to obtain accurate values of the central pore pressure for Mandel's problem of a slab of soil compressed between impervious plates by a constant load (average stress q).

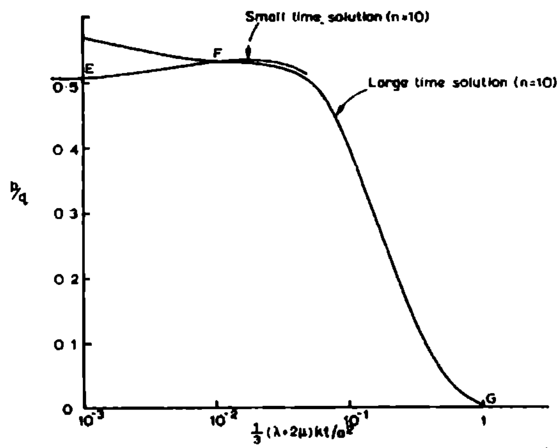

FIG. 1.

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PROBLEMS OF NON-LINEAR SOIL MECHANICS.

S.K.Chakraborty /India/.

Even though the abstract theoretical measure theory has been highly developed, it has not been suitably exploited in the domain of non-linear mechanics, and as because soils are granular unaggregated solid particles there are a number of familiar difficulties associated with the treatment of soils as engineering materials. In fact, soils are to be treated as having non-linear material properties in which are involved stress-strain history and temperature-time dependent phenomena. If in a very small interval, the number of fluctuations is very large the ordinary measure based on Riemannian integral concept fails, and measures like that of Lebesgue have to be used (Seth...1966). In like manner, generalised measures given by weighted integral representations render very satisfactory results in problems like that of plasticity and creep. An obvious generalisation of these measures is

$$\int_{lo} (lo/l)^{n+1} dl/lo = 1/n\ /1-(lo/l)^{n}/ \quad (1)$$

based on uniaxial measure (Seth...1962). The weighting function in (1) is $(lo/l)^{n+1}$. Substituting n=0,1,2...-1,-2 etc known measures can be obtained.

Again, since Norton's law deals with only secondary creep, one dimensional measures are taken as

$$\epsilon^{m}=1/n\ /-(lo/l)^{n}/=\alpha t \text{ (say)}$$

where α is the average rate of measure with respect to time. Differentiating with respect to t, creep strain rate is

$$\gamma=1/l(dl/dt)=\alpha(l/lo)^{n}$$

but the constancy of applied load and the incompressibility condition give $\tau A=\tau_o A_o$, $lA=loAo$ where τ, τ_o =effective loads; A, Ao=cross-sections of the test specimen, l, lo=deformed and undeformed lenghts. Thus, $\gamma=\alpha(\tau/\tau_o)^{n}$ .. (2)

which is Norton's law. For Andrade's law, transient creep is also taken into consideration and Green measure is used.

Thus, $\epsilon^{m}=1/n^{m}/(l/lo)^{n}-1/^{m}=\alpha t$

so that $$\gamma=1/l(dl/dt)=\alpha/m.\ \frac{(\alpha t)^{(1-m)/m}}{1+n(\alpha t)^{1/m}}$$

Substitututing m=1, n=0 (Hencky's measure) and for m=3, n=1, $\gamma=\gamma_1+\gamma_2=\alpha_1+1/3\,\alpha_2\frac{(\alpha_2 t)^{-2/3}}{1+(\alpha_2 t)^{1/3}}$ (3)

which is Andrade's law (Andrade's...1948). In like manner, any kind of creep behaviour can be visualised by suitably substituting different values of m and n.

Moreover, the derivation of the basic rate process equation is already done (Glasstone, Laidler and Eyring...1941). The basis of the relationship is that atoms and molecules participating in a deformation process (termed flow units) are constrained from movements relative to each other by virtue of their energy barriers separating adjacent equilibrium positions. The displacement of flow units to positions requires that they become activated through acquisition of sufficient energy ΔF to surmount the energy barriers. The value of ΔF is commonly known as free energy of activation. From statistical mechanics it is known that flow units are in fact not at rest but vibrate with a frequency of KT/h as a consequence of their thermal energy (Eyring...1936), where K=Boltzmann's constant. T=absolute temperature and h=Plank's constant. In fact, creep can be treated as a thermally activated process.

The author is proposing a hyperbolic strain-time dependent relationship valid for cohesive soils in the following form: (Chakraborty...1972) $\epsilon\%=t/(a+bt)$... (4) where a and b are soil constants and t is the time causing the creep strain (volumetric or uniaxial strain) under the sustained stress. The equation (4) can be obtained from the $t/\epsilon\%$ versus time plot in which b is the slope.

$$\epsilon_{max\%}=\lim_{t\to\infty}(\epsilon\%)=1/b... \quad (5)$$

The author has replotted the results obtained by Bishop and Lovenbury on drained creep tests in London clay continued upto 3 1/2 years (Bishop and Lovenbury...1969) and observed that $\epsilon_{max\%}$ can be predicted by using the equation of the form $\epsilon_{max\%}=C.1/b$ .. (6) where C varies within narrow range from 0.55 to 0.65 valid under low to high level of sustained stresses. It is possible to predict $\epsilon_{max\%}$ even by observing results of creep tests only for a period of 10 days. The equations (5)&(6) are more general than the one proposed by Singh and Mitchell, which is valid for triaxial testing conditions (Singh & Mitchell... 1968).

Based on experimental investigations the author developed a graphical method of predicting secondary consolidation as discussed below, let $Y=\log_{10}(So)^2/Sr+1$, r=0 ton where n=no. of plotted points, So=observed consolidation (primary) at the beginning of the straight line portion of the consolidation Vs.log(time) at time t, Sr+1 at r=0 at 2t and so on. The amount of secondary consolidation can be obtained corresponding to the intersection of the tangent from a plot Y Vs.t exactly under So at t(Chakraborty...1968).

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DEVELOPMENT OF GENERAL FINITE ELEMENT COMPUTER PROGRAM FOR THE ANALYSIS OF NON-LINEAR PROBLEMS IN SOIL MECHANICS. C-Y Chang and K. Nair (USA)

A single general finite element computer program which includes the following non-linear characteristics has been developed to conduct incremental analysis of plane problems in soil and rock mechanics: (i) stress dependent modulus, (ii) presence of joints and other discontinuities, (iii) inability of material to sustain tension, (iv) elasto-plastic behavior, and (v) non-linear behavior of the soil structure interface. Details of this program are given by Chang and Nair (1).

Various investigators have developed techniques for analyzing each of the above soil characteristics (2, 3,4,5,6,7,8). The development of the general program reported here can analyze a heterogeneous soil mass which has all the above mentioned characteristics. This required utilizing a consistent computational technique to include all the above methods of analysis. The stress transfer technique was utilized.

There are many design associated problems where such non-linear considerations are present. It has been found that soil behavior over a wide range of stress is non-linear, inelastic, and dependent on the magnitude of the stress; this has been most often expressed in terms of the influence of confining pressure on the stress strain behavior(9). A soil mass is generally much weaker in tension than in compression and in most design situations is not considered capable of sustaining any tension. This "no tension" behavior is manifested by numerous field observations of tension cracks near the crest of an excavation and the cracking in a composite embankment dam (10). The plastic behavior of soils under high stresses that can occur under foundation slabs has long been recognized and has been the basis of the classical bearing capacity formulae. In problems of soil-structure interaction where a soft soil lies against a stiff material the non-linear behavior of the soil-structure interface may be important in determination of stress transfer or displacement incompatibility between two materials(11). The non-linear behavior of discontinuities is known to be important in stress analysis of a soil and rock mass because of natural or induced discontinuities such as joints, bedding planes, foliations, shear zones, faults and other geologic discontinuities commonly prevalent under such conditions. The program developed can be utilized to assist in solving the design associated problems mentioned above.

The program has been verified by solving various illustrative examples and comparing the results with those obtained from closed form solutions and other numerical techniques(1). The application of the program to the solution of various practical problems is also presented in (1). It is the opinion of the authors that the program provides a powerful analytical tool for solving pertinent design boundary value problems in soil mechanics and foundation engineering.

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INFLUENCE OF NON-LINEAR SOIL PROPERTIES ON FOUNDATION STABILITY

C. Dinis da Gama (Angola, Portugal)

SUMMARY

Foundation analysis and design are dependent upon the perfect knowledge of the mechanical properties of the soil underlaying every engineering structure.
A common approach, after conducting site investigation, is to consider linear properties for the soil material, like a constant elastic modulus. It is shown in this paper that such an approach is conservative, because non-linear properties and perfectly plastic soil behavior lead to less stable conditions, as well as to larger displacements in the structures.
Using the finite element analysis, examples of a building footing and a dam foundation constructed on non-linear soil illustrate that conclusion.

INTRODUCTION

The mechanical properties of soil, represented by its stress-strain characteristic, are of great importance for the design of soil structures. Commonly, laboratory and *in situ* determination of soil properties are conducted to obtain the experimental parameters included in the equations of strength and deformation taken from the Theory of Elasticity. With the continuous progress of experimental techniques for accurate measurement of soil characteristis and the advance of computational methods in structural analysis, design in the field of Soil Mechanics is improving both on the realistic and the safety view points, because actual properties of soil can be considered for the analysis.
Although it is a well known fact that soils are not perfectly elastic materials, there is not a correct evaluation on the consequences that deviations from such elastic behavior will introduce in Soil Mechanics design.
The purpose of this study is to assess the stability conditions in structures founded over soils having different types of mechanical behavior:

a) Linear elastic soil.
b) Bilinear soil, with a 0.50 ratio between plastic and elastic moduli.
c) Bilinear soil, with a 0.20 ratio between plastic and elastic moduli.
d) Perfectly plastic soil.

Throughout the analysis, other soil properties were kept constant, namely specific gravity and the two parameters of strength (cohesion and angle on internal friction). Two concrete structures, a spread footing and a gravity dam, are considered to illustrate the analysis.

SPREAD CONCRETE FOOTING

The finite element method, with a version for non-homogeneous structures is utilized to study this problem(1),(2). The material properties had the following typical values:

PROPERTIES	CONCRETE	SOIL
Elastic modulus (kg/cm^2)	2×10^5	400
Poisson's ratio	0.25	0.20
Specific gravity (g/cm^3)	2.0	1.5

After the computation, a full description of displacements and stresses within the structure is obtained. In order to assess stability conditions, the Mohr-Coulomb criterion of soil failure was applied, using a cohesion of 0.2 kg/cm^2 and an angle of internal friction of 30°. The applied load was 50 ton, and the dead loads of both concrete footing and soil were incorporated in the state of stress calculations. The elastic limit for the non-linear soil was fixed at 1 kg/cm^2.
From the knowledge of principal stress distribution that criterion was applied in each element, providing the definition of failure zones, which size changes with the non-linear soil properties (Fig.1).

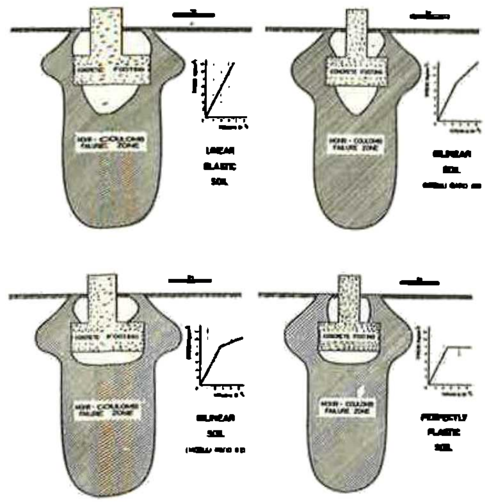

Fig.1

It can be observed that the dimensions of the rupture zones in the soil material increase as the ratio of plastic to elastic moduli decreases. Furthermore, displacements in the structure are also successively rising under such conditions.

CONCRETE GRAVITY DAM

With this type of structure, and using identical material properties as before, a study of settlements and displacements was conducted. The acting forces for the computation were the dead load of both concrete and soil, and the water pressure in the reservoir. Plane stress analysis was achieved, using a finite element mesh such as the one represented in Fig.2(3).
Results of the computation are depicted in Fig.3, considering the four types of soil behavior.

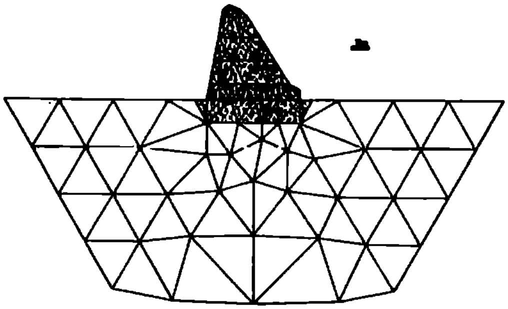

Fig.2

Noting that the scale of displacements is the same as the whole dam, it can be concluded that as the elastic behavior is substituted for bilinear and plastic properties, the stability of the structure is drasticly reduced.

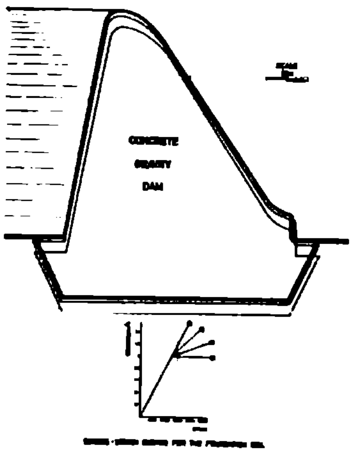

Fig.3

Because the computed settlements and displacements have quantitative meaning, engineering judgment is called to dictate what settlements are too large to be tolerated, using the results of the non-linear analysis.

CONCLUSIONS

The two examples have shown that non-linear soil properties considerably changes the situation expected in linear elastic soil behavior.
Because structures become less stable, it is recommended that correct determination of *in situ* soil parameters should always take in account non-linear properties, so that design can be properly and safety conducted, with soil-foundation interaction correctly evaluated.

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PROFONDEUR CRITIQUE D'EBOULEMENT D'UN SOUTERRAIN.

Y. d'Escatha et J. Mandel (France).

Les auteurs utilisent la méthode des lignes caractéristiques pour déterminer la pression de soutènement nécessaire à la tenue d'une cavité cylindrique peu profonde.

Le sol supposé homogène, isotrope, de poids volumique γ, contient une cavité cylindrique de rayon a, dont l'axe est à la profondeur h. Il obéit au critère de Coulomb (angle de frottement interne ϕ, cohésion $C = H \operatorname{tg}\phi$; cas particulier $\phi=0$, $C=k$: critère de Tresca). On envisage deux types de conditions à la limite sur le contour du souterrain :

1 : pression normale uniforme p_o (la galerie contient un gaz),

2 : pression d'un fluide de même densité que le sol : $p(a,\theta) = p_o + \gamma a(1-\cos\theta)$, où θ est l'angle polaire à partir de la verticale ascendante.

On cherche la relation entre la profondeur relative $Y = h/a$, la pression réduite $P_o = p_o/\gamma a$ et les caractéristiques réduites du sol : ϕ et $\bar{H} = H/\gamma a$, dans le cas de l'éboulement du terrain de la surface libre vers la cavité. On utilise à cet effet une solution "statique" qui fournit une approximation par excès de P_o pour Y, ϕ, $\bar{H}$ donnés.

A. Caquot (1934) a donné pour le cas 2 une solution statique explicite, en supposant l'équilibre limite atteint sur la verticale ascendante $\theta=0$, mais non atteint (sauf lorsque $\phi=0$) pour $\theta\neq 0$. Ici nous supposons l'équilibre limite atteint dans un domaine entourant complètement la cavité, ce qui conduit à une pression P_o inférieure (égale pour $\phi=0$) à celle calculée par Caquot.

A partir des données sur la cavité nous déterminons le réseau des lignes de glissement et calculons de proche en proche les contraintes le long de la ligne $\theta=0$. Le calcul est arrêté lorsqu'on obtient pour $\theta=0$ une contrainte verticale nulle correspondant à la surface du sol (non chargée).

Les résultats du calcul, exécuté sur ordinateur, sont donnés par les figures ci-contre pour $\phi=10°$, 20° 30°, 40°. Pour $\phi=0$, on a la relation explicite (Caquot, 1934) : $P_o = Y - 1 - 2(k/\gamma a)\log Y$ dans le cas 2, et des résultats très voisins dans le cas 1. On note les points suivants :

1°) Dans la solution explicite de Caquot, lorsque $\phi > 19°1/2$, P_o tend vers une limite finie quand Y tend vers l'infini. Ce résultat très important est confirmé par notre calcul.

