RECTILINEAR EXTENSION OF DRY SAND: TESTING APPARATUS AND EXPERIMENTAL RESULTS

DILATATION RECTILINEAIRE DES SABLES SECS. APPAREIL D'ESSAI ET LES RESULTATS EXPERIMENTALS

ДИЛАТАНСИЯ СУХОГО ПЕСКА – ПРИБОРИ И РЕЗУЛЬТАТЫ ИСПЫТАНИЙ

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Introduction

It is now generally accepted that quantitative informations about the stress-strain behaviour of soils can only be obtained in a testing apparatus producing uniform stresses and strains in the sample. Furthermore, the apparatus must have a sufficient number of degrees of freedom to allow for the three-dimensional nature of deformations in soils. Since the first device of Kjellman (1936) a lot of truly triaxial apparatuses were built to satisfy the above requirements. This contribution deals with such an apparatus (that can be correctly named as cuboidal deformation apparatus) and some results obtained with it in Karlsruhe.

The apparatus was constructed starting from an idea of Hambly (1969). It was devised for purely strain controlled tests for theoretical reasons. The cuboidal deformation produced by this device is a rectilinear extension according to the definition by Coleman (1968). In the frame of modern non-linear field theories this class of motions can be used to study experimentally certain classes of constitutive equations. Two types of constitutive equations, finite and elastic-plastic ones, were developed by the authors to evaluate truly triaxial tests carried out with dry granular materials. As the apparatus allows a very great variety of tests, the deformation programmes were worked out in advance based on certain constitutive hypotheses.

In the following sections apparatus, performance of tests and some results are outlined only very briefly. A detailed description of the apparatus and preliminary tests will be given in a report by the authors (1972 a). Mathematical details of the proposed constitutive equations were published by Goldscheider (1972) and Gudehus (1972).

Description of apparatus

The system of six plates "nesting" the cuboidal sample is represented in Fig. 1. To allow net side lengths of the sample between 5.5 cm and 14.5 cm each plate (1) is 16 x 16 cm large. Each plate is connected with two other ones by translatory guides (2), (4), (5), (6). The three pairs of opposite plates are moved past each other by three pairs of rams (7), designed for a maximal force of 2 tons. The whole system has only three degrees of freedom. The center of the sample remains fixed in space.

One of the main requirements of this device is to reduce friction in three places. First, the sample is to bear only normal surface forces. This is necessary for coincidence of principal axes of stress and strain. For this aim a silicone grease is used between the plates and the rubber mould of about 0.2 mm thickness around the sample. The lubrication is not squeezed out if the rubber mould is not too thin. Upon each of the six plates (1) that are made from cast aluminum is glued a sheet of polished steel (Fig. 2). Preliminary tests have shown that this lubrication reduces friction to below 2% for stresses below 1 kp/cm^2.

Secondly, the translatory guides consist of two parallel steel cylinders (Fig. 1, (2)), cages for 3 mm steel balls (6) and steel shells. Each pair of shells is fixed within a guide cell (5) that is mounted upon a socket (3), (4) of a plate (Figs. 1 and 2). The steel surfaces are lubricated by oil and protected against dust.

Thirdly, no friction can be admitted between rams (7), (8) and plates. This is achieved by pairs of needle bearings (Fig. 1, (9)). All polished steel surfaces are repeatedly inspected and cleaned.

The three pairs of rams are supported and driven by six planetoidal spindles and axial ball bearings. The

