ANALYSIS OF PROGRESSIVE FAILURE IN CLAY SLOPES

ANALYSE DE RUPTURE PROGRESSIVE DES TALUS D'ARGILE

АНАЛИЗ ПРОГРЕССИРУЮЩЕГО РАЗРУШЕНИЯ ГЛИНИСТЫХ ОТКОСОВ

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SYNOPSIS. A finite element solution of the stress distribution in a slope of strain-softening material is briefly described. The results obtained may be incorporated into limit equilibrium method for stability analysis so as to take into account the effects of in-situ stresses and progressive failure. It is also shown, by analyses of three case records, that the time of failure of excavations may be predicted, provided that the rate of decrease of drained strength is known.

INTRODUCTION

The engineering perception of the problem of progressive failure dates back to 1936, when Terzaghi first postulated softening as a mechanism leading to the progressive failure of slopes in stiff-fissured, overconsolidated clays. Subsequently, further understanding of the physical and conceptual aspects of the problem has been achieved (e.g. Taylor, 1948; Skempton, 1964; Bishop, 1967, among others). While an analytical solution to the problem has yet to be developed, the pre-requisite conditions for its occurrence are now reasonably well defined (Lo, 1972). Essentially, these conditions consist of:

a) a degree of non-uniformity in the distribution of shear stresses and strains in the slope and its vicinity;

b) the prevalence of a brittle (or strain-softening) behaviour of the soil once shear failure is reached locally in any part of the slope; lastly

c) a decrease in strength with time due to such mechanisms as softening, progressive rupture of bonds, and periodical change of ground water condition.

This paper describes an approximate solution to the problem of progressive failure. The solution itself is composed of two parts:

a) A finite element analysis of the stress distribution in the slope (taking into account the strain-softening behaviour of the material), followed by an assessment of slope stability in terms of a "residual factor" (Skempton, 1964).

b) Wherever applicable, a subsequent incorporation of the time element into the analysis for an estimation of the time of slope failure.

Typical results based on the above approach and method of analysis will be given, and the effects of various factors controlling slope stability studied. Available case histories of first-time slides in London Clay are finally examined using the present approach, and the results are shown to be reasonably consistent.

STRESS ANALYSIS IN STRAIN-SOFTENING MATERIAL

The finite element method will be employed in the first stage of the solution to evaluate the stress distribution in the slope and its vicinity. A versatile method originally developed for stress analyses in a homogeneous and elastic continuum, the finite element method has, in recent years, been refined to increase its capacity for handling non-linear, non-homogeneous problems. Zienkiewicz et al (1968), for instance, have developed the Newton-Raphson approach (otherwise known as the "iterative stress release and transfer" approach) for a more realistic stress analysis in the so-called "no-tension" material. The same approach, appropriately modified, will be presently adopted to handle the strain-softening behaviour frequently associated with soils which are vulnerable to progressive failure. The development of the finite element solution adopted has been dealt with in considerable detail elsewhere. Due to space limitations, only a brief outline of the procedures will be given in the following section.

Consider the idealized stress-strain curve for strain-softening material shown in Fig. 1.

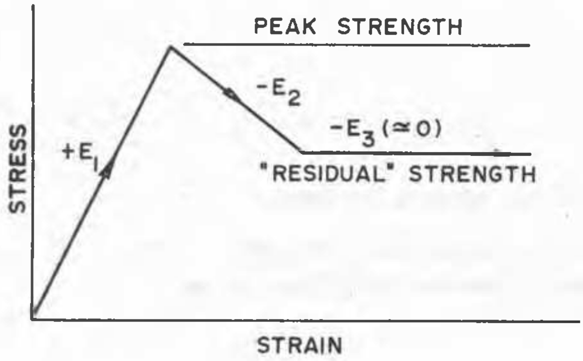


FIG. 1. SIMPLIFIED STRESS-STRAIN RELATIONSHIP FOR WORK-SOFTENING CLAYS

The curve is comprised of: (i) a linear pre-peak portion with modulus E_1, (ii) a descending linear portion from the peak to the "residual strength" level, having negative modulus $-E_2$, and (iii) a nearly horizontal linear post-peak portion with modulus E_3 whose value is close to zero (say, -0.1 psf). It is important to note that the term "residual strength" is used herein to designate the slowly decreasing strength in the post-peak range corresponding to moderately large displacement in the field, not the residual strength corresponding to extremely large displacement in the sense of Bishop et al (1971).