2°) L'écart entre la valeur de P_o calculée par Caquot et celle que nous obtenons dans le cas 2 est important (sauf pour les faibles valeurs de ϕ). De même l'écart entre les valeurs pour le cas 1 et le cas 2.

3°) Dans le cas 1 on a constaté pour des valeurs suffisamment petites de $\bar{H}$ et Y la formation de lignes enveloppes des lignes caractéristiques. Pour ces valeurs le mode d'écoulement envisagé, dans lequel la pression $p(a,\theta)$ est supposée mineure quel que soit θ, n'est pas valable. Mais seul le voisinage du point de départ de certaines des courbes en trait plein s'en trouve affecté.

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LEGENDE DES FIGURES :

—— - ——Solution de Caquot $\bar{H}$=0.

————Pression uniforme.

- - - - - - - -Pression d'un fluide de même densité.

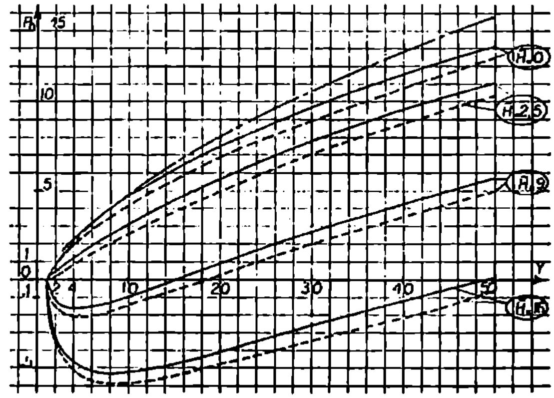


Fig. 1. — φ = 10°.

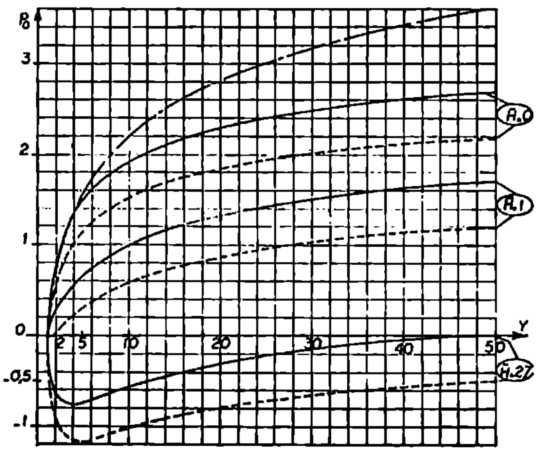


Fig. 2. — φ = 20°.

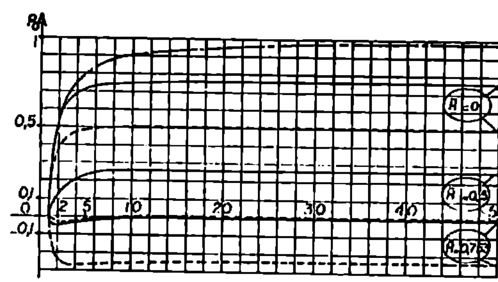


Fig. 3. — φ = 30°.

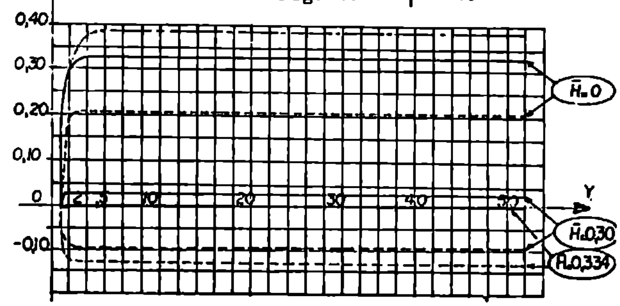


Fig. 4. — φ = 40°

INFLUENCE DU GLISSEMENT ET DU CONTENU DE GAZ SUR LE PROCESSUS DE CONSOLIDATION DES SOLS.

A.L. Goldine, L.V. Gorelik, B.M. Nuller, U.R.S.S.

C'est la nécessité de tenir compte des parametres suivants: compressibilité et solubilité de la phase gazeuse du sol, relations entre le coefficient de consolidation et les contraintes dans le squelette du sol, entre le coefficient de perméabilité et la porosité, ainsi qu'entre les perméabilités de différentes phases et l'indice de saluration d'eau, qui exige un système de deux équations différentielles non-linéaires de consolidation du sol triphasique par rapport à deux fonctions inconnues des coordonnées et du temps-charge H et volume relatif de l'air dans le sol S. Pour une couche de sol le systeme de ces équations a la forme:

$$A_1 \frac{\partial H}{\partial t} = a_1 \frac{\partial h}{\partial t} + a_2 \frac{\partial q}{\partial t} + B_1 \frac{\partial S}{\partial t} + C_1 \frac{\partial}{\partial z}\left(D_1 \frac{\partial H}{\partial z}\right), \quad (1)$$

$$A_2 \frac{\partial H}{\partial t} = N\left(a_1 \frac{\partial h}{\partial t} + a_2 \frac{\partial q}{\partial t}\right) + B_2 \frac{\partial S}{\partial t} + C_2 \frac{\partial}{\partial z}\left[D_2\left(\frac{\partial H}{\partial z} - 1\right)\right] + E\left(\frac{\partial H}{\partial z} - 1\right)^2 \quad (2)$$

(où $\frac{\partial h}{\partial t}$ - vitesse de la mise en place de la couche; $\frac{\partial q}{\partial t}$ - vitesse de l'elevation de la charge; $A_1, A_2, B_1, B_2, C_1, C_2, D_1, D_2, N, E$ - fonctions des z, t, H et S).

On a déjà donné la résolution numerique du problème de la consolidation du sol triphasique sur la base d'une seule équation différentielle de consolidation. Pour fermer cette équation on avait recours à une relation approchée entre le volume d'air relatif et la pression, obtenue à condition du systeme fermé. La solution du systeme (1)-(2) est obtenue à l'aide de la méthode des différences finies. La fig.1 donne les resultats du calcul de la consolidation d'une couche de sol triphasique progressivement mise en place. Dans ce cas $a_1 = \gamma/\gamma_\omega$; $a_2 = 0$. Les conditions aux limites du probleme sont les suivantes: à $z = 0$ $\frac{\partial H}{\partial z} = 0$ et $\frac{\partial S}{\partial z} = 0$, à $z = h$, $H = h$. A $0 \leq t \leq T_0$ la courbe de la mise en place de la couche est approximativement exprimée par l'expression $h = \theta t$, à $t > T_0$ par $h = \theta T_0$. La vitesse de l'élévation de la couche est égale à 50 m par an. Les courbes 1, 2 correspondent au moment de temps $t = 2$ ans et aux différentes valeurs initiales des coefficients de saturation d'eau S_γ. A $S_\gamma = 0.95$ les surcharges $H_s = H - h$ sont positives et voisines de la valeur $(\gamma/\gamma_\omega - 1)h$. A mesure de la diminution de S_γ dans la partie supérieure de la couche apparaissent les valeurs négativent de H_s (courbe 2, $S_\gamma = 0.80$). La diminution ultérieure de l'humidité du sol mène à ce que le domaine des valeurs négatives de H_s embrasse toute la couche (courbe 3, $S_\gamma = 0.68$).

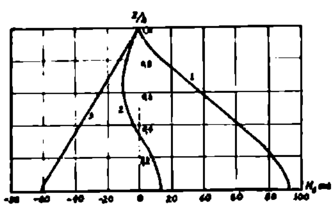


fig. 1

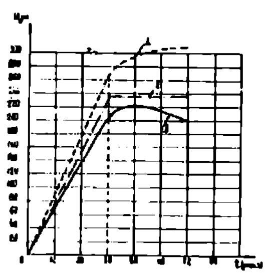


fig. 2

L'influence du glissement du squelette sur la répartition des pressions interstitielles dans la couche croissante est déterminée par l'analyse des équations du problème mixte de la théorie de consolidation. Ce problème est déjà examiné dans le cadre de la théorie de consolidation due à filtration sans tenir compte du glissement pour les caractéristiques constantes du sol. Dans le cas de la hauteur de la couche variable dans le temps l'équation de consolidation peut être écrite sous forme

$$-a_0 \gamma \frac{\partial^2 h}{\partial t^2} - \gamma \gamma_1 (a_0 + a_1) \frac{\partial h}{\partial t} + \gamma_\omega \left[a_0 + \beta'(1 + e_{mn})\right] \frac{\partial^2 H_s}{\partial t^2} + \gamma_\omega \gamma_1 \left[a_0 + a_1 + \beta'(1 + e_{mn})\right] \frac{\partial H_s}{\partial t} = k(1 + e_{mn})\left(\gamma_1 \frac{\partial^2 H_s}{\partial z^2} + \frac{\partial^3 H_s}{\partial t \partial z^2}\right) \quad (3)$$

L'équation (3) est resolue pour les conditions initiales:

$$H_s(z,0) = 0 \quad \text{et} \quad -a_0 \gamma \frac{\partial h}{\partial t} - a_1 \gamma_1 \gamma h + \gamma_\omega \left[a_0 + \beta'(1 + e_{mn})\right] \frac{\partial H_s}{\partial t} + a_1 \gamma_1 \gamma_\omega H_s = k(1 + e_{mn}) \frac{\partial^2 H_s}{\partial z^2} \quad (4)$$

et pour les conditions aux limites $H_s = 0$ à $z = h(t)$,

$$\frac{\partial H_s}{\partial z} = 0 \quad \text{à} \quad z = 0 \quad (5)$$

Dans le cas $h(t) = \alpha t + h_0$ à conditions (4) et (5) la solution approchee donne pour H_s l'expression:

$$H_s(z,t) = 2 \sum_{n=0}^{\infty} \int_0^t \exp\left[\gamma_1 (x - t)\right] \Big\{ A_n \exp\left[-\delta \frac{x}{x+m} (2n+1)^2\right] - b_n \int_0^x \exp\left[\delta\left(\frac{\tau}{\tau+m} - \frac{x}{x+m}\right)(2n+1)^2\right] d\tau \Big\} dx \cdot \cos \frac{(2n+1)\pi z}{2(\alpha t + h_0)} \quad (6)$$

où m, δ, A_n et b_n dépendent des parametres de glissement, de compactibilité et de perméabilité. Les calculs effectués ont démontré que la diminution de la pression interstitielle imputable aux déformations visqueses du squelette du sol n'a lieu qu'aux très petites vitesses du glissement. Ce sont la présence de la phase gazeuse et la valeur du coefficient de perméabilité qui exercent une influence notable sur le processus de consolidation. Dans le cas de diminution de la vitesse de mise en place d'une couche à petite vitesse de glissement la pression interstitielle dans la base de la couche devient plus grande, ce fait étant dû à la compression du sol à la suite d'une deformation visqueuse supplémentaire qui dépende du temps. Le glissement influence aussi le caractère de la variation de la pression interstitielle apres la fin de la mise en place. A petit coefficients de perméabilité la pression interstitielle continue a s'augmenter durant un certain temps apres la mise en place de la couche. La fig.2 représente les courbes de variation de la surcharge à la base d'une couche qui croisse durant 36 mois, et ensuite lorsque sa hauteur reste constante. Les paramètres de calcul sont les suivants: $a_0 = 5 \ 10^{-3} cm^2/kg$, $\beta' = 5 \ 10^{-3} cm^2/kg$, $\gamma_1 = 10^{-4}$ 1/h, $e_{mn} = 0.54$ $\gamma = 2$ t/m^3, $h_0 = 1$ m, $\alpha = 0.275$ m/jour. Pour la courbe 1: $a_1 = 2 \cdot 10^{-3} cm^2/kg$, $k = 10^{-7}$ cm/sec, pour la courbe 2: $a_1 = 2 \cdot 10^{-5} cm^2/kg$, $k = 10^{-7}$ cm/sec; pour la courbe 3: $a_1 = 2 \cdot 10^{-3} cm^2/kg$ $k = 10^{-5}$ cm/sec.

SOLUTION OF A COMBINED PROBLEM OF THE ELASTICITY AND PLASTICITY THEORIES OF SOILS.

M.I.Gorbunov-Possadov. (USSR)

The application of the theory of elasticity in calculating settlements and reaction pressures under a structure erected on an elastic base (foundation bed) proves difficult because in calculating the stresses in the soil under the edges of the foundation according to this theory, regions are found in which the following condition is not complied with:

$$\left(\frac{\sigma_1 - \sigma_2}{\sigma_1 + \sigma_2 - 2c \cot \varphi} \right)^2 \leqslant \sin^2 \varphi \qquad (1)$$

where σ_1 and σ_2 =principal normal stresses

c = cohesion

φ = angle of internal friction

Consequently, in these regions, the soil must pass over to the plastic state. This raises the necessity of solving a combined problem, assuming that both elastic and plastic regions exist simultaneously in the soil. Here the sign < is taken in equation (1) for the elastic regions and the sing = inside the plastic regions. The boundaries between the regions are determined in such a way that all the stress components remain continuous in passing through a boundary.

So far we have obtained an approximate solution of this problem for the simplest case of a strip load (Fig.1). The whole half-plane is divided up into elementary squares with sides equal to 1/10 of the halfwidth of a load strip. As a first approximation, the boundary of the plastic regions was taken as the locus of points at which relationship (1) with an equal sign is complied with in determining the stress according to the theory of elasticity. Then, a fictitious self-balanced load made up of combinations of two mutually perpendicular double forces is uniformly distributed among the areas of the elementary squares whose centres are within a plastic zone of the first approximation. The intensity of the loads in each square was determined on the basis of the requirement that equation (1) is exactly complied with at the centre of the square, neglecting the action of the loads distributed over the remaining squares. The total action of the loads violates equation (1), and some of the squares that were previously elastic become plastic. The operation is repeated until the mean error in complying with equation (1) becomes sufficiently small.

If the fictitious load diagrams along the boundaries of the elementary squares are of a stepwise nature, the condition of continuity is violated. A rigorous solution can be obtained only at the limit upon reducing the size of the elementary squares, when the curve of fictitious loads becomes smooth at zero intensity on the boundary. The combinations of two mutually perpendicular double forces are insufficient to achieve convergence of the solution. It is necessary to additionally employ double forces with moments.

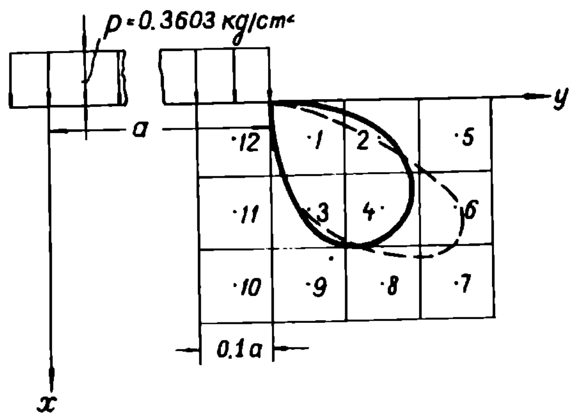
Shown in Fig.1 are the results of calculations for a strip load distributed along a strip of a width 2a=2M for a sand base with the characteristics: $\varphi=36^\circ$, c=0.01 kg/cm^2 and a unit weight of γ =1.8 tons/m^3. The intensity of the load is p=0.36 kg/cm^2. The calculations were completed at the seventh iteration. To evaluate the error of the solution the quantity $\delta_k = \frac{1}{n} \sum (T^2_{ik} - \sin^2 \varphi)$ was used. Here T^2_{ik} denotes the value of the left-hand side of equation (1) at the centre of square i for the k-th iteration; n- is the number of plastic squares considered in the given iteration. If innitially δ_1=0.0560, then after the seventh iteration it is δ_7= 0.0065. The boundary conditions at x=0 are always rigorously complied with. With the aid of programs for an electronic computer, the calculations were rapidly done.

The solution enables the additional displacements of the middle or edges of the strips, due to fictitious loads, to be determined using formulas of the theory of elasticity. In our example they were negligible. Upon an increase in load, the additional displacements increase sharply. Evidently, however, the main role in the nonlinear dependence of settlements on pressure is played by gravitation.

The solution is generalized for the case of a rigid test plate where, instead of the known Sadovsky diagram, a diagram is obtained with finite values of the stresses under the edges of the plate.

The author wishes to thank V.F.Alexandrovich and N.S.Rivkin for their aid in carrying out the calculazions.

Boundaries of the region of plastic deformation: the solid line was obtained according to the theory of elasticity; the dash line was obtained from the solution of the combined problem.