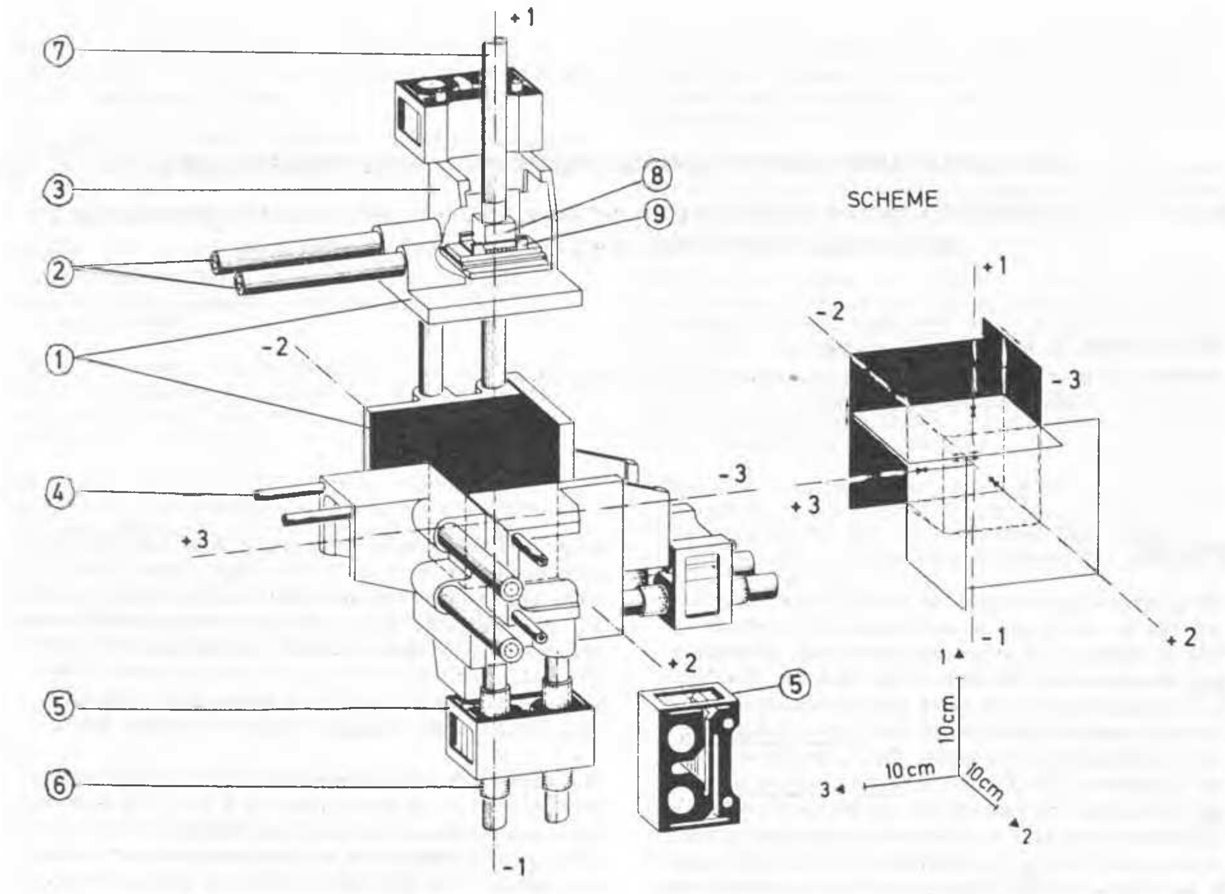

Fig. 1. Cuboidal deformation apparatus: System of plates.

bearings are connected by a stiff octahedral frame that is carried by a frame (Fig. 3). Three motors drive the spindles. The motors are steered viathree thyristor units and handle potentiometers in a steering desk. The apparatus is protected against vibrations by rubber supports.

