PREDICTION OF STRAIN RATES USING E_G, E_K PARAMETERS

LA PREDICTION DE LA VITESSE DE DEFORMATION PAR DES PARAMETRES E_G ET E_K

ПРОГНОЗ СКОРОСТИ ДЕФОРМИРОВАНИЯ ПРИ ПОМОЩИ ПАРАМЕТРОВ E_G И E_K

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SYNOPSIS. The paper shows how the method of the e_g, e_k parameters may be applied to the problem of forecasting the stress-strain relationship for specimens of silt which are being sheared to failure in a triaxial cell in both drained and undrained conditions, and under different rates of strain. Good agreement is found between computed and observed values.

The method has been in use in the authors' laboratory for a number of years and has been applied successfully to a number of problems. The e_g, e_k parameters are principal strains in cylindrical specimens (with provision for radial drainage) brought about, respectively, by (i) principal stress-difference and (ii) volumetric stress.

INTRODUCTION

The relationship between stress and strain obtained in the laboratory by testing small specimens of soil is widely used as the basis of quantitative analyses of full-scale structures. It is not unusual for factors of safety, evaluated to the second place of decimals, to be assigned to a project on the assumption that an element in the ground will exhibit the same behaviour as a small laboratory specimen, albeit that the laboratory test may have been executed at an arbitrary rate of deformation.

Bjerrum (1969) stated that "the undrained shear strength analysis is a hopeless approach to natural slopes in soft clay". The time-dependency of the stress-strain relation ship has long been recognised, e.g. (Casagrande and Wilson 1951). The profound effect on the strength of a soil caused by variations in the duration of the laboratory test has been shown by Parry (1971). A correction for the effect of strain rate was proposed by Brown (1969).

The size of the test specimen is a further important variable. A great deal of work has not been done on this topic, and the results available are frequently complicated by further variables such as presence of fissures, rate of testing and type of apparatus employed.

AUTHORS' METHOD

In the authors' laboratory the aim has been pursued for a number of years of finding a simple regime of laboratory measurements, from which a general relationship between the three quantities, total principal stress, total principal strain, and time, could be formulated, and which would be applicable to specimens of dimensions other than those of the test specimen (or to any element in a continuum or deposit). The value of such a relationship is considerable, since, when two of the three quantities are known, or specified, the third is automatically defined.

Problems involving the lateral pressures in operation on specimens of different sizes while undergoing one-dimensional consolidation (Hanrahan and Mitchell 1969), (Hanrahan 1971), and the forecasting of settlements of loaded deposits of peat (O'Sullivan 1970) are among those which have been solved satisfactorily using the proposed system of laboratory measurement of strains which are termed the e_g and e_k parameters.

DEFINITION OF e_g AND e_k PARAMETERS.

The relationship on which the proposed method is based is expressed by the equations of principal strains which are produced by an axi-symmetric stress system:

$$e_1 = e_{k1} + e_{g1} \quad \ldots\ldots \quad 1$$

and $$e_3 = e_{k3} - e_{g3} \quad \ldots\ldots \quad 1a$$

where e_1, e_3 = nett total principal strains

e_{g1}, e_{g3} = major and minor principal strains, respectively, caused by a principal stress-difference ($\sigma_1 - \sigma_3$).

N.B. (Strains due to volume change are excluded, except when this accompanies shear, as, for example, in connexion with dilatant soils).

e_{k1}, e_{k3} = major and minor principal strains, respectively, caused by a volumetric stress $\frac{\sigma_1 + 2\sigma_3}{3}$.

N.B. (All principal strains associated with shear are excluded).

The usual convention of signs, compressive strains positive, tensile strains negative, is adopted in equations 1.

COMPARISON WITH THEORY OF ELASTICITY

Equations 1 may be compared with the corresponding equations for a perfectly elastic material subject to a similar axi-symmetric stress system, viz.

$$e_1 = \frac{\sigma_1 + 2\sigma_3}{9K} + \frac{\sigma_1 - \sigma_3}{3G} \quad \ldots\ldots 2$$

$$\text{and } e_3 = \frac{\sigma_1 + 2\sigma_3}{9K} - \frac{\sigma_1 - \sigma_3}{6G} \quad \ldots\ldots 2a$$

where K = bulk modulus

$$= \frac{\sigma_1 + 2\sigma_3}{3(e_1 + 2e_3)} \quad \text{and}$$

G = shear modulus

$$= \frac{\sigma_1 - \sigma_3}{2(e_1 - e_3)}$$

In both equations 1, 2, the first term on the RHS refers to the principal strains caused by the volumetric stress $\frac{\sigma_1 + 2\sigma_3}{3}$ and the second to those caused by the principal stress-difference $(\sigma_1 - \sigma_3)$. Also there is agreement as regards the sign of the resulting strains. These similarities apart, there are fundamental differences between the behaviour of a soil and that of a linearly-elastic material. For the latter condition, the moduli K, G, are single-valued and independent of time. Thus, both components of principal strain are determined, simply by specifying values of the moduli and the principal stresses. On the other hand, in a soil the components (e_g, e_k) are influenced by time, and the value and range of the stress increment. They are also influenced by consequential changes in the soil properties. Furthermore, both components develop at different rates. Thus, e_k, which is brought about largely by change of water content is governed by the rate of consolidation, which, in turn, is determined by the drainage path. For this reason e_k is said to be dimension-dependent.

