ANISOTROPY AND NON-LINEARITY IN SAND PROPERTIES

L'ANISOTROPIE ET LA NON-LINEARITE DANS LES PROPRIETES D'UN SABLE

АНИЗОТРОПИЯ И НЕЛИНЕЙНЫЙ ХАРАКТЕР ДЕФОРМИРОВАНИЯ ПЕСКА

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SUMMARY.- Numerical methods of analysis for soil engineering structures have placed emphasis on elastic models of soil behaviour. In order to take into account the observed non-linear and anisotropic response of soils a model is proposed based on cross-anisotropic elastic behaviour, the non-linearity being accounted for by a series of linear increments. Volume strains may be of opposite sign to stress changes without violating strain energy requirements, this being in accordance with observed soil behaviour. The cross-anisotropic model is considered to be a reasonable approximation to the mechanical anisotropies of lower order that generally occur in nature. Adoption of the model requires the definition of five parameters, four relating to direct stress and the fifth to shear stresses. The use of triaxial, three-axial, and torsion triaxial tests to measure these is discussed with reference to symmetry and strain energy. The influence of stress level on moduli and Poisson's ratios is examined.

INTRODUCTION

Current methods of soil engineering analysis are increasingly directed at prediction of behaviour under working loads. This is often achieved by allotting the soil the properties of an elastic body and applying the exact or approximate analyses available for the particular loading case. The reliability of the results will depend on the correspondence in the behaviour of the soil and the elastic body used to represent it. This paper proposes a model of sand behaviour as a non-linear anisotropic material and suggests how the material properties may be obtained.

Previously, sand has been shown to exhibit non-linear stress-strain response with irrecoverable deformation (energy loss) (Trollope *et al.*, 1962; Morgan, 1966) and mechanical anisotropy that depends both on the soil formation processes and subsequent applied stress paths (Biarez, 1961; Gerrard, 1967). The exact modelling of such behaviour presents considerable problems so that the following realistic approximation is suggested for cases of loading and unloading. The sand behaviour is modelled by considering the non-linearity to be composed of several linear increments, where, in any increment, the sand is assumed to behave as a linear elastic (no energy loss) anisotropic material whose properties are such as to develop exactly the same strains as the sand when subjected to the same stress path.

The ultimate aim is to model the behaviour of half-spaces, slopes and excavations subject to stress changes. In these real situations the mechanical response of the material is non-linear, anisotropic and with energy loss. Initial non-homogeneity in the soil mass can be allowed for by testing a series of samples chosen to represent relatively homogeneous zones. The field stress paths occurring in these zones is estimated and duplicated as closely as possible as a series of increments in the loading of these samples (Lambe, 1967). This incremental stress change approach, in which each increment is characterized by linear anisotropic elastic properties that depend on the stress level of the increment, implies that the initial pattern of non-homogeneity is significantly modified by subsequent stress changes.

Previously workers have attempted to model soil behaviour by isotropic elastic behaviour. However such material cannot undergo volume changes of opposite sign to the applied stresses, unless Poisson's ratio exceeds 0.5, which is the maximum possible for the strain energy to remain positive. Such volume change restrictions do not apply to anisotropic materials and it appears that parameters satisfying strain energy can always be found provided the symmetry of the mechanical anisotropy of the model approximates to that of the soil. Thus these models can be expected to be more meaningful in analysis.

The simplest type of anisotropy relevant to soils is cross-anisotropy, i.e. having an 'n' fold axis ($n \to \infty$) of symmetry which is usually assumed vertical. This involves the elastic parameters E_h, E_v, ν_h, ν_{vh}, ν_{hv} and

$F_v(=2G_v)$ of which only four of the first five are independent (Gerrard and Wardle, 1972)*. By analysing the response to changes of one principal stress only two of these four can be directly found in the conventional triaxial test, and in a three-axial test†, only three. Later it is shown how the possible range of the other parameters may be defined provided more than one principal stress varies. The shear modulus ($F_v=2G_v$) can only be found by applying a horizontal-vertical shear stress coupled with the direct determination of the resultant corresponding shear strain.

The cross-anisotropic model suggested is simple and can be expected to hold for cases where its form of symmetry is approached both in the initial structure of the soil and in the stress patterns produced by the applied load. In many cases the anisotropy of the initial structure or of the applied stress path may be of a lower order of symmetry than that of cross-anisotropic, e.g. monoclinic symmetry with a 2-fold vertical symmetry axis. However, attempts to model this would require many more parameters so that unless there is a significant departure from an 'n' fold axis of symmetry the gain in accuracy over the simpler model will be small. The cross-anisotropic model is the simplest possible after the isotropic model, but has the following features that more realistically model soil behaviour;

a) $E_h \neq E_v$

b) $G_v \neq E_h \div (1+\nu_h)$

c) Volume change can be of opposite sign to applied stress.