In order to stabilize the array in arbitrary positions the plates are dragged against the rams by counter weights. The top plate and spindle are counter-balanced and can easily be moved upwards. Six dial gauges attached to the frame are used to adjust the entire device in centered position. Strains in the sample are measured indirectly by three inductive transducers (0.02 mm accuracy) measuring the distances between opposite plates. Normal forces are measured by strain gauges glued upon slender hollow cylinder shafts (of refined steel) in three rams. This device was calibrated within the octahedral frame by use of a c alibrated load cell. The characteristic is linear and holds with an accuracy of ± 1.5 kp. Aside these six fundamental quantities various control quantities can be measured. Each guide body is constructed as a load cell for two force and three moment components (Fig. 1 (5) and Fig. 2). The complicated cell body (of refined steel)

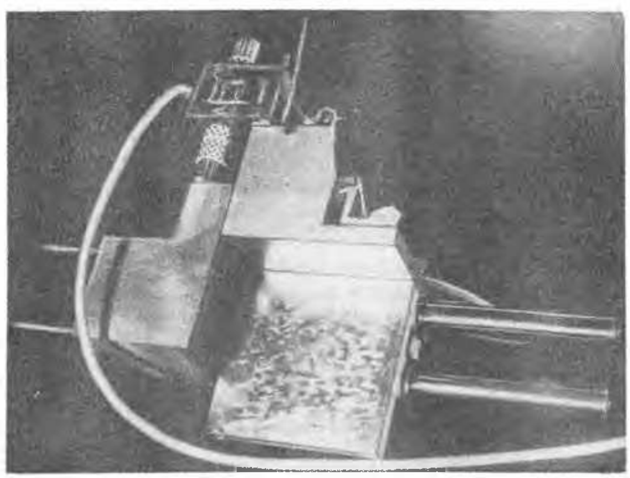

Fig. 2. Combination of 3 plates (with guides).

was eroded electrically. The guide cells were calibrated in separate devices. The three hollow cylinder shafts carry additionate strain gauges to measure the bending moments.

All measured quantities are registered on punched tape by means of a compact data logger. One complete set of quantities representing one state of the apparatus can be registered within 1 minute.

Execution of tests and data processing

In its present state the apparatus is equipped for dry granular materials with a grain size between 0.1 and 1 mm. The sample is prepared in the apparatus with removed top plate and ram. The grease is distributed with a brush in an expanded position of the plates. The plates brought back into a suitable initial position (normally 10 x 10 x 10 cm), the rubber mould with a 8 x 8 cm top opening is placed; it is kept in position by the grease. A shaft of 30 cm height is attached on the opening of the rubber mould. The opening is spread to 9.2 x 9.2 cm for filling in the material. A steady sand rain from a funnel, distributed over the shaft, fills the mould. Speed is adjusted by nozzles to obtain different densities. The material is electrically discharged in advance and the filling device is earthed. A slight vacuum is maintained in the shaft to avoid dust clouds. After removal of the shaft the sample surface is planed by suction to an accuracy of 0.1 mm. During the filling procedure all sensitive parts of the apparatus are protected against dust.

After placing a rubber lid upon the sample (without glue) the greased top plate is brought into position. One of the guide cells, previously removed, is shifted upon its socket and the top ram is lowered down to the top plate. Now the apparatus is in the working position. This initial state is controlled by observing the ram forces and the dial gauges.

Next an isotropic compression is carried out to bring the sample into full contact with the plates and to squeeze out abundant grease. Then the prescribed deformation programme is executed from the steering desk. Limits for loads and deformations are regarded. The drive is stopped automatically if certain bounds are reached. The 12 gaps between the plates are observed; if these close or open beyond an (empirically) admissible amount, the test is interrupted as this indicates an inadmissible increase of boundary friction due to damage of the rubber mould. After completion of the programme the sample is unloaded to remove the top plate. The tested material is succed out and weighed.

All measured quantities are punched every one or two minutes. The tape that may contain up to 25.000 data is transformed into a readible record by a computer programme. The programme first notes and eliminates data blocks that are wrong due to failure of the registration units. Then the principal stresses and natural strains are calculated, printed

