OEDOMETER TESTING OF VISCOUS SOILS

ESSAIS OEDOMETRIQUES DES SOLS VISQUEUX

ИСПЫТАНИЕ ВЯЗКО-ПЛАСТИЧНЫХ ГРУНТОВ В ОДОМЕТРЕ

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SYNOPSIS. In order to avoid structural disturbance, a modification of the classical oedometer - test procedure has been suggested. The possibility of determining the coefficient of permeability of viscous soils from consolidation lines observed after a slow load application has been analysed. The presentation of experimental data by void ratio versus effective stress plots corresponding to different consolidation speeds has been given support.

INTRODUCTION

If the general differential equation of consolidation (Biot 1938) of saturated soils is applied to the one-dimensional seepage conditions, it gets the form

$$\frac{\partial e}{\partial t} = \frac{1+e}{\gamma_w} \frac{\partial}{\partial z} \left(k \frac{\partial u}{\partial z} \right) \qquad (1)$$

e is the void ratio, u the excess pore - pressure, z coordinate in the seepage direction and k the coefficient of permeability as defined by Darcy's law. The classical form of equation (1) (Terzaghi 1923) has been based on the assumptions of linear stress - strain relationship and of constant permeability coefficient. Substituting

$$\frac{\partial e}{\partial t} = \frac{\partial e}{\partial \sigma'} \cdot \frac{\partial \sigma'}{\partial t} = -(1+e) m_v \frac{\partial(\sigma - u)}{\partial t} \qquad (2)$$

with σ for total and with σ' for effective pressures we get Terzaghi's equation

$$\frac{\partial u}{\partial t} = \frac{\partial \sigma}{\partial t} + c_v \frac{\partial^2 u}{\partial z^2} \qquad (3)$$

$$c_v = \frac{k}{\gamma_w m_v} \qquad (4)$$

The coefficient of consolidation can be obtained from the curve of consolidation e = e(t) corresponding to a sudden load increment ($t = 0 \rightarrow \Delta\sigma = \sigma - \sigma_o$, $t > 0 \rightarrow \frac{\partial \sigma}{\partial t} = 0$), in the well-known way. The coefficient of compression m_v is defined by the strain and stress increase ($m_v = \frac{-\Delta e}{(1+e_o)\Delta\sigma}$) and the coefficient of permeability can then be determined from equation (4).

However, if the structural resistance of the soil skeleton is important, the consolidation curve can appreciably decline from Terzaghi's theory, even when the load increases suddenly at large intervals (see e.g. Vidmar 1956, Berre and Iversen 1972). In this paper we shall present a modification of the classical oedometer test in order to get appropriate rheological parameters accounting for viscous soil properties; such test data can serve for predicting the consolidation of thick layers in linear strain and seepage conditions.

OEDOMETER TEST FOR VISCOUS SOILS

In linear strain conditions the deformability of viscous soils is represented by the rheological equation

$$R(e, \dot{e}, \sigma', \dot{\sigma}') = 0 \qquad (5)$$

$\dot{e}$ is the speed of the void-ratio change, σ' is the effective stress in the sample axis and $\dot{\sigma}'$ its speed.

By neglecting the influence of the stress rate $\dot{\sigma}'$, equation (5) gets simplified:

$$R(e, \dot{e}, \sigma') = 0 \qquad (6)$$

The stress-rate is expected to be the more effective, the greater its value. Furthermore, thixotropic effects destroying or diminishing the viscous structural resistance of the soil skeleton, can appear at large stress rates as occurring at a sudden load application (Šuklje 1957). In order to avoid such effects and to reduce the influence of the stress-rate as much as possible, a slow, continuous load increase is to be recommended. At appropriate stress intervals the continuous total stress increase has to be stopped; after the pore-pressures had dissipated to very small values, the so called secondary compresion must be observed during a sufficiently long period (see example in Fig.1).

From the secondary consolidation branches $e = [e(t)]_{\sigma' \cong const}$ the values of the void ratio e and of the rate of its change $\dot{e}$ can be ascertained at any time t. By using $\dot{e} = \dot{e}(t)$ plots, the corresponding values of e and σ' can be obtained for different chosen values of $\dot{e}$. The