The concept of modelling is stressed in this work. The parameters E_h, E_v, ν_h, ν_{vh}, ν_{hv} and G_v are not intrinsic properties of the soil but refer to the mathematical model used to represent the soil mass response and thereby predict its behaviour.

STRAIN ENERGY AND VOLUME CHANGE

The requirements for positive strain energy for a cross-anisotropic material are (Pickering, 1970):

$$\left.\begin{array}{c} E_h > 0; \quad E_v > 0; \quad F_v > 0 \\ 1 - \nu_h > 0 \\ 1 + \nu_h > 0 \\ 1 - \nu_h - 2\nu_{hv}\,\nu_{vh} > 0 \end{array}\right\} \qquad (1)$$

The last three may be expressed as:

$$\left.\begin{array}{c} -1 < \nu_h < 1 \\ 1 - \nu_h - 2(E_h \div E_v)\nu_{vh}^2 > 0 \end{array}\right\} \qquad (2)$$

(It is convenient here to use the relationship $\nu_{hv} = \nu_{vh}.E_h \div F_v$).

Equations 2 define a region in ν_h, ν_{vh}, $E_h \div E_v$ space in which strain energy is positive and therefore must contain the parameters for the cross-anisotropic elastic model.

When the model is subject to an axi-symmetric stress change of $\widehat{zz}$ (vertical) and $\widehat{rr} = \widehat{\theta\theta}$, the volume change is:

$$\varepsilon_{zz} + \varepsilon_{rr} + \varepsilon_{\theta\theta} = \frac{\widehat{zz}}{E_v}(1-2\nu_{vh}) + \frac{\widehat{rr}+\widehat{\theta\theta}}{E_h}(1-\nu_h-\nu_{hv}) \quad (3)$$

Since E_v and E_h must be positive, and assuming that all stresses are compressive, the volume change characteristics may be examined by considering only the signs of $1-2\nu_{vh}$ and $1-\nu_h-\nu_{vh}(E_h \div E_v)$. This is most conveniently done by plotting the behaviour in a ν_h, ν_{vh} plane for particular values of $E_v \div E_h$ and in conjunction with the strain energy limitations (equations 2). This is shown in Fig. 1 where the inverted parabola between $\nu_h = \pm 1$ contains the positive strain energy region; the line labelled 'a' is the relationship $1 - \nu_h - \nu_{vh}(E_h \div E_v) = 0$ and 'b' is $1 - 2\nu_{vh} = 0$.

The lines 'a' and 'b' always intersect on the parabola creating either two or three zones depending on whether $E_v \div E_h$ is smaller than ¼ or greater than ¼.

These zones are labelled in Fig. 1 and have the following bounds and volume change characteristics.

Zone I: In this zone the volume change is of the same sign as the applied stress. The zone is to the left of line 'b' and when $E_v \div E_h > ¼$ it must also be to the left of line 'a'.

Zone II: $1-\nu_h-\nu_{vh}(E_h \div E_v)$ is negative but 244
$1-2\nu_{vh}$ is positive. Volume changes of opposite sign to the applied stress may occur if $(\widehat{rr}+\widehat{\theta\theta}) \div E_h$ is sufficiently large compared with $\widehat{zz} \div E_v$. This zone is to the right of line 'a' above its intersection with line 'b'.

Zone III: $1-2\nu_{vh}$ is negative and $1-\nu_h-\nu_{vh}(E_h \div E_v)$ is positive. Volume changes of opposite sign to the applied stress may occur if $\widehat{zz} \div E_v$ is sufficiently large compared with $(\widehat{rr}+\widehat{\theta\theta}) \div E_h$. This zone only occurs if $E_v \div E_h > ¼$ and is

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* E_h and E_v are the Young's moduli in the horizontal and vertical directions respectively. ν_h, ν_{vh}, ν_{hv} are Poisson's ratios, ν_h indicating the effect of strain in a horizontal direction on the strain developed in the complementary horizontal direction, ν_{vh} indicates the effect of strain in the vertical direction on that developed in a horizontal direction and ν_{hv} indicates the effect of strain in a horizontal direction on that developed in the vertical direction. F_v is a shear modulus relevant to shear strain occurring within vertical planes.

† A test in which three independent normal stresses are applied to the faces of a soil cube.