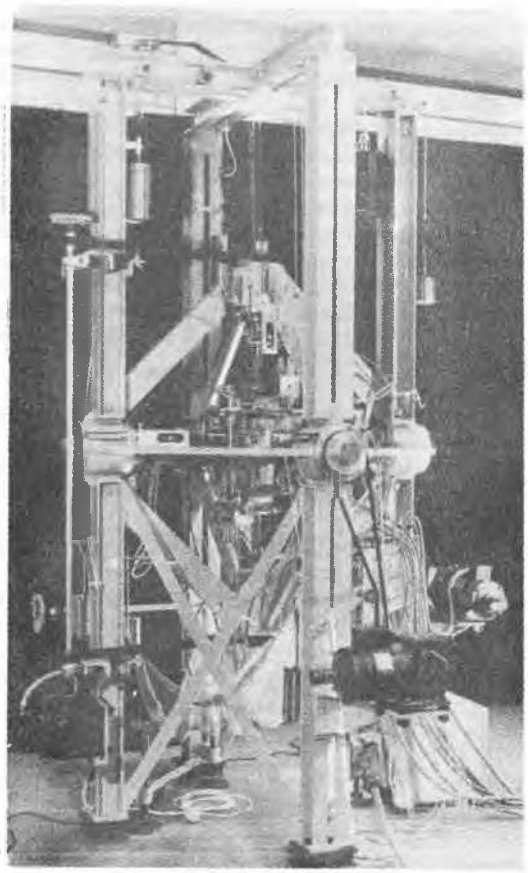

Fig. 3. Cuboidal deformation apparatus, total view.

and punched on cards for further evaluation. Compressibility of the rubber mould and forces in the guide cells are allowed for. Nine quantities are calculated from the control measurements: percentages of friction upon the six plates and resultant eccentricities of the three principal forces. Only test sections with tolerable control quantities (usually friction below 4 % and eccentricity below 1 cm) are further evaluated.

The dependency of three stresses on three strains cannot be represented in usual diagrams. Independent graphs of all six quantities versus time are plotted to discover possible irregularities. A more (although not easily) perceptual representation is obtained by axonometric plots of stress and strain paths in principal component space and their projections to the deviator planes through the origin (Fig. 4, e.g.). This representation is coordinate-invariant, and some invariant quantities can be measured geometrically.

Execution and data processing of one test requires about 8 man-hours.

Monotonous deformation tests

For planning and evaluation of the test programmes certain hypotheses must be made (Gudehus, 1969):

i) The sample consists of a simple material with memory;
ii) the sample is homogeneous;
iii) the material is rate-independent and isotropic.

These assumptions are by no means trivial for granular materials but inevitable (Goldscheider, 1972). Invariance requirements must also be allowed for in all mathematical representations: coordinate-invariance, frame-indifference, dimensional invariance. In addition only such constitutive equations are useful that can be fully specified by rectilinear extensions.

To obtain finite stress-strain laws the following restrictions must be made

i) among all possible deformation histories only motions of extension (Coleman, 1968) are admitted;
ii) the deformation paths in strain space are monotonous and linear.

Under these conditions the stress tensor is exactly an isotropic function of the strain tensor. This function was approximated by a tensor polynomial of 14 terms. The 14 coefficients can be determined by a Chebyshev method.

For this aim a series of 80 tests was carried out with a dense dry sand (grain size between 0.1 and 1 mm). For each test a ratio $\dot{\varepsilon}_1 / \dot{\varepsilon}_2 / \dot{\varepsilon}_3$ was prescribed. Starting from an isotropic state of $\sigma_1 \doteq \sigma_2 \doteq \sigma_3 \doteq 1.8$ kp/cm^2 a deformation with constant speed was executed until a prescribed force limit of 1 000 kp was reached (usually after less than 4 % strain). Upon reversal of all three speeds an approximately isotropic stress state was obtained. This was adjusted manually to get $\sigma_1 \doteq \sigma_2 \doteq \sigma_3 \doteq 1.8$ kp/cm^2 again. Now the deformation was repeated after an interchange of the components. Thus all six possible interchanges were realized. The test series results from systematic variation of the two dimensionless invariant quantities derivable from $\dot{\varepsilon}_1 / \dot{\varepsilon}_2 / \dot{\varepsilon}_3$.