NONLINEAR RESPONSE ANALYSIS OF SOIL DEPOSITS. H. Dezfulian (Iran)

Response of soil deposits which is nonlinear during strong-motion excitations may be analyzed by employing either the lumped mass or wave propagation procedures for profiles with horizontal boundaries and the finite element method for profiles with irregular boundaries. This paper presents a general discussion and some results obtained on the nonlinear aspects of these methods.

For analysis purposes, the nonlinear stress strain characteristics of soils may be represented by bilinear or multilinear relationships. Alternatively, an equivalent linear visco-elastic procedure might be employed in which the soil moduli and damping characteristics are selected to be compatible with the strains developed in the deposit. In this procedure modal superposition is utilized for the solution of the equations of motion which require the use of a single damping value for all segments of the deposit. In the lumped mass method, the uniform damping is usually the weighted average of the damping ratios along the depth of the deposit. This can lead to significant errors which can be reduced by utilizing some form of weighted modal damping.

A linear visco-elastic model has been proposed by Dobry et al (1971) in which viscosity is chosen in such a way as to simulate the nonlinear hysteretic behavior of soils. Lysmer et al (1971) have used the wave propagation method to study response of soil deposits subjected to seismic deformations. In this study, the nonlinear characteristics of the deposits were taken into account by using an iterative procedure to ensure that the moduli and damping factors of the soils at different depths would be compatible with the strains induced in the soils at those depths.

In the finite element analysis, in order to take the strain dependency of damping in soils into consideration, Dezfulian and Seed (1970) have proposed a correction procedure to account for variation of damping in different parts of a soil deposit underlain by an inclined rock surface. The computed response values using the constant damping finite element solution and the corresponding corrected values are presented in Fig. 1.

More recently, Idriss and Seed (1973) utilizing a step-by-step integration method have described an analystical procedure that permits the use of a different damping ratio for each individual element based on the strain developed in the element. This procedure was used to evaluate the response of the soil deposit shown in Fig. 1. The results in this figure show that the values obtained by the above correction method are in good accord with the response values computed by the variable damping solution and the two solutions have been found to be in somewhat better agreement with recorded values than those obtained by a constant damping solution.

The step-by-step integration method has been employed for the analysis of nonlinear systems in which linear behavior is assumed throughout each successive time step and proper modifications to the linear properties of the system are made prior to each step. In such cases Raleigh's damping is generally utilized for the entire finite element representation and thus the same damping is assigned to all elements.

The theory of plasticity has been used to describe criteria which include the hysteretic characteristics of the stress-strain relationships of soils. Results of several such nonlinear analyses have indicated that the dynamic response of earth structures are significantly affected by the nonlinear properties of the materials distributed within the structure and that realistic response predictions of dynamic deformations can be achieved only by nonlinear analysis procedures.

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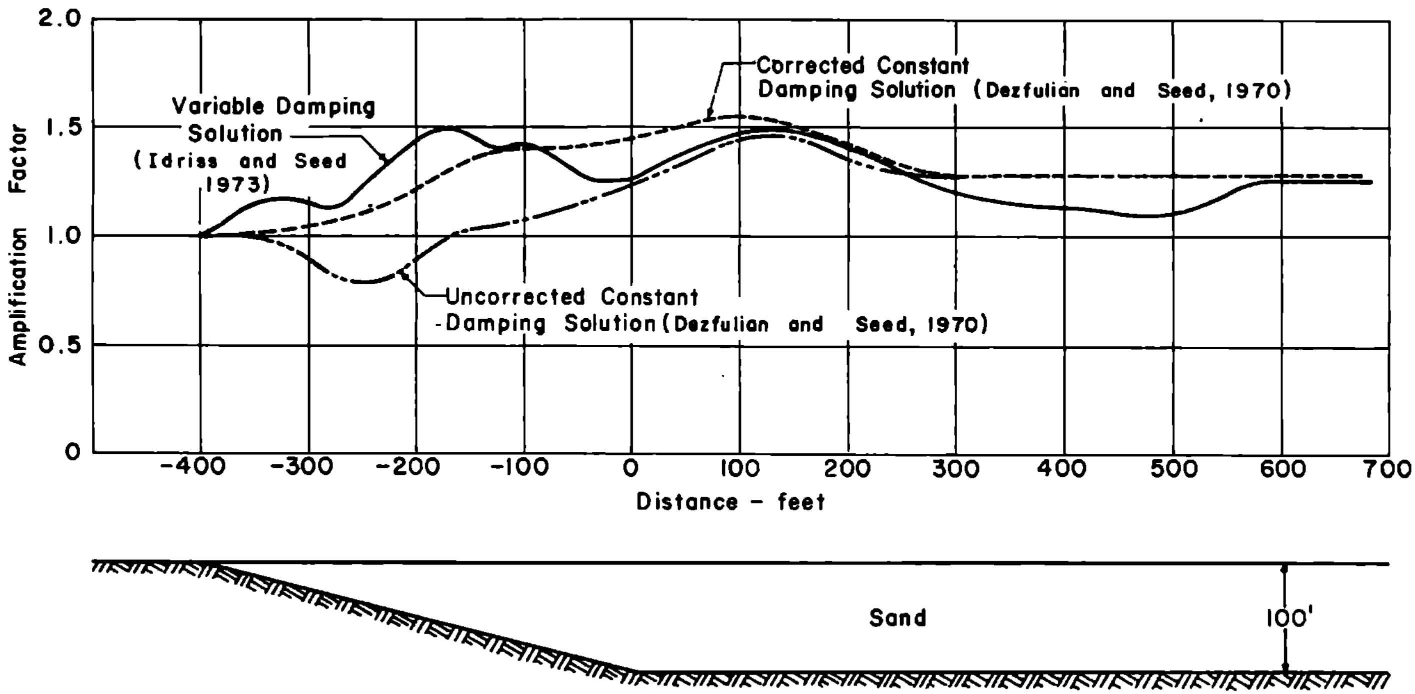

Fig. 1 RESPONSE OF SOIL DEPOSIT UNDERLAIN BY SLOPING ROCK BOUNDARY

CONSTITUTIVE RELATIONS FOR SOILS.

J. Feda (Czechoslovakia)

Owing to the recent progress of numerical methods more complicated constitutive relations (i.e. relations between the stress and strain tensors and their increments or derivations) can be used for the description of the mechanical behaviour of soils. It seems therefore useful to investigate the nature of those relations which are acceptable for soils on a larger basis than the one formed by the special case of nonlinear mechanics.

Every constitutive relation analytically describes the mechanical behaviour of an ideal material where some aspects of the actual behaviour were suppressed in order to enable the appropriate mathematization. The author holds for mistaken the considerations of the nature of soils being elastic,plastic,isotropic etc.One may only discriminate the reliability of the mechanical description of the soil behaviour inside some of its state intervals by the constitutive relations of one or another ideal material.Every extrapolation beyond the experimentally verified limits may be and often is considerably unreliable.

To derive the constitutive relations one may apply either structural or phenomenological approach.The existing evidence seems to the author to provè that the constitutive relations of soils should basically be constructed phenomenologically,with a necessary and plausible structural interpretation.

There are time-independent and time-dependent (rheological) constitutive relations,the first one being much more worked out.From the formally mathematical standpoint the actual soil behaviour may be described by almost any constitutive relation.The only difference between individual relations would be the size of the range of stress and the type of stress and state paths where the description of the actual behaviour will be satisfactory.

To achieve the maximum generality,i.e. the widest range of applicability,the purely mathematical view must be narrowed by the requirement of the physical match and interpretation.This will for soils doubtless mean that the ideal material of any acceptable constitutive relation should be dilatant and prevalently plastic.

Among time-independent finite constitutive relations at least partially suitable are those of isotropic physically and/or tensorially nonlinear materials and linear anisotropic materials.These materials are elastic (dilatancy is therefore elastic too,e.g. Kelvin effect).Since actual behaviour is plastic,only monotonous loading is admissible.

On contrary to the above relations,thermodynamically correct are incremental theories of plasticity,state equations of soils (theories of Roscoe,Rowe etc.) being therefore the most perfect time-independent constitutive relations for soils at present available.Their reliability may be further improved introducing the anisotropic strain-hardening.The suitability of those relations is reflected in the number of necessary mechanical parameters,highly reduced in comparison with the finite constitutive relations.

The most perfect of rheological relations in use in soil mechanics is at present the nonlinear visco-elastic theory of Green and Rivlin.Being nonspecific for soils its application in soil mechanics is not fully satisfactory.It seems appropriate to divide the creep deformation into immediate (time-independent) and time-dependent.The author feels to be promising to try the application of the state equation of soils where the strain-hardening parameter would be time-dependent.Such a formulation of the rheological equation of the solid skeleton could form a base for a more reliable theory of (primary, filtration) consolidation. The solution has to be certainly a numerical one only.

Selecting the appropriate constitutive relation one must further account for two important criteria: how accurate prognosis is required and what is the reliability of the mechanical parameter measurements.Aiming to the qualitative forecast only or measuring soil parameters in the field one is fully authorized to use even very simple constitutive relations,e.g. nonlinear elasticity.

5/6/4

NON-LINEAR ANALYSIS OF FOUNDATIONS. V.G.Fedorovsky, S.E.Kaganovskaya, V.A.Barvashov, (USSR)

In the recent time some problems of soil mechanics have been solved with the help of electronic computers in which rather complex experimental stress-strain relations / 1, 3 / have been used. The complexity of such relations makes it impossible to obtain analytical solutions even for the simplest problems. As a result of this fact the role of numerical solutions increases. Usually non-linear problems are solved on the base of finite element or finite difference approximation by one of the three methods: incremental, iterational or combined one. In the realm of small deformations all these methods give about the same results, but for large deformations they often become unstable. At the same time variation methods of direct minimization of energy functionals are more cumbersome but more stable. Here below a variant of a combined incremental- iterational method of solution of non-linear problems is given which posesses increased stability.

Let us consider this method as applied to the problem of a rigid plate on the non-linear base. The problem is solved for displacements and so the Euler's equations for variation problem as usually correspond to the equilibrium equations expressed in displacements considering a given relation $\varepsilon = \varepsilon(\sigma)$ (for example a hyperbolic or a spline one). Let us assume a small fixed increment ΔW for the settlement of the plate. The fact that the increment of the settlement has been assumed and not of the load applied to the plate (ΔP) in other words the inverted problem has been solved increases stability of the method. At the every step values of displacements are variated to satisfy the equilibrium equations inside of the considered region. At the same time finite difference equilibrium equations are linearized in regard to the increments of displacements in the node and corresponding increments are found. To increase stability of computation process the absolute value of the increments are limited as it is made in the method of local variations /2/. Modified values of displacements in a given node are considered in the equations for subsequent neighbouring nodes. This accelerates in the large extent the convergence of iterational process. When a necessary accuracy is achieved the process is stopped and the next increment of the plate settlement is assumed or more fine net is adopted for a chosen sub-region of the base. It appears that for a certain value of increments of the plate settlement and limitations on displacements of internal nodes the iterational process converges even for large values of plate settlements i.e. in the phase when stability is going to be lost, which fact permits to estimate rather accurately the critical load. The method may be applied to the case of finite stress-strain relations and also for differential (non-holonomic) relations.

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ON APPLICATION OF FLOW THEORIES FOR SOILS

I.N.Ivastchenko, M.N.Zakharov (USSR)

Investigations of soils under three-dimensional states of stress have shown that applications of deformation theories of plasticity for an evalution of strain-stress states of soil massifs are often invalid (Lomize, G.M., Ivastchenko,I.N.,Zakharov, M.N.,1970). In continuum mechanics there were suggested flow theories in which the relationship between plastic strain increments and stresses was established (Sedov,L.I.,1970):

$$de^{p}_{ij} = G\left[\partial f/\partial \sigma_{ij}\right] df \qquad (1)$$

Where de^{p}_{ij} the tensor of the plastic strain increment; f-the function of loading that is the equation of loading locus separating an elastic domain from a plastic one in the space of stresses; σ_{ij} -the tensor of stress; G-the strain-hardening function.

According to (I), the vector of the plastic strain increment is orthogonal to the loading locus. The loading locus is usually postulated to be closed,convex,with the origin of coordinates being within the locus. Recently,the flow theory has been applied to the investigations of soil massifs with limit analysis (for evaluting "upper bound"). In this case the loading locus was assumed to be fixed together with the aforementioned properties of one. There are few investigations which are necessary for checking the basic principles of the flow theory and for making one's equations more concrete (Drucker,D.C.,Gibson, R.E.,Henkel, D.J., 1957).

The experimental program presented in the report deals with the following problems: the plastic deformation of

1) the research of a loading locus shape and evolution when the process of the plastic deformation of clay or sand is taking place;
2) the investigation of orientation of the plastic strain increment vector de^{p}.

The conditions of the three-dimensional stress-controlled states were due to loading a hollow cylindrical soil specimen (80 mm in high, 35 mm and 60 mm in diameter) with axial and hydrostatic pressures. The experiments have been carried out in the special apparatus permitting the three principal stresses σ_1, σ_2, σ_3 to be applied with negligible deviations and the stabilized principal strains e_1, e_2, e_3 to be measured.

Remoulded kaolinitic loam and sand were tested. The loam had water content 12,3%, void ratio 0.76, plastic limits 20% and 30% and degree of saturation 0.44. The sand was medium-grained, uniform and of medium density corresponding to void ratio 0.57.

In the tests the hydrostatic pressure, up to σ =5 kg/cm^2, was applied to the specimen and only then was varied the stress intensity:

$$\sigma_i = \frac{\sqrt{2}}{2}\sqrt{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2} \qquad (2)$$

In the process of the primary loading with a fixed value of the Lode's parameter μ the given magnitude of plastic strain e_i;

$$e_i = \frac{\sqrt{2}}{3}\sqrt{(e_1-e_2)^2+(e_2-e_3)^2+(e_3-e_1)^2} \qquad (3)$$

was reached. Then the specimen was unloaded (σ_i =0, σ =const) and loaded again but with other μ. The increment of the axial plastic strain of the specimen 0.1% was considered to be the condition of reaching the loading locus.

In fig.1 one of the investigated loading

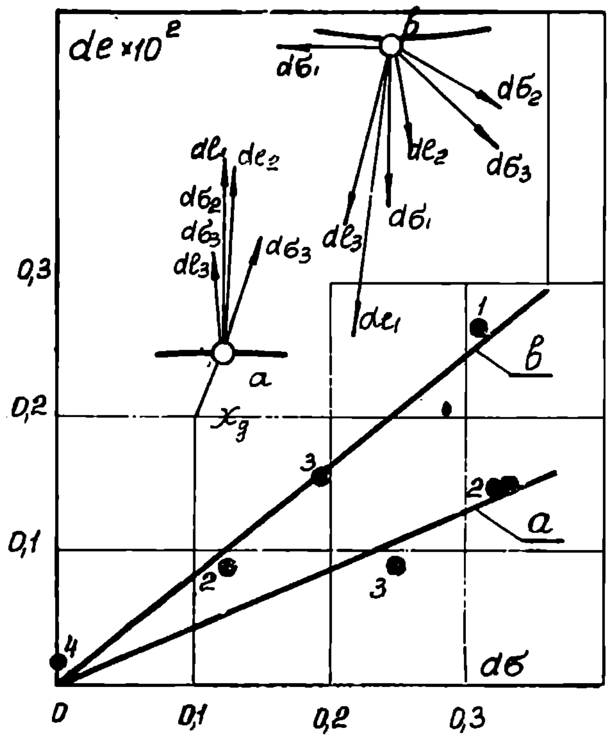

Fig.1. The loading locus (the trace of the loading surface on the plane =5kg/cm^2
1-for the loam (=8,5%);
2-for the sand (=2.1%)

locus traces (with μ =-1) on the plane σ =5 kg/cm^2 are presented for the loam (e_i =8.5%) and for the sand (e_i =2.1%).Dotted lines represent the stress paths along which values $2/3\,\sigma_i$ corresponding to the given values of the plastic strains are plotted.

The essential features of the obtained loading locus involved in convexity,smooth (regularity)and stretch in the primary loading direction.The shape of the loading locus differs significantly from the round one especially for the loam and unsymmetric to the axis $\sigma_1 = \sigma_2 = \sigma_3$ .The experiments have shown that the locus expends as the strain is increasing in all directions and the locus doesn't translocate as a whole.

The results of the investigation concerning to the orientation of the vector de^{p} are presented in fig.2 as a family of the vectors de^{p}, $d\sigma$ which proceed from the points a,b of the loading locus. The direction of the vector de^{p} coincides rather exactly with the normal one to the loading locus. The relation between modulus of the de^{p} and the normal component of the stress increment vector $d\sigma$ also shown for the same points in the fig.2 may be approximated with a linear function:

$$de^p = \mathcal{F} d\sigma \quad (4)$$

Component of the $d\sigma$ tangential to the loading locus doesn't cause the plastic strain (neutral loading). The practical application of the relationship (4) demands the concretization of the function F along the stress path.