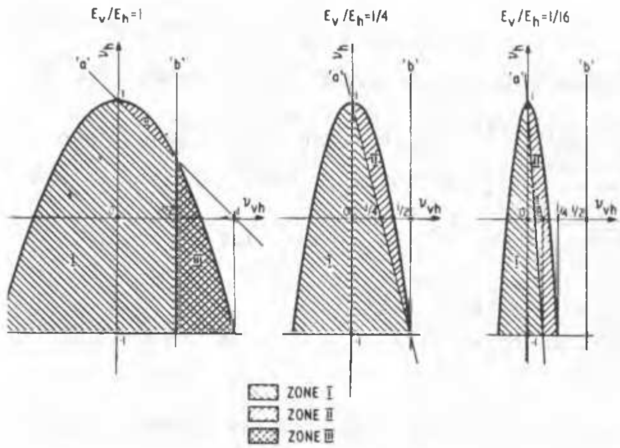

FIG. 1. STRAIN ENERGY AND VOLUME CHANGE ZONES

to the right of line 'b' below its intersection with line 'a'.

EXPERIMENTAL MEASUREMENTS

All the results refer to measurements on a closely graded medium-fine sand (d_{50} = 0.35mm d_{10} = 0.16mm). Samples were prepared by raining in air to a relative density of about 75 per cent. Two types of apparatus were used, conventional triaxial apparatus incorporating low friction ends for testing 102mm dia. by 203mm high samples, and three-axial apparatus for 102mm cube samples. In both cases vertical deflection measurements were obtained by vernier microscope, and horizontal by transducers (Gerrard, 1967).

Separate fabric analysis of the prepared samples showed that the mode of deposition produced a vertical 'n' fold axis of symmetry. Ambient stress tests (Gerrard, *op. cit.*) demonstrated that, as expected, the mechanical response also exhibited anisotropy of an identical order of symmetry.

TABLE I

Increment No.	1	2	3	4	5
$\Delta\widehat{zz}$ (kPa)	35.8	42.0	42.0	42.0	42.0
$\Delta\widehat{rr}$ (kPa)	12.4	14.7	14.6	14.7	14.6
Triaxial					
$\Delta\varepsilon_{zz} \times 10^{-3}$	1.63	1.52	1.14	0.89	0.70
$\Delta\varepsilon_{rr} \times 10^{-3}$	1.30	-1.81	-1.52	-1.56	-1.17
Triaxial with torsion					
ΔT (Nm)	1.58	2.28	2.28	2.28	2.28
$\Delta\varepsilon_{zz} \times 10^{-3}$	1.98	2.06	1.60	1.15	1.02
$\Delta\varepsilon_{\theta z}$* $\times 10^{-3}$	1.04	1.89	1.40	1.03	0.65

* where $\varepsilon_{\theta z} = \frac{1}{2}\frac{\partial v}{\partial z}$

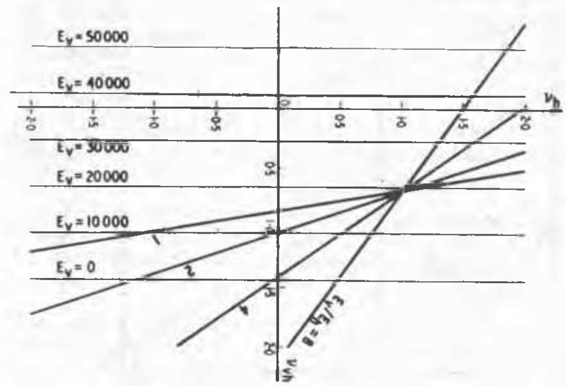

FIG. 2. E_v AND $E_v \div E_h$ CONTOURS FOR INCREMENT 3 IN THE TRIAXIAL TEST ($\widehat{zz} \div \widehat{rr}$ = 2.8)

A. TRIAXIAL AND TORSION TRIAXIAL TESTS

Two types of drained tests are considered here, both at a constant stress ratio $\widehat{zz} \div \widehat{rr}$ = 2.8, with one having in addition an applied torsion T to give a shear stress as a fixed proportion of $\widehat{zz}$ (Gerrard, *op. cit.*). The readings can be expressed as a series of increments over each of which a linear model may be fitted, the results being shown in Table I.

For both types of tests the increments in $\Delta\widehat{zz}$ and $\Delta\widehat{rr}$ were made the same and the results given represent the average of five tests.