The other component, e_g, is time dependent but largely independent of dimension, a similar rate being observed on large and small specimens which are subject to the same value of principal stress-difference.

Lastly, in equations 1, the principal strains are divided into two distinct categories, viz. (i) caused by change of volume and (ii) caused by change of shape. In a soil, such a clear-cut distinction is not possible. The e_g strains, while largely caused by change of shape, are liable to be affected by change of volume, if the soil is dilatant. Similarly, the e_k strains are usually accompanied by some change of shape (Hanrahan 1971a).

ILLUSTRATIVE EXPERIMENT

These various points may be observed in the following simple illustrative experiment. A cylindrical specimen of saturated cohesive soil is prepared, with provision for radial drainage. It is then placed in a triaxial cell and subjected to an axi-symmetric stress system $(\sigma_1 > \sigma_2 = \sigma_3)$ which is kept at a constant value throughout the test. Observations are made of the principal strains which develop with time, during
(i) Undrained Phase. The strains in this phase are the e_g strains, which are determined by the chosen value of principal stress-difference and the soil characteristics. This phase is followed by
(ii) Drained Phase. Since most of the strains caused by principal stress-difference will have been completed during the previous phase, and since the soil is likely to become stiffened by drainage, it follows that the observed principal strains during the drained phase are the e_k strains.

It will be confirmed that the net principal strains are the algebraic sum of the e_g and e_k strains in accordance with equations 1.

APPLICATION TO PROBLEM OF PREDICTION OF STRESS-STRAIN RELATIONSHIP.

The purpose of this paper is to demonstrate how the measured parameters e_g, e_k may be combined so as to enable a prediction to be made of each of the several stress-strain relationships which are obtained when the mode of testing in the triaxial apparatus is altered. The soil used was a remoulded organic silt (W_L = 80, W_P = 36) in a near liquid condition. To enable the soil to be handled, it was given a slight initial air-drying during which the water content was lowered to 56%.

BASIC DATA
MAGNITUDE AND RATE OF e_{g1}

The measured relationships between various values of principal stress-difference, time, and e_{g1} are set out in Fig.1. These

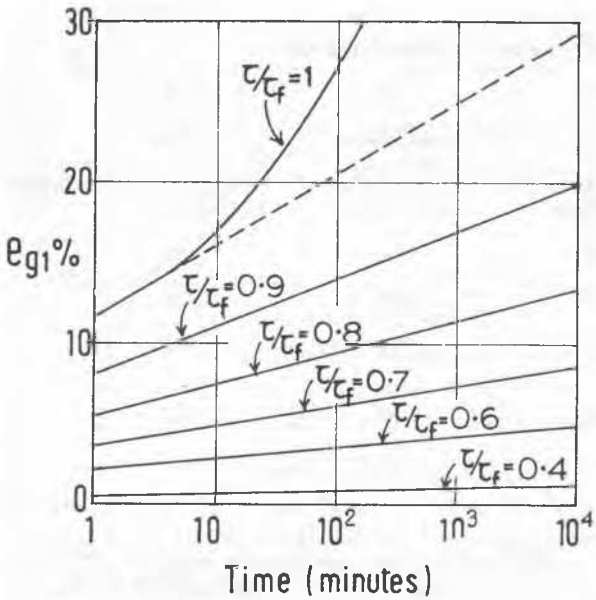

FIG. 1. MEASURED VARIATION OF e_{g1} STRAIN WITH STRESS AND TIME

measurements were made on undrained cylindrical specimens 38mm.diam. by 76mm. high. This family of curves may be described by the equation

$$e_{g1} = 11.5(\tau/\tau_f)^{3.31} + 4.3(\tau/\tau_f)^{3.54}\log t \quad \ldots 3$$

where $\tau = (\sigma_1 - \sigma_3)/2$

$$\tau_f = (\sigma_1 + \sigma_3)\ /2$$

In some of the tests subsequently described, the water-content changed during the test. In such tests the value of e_{g1} alters for the same value of principal stress-difference. However, it has been shown (Shahrour 1972) that by using the dimensionless quantity, τ/τ_f substantial changes of water-content have little effect on the value of e_{g1}.