Thus the results presented in the report have confirmed the principal hypothesis of the flow theory and the validity of the basic relationship in the form (I) for soils.

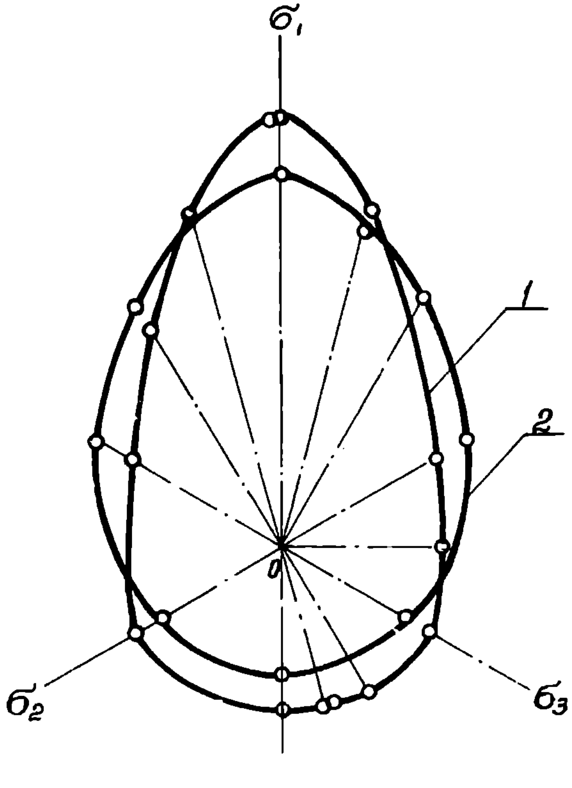

Fig 2

Fig.2 The modification of the plastic strain increment vector de^p with the loading increment

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NON-LINEAR ANALYSIS FOR RIGID POLES SUBJECTED TO LATERAL LOADS.

Dr. R.Kapur (India)

Since Winkler (1867) presented the concept of subgrade modulus, it has been used extensively in the field of soil mechanics. A linear variation of subgrade modulus with depth has been found to fit well for sandy soils. Although it is understood that the subgrade modulus is not a linear function of strain or displacement, this simplifying assumption continues to be made. Recently, exponentional and hyperbolic variation of subgrade modulus with strain or displacement has been found to fit experimental results.

The author has used a linear variation with depth and a rectangular hyperbolic relationship with respect to displacement to solve the problem of a rigid pole subjected to lateral loads.

The equation of the rectangular hyperbola is:

$$\frac{y}{p} = \frac{1}{K_h} = \frac{1}{K_1} + \frac{1}{p_f} \cdot y \quad \ldots\ldots\ldots\ldots\ldots(1)$$

where y is the horizontal displacement at any depth z.

p is the soil pressure mobilized at depth z.

K_h is Coefficient of horizontal subgrade reaction.

K_1 is the tangent modulus at small displacement.

p_f Value of pressure p at failure.

A linear variation with depth is assumed.

Therefore, equation (1) becomes:

$$\frac{1}{K_h} = \frac{1}{A_1 . z} + \frac{1}{B_1 . z} \cdot y$$

or

$$K_h = \frac{A_1 . B_1}{B_1 + A_1 . y} . z \quad \ldots\ldots\ldots\ldots\ldots(2)$$

Displacement, y, of a pole at any depth z can be represented in terms of the displacement, y_g, at ground level and the depth of point of rotation, $n.D$, from ground level in the form $y = y_g\ (1 - z/n.D)$ ....(3)

Therefore, Eq. (2) reduces to:

$$K_h = \frac{A_1 . B_1}{B_1 + A_1 . y_g - A_1 \frac{y_g}{n.D} . z} . z$$

$$= \frac{K_1}{K_2 z + K_3} . z \quad \ldots\ldots\ldots\ldots\ldots(4)$$

Where

$$\left.\begin{array}{l} K_1 = A_1 . B_1 \\ K_2 = A_1 . y_g / n.D \\ K_3 = B_1 + A_1 . y_g \end{array}\right\} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(5)$$

The horizontal soil pressure, p, at any depth, z, is given by
$p = K_h . y$

Substituting for y and K_h from eqns.(3)&(4), we get

$$p = \frac{K_1}{K_2 z + K_3} z . y_g - \frac{K_1}{K_2 z + K_3} . z . z \frac{y_g}{n.D} \quad \ldots(6)$$

The resultant of the soil pressures acting above and below the point of rotation can be obtained by integrating expression for p, as given in eqn.(6), from 0 to nD and nD to D respectively. Once the two resultants and the external lateral load is known, the conditions of statical equilibrium, viz. sum of horizontal forces equal to zero and sum of moments about the point of rotation equal to zero, can be applied. Finally two equations would result which have to be satisfied simultaneously to arrive at the required solution.

The two equations are:

$$Q_h - (C_{p1} - C_{p2}) . D^2 = 0 \quad \ldots\ldots\ldots\ldots\ldots(7)$$

and

$$Q_h . nD - (\overline{C}_{p1} + \overline{C}_{p2}) . D^3 = 0 \quad \ldots\ldots\ldots\ldots\ldots(8)$$

where

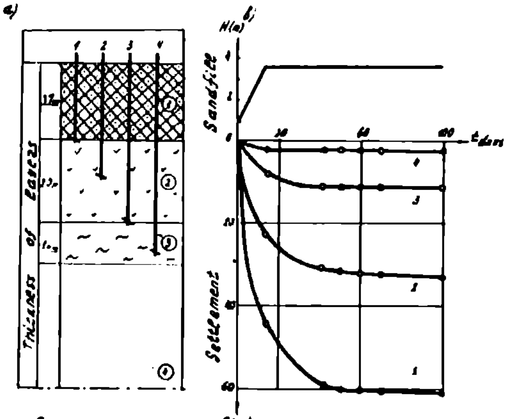
$$Cp_1 = B.A_1.B_1.n^2 \left\{ -\frac{1}{2A_1} + \frac{(B_1 + A_1 . y_g)}{A_1^2 . y_g} + \left[\log_e (B_1 + A_1 . y_g) - \log_e (B_1) \right] . \left[\frac{B_1 + A_1 . y_g)}{A_1^2 . y_g} - \frac{(B_1 + A_1 . y_g)^2}{A_1^3 . y_g^2} \right] \right\} \ldots(9)$$

$$C_p 2 = B.A_1.B_1.n^2 \left[\frac{1}{A_1} - \left(\frac{1}{n} + \frac{1}{2n^2} + \frac{1}{2} \right) + \left\{ \frac{(B_1 + A_1 . y_g)}{A_1^2 . y_g} \right\} \left\{ \frac{1}{n} - 1 + 1 \left[\frac{(B_1 + A_1 . y_g)}{A_1 . y_g} \right] \left[\log_e B_1 - \log_e (B_1 + A_1 . y_g\ 1 - \frac{1}{n}) \right] \right\} \right] \ldots(10)$$

THE EXPERIMENTAL INVESTIGATION OF A WEAK SOIL NONLINEAR CONSOLIDATION. Konovalov (USSR)

Experiments accomplished by authors on building sites were aimed to the investigation of the deformation nature and its dimensions of an unconsolidated peat soil after its loading by a layer of sand (one-dimension problem) and of a two-layered (sand and peat)soil after its loading by a round rigid plate (symmetrical to an axe problem).For the experimental needs four sites were chosen with almost complex geological conditions and before the hydrofill depth marks were installed.

For the determination of overall deformations developed at the time of loading and after it,the movements of the surface of the hydrofill,on the contact zone with the layer of peat,by the depth of the peat layer,the surveying was carried out by geodezical method. The deformations of the peat layer of one of sites is shown on fig.1.



The deformations of the sublaying (cohesive soil in plastic condition) of 1m thickness and initial modulus from 3 to 30 kg/cm^2 gave the amount of 10 to 15% of the deformation for the same period of time.The general value of the settlement of the peat and cohesive soil layers (the overall thickness is of 3,0m) under the load of the hydrofill 2.7m layer in a time of 4 months was a little more than 60 cm.

For the calculation of design deformations of the weak sublayer the Zaretski nonlinear theory was used.

$$\psi = \exp(-aq) \quad (1)$$

where

$$\psi = (\varepsilon - \varepsilon^{(k)}) / (\varepsilon^{(o)} - \varepsilon^{(k)})$$

$\varepsilon^{(o)}$ and $\varepsilon^{(k)}$ - the enitial and the end value of the porosity coefficient,
a - parametre of the compression curve,
q - unit pressure (kg/cm^2)

The overall compressibility of the peat under the load of 0.7kg/cm^2 calculated by means of the formula (2) were near to the real deformations.

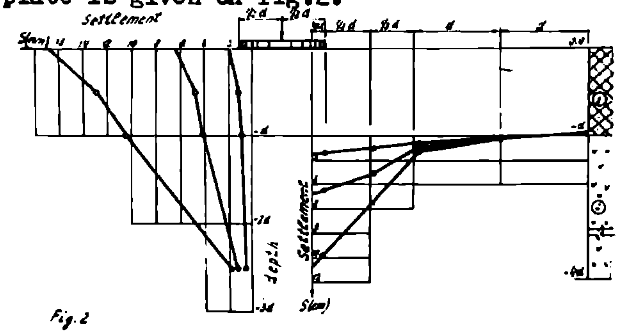
where

$$\delta_i = \frac{\varepsilon_i^{(o)} - \varepsilon_i^{(k)}}{1 + \varepsilon_i^{(o)}} a^{(n)} q ; \quad S = \sum h_i \delta_i \quad (2)$$

The investigations of vertical deformations of a two-layered subsoil system "sand and peat" under loaded rigid plate were done for plates of 1000, 5000, 10000 sq.cm area.The thickness of the sand hydrofill on peat stratum was from 1 to 3d (where "d" plate diametre). The peat stratum thickness was of 1.5 to 2.0.m.

The plate investigations were done in 6-15 months time after the sand hydrofill.So the two-layered soil was taken in account as a fully consolidated.

An example of a curve of the vertical movements of a two-layered subsoil under loaded plate is given on fig.2.



The analyse of investigations permits ut to result:

1.The settlement of the plate increases up from 3 to 2.5 times as the thickness of the upper sand layer decreases from 3 to 1 diametre of the loaded plate.The deformations at the contact zone increases up to 7 to 10 times.

2.The deformation at the contact zone in relation to the settlement of the plate is of 50% for the thickness of the sand hydrofill equal and is of 10-20% for thickness of three diametres.

3.For small thickness of the upper dense sand hydrofill (equal to 1 diametre of the plate)as the pressure increases the deformations concentrate in the weak stratum in the zone little bigger than the surface of the plate in plan.In this zone the vertical deformations of the soil are 10 times bigger then the some deformations at the distance of 1-1.5 diametre from the edge of the plate.

Fig.1. A geological cut with depth marks (a') Legend: 1-sand hydrofill, 2-peat, 3-cohesive soil in plastic condition, 4- sand.

Curves of sand hydrofill and soil layer deformation under load (b'): The Nos. of curves (on the right correspond to the Nos. of the depth marks (on the right).

Fig.2. Curves of vertical deformations under the plate and at the contact zone (sand and peat).

STRESS-STRAIN BEHAVIOUR OF COHESIVE SOILS.

B. LeLievre, E.L. Matyas, B. Wang, (Canada)

Introduction

The rapid increase in the capacity of computers has emphasized the desirability of determining better constitutive models for soils. Consequently, a considerable amount of testing has been done in the hope of producing a relatively simple constitutive relationship capable of modelling soil behaviour quite closely. Unfortunately, the behaviour of soils has so far proved too complex to achieve this aim.

This paper presents the principal observations from an extensive testing programme on saturated cohesive soil.

Limitations of space do not permit a complete review of work in this area and a significant knowledge of previously published material has necessarily been assumed.

Stress and Strain Parameters

The stress parameters used under 'triaxial' conditions are the hydrostatic stress component $p = \frac{1}{3}(\sigma_1' + 2\sigma_3')$ and the deviatoric stress component $q = (\sigma_1' - \sigma_3')$, where σ_1' and σ_3' are the principal effective axial and radial stresses respectively. The associated volumetric and distortional strain increments are respectively defined as $\delta v = (\delta\epsilon_{11} + 2\delta\epsilon_{33})$ and $\delta\epsilon = (\delta\epsilon_{11} - 1/3\delta v)$ when $\delta\epsilon_{11}$ and $\delta\epsilon_{33}$ are the natural axial and radial strains respectively. Compressive strains are considered positive.

Representation of State

According to the hypothesis regarding the existence of a unique (p, q, w) 'state' domain bounded by surfaces of limiting state which has been developed in successive works by Roscoe, Schofield and Wroth (1958), Roscoe and Poorooshasb (1963) and Roscoe and Burland (1968), the successive changes in state of a soil element which continues to yield volumetrically when loaded from an initial state on the virgin isotropic consolidation curve will trace out a path on the surface EFCD shown in Fig.(1). State paths of drained tests which remain on the surface EFCD will be consequently associated with large irrecoverable volumetric strains. The volumetric strains associated with state paths within the state domain are largely recoverable.

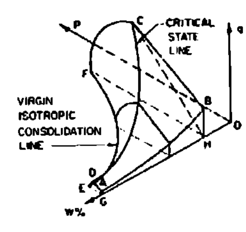


FIG 1 ISOMETRIC VIEW OF STATE DOMAIN (AFTER ROSCOE AND POOROOSHASB)

The three-dimensional representation shown in Fig. (1) may be transformed to the more convenient two-dimensional form shown in Fig.(2) as described by Roscoe and Burland (1968). In Fig.(2), p_e is the particular stress on the virgin isotropic consolidation curve corresponding to the current water content of the soil element; the curve AB represents the surface EFCD of Fig.(1) and the line BC represents the surface DCBA. The surface represented by the curve AB of Fig.(2) will be termed the Normally Consolidated State Surface.

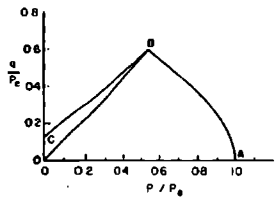


FIG 2 TWO-DIMENSIONAL REPRESENTATION OF STATE DOMAIN FOR KAOLIN

Sample Preparation and Test Procedures

Powdered kaolin (LL = 70%, PL = 38%, % clay < .002 mm = 65%, Activity = 0.5) and desired water were mixed under vacuum to give a slurry with a water content approximately twice the liquid limit. The slurry was then consolidated one-dimensionally to a water content just below the liquid limit. The detailed testing procedures have been described by Wang (1972). An important feature of the programme was that most drained tests were performed in a load-controlled manner in which load increments of about 5 psi were applied sufficiently slowly to restrict the rise in pore pressure at the centre of the sample to about 1.0 psi and then each increment was maintained at a constant value for more than 24 hours until the creep rate was very low. In this way, all measured deformations referred essentially to equilibrium states.

Strain-controlled tests were tested at a deformation rate of .00012 in/min which was found to be sufficiently slow to provide results comparable to those from load-controlled tests though some slight deviation could be noted.

Tests Results

(1) Uniqueness of the Normally Consolidated State Surface: The existence of the Normally Consolidated State Surface was demonstrated by a variety of compression and extension tests with different stress paths. The stress path for a typical test is shown in Fig.(3). The associated volumetric strains and state path are shown in Figs.(4) and (5) respectively. It can be seen that the state path remained on the state surface during the entire test from the compression zone to the extension zone.

Space does not permit other tests to be described; however, the results of all tests performed, including those in which a reversal of the stress system was involved, indicate that the shape and location of the Normally Consolidated State Surface was remarkably consistent.