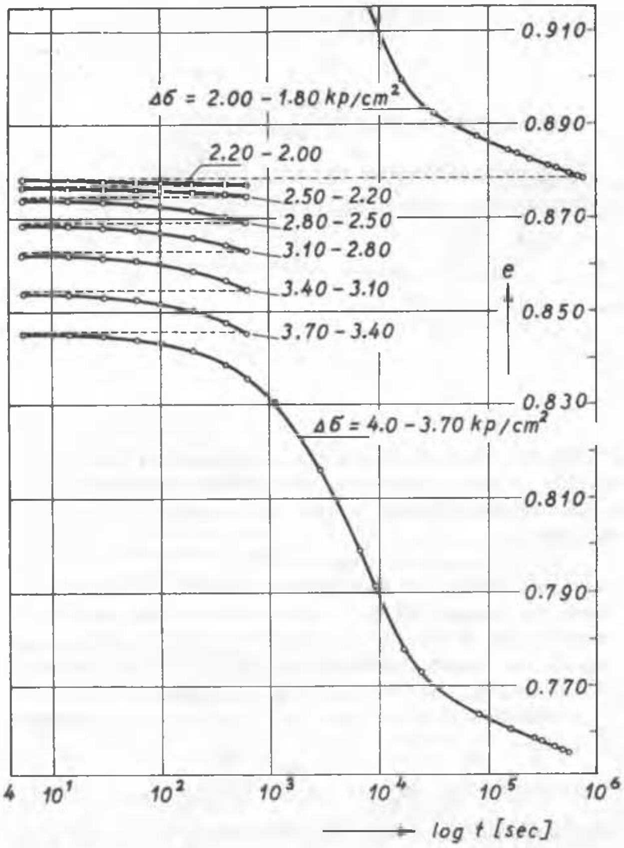


FIG.1. Consolidation lines of a clay of high compressibility corresponding to a slow load application

resulting plots $e = [e(\sigma')]_{\dot{e}=const}$ (see example in Fig.2) have been called isotaches (Šuklje 1957). They can be expressed by analytical functions:

$$e = e\ (\sigma',\ \dot{e}) \qquad (7)$$

or $$\dot{e} = \dot{e}\ (\sigma',\ e) \qquad (8)$$

During the compression, permeability coefficients k are recommended to be determined from direct permeability tests, preferably in the last phase of each secondary consolidation. By inserting the equation (8) for $\dot{e}$ and relation (9)

$$k = k\ (e) \qquad (9)$$

into the differential equation of consolidation (1), we can solve it in a numerical way taking into account the boundary conditions of the case treated (examples: Šuklje and Kogovšek 1968, Šuklje 1969-b). The use of isotaches in analysing layers of thickness of the order of magnitude 1 m or 10 m, has proved that the consolidation of natural layers occurs at small consolidation speeds; their maximum values usually do not overpass the rates appearing in the early phase of the secondary consolidation of oedometer samples of ordinary thickness (about 2 cm with bilateral drainage).

Now, for the use in predicting consolidation development of natural layers, the isotaches have to be given also for volume strain rates greater than those observed in the period of duration of the oedometer test. Consequently, the extrapolation of secondary consolidation curves of oedometer tests is inevitable. If the void ratio decreases with the logarithm of time, the extrapolation is simple, however, it is generally Hazardous. In some cases it is facilitated by knowing the porosity of undisturbed samples as well as the stress history of the natural layer (cf. Bjerrum 1967). If we are interested in knowing isotaches for greater rates appearing during the primary consolidation of oedometer samples, the permeability k = k (e) should be known and the slope of isotaches assumed. For parabolic isochrones of degree n, the mean value of the excess pore-pressure u is given by the equation (Šuklje 1969-a):

$$\bar{u} = \frac{\gamma_w\, h^2}{(n+1)\ (1+\bar{e})} \cdot \frac{\dot{\bar{e}}}{k} \qquad (10)$$

h being the length of the seepage path (half-thnickness of the sample at bilateral drainage), $\bar{e}$ and $\dot{\bar{e}}$ the respective mean values of the void ratio and its rate at time t (see Šuklje 1957). Isochrones resulting for uniform stress fields by applying, in the consolidation equation (1) the experimentally obtained rheological relationships (7) (Šuklje and Kogovšek 1968, Šuklje 1969-b) or the relationships corresponding to certain rheological models (Taylor 1942, Barden 1965), prove that, on average, their shape is similar to parabolas of 2^{nd} degree (n = 2) (in Barden's isochrone chartes $n > 2$ at the beginning and $n < 2$ at the end of the pore-pressure dissipation).