A typical test result is represented in Fig. 4 (equal letters in both graphs refer to the same state and upper bars belong to points projected in the direction of the space diagonal onto the zero deviator plane). The stress paths produced by linear strain paths are curved except for unloading. The graphs represent a considerable degree of isotropy:

- the stress paths can be transformed into each other by interchange of components;

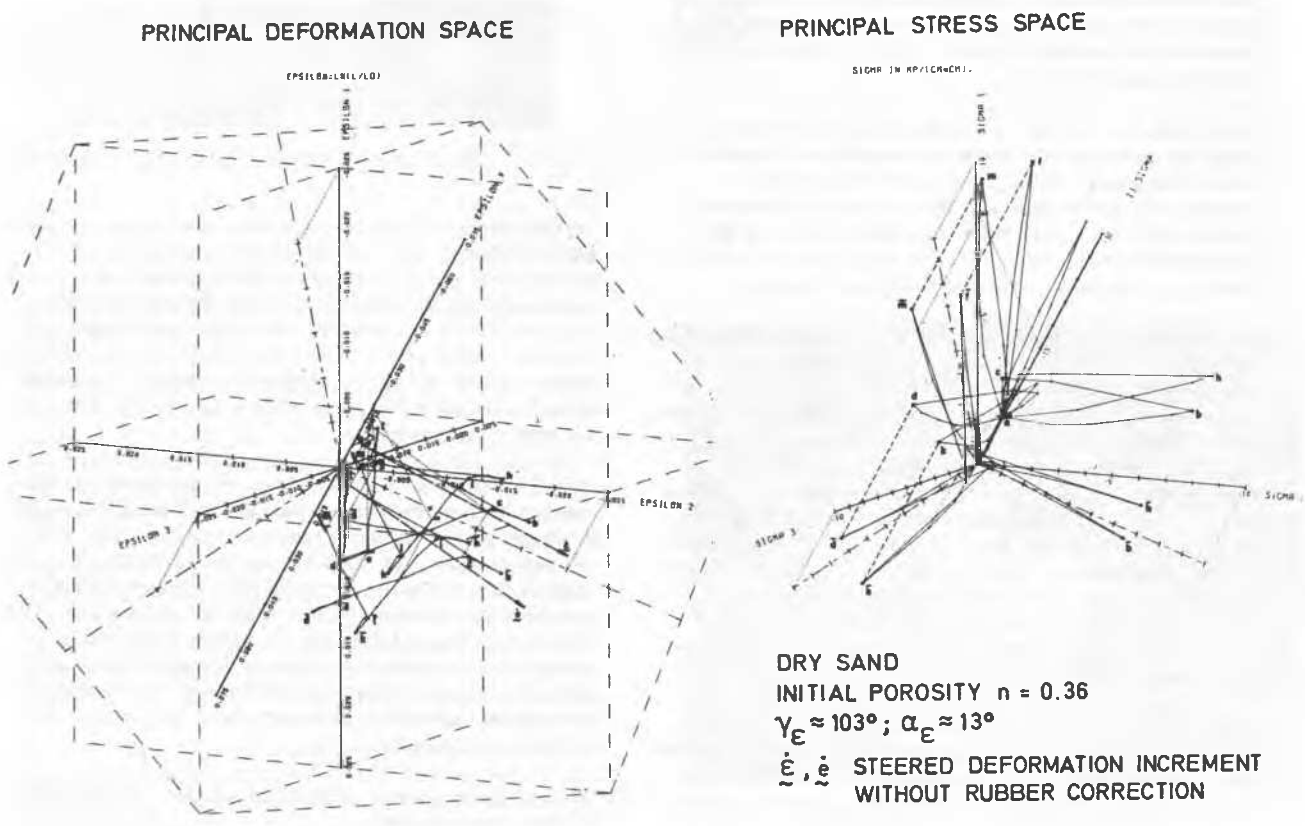

Fig. 4. Axonometric representation of six linear deformation tests. Dense dry sand.