The method used in the finite element solution consists essentially of the following steps of operation.

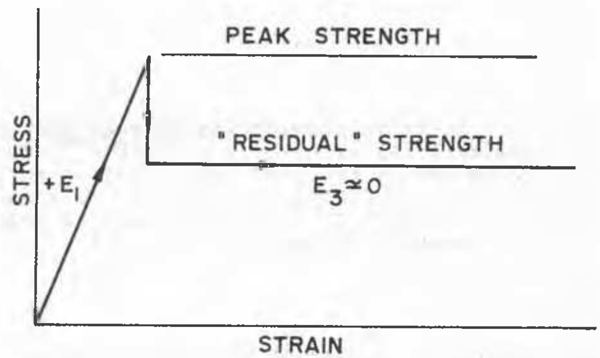
a) An elastic analysis is performed with modulus E_1.

b) The state of stresses determined are compared with a chosen failure criterion (fully drained or undrained). "Excess" stresses are removed from "overstressed" elements by applying systems of stresses equal in magnitude but opposite in sign. Simultaneously, the strains for these "overstressed" elements are brought back to the peak condition.

c) New stiffness matrices are generated in each subsequent step of stress release and transfer and the post-peak relationship is obeyed in elements where the peak strength is exceeded.

d) The process is repeated until convergence is finally obtained (i.e. until no significant amount of "excess" stress can be detected in any element).

While it is possible to obtain the above numerical solution with the immediate post-peak portion of the idealized stress-strain curve taken as a descending line with a negative modulus $-E_2$, the measurement of the post-peak stress-strain or load-deformation relationship often requires a stiff testing system to ensure accurate determination of the rapid post-peak decrease in strength. Experimental results meeting this requirement are scanty. At present (1972), it is appropriate and conservative to replace the descending portion by an abrupt drop from the peak to the residual strength level as shown in Fig. 2. For this post-peak stress-strain relationship, the procedures for a stress analysis are essentially the same as those described previously. The "overstressed" elements are again identified after the elastic stresses are computed. The shear stress level will be brought down to the "residual" state instead of the peak state, and a new value of modulus E_3 will be assigned to the failure planes of these elements, the rest of the computing process being unchanged.



TYPICAL RESULTS OF STRESS ANALYSIS

To illustrate the application of the finite element programs developed, some of the results of a parametric study of the stress distribution around excavated slopes will be presented. Consider the excavated slopes shown in Figs. 3(a),(b) and (c), each of height 30 feet and inclination 25°, having a water table 5 feet below the ground surface under the long term condition. The soil parameters chosen are: $c'=300$ psf., $\phi'=30°$ for peak strength, $c_r'=0$, $\phi_r'=15°$ for residual strength, pre-peak modulus $E_1=2000$ psi., Poisson's ratio $\mu=0.35$ for the fully drained condition, and bulk density $\gamma=125$ pcf. To study the effect of in-situ stresses, three values of the effective coefficient of earth pressure at rest are being used: $K_0=0.5$, 1.0 and 2.0 respectively. With the stress-strain relationship shown in Fig. 2, and the procedures outlined in the previous section, analyses were performed. The results are shown in Figs. 3(a),(b),(and (c) respectively, in the form of "shear stress level" contours. They represent the state of stresses in the slopes when the fully drained condition is being reached.

The shear stress level "λ" is herein defined as the ratio of the shear stress to the peak strength for elements in which the peak strength has not been reached. For elements that have failed, it is the ratio of the shear stress to the "residual strength". It follows that $\lambda=1$ indicates regions in which the strength has fallen off to the "residual" state.