The relationship between the peak shearing strength τ_f in equation 3 and the effective consolidation pressure $\bar{\sigma}_c$ was determined by a separate series of consolidated-undrained triaxial tests, and is described by the equation

$$\tau_f = 0.62\ \bar{\sigma}_c \quad \ldots\ldots 4$$

The e_g strains may develop in the absence of drainage. As expected, it has been found that these strains are relatively independent of scale effects. Thus the relationships of equation 3 may be applied to specimens of all dimensions.

MAGNITUDE AND RATE OF e_{k1}

As a first approximation, e_{k1} may be obtained from the equation

$$e_{k1} = \frac{M_v}{3}\Delta\bar{\sigma}\ \text{oct} \quad \ldots\ldots 5$$

where M_v is the coefficient of unit volume change determined by a triaxial consolidation test (N.B. not Odometer test) giving a relationship between equilibrium void-ratio and octahedral effective stress ($\bar{\sigma}_{oct}$).

However, because the e_k strains are generally accompanied by changes in shape it has been found preferable (Hanrahan 1971a) to apply a correction factor, which alters equation 5 to

$$e_{k1} = \frac{M_v}{3}\Delta\bar{\sigma}\ \text{oct}\cdot\frac{\bar{\sigma}_1}{\bar{\sigma}_{OCT}} \quad \ldots\ldots 6$$

The rate at which the e_k strains develop (i.e. $\frac{\partial e_K}{\partial t}$) is determined by the rate of dissipation of excess porewater pressure (i.e. $\frac{\partial u}{\partial t}$), the latter, in turn, being calculated from the standard equations of consolidation.

The foregoing proposed methods of determining magnitude and rate of e_{k1} are applicable to field conditions in which hydraulic gradients in any small elemental volume are not unduly steep. In laboratory specimens, in which, for example, water may drain radially but not through the ends, the variations in hydraulic gradient in any preferred direction may be severe. In such instances, as in the present, it is preferable to make direct observations of the relationship between e_{k1} and effective consolidation pressure ($\bar{\sigma}_c$). Radially drained cylindrical specimens 39mm. diameter by 78mm. high were observed, and the resulting relationship is expressed.

$$e_{k1} = 1.54\ \bar{\sigma}_c \quad \ldots\ldots \quad 7.$$

CONFIRMATORY TESTS

Four triaxial tests are reported in Figs 2-4 inclusive which show comparisons between observed and computed values of the relationship between stress and deformation (or time)

Details of the tests and soil characteristics are shown on Table No.1

Table 1.

Details of confirmatory triaxial tests on remoulded silt.

Test No.	1.	2.	3.	4.
Rate of Strain (%per min.)	2.0	0.016.	variable	0.016
Type of Test	Consolidated-undrained	Consolidated-undrained	Consolidated undrained	Partly Consolidated-drained
w/c after preparation %	56	56	56	56
Cell pressure kN/m^2	220	220	220	220
w/c after consolidation in cell.	30	30	30	31
Effective consolidation pressure ($\bar{\sigma}_c$) at start of shearing phase kN/m^2	159	159	159	141

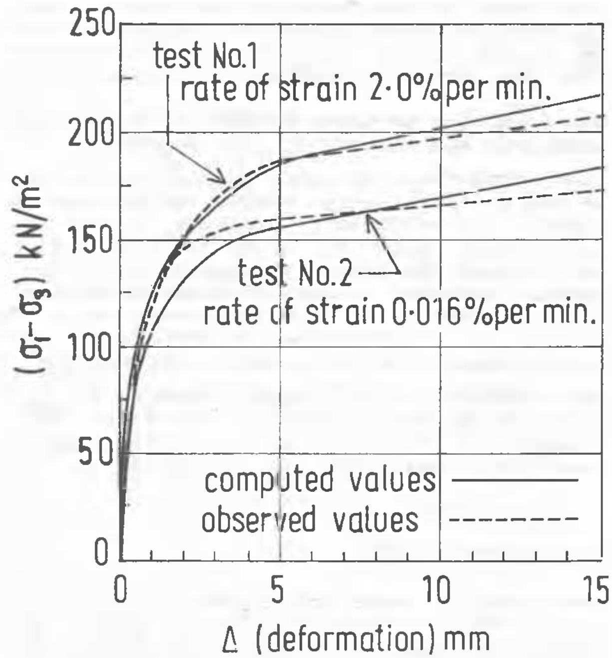

FIG.2. COMPUTED AND OBSERVED EFFECT OF STRAIN RATE ON UNDRAINED COMPRESSION TESTS.