- during pure compression the stress paths are very close to the space diagonal.

However, in the course of each test some strain hardening was observed: the same upper stress bound was reached by a decreasing amount of strain. Nevertheless, repeatedly reached states of $\sigma_1 \doteq \sigma_2 \doteq \sigma_3 \doteq 1.8$ kp/cm^2 were looked upon as isotropic initial states for the determination of one approximation.

The approximation of all monotonous deformation tests reads

$$(1)\quad \sigma_{ij} = (p_o + 990\, I_{1\varepsilon} - 4.66 \cdot 10^4 \cdot I_{1\varepsilon}^2 - 631 \sqrt{J'_{2\varepsilon}} \cdot \cos 3\alpha_\varepsilon - 1.33 \cdot 10^4\, I_{1\varepsilon} \sqrt{J'_{2\varepsilon}} - 417 \sqrt{J'_{2\varepsilon}} + 1.77 \cdot 10^4 \cdot J'_{2\varepsilon})\, \delta_{ij} + (892 - 2930 \cos\gamma_\varepsilon + 3.54 \cdot 10^5\, I_{1\varepsilon} - 3.32 \cdot 10^4 \sqrt{J'_{2\varepsilon}} - 1.21 \cdot 10^7 \cdot I_{1\varepsilon} \sqrt{J'_{2\varepsilon}})\, \varepsilon_{ij} + (4.98 \cdot 10^4 + 1.92 \cdot 10^6\, I_{1\varepsilon} - 2.60 \cdot 10^6 \sqrt{J'_{2\varepsilon}})\, \varepsilon_{ik}\, \varepsilon_{kj},$$

wherein

$$(2)\quad \begin{cases} I_{1\varepsilon} = \varepsilon_1 + \varepsilon_2 + \varepsilon_3; \quad J_{2\varepsilon} = \varepsilon_1^2 + \varepsilon_2^2 + \varepsilon_3^2; \\ e_i = \varepsilon_i - I_{1\varepsilon}/3 \text{ (deviator components)}; \\ J'_{2\varepsilon} = e_1^2 + e_2^2 + e_3^2; \quad J'_{3\varepsilon} = e_1^3 + e_2^3 + e_3^3; \\ \cos 3\alpha_\varepsilon = +\sqrt{6}\, J'_{3\varepsilon} / J'^{\,3/2}_{2\varepsilon} \text{ (angle in the deviator plane)}; \\ \cos\gamma_\varepsilon = I_{1\varepsilon} / \sqrt{3 J_{2\varepsilon}} \text{ (angle with the space diagonal)}. \end{cases}$$

The obtained stress-strain equation can be shown to hold approximately also outside the restrictions for certain bended strain paths and motions with rotation of principal axes.

Cyclic deformation and strength tests

For markedly non-monotonous strain paths finite constitutive equations cease to hold. A more realistic description is achieved by incremental laws of an elastic-plastic type. As with metal plasticity, one needs an elastic law, a flow condition, a flow rule, and a hardening rule. Various tests were made to specify these rules.

As the elastic deformations are very small only crude estimates of the elasticities can be obtained from the tests. Unloading and reloading tests at least indicate (as can be seen from Fig. 4, e.g.) that the elastic behaviour is isotropic and can be looked upon as linear in good approximation. The measured elasticities scatter around 2 000 kp/cm^2 for E and 0.3 for the Poisson's ratio. More precise values are expected after an improvement of the deformation measurements.

Due to the linear law of dry friction, both flow condition and flow rule can be expected to be homogeneous functions of the stress components. Therefore a dimensionless representation of test results is advisable. This is obtained by projecting the stress point $\{\sigma_1; \sigma_2; \sigma_3\}$ and the strain increment $\{\delta\varepsilon_1; \delta\varepsilon_2; \delta\varepsilon_3\}$ (neglecting the elastic portion) centrally onto the unit deviator plane $\sigma_1 + \sigma_2 + \sigma_3 = 1$ (Fig. 5). The coincidence of axes of σ_{ij} and $\delta\varepsilon_{ij}$ is provided. In the following we regard some of such projections.