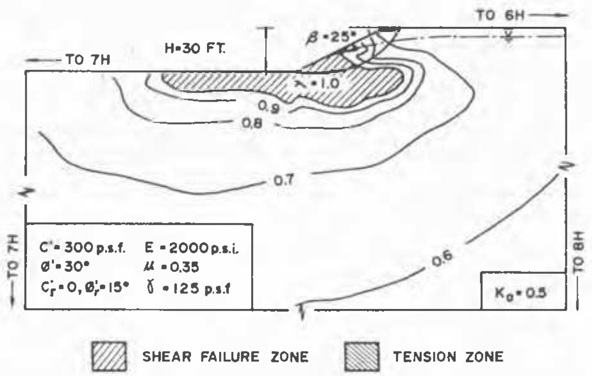


FIG. 3(a). SHEAR STRESS LEVEL CONTOURS FOR K_0=0.5

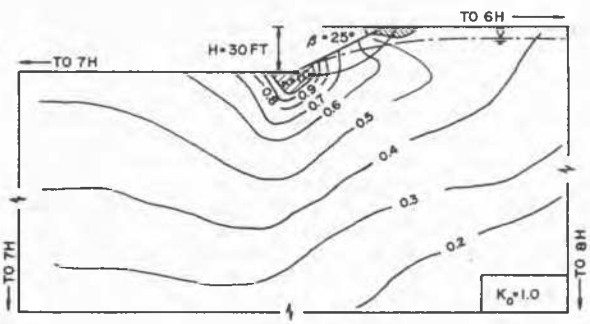


FIG. 3(b). SHEAR STRESS LEVEL CONTOURS FOR K_0=1

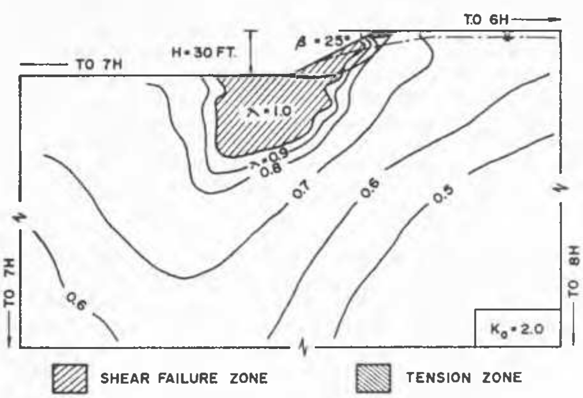


FIG. 3(c). SHEAR STRESS LEVEL CONTOURS FOR K_0=2.0

For each value of K_0, it may be seen that the portion of the slope around the toe has passed into the residual state. The physical existence of a failure zone at the toe is consistent with field observations in brittle clays. Skempton and La Rochelle (1965), for instance, have described the phenomenon of "clay burst" in some of the slopes in London Clay in the Bradwell area. However, the extent of the failure zone below the bottom of the excavation is probably overestimated in tne examples given here, since both the modulus of deformation and peak strength parameters often increase with depth in reality. For the cases studied it can be seen that the minimum failure zone is associated with K_0=1.0 [Fig.3(b)], (zero initial shear stress). Results for the K_0= 0.5 [Fig. 3(a)] suggest that a significant failure zone may also be formed in a slope of normally consolidated or slightly over-consolidated clays. For values of K_0 greater than unity, it appears that both the shear stress level and the size of the failure zone increase with K_0, lending support to Bjerrum's hypothesis (1967) that high lateral stresses play an important role in the initiation of progressive failure in over-consolidated clays and clay-shales.

AN INTEGRATED APPROACH TO STABILITY ANALYSIS

While it is possible to determine the state of stress in a slope by the finite element method, the latter does not give any direct measure of the overall stability of a slope. Limit equilibrium methods, on the other hand furnish a value of the factor of safety but fail to take into account both the constitutive relationships of the material and the effect of initial stresses. It is therefore desirable to combine the advantages of both approaches by incorporating the results of finite element analysis into the limit equilibrium analysis. For this purpose, the concept of Skempton's residual factor appears to provide the crucial link.

The residual factor R, as originally defined by Skempton (1964), is "that proportion of the total slip surface in the clay along which its strength has fallen to the residual value". This physical definition will be presently retained, and its numerical value is now given by R=D/L, where L=length of the slip surface, and D = that portion of L along which residual strength operates.

Now, if it is accepted that the stress-strain (or load-displacement) relationship measured in the laboratory may be applied directly to the field, then D (and hence R) may be determined by the finite element analysis previously described.