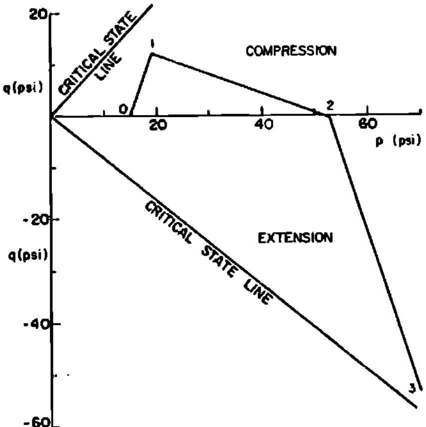


FIG.3 STRESS PATH

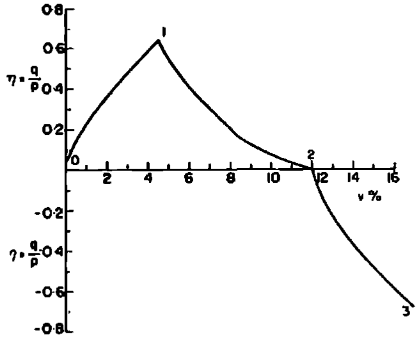


FIG.4. VOLUMETRIC STRAIN

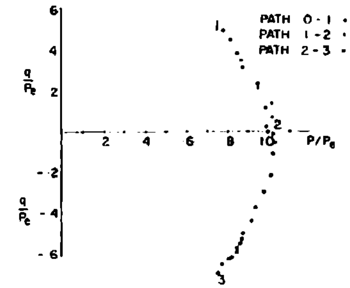


FIG 5. STATE PATH

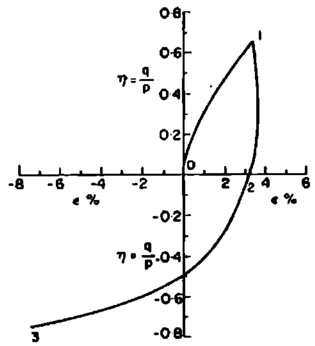


FIG.6. DISTORTIONAL STRAIN

Some inconsistency occurred, however, in the results of tests with stress paths in which stress-reversal was simultaneously combined with increasing values of the mean normal stress as discussed by LeLievre, Matyas and Wang (1972).

(ii) <u>Volumetric and Distortional Strain Yielding</u>: The test results appear to indicate that volumetric strain yielding and distortional strain yielding are governed by different criteria. As noted above, the state boundary surface appears to define the (p, q, w) condition for volumetric straining to occur for most of the stress paths studied. However, referring to Fig.(6) which shows the distortional strains associated with the stress paths given in Fig.(3), it can be seen that yielding is occurring during the stress path portions 0-1 and 2-3 but yielding does not occur over the stress path portion 1-2. Furthermore, the magnitude of the distortional strains which occur during the stress path portion 2-3 are almost identical to those which would be obtained from the sample had it been first isotropically consolidated to point 2 of Fig.(3) before being loaded in extension along the path 2-3.

To illustrate this behaviour, Fig.(7) provides the stress path for a test which was loaded along the path 0-1-2-3. The associated distortional strains are shown in Fig.(8) and the observed state path for this test is given in Fig.(9). The distortional strains for the path 0-1 (Fig.(8)) agree closely with those for the path 2-3 of Fig.(3). Also the distortional strains for path 2-3 of Fig. (7) agree closely with the distortional strains which would be obtained from tests in which the sample was first isotropically consolidated to point 2 of Fig.(7) and then loaded along the path 2-3. The distortional strains observed for these standard compression tests are also given in Fig. (8) for purposes of comparison.

The result of other tests also confirmed that volumetric and distortional strain yielding can occur independently but space limitations prevent their inclusion.

A second conclusion which has been tentatively made regarding the observed distortional strains is that the soil's memory of it's previous loading history is effectively erased as soon as the stress system is reversed from a compression state to an extension state and vice-versa. This phenomenon is illustrated diagrammatically in Fig.(10), where the actual stress paths followed by typical tests are shown on the left hand side of the figure with the final stress path portion being indicated by the continuous line. The distortional strains associated with these final stress path portions were observed to be respectively similar to tests with the stress paths shown on the right hand side of Fig.(10)

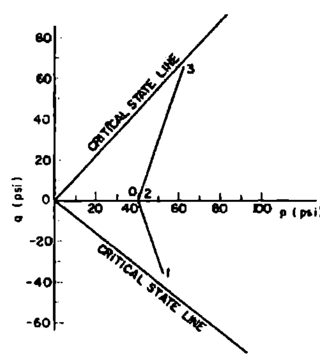


FIG 7. STRESS PATH FOR DRAINED EXTENSION - - COMPRESSION TEST

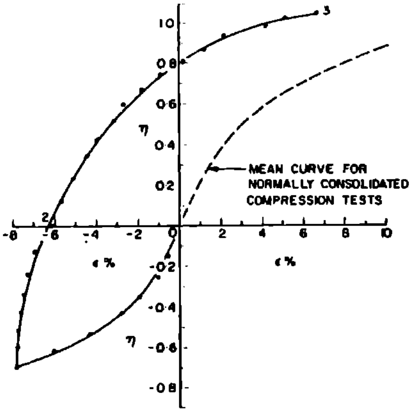


FIG 8. DISTORTIONAL STRAIN FOR DRAINED EXTENSION-COMPRESSION TEST

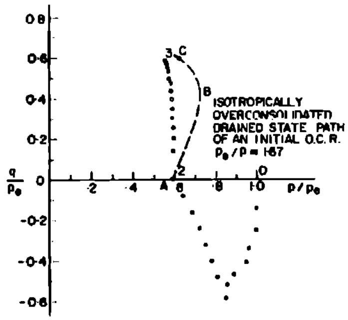


FIG.9 STATE PATH FOR DRAINED EXTENSION-COMPRESSION TEST

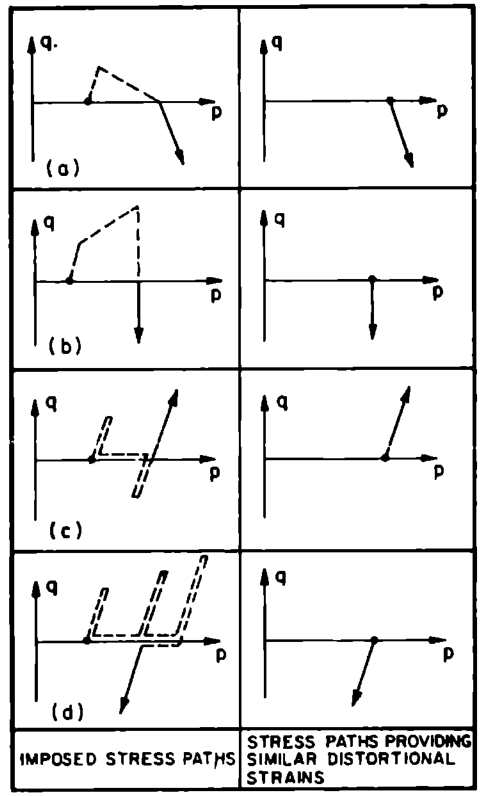


Fig. 10 DIAGRAMMATIC ILLUSTRATION OF 'MEMORY' BEHAVIOUR WITH RESPECT TO DISTORTIONAL STRAINS

Stress-Strain Models

The soil behaviour which has been briefly discussed in this paper has so far proved too complex to model in any simple form. Investigations at the University of Waterloo are in progress using finite-element analyses and a modified 'Cam-Clay' model of the soil behaviour (Roscoe and Burland (1968)) with the aim of investigating field stress paths to determine what degree of complexity of constitutive relationships is actually required for practical application.

Acknowledgements

The experimental results on which this study is based was supported by the National Research Council of Canada.

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EARTH PRESSURE ON TIED QUAYWALLS. A.A. Loizoz (Greece) J. Forati (Iran). H.B. Poorooshasb (Iran).

Development of pressure acting on smooth vertical faces of the earth retaining flexible walls of a quaywall tied at the top is discussed taking into consideration non linear soil deformation, wall deformation and their compatibility.
In figure 1 flexible wall AB of height H is shown and is assumed to be firmly embede in the underlying strate i.e. at point A both deflexions and votations are assume to be zero. At point B the wall is assume to be tied (say by non yeilding tie bars) so that at this point only deflexion is zero.

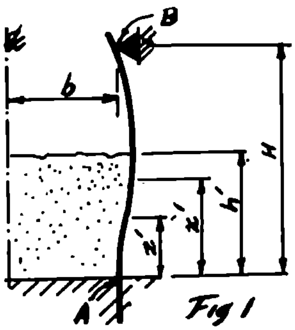

Fig 1

Three space variables are employed. Parameter h' representing the height of filling assumed to be a non decreasing function of time, variable x' representing a typical point on the wall and variable z' such that $z' \leqslant x'$. It is convenient to use normalized space parameters $h=h'/H'$, $x=x'/H$ and $z=z'/H$.
(i) Compatibility- Let w' represent deflexion of wall at point x' at the time when height of filling is h'. Also let s' represent the corresponding extension of the soil layer in the horizantal direction. Since these deformations must be compatible.

$$\frac{\partial w}{\partial h} = \frac{\partial s}{\partial h}$$

or $w(h,x) = s(h,x) + f(x)$
where again $w = w'/H$ and $s = s'/H$ have been used for convenience. Obviously $f(x)$ represents the normalized wall deflexion at the time when h was equal to x. In other words the above equation may be written as

$$w(h,x) = s(h,x) + H\int^{x} p(x,z)i(x,z)dz \qquad 1$$

where $p(x,z)$ represents the pressure distribution on the wall at the time that height of filling had reached level x. Parameter i is the deflexion influence value (also normalized) at point x of a unit horizantal knif edge load acting at point z. Its value is obtained from

$$i(x,z) = \{(1-z)\left[(1+z-z^2/2)x^3 - 3(1-z/2)zx^2\right] - (x-z)^3\}\,(H^2/6EI)$$

where EI is the flexural rigidity per unit lenght of the wall.
(ii) Soil Deformation- Assuming the soil to be rigid strain hardening plastic and following a stress-strain law as proposed by Poorooshasb et all (1966-1967), Forati (1971) has demonstrated that the relationship between the total lateral principle strain ϵ_3 and the ratio of σ_3/σ_1 is unique provided that unloading does not take place and also that no rotation of the principle stress axes are encountered. Thus according to Forati $\epsilon_3 = \epsilon_3(\sigma_3/\sigma_1)$. In this analysis this functional dependency is assumed to be linear for simplicity. Thus if the distance between the two supporting walls of the structure is assumed to be 2b the extension is the horizantal direction of the soil layer may be obtained from the relationship

$$s = s'/H = (\lambda b/H)\left[k_0 - p(h,x)/H\gamma(h-x)\right] \qquad 2$$

where λ and k_0 are soil constants, γ its density and $p(h,x)$ the horizantal pressure at point x when the height of filling is equal to h.
(iii) Wall Deformation- Assuming the supporting walls to be constructed of linearly elastic material, then deflexion at any point x may be related to pressure p by equation

$$\partial^4 w(h,x)/\partial x^4 + H^3 p(h,x)/EI = 0 \qquad 3$$

Thus the solution to the problem cited may be obtained through the solution of the set of equations stated below:

$$w(h,x) = s(h,x) + H\int^{x} p(x,z)i(x,z)dz$$
$$s(h,x) = (\lambda b/H)\left[k_0 - p(h,x)/H\gamma(h-x)\right] \qquad I$$
$$\partial^4 w/\partial x^4 + H^3 p/EI = 0$$

Solution of Set I. The method of solution adapted by the Authors uses a process in which the rigidity of wall is "relaxed" in stages. Thus starting from a rigid wall $p(x,z)$ is evaluated and the integral in in first equation of set I obtained. Now using the value thus obtained and employing a flexural rigidity value corresponding to a semi rigid wall w is obtained from the first and second equations by ellimination of variable s. The resulting expression is inserted in third equation to obtain a differential equation in terms of p the pressure distribution. The process may be repeated if need be. The first step of this analysis is described here for clarity. For a rigid wall $EI = \infty$, $w = s = 0$ and hence from the second equation $p(x,z) = k_0 H\gamma(x-z)$ which is as expected. Now substitution for this value of $p(x,z)$ and from second equation for $s(h,x)$ results in

$$w = \frac{\lambda b}{H}\left[k_0 - \frac{p}{H\gamma(h-x)}\right] + \frac{H4k_0\gamma}{EI}F_1(x)$$

where $f(x)$ is a function of x only and is obtained from the integral of the first equation. Now let a solution for p in the form $p = k_0\gamma H(h-x)\phi$ be sought. Then substituting for this value of ϕ in the expression for w and using the third equation of the set, also employing a new parameter $y = x-1$ we obtain

$$\beta\frac{d^4\phi}{dy^4} - y\phi = 7y^4 - 19.5y^2 - 15y - 2.5 \qquad 4$$

where $\beta = (\frac{b}{H}).(\frac{\lambda EI}{\gamma H4})$. The solution to equation 4 is in the form

$$\phi = \Sigma a_n y^n$$

with a recurrence formula for a_n given by

$$a_n = \frac{2a_{n-5}}{\beta(n)(n-1)(n-2)(n-3)}$$

The constants a_0, a_1, a_2 and a_3 are determined by the boundary conditions that at $y = -1$ and $y = 0$, $\phi = 1$ and $d\phi/dy = 0$.

R E F E R E N C E S :

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GENERAL ONE-DIMENSIONAL CONSOLIDATION THEORY FOR HIGHLY COMPRESSIBLE CLAYS. M. Mikasa (Japan)

A general theory for the one-dimensional consolidation of homogeneous saturated highly compressible clay, published in 1963, is summerized here as concisely as possible.

Used assumptions are the following three.
1) Soil grains and water are incompressible.
2) Pore water flow obeys Darcy's law.
3) The compressibility of soil structure is not time-dependent.

By using assumption 1) the following equations are derived from continuity condition of pore water flow.

Non-stationary (during consolidation) $$\frac{\partial \varepsilon}{\partial t} = \frac{\partial v}{\partial z} \quad (1)$$

Stationary (after consolidation) $$v = v_0(\text{const.}) \quad (2)$$

where $d\varepsilon(= -df/f)$ is the increment of compression strain ($f=1+e$; volume ratio) and v is the superficial velocity of the pore water flow.

Assumption 2) gives
$$v = k \cdot i \quad (3)$$
The seepage force of pore water upon the clay structure is
$$j = i \cdot \gamma_w \quad (4)$$
The body force acting upon the clay structure downwards is
$$\partial \bar{p}/\partial z = j + \gamma' \quad (5)$$
where is the submerged unit weight of clay, and $\bar{p}$ is the effective stress.

Assumption 3) allows us to define the compressibility m_v by
$$m_v = d\varepsilon/d\bar{p} \quad (6)$$
Combining Eqs(3), (4), (5), and (6) we obtain
$$v = c_v\left(\frac{\partial \varepsilon}{\partial z} - m_v \gamma'\right) \quad (7)$$
where c_v $(=k/m_v\gamma')$ is the coefficient of consolidation. Inserting Eq(7) in Eqs(1) and (2), and taking c_v as a function of ε, we obtain the following consolidation equations.

Non-stationary
$$\frac{\partial \varepsilon}{\partial t} = c_v\frac{\partial^2 \varepsilon}{\partial z^2} + \frac{dc_v}{d\varepsilon}\left(\frac{\partial \varepsilon}{\partial z}\right)^2 - \frac{d}{d\varepsilon}(c_v m_v \gamma') \text{—} \quad (8)$$
Stationary
$$\frac{d\varepsilon}{dz} = m_v\gamma' + \frac{v_0}{c_0} \quad (9)$$

Now in the case of compressible clay, z coordinate of each clay element changes its value during consolidation process, and Eqs (8) and (9) cannot be duly integrated for finite strain. To overcome this difficulty a new notion of original coordinate z_0(z coordinate in the original state in which the clay is supposed to have the same volume ratio $f_0(=1+e_0)$ everywhere), together with the following three quantities concerning compression strain, are introduced.

Compression ratio $$\zeta = f_0/f \quad (10)$$

Natural strain $$\varepsilon = \int_{f_0}^{f}\frac{-df}{f} = \log_e\left(\frac{f_0}{f}\right) = \log_e \zeta \quad (11)$$

Nominal strain $$= \int_{f_0}^{f}\frac{-df}{f_0} = \frac{f_0 - f}{f_0} = 1 - \frac{1}{\zeta} \quad (12)$$

Using these, Eqs(8) and (9) are transformed into the following equations valid for finite strain.

Non-stationary
$$\frac{\partial \zeta}{\partial t} = \zeta^2\left\{c_v\frac{\partial^2 \zeta}{\partial z_0^2} + \frac{dc_v}{d\zeta}\left(\frac{\partial \zeta}{\partial z_0}\right)^2 - \frac{d}{d\zeta}(c_v m_v \gamma')\frac{\partial \zeta}{\partial z_0}\right. \quad (13)$$
Stationary
$$\frac{d\zeta}{dz_0} = m_v\gamma' + \frac{v_0}{c_v} \quad (14)$$

These equations have very wide generality being free from the following six assumptions, which are necessary for the Terzaghi theory: 4) Finite strain and 5) self-weight of clay do not affect the consolidation. 6) c_v, 7) p_c(consolidation stress), 8) k, and 9) m_v remain constant during the consolidation process.