From the basic elastic stress-strain relationships one may write for each increment (omitting the Δ notation)

$$\nu_{vh} = \left[\frac{\widehat{zz}}{\widehat{rr}} - (1-\nu_h)\frac{E_v}{E_h}\frac{\varepsilon_{zz}}{\varepsilon_{rr}}\right] \div \left[2 - \frac{\widehat{zz}}{\widehat{rr}}\frac{\varepsilon_{zz}}{\varepsilon_{rr}}\right] \quad (4)$$

This represents a series of straight lines in ν_{vh}, ν_h plane each having a different value of $E_v \div E_h$. For increment 3 of the triaxial test these lines are shown in Fig. 2.

Additionally, the following relationship exists:

$$E_v = \frac{\widehat{zz}}{\varepsilon_{zz}}\left(1 - 2\nu_{vh}\frac{\widehat{rr}}{\widehat{zz}}\right) \quad (5)$$

so that lines representing E_v as a linear function of ν_{vh} may also be drawn and are shown in Fig. 2.

Now, for the particular increment, allowable combinations of values of $E_v \div E_h$, E_v, ν_h, ν_{vh} may be obtained by determining the space common to Fig. 2 (the test measurements) and Fig. 1 (the strain energy limitations). In

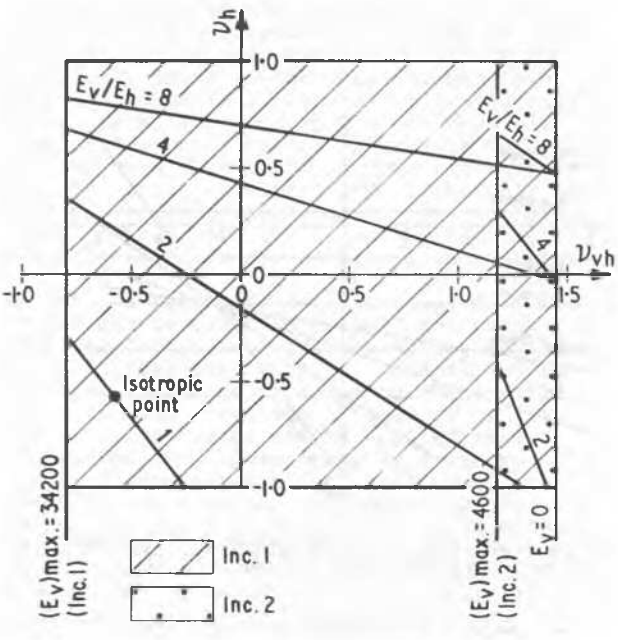

FIG. 3. ZONES OF POSSIBLE VALUES OF THE CROSS-ANISOTROPIC PARAMETERS FOR THE TRIAXIAL TEST ($\widehat{zz} \div \widehat{rr}$ = 2.8)

practice this is most easily done using a transparent overlay similar to Fig. 1 having the parabolas for various values of $E_v \div E_h$ constructed on a common set of axes.

The allowable space for the triaxial tests is shown in Fig. 3. In contrast to subsequent increments, the region for increment 1 is not usefully bounded and is not shown. This difference is attributed to the change in stress state from isotropic during sample preparation to anisotropic for all subsequent increments. The results show that ν_{vh} for the model can only lie in a very restricted range. For the later increments ν_h is also restricted, negative values being indicated. For these increments too, $E_v \div E_h$ is substantially greater than 1, confirming the expectations of the fabric analysis. Although these latter two parameters can take a much wider range of values than ν_{vh}, they are not independent and the selection of a particular value of one will restrict the other to a very narrow range.

The results for the torsion triaxial tests lie in an almost identical region in ν_h, ν_{vh} space. However, the maximum possible values of E_v are much lower than for the triaxial results, the values for both types of test being shown in Table II. By assuming shear strain to be a linear function of radius, values of G_v may be directly calculated in the torsion tests and are shown.

The non-linearity of the model, i.e. stress-dependence of stiffness, is well shown in Table II. It is of considerable interest

TABLE II

Increment No.	1	2	3	4	5
$(E_v)_{max}$ (kPa) (Triaxial)	48000	8300	6900	4800	4800
$(E_v)_{max}$ (kPa) (Torsion triaxial)	12400	855	725	1730	1730
$F_v(=2G_v)$ (kPa) (Torsion triaxial)	1080	850	1140	1550	2450

that, for the same increases in direct stress levels, $(E_v)_{max}$ decreases in the triaxial tests but increases in the torsion triaxial tests (omitting as before the first increment). For these latter tests $F_v(=2G_v)$ also increases with stress level. These different patterns of behaviour between the triaxial and torsion triaxial tests may be partially caused by the existence of the applied shear stresses, producing possible non-homogeneity of the samples and a lower order of symmetry in their mechanical properties.