In Figs. 3(a),(b), and (c), the "critical circles" defined by limit equilibrium method are shown. It may be observed that if the strain-softening effect has been taken into account, a portion of the potential slip surface would have passed into the "residual" condition. The value of the residual factor R can be directly measured from these figures. A typical plot of R versus slope height and slope inclination is shown in Fig. 4, for K_0=1.0. Fig. 5 shows typical relationship between R and K_0 for a slope of height 30 feet and inclination 25°. The effects of slope geometry and K_0 on R are apparent from these figures.

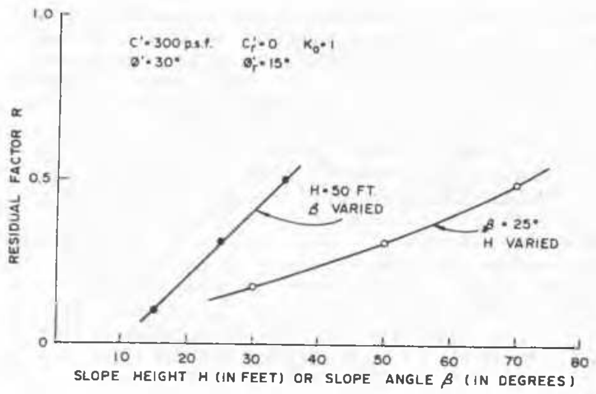


FIG. 4. EFFECTS OF SLOPE GEOMETRY ON RESIDUAL FACTOR

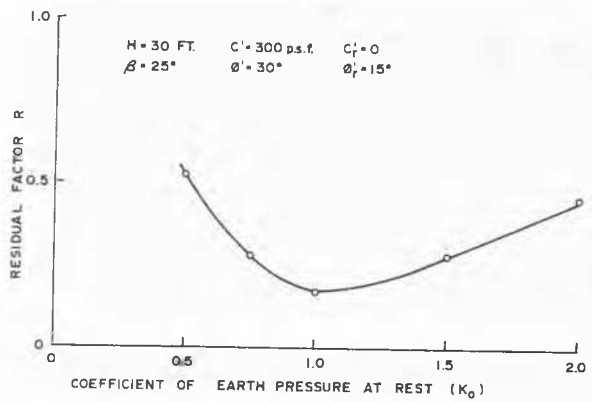


FIG. 5. EFFECT OF K_o ON RESIDUAL FACTOR

To take into account the effect of the "overstressed" zone on slope stability, it is convenient to define a "corrected" factor of safety F_c such that the "residual strength" governs the "overstressed" portion of the potential slip surface, and the peak strength governs the pre-peak portion of it. This correction for the post-peak strain-softening effect has been made for the slopes shown in Figs. 3(a),(b) and (c). The factor of safety with respect to peak strength F_p is 1.58 and that with respect to residual strength is 0.44 for all of the three slopes concerned, since the effect of in-situ stresses does not enter into computation. From finite element analyses, the residual factors for these slopes are 0.52, 0.17 and 0.45 respectively. Hence, taking into account the strain-softening effect, the factor of safety F_c falls from 1.58 to 1.06, 1.45 and 1.16 respectively for the three values of K_0 of 0.5, 1.0 and 2.0. It is obvious therefore, that by using this approach the engineering dilemma of choosing peak or residual strength parameters for design may be resolved.

The above approach may be extended to treat the time dependent phase of progressive failure. For clays that exhibit decrease in drained strength with time, the overstressed zone will propagate, hence the residual factor increases and the factor of safety decreases. It is clear, therefore, that both the residual factor and factor of safety are functions of time after slope formation. For a complete quantitative analysis of the problem, it is convenient to establish the following relationships:

a) a relationship between the residual factor and factor of safety,

b) a relationship between the residual factor and time, and finally

c) a relationship between factor of safety and time.

To establish relationships (b) and (c), the rate of decrease of drained strength with time must be known. The effect of time on the drained strength has been discussed by Lo (1972) who suggested a linear relationship between the drained strength and logarithm of time to failure, with the drained residual strength as the lower limit. Given this (or other) rate of drained strength decrease, then for each time interval "t" after slope formation, the finite element analysis is made using the drained strength which is operative at t. The extent of the "overstressed" zone and the residual factor R may therefore be determined. The factor of safety at t (F_t) may be computed by an appropriate limit equilibrium analysis.taking the overstressed zone into account. The procedure is repeated for several time intervals, from which R and F_t are determined for different times. The relationships of R versus F_t, R versus t, and F_t versus t are therefore established. This entire procedure of predicting time to failure of a slope is illustrated by the analyses of case records in a subsequent section. Before proceeding to the case histories, however, it is necessary to recapitulate the simplifying assumptions made in the approach expounded so far, thus enabling an assessment of the errors involved in the prediction of the life time of a slope.