Assumptions 4), 5), or 6) may effectively be used to simplify Eqs(13) and (14), when the effects of such factors are considered small. A series of equations derived in this way were calculated for many cases and showed good agreement with the experimental results. (MIKASA,1965; MIKASA and TAKADA,1973). The effects of finite strain and self-weight of clay, for example, were both found to accelerate the consolidation speed of compressible clays considerably.

If assumptions 4), 5), and 6) are applied together, consolidation equations are reduced to such simple forms as follows:

Non-stationary $$\frac{\partial \varepsilon}{\partial t} = c_v\frac{\partial^2 \varepsilon}{\partial z^2} \quad (15)$$

Stationary $$\frac{d\varepsilon}{dz} = \frac{v_0}{c_v} \quad (16)$$

Eq(15) being the same in form as the Terzaghi equation, it would not be worthwhile to transform the former into the latter by applying three additional assumptions 7), 8), and 9) restricting unduly the applicability of the theory. Note here also that Eq(15) has a vast stock of evidences to support it already in the numerous consolidation test data on compression strain, that have long been considered to support the Terzaghi consolidation theory.

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A NOTE ON THE STRAIN CONDITION FOR SAND AT FAILURE.

P. W. Mitchell (Australia)

To understand clearly the stress-strain behaviour of sand, it is necessary to investigate its behaviour at the failure state.

Numerous investigations have found that the Mohr-Coulomb failure criterion, with parameters determined from conventional solid cylinder compression tests, provides an adequately safe estimate of the strength of sand. While there has been considerable investigation of the failure stress envelope, the strain condition for sand at failure under generalized stress conditions has not received the same attention. An investigation carried out at the University of Adelaide has examined the drained deformation behaviour of a medium dense sand under three-dimensional states of stress by means of the hollow cylinder compression test.

In classical plasticity, the plastic potential is identical with the yield surface. However, taking the Coulomb criterion as the yield surface at failure, an unacceptable dilation rate for sand is predicted, and hence the association of the plastic potential with the yield surface is not valid for this material (Poorooshasb et. al. 1967). To define the observed plastic potential at failure, recent investigators (Davis 1968, Roscoe 1970, James & Bransby 1971), have substituted a parameter ψ for the Coulomb friction angle ϕ, and have obtained strain rates at failure not associated with the Coulomb stress criterion.

For an accurate determination of the strain rates occurring in a deforming soil mass under generalized stress conditions in the field, it is of importance to examine the shape of the plastic potential surface in three-dimensional principal stress space. If the shape of the plastic potential function is of the Mohr-Coulomb hexagonal pyramid type, then the strain rates at failure are:

$$\dot{\varepsilon}_1 + \dot{\varepsilon}_3 = -(\dot{\varepsilon}_1 - \dot{\varepsilon}_3)\sin\psi \quad \ldots\ldots\ldots\ldots \quad (1)$$

Equation (1) takes no account of the strain in the direction of the intermediate principal stress. Its appropriateness in describing the strain rate behaviour of sand at failure under arbitrary stress conditions, can be established from its observed behaviour in three-dimensional laboratory shear tests.

On the basis of results from the hollow cylinder compression test, it has been found that the plastic potentials in three-dimensional stress space are bullet-shaped surfaces of revolution about the hydrostatic axis. Hence at failure, the plastic potential surface is of the extended von Mises conical shape, and the strain rates are defined by

$$\dot{\varepsilon}_{ij} = \lambda\{\alpha\delta_{ij} + (\sigma_{ij} - (J_1/3)\,\delta_{ij})/2J_2^{\frac{1}{2}}\} \quad \ldots\ldots \quad (2)$$

where α is a soil constant related to ψ, λ is a non-negative parameter related to the state of plastic strain, and J_1 and J_2 are respectively the first and second invariant of stress. Equation (2) describes the rate of deformation of a sand mass under general stress conditions at failure. Only for the case of plane strain do these strain rates, associated with a plastic potential of the extended von Mises shape, reduce to the strain rates associated with a plastic potential of the Mohr-Coulomb type (equation 1). If ε_1, ε_2 and ε_3 are the principal strains, then from (2)

$$\dot{\varepsilon}_1 + \dot{\varepsilon}_2 + \dot{\varepsilon}_3 = 3\alpha\lambda \quad \ldots\ldots\ldots\ldots\ldots \quad (3)$$

$$\frac{1}{\sqrt{6}}\left[(\dot{\varepsilon}_1 - \dot{\varepsilon}_2)^2 + (\dot{\varepsilon}_1 - \dot{\varepsilon}_3)^2 + (\dot{\varepsilon}_2 - \dot{\varepsilon}_3)^2\right]^{\frac{1}{2}} = \frac{\lambda}{2} \quad (4)$$

Equating (3) and (4) through the parameter λ, and substituting $\dot{\varepsilon}_2 = 0$ for plane strain, it can be shown, on rearranging, that

$$\frac{\dot{\varepsilon}_1 + \dot{\varepsilon}_3}{\dot{\varepsilon}_1 - \dot{\varepsilon}_3} = \frac{3\alpha}{\sqrt{1 - 3\alpha^2}} \quad \ldots\ldots\ldots\ldots\ldots \quad (5)$$

Equation (5) becomes identical with equation (1) if $-\sin\psi$ is substituted for the constant on the right hand side of equation (5).

The use of a plastic potential not associated with a failure criterion has the advantage that it can accurately model the behaviour of sand at the failure state. However, whereas the failure stress conditions can be safely defined by the Mohr-Coulomb criterion, experiments have found that the strain rates at failure are associated with a plastic potential surface of the extended von Mises conical shape. Only for the case of plane strain, does a plastic potential of the Mohr-Coulomb shape describe the strain rate behaviour of sand at failure.

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PREDICTED AND MEASURED SETTLEMENT OF ROAD EMBANKMENTS, R T Murray (England)

INTRODUCTION

In recent years there has been a considerable upsurge in the development of non-linear theories of consolidation. Although such theories are extremely valuable in leading to a greater understanding of the behaviour of soil, research being carried out by the Transport and Road Research Laboratory and elsewhere has indicated that a major source of error in calculating the rate of settlement arises from differences between the field and laboratory compressibility parameters.

The Laboratory has been carrying out a comprehensive programme of field studies of the settlement of road embankments constructed on compressible subsoils. Computer programs have been developed (Murray 1972) which take account of both the stratified nature of the subsoil profile and the non-linear behaviour of the consolidation parameters.

This Report gives a brief description of the results at two recent settlement studies and supplements earlier studies which have already been published.

GENERAL

The subsoil conditions at both sites consisted of highly compressible alluvial deposits of recent origin. In Fig 1 are shown the coefficients of consolidation (Cv) and volume compressibility (Mv) for the site at Oxford. These values were determined from laboratory tests, for the mean of the initial and final effective stresses. Also shown are the coefficients of consolidation determined from measurements of in-situ permeability used in conjunction with the laboratory values of Mv. The corresponding consolidation parameters for the site at Over are shown in Fig 2. The field coefficients of consolidation at both sites are generally much greater than the corresponding laboratory values.

MEASURED AND CALCULATED SETTLEMENTS

The settlement records for the two sites are shown in Figs 1 and 2. The results of the settlement analyses, shown in these figures, were based on a multi-layer theory of consolidation assuming one-dimensional drainage as the embankments were wide in relation to the depth of compressible strata.

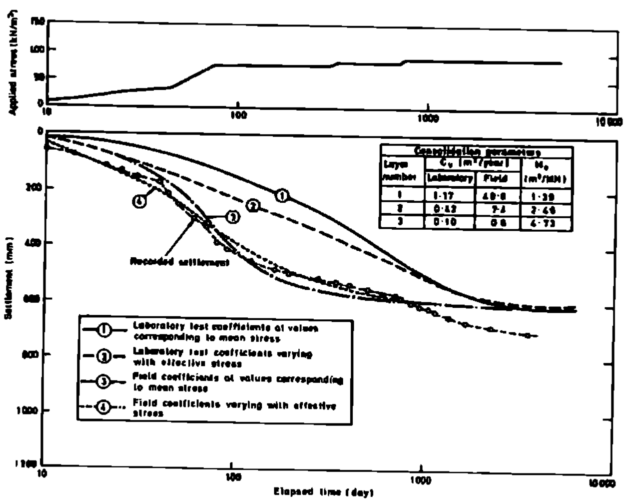

Fig 1. RELATION BETWEEN SETTLEMENT AND TIME AT THE OXFORD SOUTHERN BY-PASS EXTENSION

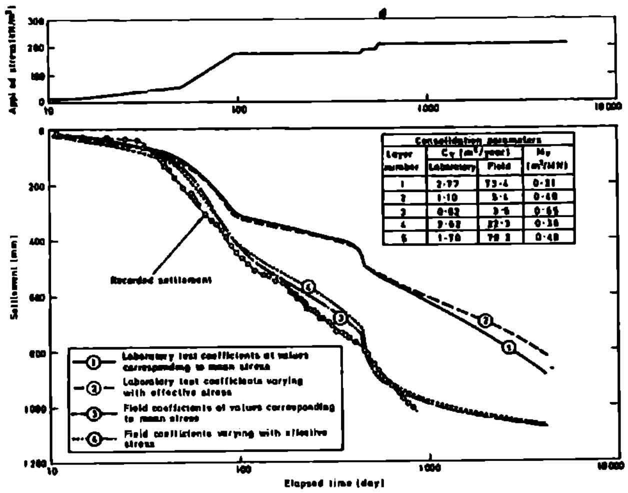

Fig 2 RELATION BETWEEN SETTLEMENT AND TIME AT THE OVER CAUSEWAY BY-PASS

The records indicate that the rates of movement were much greater than was calculated on the basis of the laboratory coefficients. However, the magnitudes of the measured and calculated ultimate primary settlements are likely to be in reasonable agreement. The relations between settlement and time based on the measurements of in-situ permeability show much closer agreement with the measured settlement. To study the influence of non-linear behaviour of the soils during consolidation, further analyses were carried out in which the consolidation parameters were varied with changes in effective stress (Figs 1 and 2).

The calculated relations between settlement and time using the non-linear method of analyses (Curves 2 and 4, Fig 1) at the Oxford site show slightly closer agreement with the measured settlement than was produced by the analyses which involved the use of constant parameters (Curves 1 and 3). However, no significant improvement was obtained at the site at Over (Fig 2), confirming that the differences between the field and laboratory coefficients had the greater influence on the settlement calculations.

CONCLUSIONS

The results of recent settlement studies at two road embankment sites indicated that non-linear analyses did not significantly improve the settlement predictions. It is much more important to employ coefficients of consolidation which are representative of the mass of the soil in-situ rather than values derived from small-scale laboratory tests. These findings are consistent with the results obtained from other similar studies carried out by the TRRL.

ACKNOWLEDGEMENTS

The settlement studies reported were undertaken in the Earthworks and Foundations Division of the Highways Department of the Transport and Road Research Laboratory. This paper is contributed by permission of the Director, Transport and Road Research Laboratory.

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AN INCREMENTAL THEORY OF THREE DIMENSIONAL CONSOLIDATION. H.Ohta and S.Hata (Japan)

A theory of three dimensional consolidation is presented based on the incremental stress-strain relations generally applicable to the clay which had been anisotropically preconsolidated with the application of both the effective mean principal stress σ'_{m0} and octahedral shear stress $\tau_{oct}=k\sigma'_{m0}$ and then has been swollen up with the reduction of τ_{oct} and σ'_m from the values at the virgin consolidation to τ_{octi} and σ'_{mi} where the suffix i is denoted to *initial* in the sense that is the stress state of clay prior to the application of three dimensional load. The void ratios corresponding to the virgin consolidated state and the *initial* state are respectively represented by e_0 and e_i. When $\sigma'_{mi}=\sigma'_{m0}$ and $\tau_{octi}=k\sigma'_{m0}$, it becomes $e_i=e_0$ and the clay is said to be in normally consolidated state.

The clay layer in the initial state is to be loaded in its local area and to deform immediately with the undrained condition. We continue to discuss assuming the change in the total stresses due to the application of load is known at any point in clay layer. Ohta and Hata(1971) showed that (a) The clay is in the elastic state when the octahedral shear stress after the application of load does not exceed the value of τ_{octp} given by

$$\tau_{octp} = \sigma'_{mi}\{k \mp \frac{\lambda - \kappa}{(1+e_0)\mu} \ln \frac{\sigma'_{mi}}{\sigma'_{m0}}\} \quad (1)$$

where the upper and lower signs correspond respectively to the cases that τ_{oct}/σ'_m after the application of load is greater and smaller than its virgin value k, λ, κ are the compression index, swelling index in the e-$ln\sigma'_m$ diagram and μ is the dilatancy index. The effective mean principal stress remains unchanged from σ'_{mi} in this case and the set up pore pressure is the difference between σ'_{mi} and σ_m that is the total mean principal stress after the application of load. (b) The clay is in elastic-plastic state when τ_{oct} exceeds τ_{octp} and effective mean principal stress must be related with τ_{oct} in the way that

$$e_i - e_0 + \lambda \ln \frac{\sigma'_m}{\sigma'_{m0}} \pm (1+e_0)\mu(\frac{\tau_{oct}}{\sigma'_m} - k) = 0 \quad (2)$$

The pore pressure set up at a point in clay layer is the difference between the total and effective mean principal stresses. In this way we can determine the pore pressure field in the clay layer which is to be dissipated by the expulsion of the pore water. It is noted that the effect of rotation of the principal stresses are not taken into account here.

The rate of dissipation of pore pressure depends primarily both on the relationship between the void ratio and the effective stress state and on the permeability of clay. Ohta and Hata(1971) gave the incremental stress-void ratio relations as follows

$$dv = \frac{\kappa}{1+e_i}\frac{d\sigma'_m}{\sigma'_m} \quad \textit{elastic state} \quad (3)$$

and

$$dv = \frac{(1+e_0)\mu}{1+e_i}[\frac{d\sigma'_m}{\sigma'_m}\{\frac{\lambda}{(1+e_0)\mu} \mp \frac{\tau_{oct}}{\sigma'_m}\} \pm \frac{d\tau_{oct}}{\sigma'_m}]$$

$$\textit{elastic-plastic state} \quad (4)$$

where dv is the increment of volumetric strain. Combining eqs(3) or (4) with Darcy's law, we get the fundamental equations of three dimensional consolidation

$$-\frac{\partial}{\partial x}(\frac{k_x}{\rho_w}\frac{\partial u}{\partial x}) - \frac{\partial}{\partial y}(\frac{k_y}{\rho_w}\frac{\partial u}{\partial y}) - \frac{\partial}{\partial z}(\frac{k_z}{\rho_w}\frac{\partial u}{\partial z})$$

$$= -\frac{\kappa}{1+e_i}\frac{1}{\sigma_m - u}\frac{\partial u}{\partial t}$$

$$\textit{elastic state} \quad (5)$$

$$-\frac{\partial}{\partial x}(\frac{k_x}{\rho_w}\frac{\partial u}{\partial x}) - \frac{\partial}{\partial y}(\frac{k_y}{\rho_w}\frac{\partial u}{\partial y}) - \frac{\partial}{\partial z}(\frac{k_z}{\rho_w}\frac{\partial u}{\partial z})$$

$$= \frac{1}{1+e_i}\frac{1}{\sigma_m - u}[-\lambda \pm (1+e_0)\mu\frac{\tau_{oct}}{\sigma_m - u}]\frac{\partial u}{\partial t}$$

$$\textit{elastic-plastic state} \quad (6)$$

where u is the pore pressure, ρ_w is the density of water and k_x, k_y, k_z are the coefficients of permeability in x, y, z directions. On the process of reducing eqs.(5) and (6), the unaltered total stress state during the consolidation is assumed although this is not valid for most of the consolidation problems, but acceptable by the practical engineering sense.

Solving eqs.(5) or (6) with the initial state of total stress distribution and initial pore pressure distribution throughout the clay layer and with the boundary condition of the pore pressure, dissipation process of the pore pressure can be clarified because in eqs. (5) and (6) we have constant τ_{oct} and σ_m. Unfortunately these equations are in too complicated forms to be solved analytically. Then we have to carry the numerical calculation in order to get the practical informations. With the initial values of the coefficients of permeability defined to each point in the clay layer, and with the knowledges on the initial states of pore pressure and total stresses, we can begin to solve eqs.(5) or (6) numerically. On the progress of the pore pressure dissipation, the values of coefficients of permeability may change in the way that should be clarified before the computation. In this way, the dissipation process of the pore pressure induced in the clay layer by the loading can be known.