Fig. 6 refers to a test on a dense packing of 0.25 mm glass beads and Fig. 7 to a loose sand. Numbers indicate the ordered sequences of states. It can be seen that the projected stress paths tend to a common

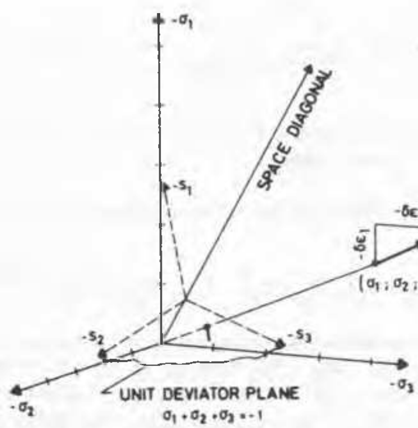

Fig. 5. Projection of stress and strain increment.

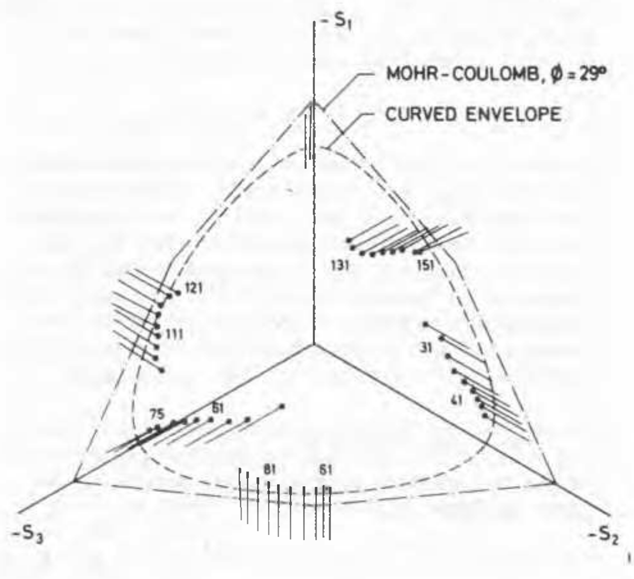

Fig. 6. Projected stresses and strain increments. Glass beads.

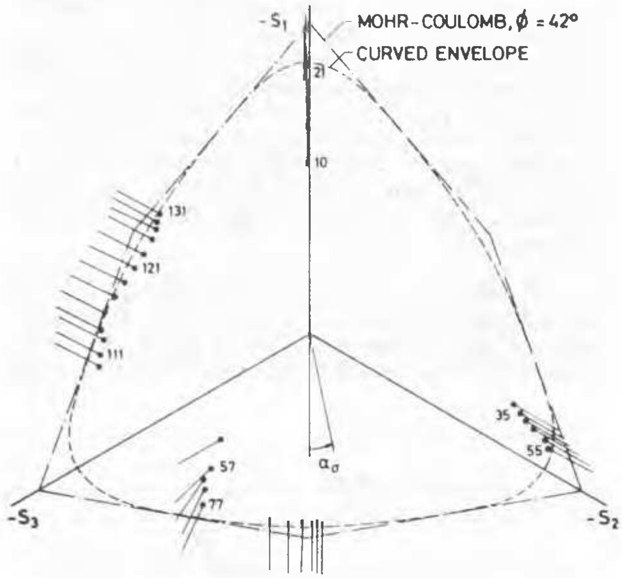

Fig. 7. Projected stresses and strain increments. Loose sand.

envelope. This can be approximated by the expression

(3) $f_\sigma = c_1 J'_{2\sigma}/I_\sigma^2 + c_2 J'_{3\sigma}/(J'_{2\sigma})^{3/2} = 0$

or by the Mohr-Coulomb hexagon (wherein the invariants I_σ, $J'_{2\varepsilon}$, $J'_{3\varepsilon}$ are given by (2) replacing ε by σ and e by s. However, (3) is not a flow condition like in metal plasticity; Markedly irreversible deformations can also occur for stresses inside the cone (3). It seems to be more realistic to look upon (3) as an envelope of yield envelopes that holds for fully developed plastic flow. For other states it may be useful to work with a mobilized ϕ_m in a Mohr-Coulomb flow condition, i. e.