a) The assumption of the abrupt decrease from peak to "residual" strength will result in an overestimate of the extent of the "overstressed" zone and therefore the residual factor at any time. From a preliminary study using the stress-strain relationship depicted in Fig. 1, the error involved appears to be not unduly large.

b) The analysis is made entirely in terms of effective stress under long term conditions and the transition from short to long term behaviour is neglected. For slope failure that occurs decades after excavation, it is reasoned that the error involved in neglecting the transition period would not be important, and is on the conservative side.

c) The assumption of constant deformation modulus and strength parameters with depth will usually, though not necessarily, lead to an overestimate of the residual factor. The assumption is easily removed if detailed experimental data are available for a particular site.

It is clear, therefore, that all these assumptions are on the conservative side and, other conditions being equal, the approach will theoretically underestimate the time to failure of a slope.

CASE HISTORIES

To illustrate the above approach to stability analysis, three case histories will be presently examined. All of these are first-time slides in slopes of London Clay, which have been chosen because:

a) Documented first-time slides in London Clay cover a wide spectrum of time to failure from shortly after slope form ation to over 80 years;

b) The soil parameters required as input data in the finite element analyses are well-defined and well-documented for London Clay. An average value of K_0=2.5 can be taken from Skempton (1961) and Bishop et al (1965). An average value of the pre-peak modulus E_1 of 2000 psi. can be measured from the stress-strain curves given by Bishop et al (1965), and be used in the stress analyses.

The case histories analysed are: the Northolt slide of 1955, the Sudbury Hill slide of 1949, and the Upper Holloway slide of 1951. These cuttings failed 19, 49 and 81 years respectively after slope formation. In view of the relative simplicity of the construction history at the Sudbury Hill site, this case record will be first discussed in detail.

The Sudbury Hill Slide 1949

Detailed description of this slide may be found in Skempton (1964), and Skempton and Hutchinson (1969). Fig. 6(a) shows the section which failed in 1949. The piezometric line shown corresponds to the highest levels recorded within the period from January 1956 to March 1957. The slip surface was not observed at depth, and its shape as drawn is based on analogy with the Northolt slide. Average values of index properties for London Clay in this area are: w=31, LL=82, and PL=28. Drained tests on brown London Clay at seven different sites from depths between 6 and 22 feet yield the peak strength parameters c'=320 psf and ϕ'=20°. These tests were made on 6 cm. square shear box 1½ in. x 3 in. triaxial specimens, typically with a time to peak failure of about one day. For residual strength parameters, the values recommended for use by Skempton are: c_r'=20 psf., ϕ_r'=13°.

Stability analyses with non-circular sliding surface yield a factor of safety of 2.27 with respect to peak strength, and 0.74 with respect to residual strength. It follows

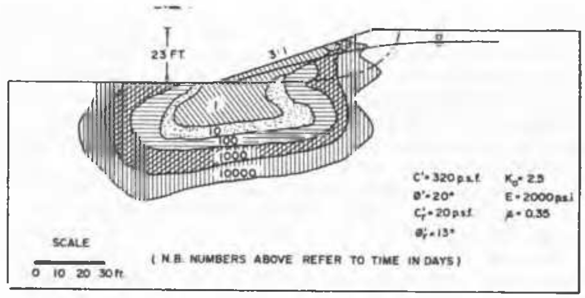


FIG.6(a). SUDBURY HILL SLIDE 1949:PROPAGATION OF FAILURE ZONE WITH TIME

that the average strength mobilized at the time of failure lies between the peak and the residual values. In other words, prior to the occurrence of the slide, some portion of the slip surface had fallen to the residual state. Obviously, the effect of time cannot be disregarded in this case. Skempton suggested that there might have been a very significant reduction in drained strength with time due to softening.