In the current analysis of consolidation process, the settlement is estimated from the void ratio change of clay induced by the expulsion of pore water. But it is the physical reality that the reduction of pore pressure causes not only the void ratio change but also the further distortional deformation of clay in elastic-plastic state. Ohta and Hata(1973) give the time derivatives of strain as follows

$$\frac{\partial \varepsilon_{ij}}{\partial t} = \frac{1}{3}\frac{\kappa}{1+e_i}\frac{1}{\sigma'_m}\frac{\partial \sigma'_m}{\partial t}\delta_{ij} \quad \textit{elastic state} \quad (7)$$

$$\frac{\partial \varepsilon_{ij}}{\partial t} = \frac{\mu}{3\sigma'_m}\frac{1+e_0}{1+e_i}[\{\frac{\lambda}{(1+e_0)\mu} \mp \frac{\tau_{oct}}{\sigma'_m}\}\delta_{ij}$$

$$\pm \frac{1}{\tau_{oct}}(\sigma'_{ij} - \sigma'_m\delta_{ij})]\frac{\partial \sigma'_m}{\partial t}$$

$$\textit{elastic-plastic state} \quad (8)$$

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THE USE OF ISOTACHES IN THE CONSOLIDATION ANALYSIS. L. Šuklje (Yugoslavia)

As early as in 1957 the suggestion had been given (Šuklje 1957) to represent oedometer test data by isotache sets

$$e = \left[e\ (\sigma') \right]_{\dot{e}=\text{const}}$$

Since the consolidation of natural layers, whose thickness normally exceeds 1 m, rarely occurs at speeds $\dot{e}$ of the void ratio change larger than 10^{-6} sec^{-1}, the isotaches can easily be obtained from secondary branches of consolidation lines. The suitability of a certain rheological scheme to the short-term primary consolidation of laboratory specimes is of minor importance and cannot be considered decisive in analysing the consolidation of thicker layers.

Rheological relationships of Kelvin's models with either linear or non-linear elasticity and/or viscosity (see Mrs. Battelino's paper in the Main Session 1), and equation

$$e = e_o - A\ (\sigma' - \sigma_o) + (-\dot{e})^a\ B\ (\sigma' - \sigma_o)^b$$

suggested by Poorooshasb (1969) and Sivapatham are examples of analytical expressions for isotache sets. In our experience, the following function has often been proved to suit the experimental data:

$$e = \sum_{i=0}^{n} \left[a_i\ (\sigma'/\sigma_o)^i \right] - \left[\ln(\dot{e}/\dot{e}_o) \right]^{1/c} .$$

$$\sum_{i=0}^{n} \left[b_i (\sigma'/\sigma_o)^i \right] \quad \text{(Šuklje 1969-b)}$$

The above equation can be applied for several sections of the entire isotache set by considering continuity conditions in the linking points between the sections.

If the isotaches as well as consolidation curves are straight lines in the respective (e, $\log \sigma'/\sigma_o$) and (e, $\log t/t_o$) coordinate systems, the analytical expression for the isotache set is:

$$\dot{e}/\dot{e}_o = \exp\left\{ \left[A + B \ln\ (\sigma'/\sigma_o) - e \right] : \left[C + D \ln\ (\sigma'/\sigma_o) \right] \right\} \quad \text{(Šuklje and Simončič 1972)}$$

If the isotaches and the consolidation lines are straight in the respective

($\log e/e_o$, $\log \sigma'/\sigma_o$) and ($\log e/e_o$, $\log t/t_o$) systems, the equation of isotaches is:

$$-\ \dot{e} = c\ A\ e^{(c+1)/c}\ \sigma'^{1/d} \quad \text{(Šuklje 1972)}$$

In Author's opinion, this equation applies also to "time lines" when they are straight, parallel and equidistant in the ($\log e/e_o$, $\log \sigma'/\sigma_o$) system. Taylor's assumption that the time lines of thicker layers coincide with those of thinner samples (Taylor 1942) requires independence of rheological relationships on the effective stress increase $\partial\sigma'/\partial t$.

In some consolidation studies, the appearance of the term $\partial\sigma'/\partial t$ in the rheological equation of the soil is due to the Hookean spring when connected in series with other elements, e.g. with a linear or non-linear Kelvin element. The unsuitability of such a connection has been discussed elsewhere (Šuklje 1969-a).

For radially symmetric space consolidation analysis, experimental data of triaxial testing can be used in a similar way (Šuklje 1963-a, Šuklje and Simončič 1972). Due to dilatation effects, different isotache sets result for different deviatoric stress components. The Author has suggested (Šuklje 1969-b) to express these effects by isotache sets corresponding to stress states whose Mohr's circles have straight envelopes

$$\tau_m = c/F + (\tan\varphi'/F), \quad F > 1 \quad :$$

$$e = \left\{ \left[e(\sigma^{o\prime}) \right]_{\dot{e}=\text{const}} \right\}_{F=\text{const}} .$$

If we assume that, at any time t, the total stresses are known and, if the boundary conditions are given, appropriate analytical expressions for rheological relationships and for the dependence of the permeability on void ratio $k = k(e)$ can be inserted into the Terzaghi-Biot differential equation of consolidation and this equation solved by a numerical method. The numerical procedure can be considerably simplified if, in the observed stress domain, the function $k(e)$ is replaced by an average value $k = \text{const}$, or if the terms containing the differential quotients of k with respect to the position coordinates are neglected, but in the remaining terms the variation $k = k(e)$ taken into account. If the variation of the coefficients k in the considered stress interval is not too large, such approximate procedures seem to be justified.

Graphical and numerical solutions based on isotache sets (Šuklje 1957, 1964-a and 1966, Šuklje and Kogovšek 1968, Šuklje 1969-a and b, Šuklje and Simončič 1972, Šuklje and Kozak 1972) have proved that initial porosity, loading speed and interval, length of seepage paths, permeability and saturation exhibit important influence onto the pore-pressure and settlement development. Our future efforts have to be directed towards finding a more appropriate approach to the consolidation analysis in statically indeterminate stress conditions.

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ONE-DIMENSIONAL PROBLEM IN THE CONSOLIDATION OF MULTIPHASE SOILS TAKING VARIABLE LOAD AND PRESSURE HEAD ON THE BOUNDARY INTO ACCOUNT.

Z.G.Ter-Martirosyan, (USSR)

Conditions are frequently encountered in engineering practice when it becomes necessary to predict processes of consolidation, taking into account the rate of application of the compacting loads, as well as the effect of time-dependent boundary pressure heads. Such cases may include the behaviour of the soils in the base (foundation bed) of hydrotechnical structures and large reservoirs, the comp etion of the clay core in dams, the consolidation of a buried clayey inclined bed overlying a pressure head level and subsidence of the earth's surface due to the pumping of underground water.

To be dealt with is the one-dimensional problem of the consolidation of a layer of soil of thickness h, lying on a fissured filtrating rock base, under the action of a time-dependent compacting load and at time-dependent boundary conditions U(o,t), U(h,t). It should be noted that the water pressure U(h,t) will give rise to additional stress in the skeleton of a multiphase soil, and should be dealt with simultaneously as a boundary pressure and as a supplementary compacting load. Then the equilibrium equation will be of the form

$$q(t)+U(h,t)=U(z,t)+\sigma(z,t) \qquad (1)$$

For the sake of generality we shall assume that the soil medium is an elasto-creeping porous medium, filled with a compressible liquid (gas-containing water). Here the differential equation of the one-dimensional problem of consolidation, taking into account the compressibility of the pore water, can be written in the form

$$\frac{\partial e}{\partial t}+a_w\bar{e}\frac{\partial u}{\partial t}=\frac{1+\bar{e}}{\gamma_w}K\frac{\partial^2 u}{\partial z^2} \qquad (2)$$

where U = pore pressure
γ_w = unit weight of pore water
K = coefficient of permeability
a_W = coefficient of volume change of the pore water, determined by the equation (proposed by the author in 1965):

$$a_w=\frac{1-J_w}{P_a}$$

where I_w = degree of saturation
P_a = atmospheric pressure.

The equation of state of the soil skeleton can be represented in the form of the hereditary creep:

$$e(\tau_1)-e(t)=\sigma(t)a_m-\int_{\tau_1}^{t}\sigma(\tau)\frac{\partial}{\partial\tau}a(t,\tau)d\tau$$

where $e(t)$ and $e(\tau_1)$ = time-dependent and initial void ratios
$\sigma(t)$ = time-dependent stress in the soil skeleton
a_m = coefficient of instantaneous volume change
$a(t,\tau)$ = coefficient of total volume change, determined by an equation of the type

$$a(t,\tau)=a_m+a_e\{1-\exp[-\eta(t-\tau)]\} \qquad (4)$$

where A_e = coefficient of long-term volume change
η = creep parameter of the soil skeleton.

Combining equations (2) and (3), and taking into account equation (4) and the equilibrium equation (1), the following is obtained:

$$\frac{\partial^2 u}{\partial t^2}+a\frac{\partial u}{\partial t}-f(t)=C_v\left[\frac{\partial^3 u}{\partial z^2\partial t}+\eta\frac{\partial^2 u}{\partial z^2}\right] \qquad (5)$$

where $f(t)=A[\ddot{q}(t)+\ddot{U}(h,t)]+B[\dot{q}(t)+\dot{U}(h,t)]$;

$$a=\frac{\eta(a_m+a_e+a_w\bar{e})}{a_m+a_w\bar{e}};\quad C_v=\frac{(1+\bar{e})K}{\gamma_w(a_m+a_w\bar{e})};$$

$$A=\frac{a_w}{a_m+a_w\bar{e}};\quad B=\frac{\eta(a_m+a_e)}{a_m+a_w\bar{e}};$$

According to the initial proposition, the boundary and initial conditions can be written in the form $U(0,t)=\chi_1(t)$; $U(h,t)=\chi_2(t)$

$$U(z,0)=\varphi(z,0);\ \dot{U}(z,0)=\psi(z,0); \qquad (6)$$

Moreover, the second initial condition should satisfy an equation of the form

$$\frac{\partial U(\tau_1)}{\partial t}+B'U(\tau_1)-f(\tau_1)=C\frac{\partial^2 U(\tau_1)}{\partial z^2} \qquad (7)$$

The solution obtained for equation (5) is of the form

$$U(t,z)=U(0,t)+\frac{z}{h}[U(h,t)-U(0,t)]+\sum_{n=1}^{n=\infty}V_n(t)\sin\frac{\pi n z}{h} \qquad (8)$$

where

$$V_n(t)=\frac{[\dot{V}_n(0)+\lambda_2V_n(0)]e^{-\lambda_1 t}}{\lambda_2-\lambda_1}+\frac{[\dot{V}_n(0)+\lambda_1\cdot V_n(0)]\cdot e^{-\lambda_2 t}}{\lambda_1-\lambda_2}+$$

$$+\int_0^t\left\{\frac{\exp[-\lambda_1(t-\tau)]}{\lambda_2-\lambda_1}+\frac{\exp[-\lambda_2(t-\tau)]}{\lambda_1-\lambda_2}\right\}[f_n(t)-F_n(t)]d\tau;$$

$$\lambda_{1,2}=\frac{1}{2}\left\{-\left[a+C_v\left(\frac{\pi n}{h}\right)^2\right]\pm\sqrt{\left[a+C_v\left(\frac{\pi n}{h}\right)^2\right]^2+4C\eta\left(\frac{\pi n}{h}\right)^2}\right\}$$

$$f_n(t)=\frac{2}{h}\int_0^h f(\xi,t)\sin\frac{\pi n\xi}{h}d\xi;\ F_n(t)=\frac{2}{h}\int_0^h F(\xi,t)\sin\frac{\pi n\xi}{h}d\xi;$$

$$F(t)=\ddot{U}(0,t)+a\cdot\dot{U}(0,t)+\frac{z}{h}[\ddot{U}(h,t)+$$

$$+a\cdot\dot{U}(h,t)-\ddot{U}(0,t)-a\cdot\dot{U}(0,t)];$$

Thus, the set problem is completely solved for the general case. Specific cases, corresponding to various loading conditions and boundary conditions, can readily be obtained from the general case.

ESSAIS DE BUTEE ET DE POUSSEE EN VRAIE GRANDEUR. Yuan Tcheng (France)

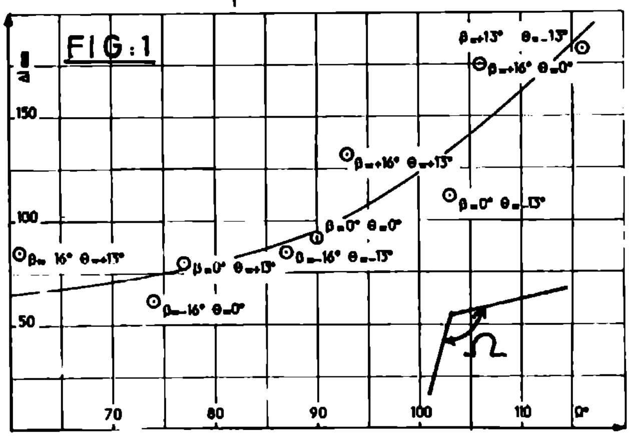
Neuf essais de butée et un essai de poussée ont été effectués à l'aide d'un mur de 3 m de haut, 5 m de large , poussé par six vérins horizontaux capables de 2000 t et deux autres verticaux de 500 t , dans un mouvement de translation horizontale régulé à 1/10 mm près .

Les contraintes ont été mesurées dans la partie centrale du mur (pour éviter les perturbations des extrèmités) et sur toute la hauteur d'une bande axiale de 20 cm de large divisée en six cellules superposées .

Le sable d'essai est un sable siliceux dont les grains ont un diamètre compris entre 0,1 et 0,2 mm ; ce sable est compacté à la densité sèche de 1,65 , pour laquelle $\varphi = 40°$. A sa teneur en eau naturelle il possède une cohésion de 3 à 5 Kp_a

A - Déplacement de rupture

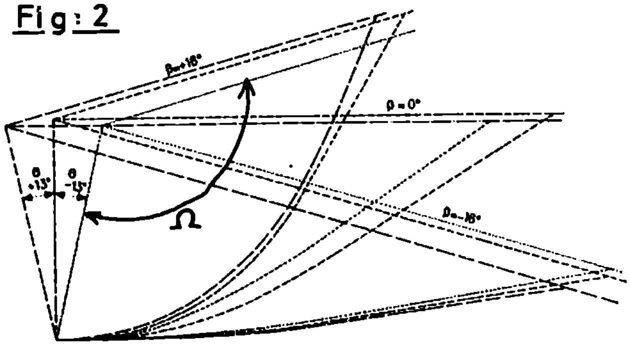
Pour chaque essai $\Delta 1_r$ augmente avec la profondeur et d'autre part avec Ω . Le déplacement de rupture correspondant à la hauteur totale de l'écran , beaucoup plus important que prévu, croît avec Ω , angle total du massif (fig. 1) . Il varie de 6 à 18 cm quand Ω croît de 61° à 116° . Sa variation est liée principalement à β .



B - Ligne de rupture

Quels que soient β et θ la longueur développée de la ligne de rupture reste du même ordre de grandeur (fig. 2)

Fig : 2



C - Variation de l'obliquité

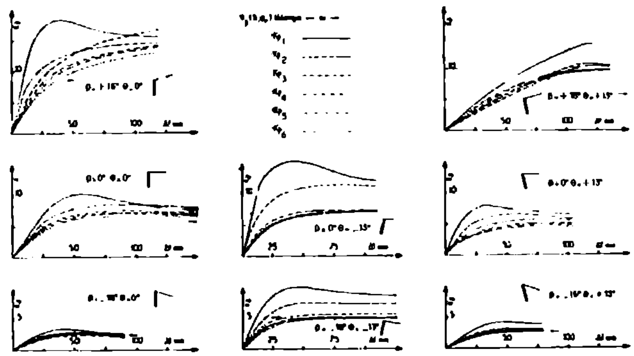
L'obliquité croît avec le déplacement de l'écran mais n'atteint jamais φ quel que soit l'état de surface de l'écran .

D - Coefficient de butée

La figure 3 donne les courbes de K_p en fonction du déplacement de l'écran .

Kp_1 et Kp_2 présentent un maximum , puis décroissent alors que Kp_3..., Kp_6 tendent vers une valeur asymptotique , Kp_n étant le coefficient K_p des cellules 1 à n comptées à partir de la surface . Pour un essai , cette valeur asymptotique est du même ordre quel que soit n . Elle est intermédiaire entre le coefficient théorique relatif à φ résiduel = 31,50° et celui correspondant à $\varphi_{max} = 40°$.