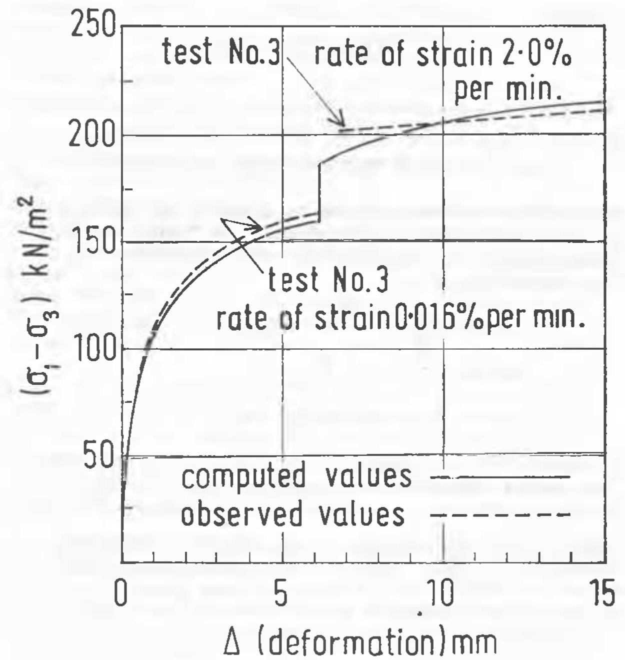

FIG.3. COMPUTED AND OBSERVED EFFECT OF VARYING STRAIN RATE DURING UNDRAINED COMPRESSION TEST.

Fig.2 shows computed and observed values of ($\sigma_1 - \sigma_3$) in tests nos 1, 2, in which undrained specimens were tested at strain rates 2.0% per minute, and 0.016% per minute, respectively.

Fig.3. refers to undrained test No.3 in which the testing rate was altered during the test in accordance with the values shown.

Fig.4. shows a partly-consolidated, drained test in which a specimen was first consolidated under an effective stress of 141 kN per m^2 (cell pressure = 220 kN per m^2, u = 79kN. per m^2) and subsequently sheared with radial drainage, keeping the pore-water pressure constant at u = 79 kN per m^2, at a strain rate of 0.016% per minute.

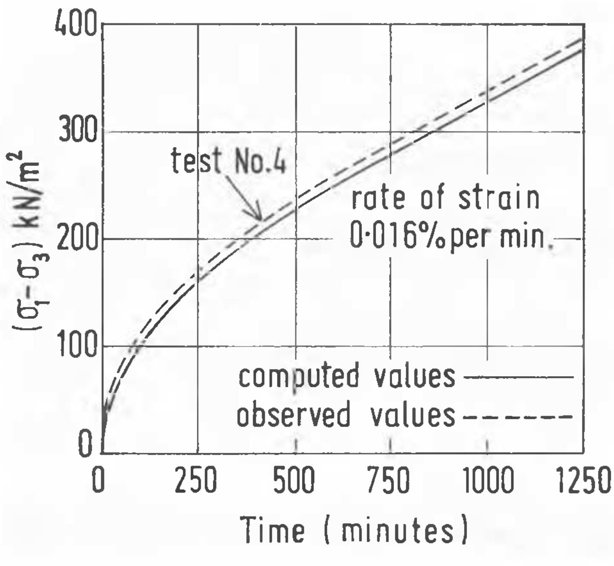

Fig.4. COMPUTED AND OBSERVED STRESS-STRAIN CURVE FOR CONSOLIDATED-DRAINED COMPRESSION TEST .

Comments.

The combination of the basic data was effected by simple superposition, using a short computer program, the object of which was to find by,iteration, the principal stress system (σ_3 was constant) which would yield values of e_{gl} and e_{kl}, respectively, to satisfy the equality,

$$e_{1(t)} = e_{gl} + e_{kl} \quad \ldots\ldots \quad 8$$

where $e_{1(t)}$ = known value of net major principal total strain at specified time t.

A problem in connexion with the basic e_g data was that in Fig.1 the stress remains constant throughout the time period, whereas, in the triaxial test, stress is applied incrementally. To make use of the basic data,therefore, it was necessary to introduce the concept of equivalent time, t_{eg}. For example, suppose that in a triaxial test the stress system and e_{gl} strain are known at some time, t, and it is required to find the increment of e_{gl} strain developed by the known stress system during a further time-increment, Δt. By inserting the known values of stress and e_{gl} in equation 3 the equivalent time, t_{eg}, is the value of t obtained. Now, by re-solving equation 3 with

$$t = t_{eg} + \Delta t$$

a new value of e_{gl} is obtained, from which the increment of e_{gl} developed during the time increment Δt is found. Iteration may now proceed as before with the object of satisfying equation 8.

CONCLUSION

It is seen from the agreement between computed and observed values of stress-strain curves shown in Figs 2-4 that the method of e_g, e_k parameters may be used to predict the effect of varying the rate of deformation. Similarly by applying the appropriate scaling factors, as described, the stress-strain curve may be constructed for any dimension of specimen or any element in a continuum.

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