For purpose of the present analysis, the rate of reduction in drained strength with time has to be determined. Let this be denoted here by "k", and let its value be expressed in % per log cycle of time (Lo, 1972). Undrained tests on brown London Clay indicate that the value of k under the undrained condition is somewhat higher than 6% (Skempton and La Rochelle, 1965). No information is available, however, as to what value should be expected under the drained condition. It seems unlikely that this value is appreciably higher than 10% Reduction in drained strength of that order of magnitude would have been detected in the long-term creep tests by Bishop and Lovenbury (1969), who suggested that k<4.8% for one test. For the present purpose, it is not unreasonable to assume a value of k=6% as the average rate of strength decrease per log cycle of time under the drained condition, but values of k of 4% and 10% are also used to complete the analysis. An alternative view is to back-figure k using this record, and the value determined will be applied to other case records in the same clay, so that the consistency of results may be examined.

Thus, in the reanalysis of the Sudbury Hill slide, a stress analysis was first made using the conventional peak strength parameters. The peak strengths were each decreased by a specified amount for each log cycle of time. Excess stresses therefore appeared in some of the pre-peak elements, and were accordingly redistributed by the iterative process of stress release and transfer. The strength reduction and stress redistribution process was repeated for four successive times (simulating the process of strength decrease in 4 log cycles of time). The propagation of the failure zone during these 4 log cycles of time was thus determined, and Fig. 6(a) shows the results for the k=6% case. The change of residual factor with time may then be deduced [Fig.6(b)]. The graphical relationship between the factor of safety and the residual factor is shown in Fig.6(c) With the residual factor at the end of each log cycle of time determined in the finite element analysis, the corresponding factor of safety can be read off from Fig. 6(c). This makes possible the final plot of factor of safety versus time shown in Fig. 6(d). On extrapolating the curve to factor of safety = 1.0, the time of slope failure can be estimated. It is interesting to note that the k=6% case gives a predicted time rather close to 40 years, whereas the k=10% case predicts failure in no more than 3 years. The very high sensitivity of the predicted time to the value of k chosen is a consequence of the logarithmic relationship between strength and time. To evaluate the admissibility of the results, additional case histories have to be analyzed on the basis of the k=6% assumption, and the overall consistency may then be verified.

The Northolt Slide 1955

Detailed description of this slide may also be found in Skempton (1964), and Skempton and Hutchinson (1969).

The Northolt cutting was first excavated in 1903 with slopes at 2.75:1. In 1936 widening of the cutting took place and the new slope was made at 2.5:1 with a small concrete toe wall (Fig. 7). Failure occurred in January, 1955, 19 years after the second excavation. Piezometers were installed in November, 1955. Portions of the observed slip surface showed a markedly non-circular shape (Fig. 7). The average values of the index properties are: w=30, LL=79, and PL=28. The peak and residual strength parameters used in the Sudbury Hill analysis are also applicable to this case. Stability analyses gave 1.63 for factor of safety with respect to peak strength, and 0.54 with respect to residual strength. Again, softening is believed to be the mechanism responsible for the strength drop.

The stress history at the Northolt site has been considerably complicated by the sequence of excavation and the construction of the small toe wall. Nevertheless, the stress change can still be allowed for in the finite element analysis by simulating the excavation process of 1903 in the first place and subsequently making the necessary strength decrease and stress redistribution. The 19 years between 1903 and 1936 (4 log cycles of time) corresponds to a strength decrease of 24% based on the k=6% assumption. The stresses obtained after making this decrease will be used as initial stresses in the simulation of the 1936 excavation process which is now the datum of time to failure. The effect of the small toe wall, however, will have to be neglected.

Analysis was thus made with the procedures previously described, and the final plot of the factor of safety versus time is given in Fig. 9. The predicted time to failure based on the k=6% assumption is approximately 14 years which compares well with the actual time to failure of 19 years.