If the coefficient of permeability as well as the pore pressure u_o at the undrained boundary (in uni-lateral drainage conditions) are measured, the approximate form of isochrones can be ascertained. Assuming that isochrones are parabolas of degree n, we get :

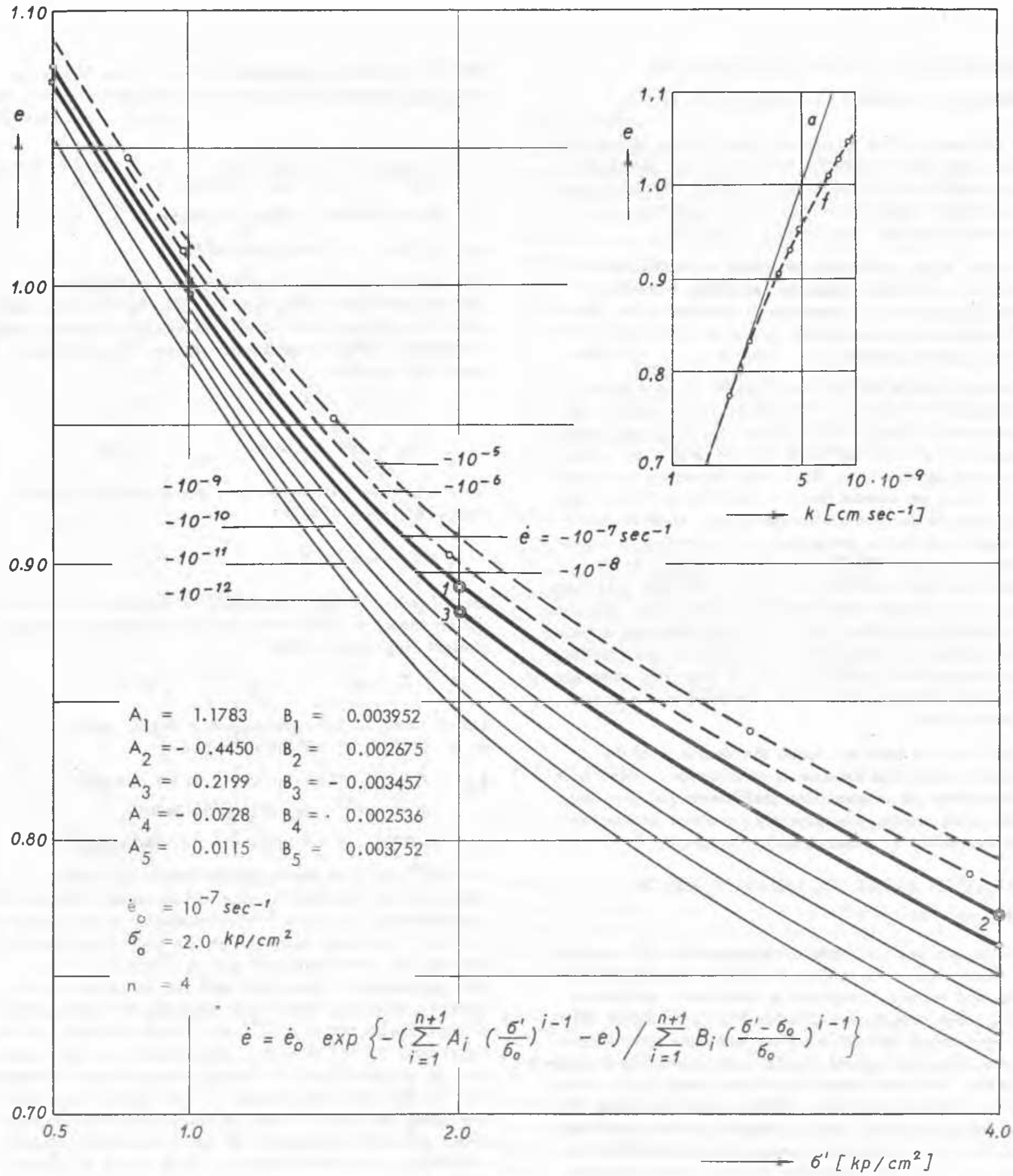
$$n = \frac{\gamma_w\, h^2\, \dot{\bar{e}}}{k(1+\bar{e})\, u_o} \qquad (11)$$

and then the everage effective pressure :

$$\bar{\sigma}' = \sigma - \bar{u} = \sigma - \frac{n}{n+1}\, u_o \qquad (12)$$

σ being the total pressure at time t. The procedure can be applied also to any stage during the continuous load-application.

It depends on the speed of the load application and on the susceptibility of the soil to the influence of larger stress-rates whether the resulting $(\sigma', e)_{\dot{e}=const}$ values lie on a single, continuous isotache or not.



$$\dot{e} = \dot{e}_o \exp\left\{-\left(\sum_{i=1}^{n+1} A_i \left(\frac{\sigma'}{\sigma_o}\right)^{i-1} - e\right) \Big/ \sum_{i=1}^{n+1} B_i \left(\frac{\sigma' - \sigma_o}{\sigma_o}\right)^{i-1}\right\}$$

FIG. 2. Isotaches of a clay of high compressibility, and coefficient of permeability versus void ratio plot corresponding to the presented isotache set: thin full line - first approximation, dashed thick line - final result.