(4) $\phi_m = \arc\sin\left[(\sigma_{max} - \sigma_{min})/(\sigma_{max} + \sigma_{min})\right]$.

As yet it can only be said from cyclic deformation tests that ϕ_m is a complicated functional of deformation history. Figs. 6 and 7 clearly indicate that glass beads generally yield a lower ϕ_m than sand. The maximal ϕ_m, corresponding to (3), is dependent on the angle α_σ (cf. (2)). For plane strain the absolute maximum is obtained ($\phi_m \leq 48^\circ$ for dense sand) and for axisymmetrical compression the absolute minimum ($\phi_m \leq 44^\circ$ for dense sand).

The deviatoric flow rule can be read from Figs. 6 and 7. It can be seen that the deviator projection of strain increment is approximately normal to the yield envelope, i. e.

(5) $\delta e_{ij}^p = \delta\varepsilon_{ij}^p - \frac{1}{3}\delta I_{1\varepsilon}^p \delta_{ij} = \delta\lambda\left(\frac{\partial f_\sigma}{\partial s_{ij}} - \frac{1}{3}\frac{\partial f_\sigma}{\partial s_{kl}}\delta_{kl}\delta_{ij}\right)$

Wherein $f_\sigma = 0$ is the flow condition (3).

medium sand, loose				medium sand, dense			
σ_{kk}	α_σ	ϕ_m	K_μ	σ_{kk}	α_σ	ϕ_m	K_μ
-10.7	-155	30.5	3.21	-15.7	-178	37.7	3.50
-10.8	-161	38.9	4.00	-13.3	-179	43.7	3.44
-15.8	-163	37.3	4.26	-12.9	-179	44.4	3.44
- 6.7	51	23.9	4.45	-15.6	68	39.6	3.58
-11.4	45	32.2	3.38	-15.3	67	40.8	3.25
-14.7	45	35.6	3.90	-15.4	66	41.0	3.35
- 6.5	- 57	20.2	4.87	-14.6	- 29	39.1	3.48
-16.2	- 65	25.7	3.54	-23.6	- 17	42.1	3.88
-17.7	- 72	35.8	4.04	-28.2	- 7	41.4	3.91

Table 1. Measured values of σ_{kk} [kp/cm^2], α_σ [deg], ϕ_m [deg], K_μ.

The volumetric flow rule is still unspecified by (5). A very simple expression specially useful for numerical boundary value problems was proposed by Gudehus (1972). For large deformation tests (up to 30%) a better approximation is obtained by the stress-dilatancy equation (Rowe, 1971):

(6) (incremental work in)/(incremental work out) = $= K_\mu = \text{const}$.

Some values of K_μ for a medium sand as determined from different monotonous deformation path sections are given in Table 1. Although K_μ is slightly variable (6) can be looked upon as a useful volumetric flow rule.

As concerns hardening rules a series of 25 cyclic deformation tests did not suffice to obtain mathematical formulations. Various types of hardening and softening were observed. Only for small deformations (below 4%) the sample remained fairly isotropic. For large deformations the samples become markedly anisotropic. This loss of isotropy can be proved theoretically (Goldscheider, 1972).

Concluding remarks

The capacity of the cuboidal deformation apparatus is still far from being exhausted. New test series are planned for getting information about
- hardening rules of dry granular materials,
- creep and grain crushing effects in sands,
- consolidation and creep of saturated silt.

Results will be published on later occasions.

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