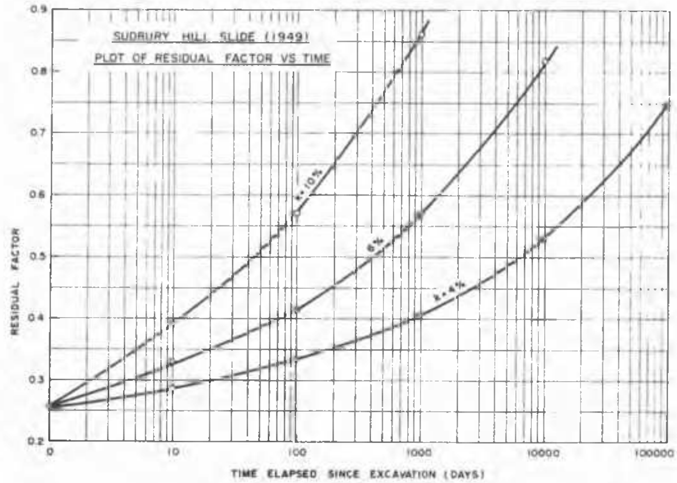


FIG.6(b). RELATIONSHIP OF RESIDUAL FACTOR

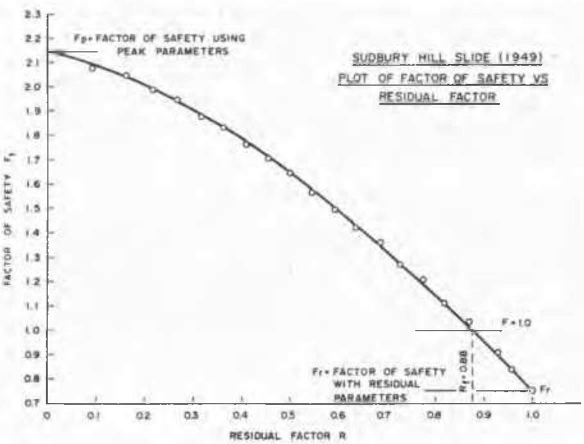


FIG.6(c). RELATIONSHIP BETWEEN FACTOR OF SAFETY AND RESIDUAL FACTOR

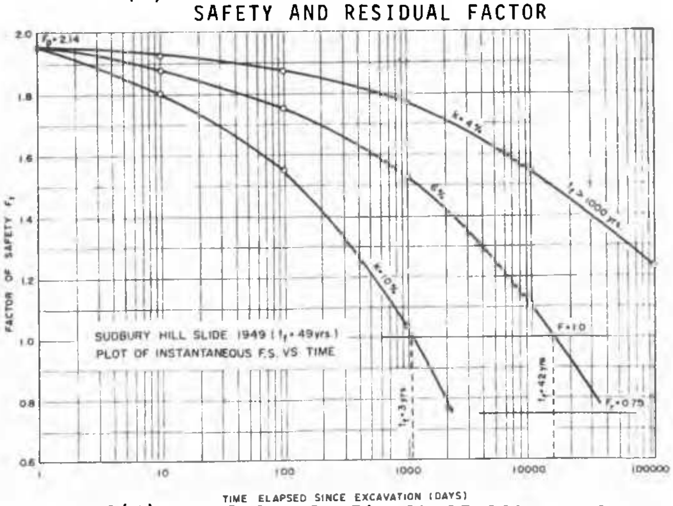


FIG.6(d). DECREASE IN FACTOR OF SAFETY WITH TIME FOR DIFFERENT k

The Upper Holloway Slide 1951

Detailed description of the slide may be found in De Lory (1957). The cutting, apparently excavated in 1870, is 18 ft. deep and entirely in weathered London Clay. The lower 4 feet of the slope was held by a brick retaining wall and the upper portions runs back at 2.75 to 1 from the wall (Fig.8). Mass movement occurred in 1951, 81 years after excavation. The slip surface was not observed, but piezometric measurements were taken over the year beginning April, 1956. The Atterberg limits of brown London Clay in the area are LL=86, PL=28. The peak strength parameters as determined by De Lory are c'= 250 psf., ϕ'=20°. The residual strength parameters for the Northolt and Sudbury Hill sites are assumed to be also applicable to the Upper Holloway site.