DETERMINATION OF THE COEFFICIENT OF PERMEABILITY FROM CONSOLIDATION LINES

The influence of the stress-rate onto the run of isotaches causes also some trouble in determining the permeability from consolidation curve of viscous soils. This procedure, interesting in cases when there is no possibility of measuring k-values, cannot be but iterative.

We start, e.g., with an appropriate extrapolation of isotaches as obtained from the secondary branches of consolidation curves. According to equation (10), the first approximations for k-values can be obtained for various isotache points $(e, \ \bar{\sigma}' = \sigma - \bar{u})_{\dot{e} = \text{const}}$. The resulting k-values are not expected to lie on a unique continuous $k = k(e)$ line. We choose a line passing the intermediate values. For the chosen $k = k(e)$ plot several couples of $(\sigma', \dot{e})$ values for certain rates $\dot{e}$ are computed from consolidation lines (Fig.1) and according to equation (10). Then, we correct the $k = k(e)$ line until the resulting isotaches become continuous lines. An example of the results of such a procedure, corresponding to consolidation curves of Fig.1, is shown in Fig.2. (In the isotache set presented in this Figure, the thick full lines have been obtained from the observed secondary branches of consolidation curves; thin full lines represent isotaches corresponding to extrapolation of observed consolidation lines obeying the logarithmic law of secondary compression, and dashed isotaches have been obtained in the above explained way).

In any case we have to choose the degree n of the parabolic isochrones for such a construction. Thus, such determination of permeability coefficients can give only approximate results. Consequently, a direct observation of permeability is recommended.

DISCUSSION ABOUT THE SIGNIFICANCE OF ISOTACHES

Isotache sets and their analytical expression (8) represent the rheological relationships for soils in one-dimensional strain and seepage conditions as obtained in oedometer testing. The analytical expression has to be conformed to the geometrical form of isotaches and their mutual distances. Some analytical functions corresponding to experimentally obtained isotache sets have been presented by Šuklje (1969) and Battelino (1970). In several cases the void ratios decrease, along a single isotache, with the logarithm of effective stress and, at a given effective stress σ', with the logarithm of time. Such isotaches have the equation

$$\dot{e} = \dot{e}_o \exp \frac{A + B \ln \frac{\sigma'}{\sigma_o} - e}{C + D \ln \frac{\sigma'}{\sigma_o}} \qquad (13)$$

The choice of constants $\dot{e}_o$ and σ_o is arbitrary, the parameters A,B,C and D can be obtained by using the $(e, \dot{e}, \sigma')$ values of four points lying on two isotaches.

For the isotache set presented in Fig. 2, the following analytical expression has been found to be suitable:

$$\dot{e} = \dot{e}_o \exp\left\{ -\sum_{i=1}^{n+1} A_i \left(\frac{\sigma'}{\sigma_o}\right)^{i-1} - e \Big/ \sum_{i=1}^{n+1} B_i \left(\frac{\sigma' - \sigma_o}{\sigma_o}\right)^{i-1} \right\} \qquad (14)$$

(cf. Šuklje 1969-b); values of parameters $\dot{e}_o$, σ'_o, A_i and B_i (for n = 4) are given in Fig.2.

The isotache sets can, naturally, be conformed to any rheological model of the type Eq. (6). For Kelvin's body with linear elastic and non-linear viscous element, used in Barden's (1965) consolidation theory, the isotaches have the equation :

$$e = C - a\sigma' - b\,\dot{e}^{\frac{1}{n}} \qquad (15)$$

$$C = e_2 + a\sigma = \text{const} \qquad (15\text{-a})$$

With n = 1 we get the linear Kelvin model of Taylor's theory B (Taylor 1942) ·

$$e = C - a\sigma - b\,\dot{e} \qquad (16)$$

and with b = 0 the Hooke model of Terzaghi's consolidation theory in which all isotache condense to a single straight line (Šuklje 1957) :