Fig : 3



E - Influence de la cohésion

Elle n'a pas pu être mise en évidence quel que soit le mode d'interprétation adopté . Il semble donc qu'une cohésion de l'ordre de 40 kp_a soit totalement négligeable pour un écran de 3 mètres de haut .

F - Contraintes dans le massif

Quinze capteurs sont incorporés dans le massif et ont fourni des résultats très intéressants.

a) Ces capteurs accusent un maximum bien avant la rupture générale du sol .

b) le long de la surface de rupture les contraintes sont les plus grandes .

En conclusion ces capteurs sont capables d'une part de signaler à l'avance l'approche de la rupture et d'autre part de localiser la surface de rupture .

G - Mouvement du massif en surface

La conclusion est encore plus nette que précédemment . La rupture peut être décelée bien avant sont apparition grâce à l'observation du mouvement superficiel .

H - Poussée au repos

Elle dépend essentiellement du compactage. La contrainte verticale peut être inférieure ou supérieure au poids des terres en raison de la formation des voûtes lors de la mise en place.

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NON-LINEAR ONE-DIMENSIONAL CONSOLIDATION OF THICK CLAY LAYERS. C. Viggiani, Italy.

The classical one-dimensional Terzaghi consolidation theory assumes both the permeability coefficient k and the volume decrease coefficient m_v to be constant. Davis and Raymond (1965) developed a theory which allows for variations of k and m_v, but is only applicable to oedometer test and thin layers in the field, since it does not allow for total stress variations with depth. A similar approach had been independently pursued by Mikasa (1965).
Janbu (1965), Raymond (1969), and Davis (1971) took into account the influense of depth in particular cases.
The purpose of this contribution is to presente some solutions of the Davis and Raymond theory, including depth effects.
Following Davis (1971) the differential equation of one-dimensional consolidation may be written:

$$\frac{\partial u}{\partial T} = \frac{\partial^2 u}{\partial z^2} - \frac{\partial \sigma'}{\partial z}\frac{\partial u}{\partial z}\frac{1}{\sigma'} \qquad (1)$$

where u is the excess pore pressure; zH is the depth below the top of the consolidating layer of total depth H; $T=c_v t/H^2$ for one way drainage and $T=4c_v\, t/H^2$ for two ways drainage; σ' is the vertical effective pressure. Eq. (1) is derived under the assumption that the consolidation coefficient c_v is constant. Assuming the well known logarithmic relation:

$$e = e_o - C_c \lg \sigma'/\sigma'_o \qquad (2)$$

and taking 1+e=const. (small strain theory), it follows that m_v=const./σ' and $k=c_v m_v \gamma_w$=const./σ'.
Truly one-dimensional consolidation may actually occur in the field due to:(i) uniform surcharge over a large area at the soil surface; (ii)lowering of the water table.
In the first case the initial excess pore pressure is constant throughout the layer depth and the clay layer may be draining at one or both boundaries. In the second case the initial excess pore pressure has a nearly triangular distribution with the vertex at the top or bottom surface; the layer must be draining at both boundaries.
The four possible cases are represented in

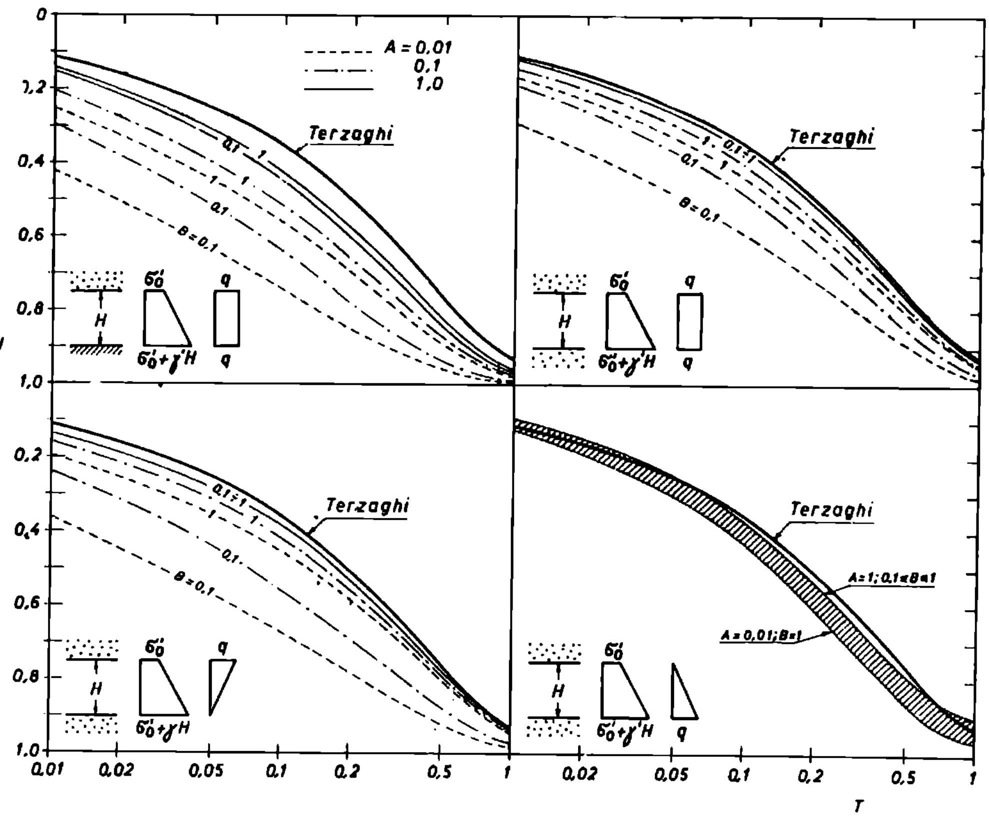

Fig. 1. Degree of consolidation vs. time factor. $A=\sigma'_0/\gamma' H$; $B=q/\gamma' H$; $T=c_v t/H^2$ for one way drainage; $T=c_v 4t/H^2$ for two ways drainage.

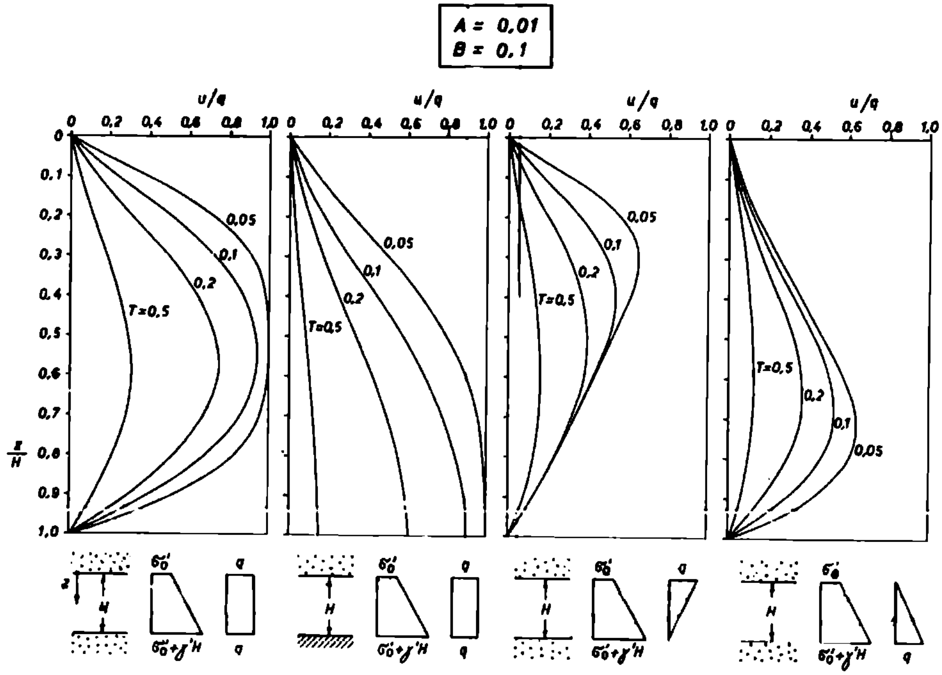

Fig. 2. Excess pore pressure isocrones for typical values of A and B.

fig. 1; they have been solved by numerical methods in the following range of parame= ters:

$A=\sigma_0'/\gamma'H= .01-1$; $B=q/\gamma'H= .1-1$

Some results are shown in fig. 1 as dia= grams of the degree of consolidation U versus T; for sake of comparison the cor= responding curve obtained by Terzaghi the= ory is shown. In fig. 2 some excess pore pressure isocrones are reported.
It may be seen that in some instances non-linear effects are likely to be of great practical importance. Diagrams pro= vided herein may guide in estimating these effects when interpreting field records of settlement and pore pressure caused by the consolidation of deep beds of soft normally consolidated clay produced, for instance, by sand reclamation filling or ground water lowering.

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COMPUTER MAKES CALCULATIONS FOR ELASTIC ANISOTROPIC BASES AND STRUCTURES OVERLYING.

Ye.F.Vinokurov, V.A.Kuzmitsky, L.G.Shulika

At present strain calculations of natural bases of buildings and structures are made on the basis of the linear strained isotropic body theory. Very often, however, anisotropic bases laminated and heterogeneous in plan and depth may be met in construction practice.

Soil anisotropy, lamination and heterogeneity change the general picture of the stress-strain state of the base but the existing calculation methods do not take into account these changes. The iteration method for solving anisotropic linear problems of soil mechanics worked out by Ye.F.Vinokurov /I/ made it possible to develop a number of working programms for the "Minsk-22" computer. The programmes may be used in solving certain plane, three-dimensional and axially symmetric problems to define transposition components of different points of homogeneous anisotropic laminated and heterogeneous bases. These may, likewise, be used when calculating reaction pressure epure configurations and foundation settlements taking into consideration (or without it) the influence of the adjacent foundations /2/. The foundations may be absolutely rigid, flexible and of ultimate rigidity.

Analysing the results of calculations one might draw a conclusion that vertical transpositions of single base points are essentially dependent on the ratio of modulus numerical values in horisontal /E_x/ and vertical /E_z/ directions. Transpositions obtained with the ratio of indicated modules equal to zero and corresponding to quasi-isotropic medium differ several times from the same transpositions determined with other module ratios (Fig.1, I). The change diagram of foundation settlements with different module ratios and equal Poisson's ratio values (Fig.1 III) as well as the dependence of the foundation settlement with different Poisson's ratios and constant value E_x/E_z=1/2 (Fig.1 IV) prove that one should not use the quasi-isotropic body theory calculating foundation settlements.

While determining foundation settlements it is also necessary to take into account boundary conditions where the foundation is in contact with the anisotropic base. If in the point of contact the tangential stresses are equal to zero /the case of slip/ or the horisontal transpositions are equal to zero (the case of adherence), the difference in the degree of settlement may be more than 10%.

Compressive stresses of anisotropic bases do not coinside with compressive stresses of quasi-isotropic bases. The greatest difference is observed within the zone from the base of the foundation to the depth equal to two width of the foundation (Fig.1, II). With equal ratios E_x/E_z but different values E_x and E_z the epures of compressive stresses coinside.

Stress and transposition components of homogeneous anisotropic bases don't coinside with the same values for laminated anisotropic bases. That is why calculating anisotropic bases one should take into consideration their heterogeneity and lamination.

While solving soil mechanics problems with anisotropic bases it is necessary to know soil stress-strain property parameters being different in various directions.

A new method /2/ of experimental determination of five basic parameters of anisotropic soil deformation properties, having, with respect to stress-strain properties, an axis of symmetry has been worked out. Tests were carried out on undisturbed samples chosen in parallels and normally to the isotropy plane.

The method is based on the use of an odometer for determining considering the results of compression,-linear deformation modules in two principal directions /E_x, E_z/, a three-axial compression device for obtaining Poisson's ratio values being used as well.

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EXPLANATIONS for FIGURES

Fig.I. Change principles of a stress-strain state of anisotropic bases depending on the ratio of E_x/E_z and

- I. Vertical transpositions with different module ratios and
- II. The same for vertical pressures;
- III. Values of foundation settlements with different module ratios and
- IV. Foundation settlement value with different ratios of and E_x/E_z=1/2
  1. for the case when Ex/Ez =I
  2. for the case when Ex/Ez =1/0.5
  3. for the case when Ex/Ez =1/3
  4. for the case when Ex/Ez =2/1

Si- foundation settlements with different module ratios

Sh- foundation settlements with E_x/E_z=I

Fig.2 Diagrams of the dependence of an anisotropic soil sample lateral expansion upon a compressive strain and a compressive pressure.

RESUME

The state of stress and strain of anisotropic bottom is considerably differed from the analogous one. The experimental technique and calculation of the five mechanical parameters of anisotropic medium is proposed.

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PROBLEMS OF NON LINEAR SOIL MECHANICS

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As part of the continuing programme of research in soil mechanics at the University of Cambridge, Roscoe (1970), work has been in progress on developing computational methods which allow solutions to be obtained to boundary value problems using non-linear models of soil behaviour. This contribution outlines some of the recent developments.

Simpson (1973) has carried out finite element computations which have been based on the family of soil models developed at Cambridge. The approach adopted by Simpson has been to keep the computational side of the work as simple as possible by concentrating on two-dimensional problems and employing constant strain triangular elements. In contrast, however, a great amount of effort has been put into the development of mathematical models which provide an adequate description of soil behaviour; these models need to be complex in order to obtain satisfactory solutions to real boundary value problems for real soils.

In some circumstances an elastic model may be adequate, but in others it will not, and a complex elastic/plastic model will be necessary to arrive at an acceptable prediction. An example of the former would be an excavation in an overconsolidated clay for which the stress changes and deformations experienced by the soil are small enough for the behaviour to be considered quasi-elastic. Account may need to be taken of the marked increase of Young's modulus with mean effective stress-that is with increase of depth-and of anisotropy along the lines suggested by Wroth (1972) and Atkinson (1973). But these features of elastic behaviour can readily be incorporated in a finite element computation.

In contrast, the problem of the construction of an embankment or structure on soft normally consolidated clay can only be satisfactorily solved if an elastic/plastic model is used. This not only means that incrementally a bilinear response is obtained from the soil (with the type of response depending on the state of the element in question and whether it is being loaded or unloaded) but also that proper account is taken of the rotation of the principal stress directions. Wroth and Simpson (1972) have used such model with soil parameters taken from a routine site investigation, in an attempt to match the field data of a trial embankment reported by Wilkes (1972). The model is such that for every increment each element of soil experiences both an elastic and a plastic strain-increment; the principal axes of the elastic component coincide with those of the associated stress-increment whereas the axes of the plastic component coincide with those of stress for the beginning of the increment. Use of a piecewise linear approximation to a non-linear but elastic stress-strain curve, such as the model adopted by Clough and Duncan (1971), would lead to a very different displacement field computed for the ground under the embankment.

Another area of great difficulty is the modelling of the strain-softening behaviour of soils. Most stress-strain curves for real soils display a peak with a subsequent loss of strength as the soil approaches a critical or residual state. The strains will vary considerably along any incipient rupture surface in a soil mass; some elements will have been strained beyond their peak strength and will have 'failed' (although they are being held in equilibrium by neighbouring unfailed elements) while other elements will not have been strained sufficiently to mobilise their peak strength. Overall failure of the soil mass will occur in a progressive manner.

One method of overcoming these difficulties has been developed by Simpson whereby the computer generates extra elements by subdivision of the mesh at various stages of the computation. The choice of elements to be divided is governed by tho amount of either strain or displacement that they have experienced. In this way, extra, small elements become concentrated around an incipient rupture surface, and progressive failure of the real situation is directly modelled in the computation without previous guesswork on the part of the operator. Simpson and Wroth (1972) show that for the case of a rigid retaining wall being rotated about its top into a sand bed in the passive mode, the peak in the load-rotation curve observed experimentally for the wall can be reproduced in the computation by using this mesh-forming technique. Without mesh-forming a peak is not obtained.

In parallel with finite element computations an alternative approach using finite difference calculations based on the method of characteristics has been under development at Cambridge and Madrid. This method links together associated fields of stresses and velocities (or strain-increments) so that the stress-strain properties of the soil are satisfied throughout the soil mass. The important development has been the modification of the basic differential equations to take account of varying stress ratio throughout the stress field and varying amount of dilatation throughout the strain-increment field. Some solutions to particular problems of retaining walls have been reported by Serrano (1972) and James, Smith and Bransby (1972) and the general method described by Wroth (1972).

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