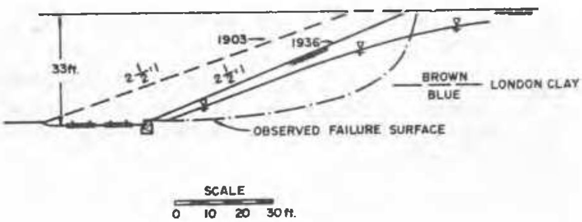


FIG. 7. SECTION THROUGH SLIDE IN NORTHOLT CUTTING AFTER SKEMPTON (1964)

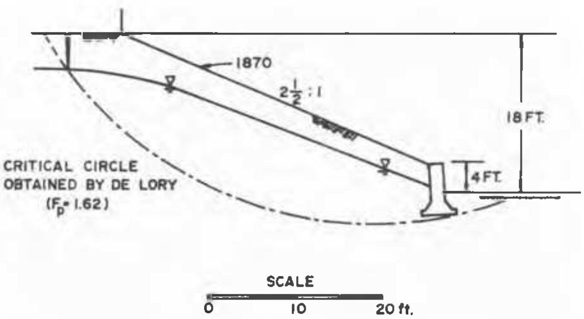


FIG. 8. SECTION THROUGH SLIDE IN UPPER HOLLOWAY CUTTING (AFTER DE LORY 1957)

In the finite element analysis performed, the small retaining wall has been ignored. Since the actual slip surface was not observed, the critical circle as determined by De Lory (1957) has been used. For this critical circle, the factor of safety is 1.62 with respect to peak strength, and 0.55 with respect to residual strength.

Analysis was made employing the procedures previously described, and the final plot of the factor of safety versus time is shown in Fig. 9. The predicted time to failure based on the k=6% assumption is about 55 years, as compared with the actual time to failure of 81 years.

From the results of analyses of the three case records, it is obvious that overall consistency in the prediction of time to failure has been obtained. The predicted times are 15% to 30% lower than the actual values, for reasons that have been theoretically expected. Taking the actual time to failure, the calculated factors of safety at the instant of failure are 3% to 5% below unity. This close agreement substantiated the validity of the approach used herein.

It is also of interest to note that the horizontal displacements at the toe (excluding short term movement) up to the time of failure determined by the above analyses are 18, 17 and 15 in. respectively for the Northolt, Sudbury Hill and Upper Holloway slides. No direct measurement is available at these three sites. At Kensal Green, which was also a first-time slide in London Clay, Skempton and Hutchinson (1969) quoted that the movement was 14 in. before failure.

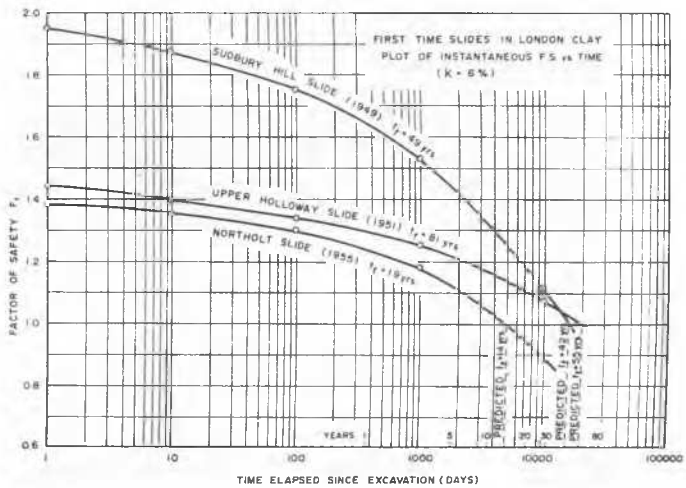


FIG. 9. DECREASE IN FACTOR OF SAFETY WITH TIME FOR THREE CUTTINGS IN LONDON CLAY

CONCLUSIONS

A finite element solution for the determination of stresses and displacements in slopes of strain-softening soils has been developed. An investigation of the various factors that affect the extent and propagation of the overstressed zone has been carried out. Typical results obtained with slope geometry and soil properties commonly encountered in practice indicate that the extent of the overstressed zone, defined by the residual factor:

a) increases with the inclination and height of the slope,

b) increases with values of the coefficient of earth pressure at rest departing from unity.

A method for stability analysis of slopes taking progressive failure into account has been developed by combining the finite element solution and limit equilibrium method, through the use of the residual factor. The method enables the effect of in-situ stresses on stability to be quantitatively evaluated.

The analysis is further developed to treat the time-dependent phase of progressive failure, provided the rate of decrease of drained strength with time is known. This rate may be determined from long term tests in the laboratory and/or backfigured from previous first-time slides. The detailed procedure for computing the decrease in Factor of Safety with time is illustrated by the reanalysis of three case records of cuttings in London Clay. The results showed that the time to failure may be reasonably estimated. The displacements are also consistent with available observations.

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