$$e = C - a\sigma' \qquad (17)$$

Let us take, in the isotache set in Fig.2, points (e_1 = 0.8919, σ' = 2 kp/cm^2) and (e_2 = 0.7691, σ' = 4 kp/cm^2) on the isotache $\dot{e} = -10^{-7}$ sec^{-1} as well as the point (e_3 = 0.8828, σ' = 2 kp/cm^2) on the isotache $\dot{e} = -10^{-8}$ sec^{-1} as starting points which have been obtained from the directly observed secondary branches of consolidation curves and whose reliability is beyond doubt. In Fig. 3 we have presented the corresponding isotache sets for the coefficients n = 10, 5, 1 and for b = 0. The comparison of these sets with the complete experimentally obtained isotache set (assuming the extrapolation of isotaches beyond $\dot{e} = -10^{-7}$ sec^{-1} value according to the logarithmic law of secondary compression , Fig. 3) shows that the parallel lines of Barden's theory do not correspond well to the observed isotaches. It can be seen also from the above sets that in cases of low n-values the viscous effect practically disappears at low consolidation speeds occurring at the consolidation of thick layers (cf. Šuklje 1969-a).

Rheological bodies connected in series are not governed by equation (6) because the stress speed appears in their stress-strain-time relationships. E.g. Merchant-Taylor's (1940) rheological model consisting of the Hookean spring in series with a linear Kelvin body, has the equation:

$$e = A\,\dot{e} + B\,\sigma' + C\,\dot{\sigma}' + e_o \qquad (18)$$

It can be presented by several isotache sets corresponding to different stress speeds.

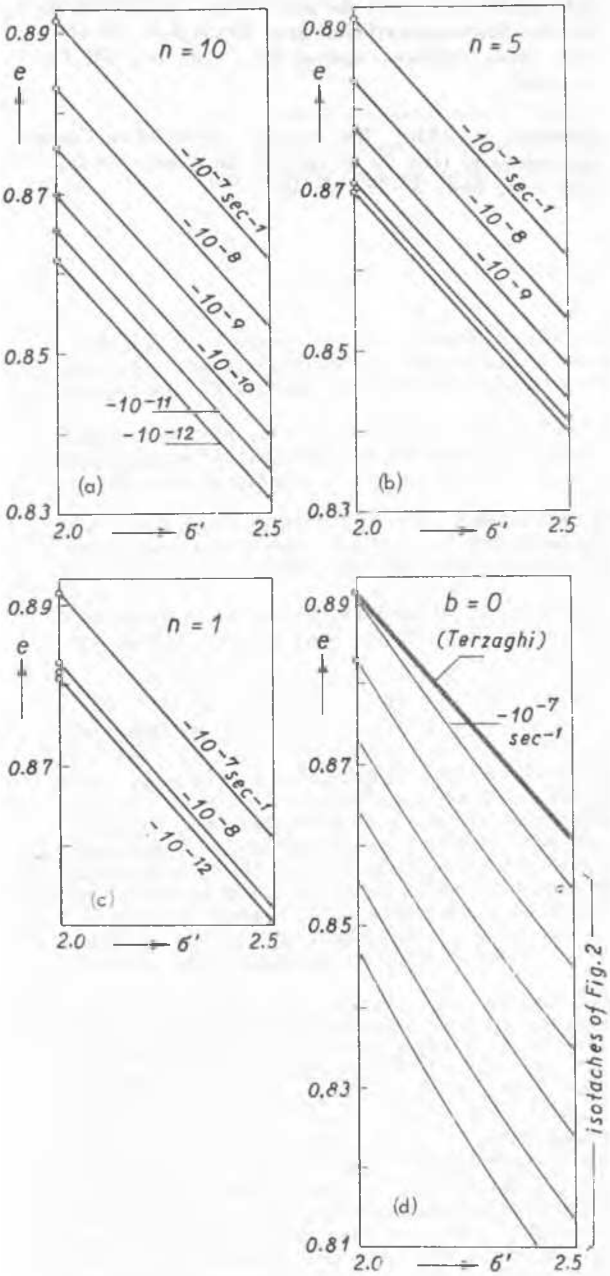


FIG.3. Isotaches according to points 1,2,3 of the observed isotaches and to Barden's rheological sheme: (a) for n = 10, (b) for n = 5, (c) for n = 1 (Taylor's theory B), (d) thick line: for b = 0 (Terzaghi); thin lines: isotaches presented ing Fig.2.

CONCLUSION

The modification of the classical oedometer test by applying slow, continuous loading has been recommended; the loading has to be interrupted by long-term observation of the secondary consolidation at certain values of total pressures, and accompanied by permeability measurement during the end phase of the observed secondary consolidation. Šuklje's proposal (1957, 1969) to express the deformability of viscous soils by isotache sets has been given support. Some isotache sets corresponding to consolidation theories based on rheological models connected in parallel have been presented and compared with the experimentally obtained set. The possibility of determining the coefficient of permeability from the observed consolidation lines of viscous soils has been analysed.

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