B. THREE-AXIAL TESTS

The above analysis of triaxial tests has indicated a way in which values of moduli and Poisson's ratios may be found for cases where the anisotropy of both the initial structure and the applied stress path (where $\Delta\widehat{rr} \neq 0$) approximate to the same vertical 'n' fold **axis** of symmetry. In the three-axial apparatus, where three principal stress $\widehat{zz}$ (vertical), $\widehat{xx}$ and $\widehat{yy}$ may be independently applied and the corresponding strains found, it may be expected that all parameters except G_v can be deduced.

Three series of tests will be used here to 843
illustrate the determination of elastic parameters. In each of these, two of the principal stresses were maintained constant while the third was increased. This means that the modulus in the direction of the increasing stress may be directly calculated together with at least one Poisson's ratio.

The results are shown in Table III, the key being:

Test No.	Initial Stress State (kPa)	Final Stress State (kPa)
A1	$\widehat{zz}$ = 10.3 $\widehat{xx} = \widehat{yy}$ = 10.3	$\widehat{zz}$ = 71.7 $\widehat{xx} = \widehat{yy}$ = 10.3
A2	$\widehat{zz}$ = 69.0 $\widehat{xx} = \widehat{yy}$ = 69.0	$\widehat{zz}$ = 380.0 $\widehat{xx} = \widehat{yy}$ = 69.0
B1	$\widehat{xx}$ = 10,3 $\widehat{yy} = \widehat{zz}$ = 10.3	$\widehat{xx}$ = 71.7 $\widehat{yy} = \widehat{zz}$ = 10.3
B2	$\widehat{xx}$ = 69.0 $\widehat{yy} = \widehat{zz}$ = 69.0	$\widehat{xx}$ = 276.0 $\widehat{yy} = \widehat{zz}$ = 69.0
C	$\widehat{zz}$ = 10.3 $\widehat{yy}$ = 10.3, $\widehat{xx}$ = 31.0	$\widehat{zz}$ = 91.0 $\widehat{yy}$ = 10.3, $\widehat{xx}$ = 31.0

TABLE III

Test	Increment	1	2	3	4	5
A1	E_z (kPa)	16400	13100	7300	4700	2480
	$\nu_{zx} = \nu_{zy}$	0.38	0.90	0.76	0.82	1.30
A2	E_z (kPa)	72500	46700	28500	18200	13800
	$\nu_{zx} = \nu_{zy}$	.44	.50	.52	.69	.92
B1	E_x (kPa)	8750	6550	2410	1380	1790
	ν_{xz}	.33	.30	.64	.63	.71
	ν_{xy}	.33	.30	.49	-	-
B2	E_x (kPa)	44800	19700	7050	-	-
	ν_{xz}	.25	.44	.67	-	-
	ν_{yy}	.21	.23	.38	-	-
C	E_z (kPa)	25400	15600	9450	6950	-
	ν_{zx}	.12	.28	.18	.39	-
	ν_{zy}	.29	.47	.58	.66	-

In test C the initial, applied stress pattern, with $\widehat{xx} > \widehat{zz} = \widehat{yy}$, has an 'n' fold axis of symmetry in the x direction. However, the pre-existing soil structure, produced during sample formation, has an 'n' fold axis of symmetry in the z direction. As suggested by Lafeber and Willoughby (1971) resulting mechanical anisotropy of the soil is likely to possess only those symmetry elements that are common to both the initial applied stress pattern and the pre-existing structure, i.e. orthorhombic symmetry. This is confirmed to some extent since in loading beyond the initial stress state by increasing only $\widehat{zz}$ it is found that $\nu_{zx} \neq \nu_{zy}$. Equality of these two quantities would be required if there was a vertical 'n' fold axis of symmetry in the mechanical anisotropy.

This suggests that attempts to use the three-axial apparatus to find parameters of any anisotropic model by making individual movements of $\widehat{zz}$, $\widehat{xx}$, $\widehat{yy}$ about a required stress path can only be successful when the magnitude of these movements is very slight so that the symmetry of the anisotropy is not greatly altered. Such small movements require specialized experimental techniques to determine the associated strains. This philosophy can be applied to the determination of $E_v \div E_h$ from the results of the A series and B series tests. It is reasonable to assume that the initial portion of increment 1 in test B1 applies to the same cross-anisotropic material as corresponding portion of increment 1 in test A1, the results giving an $E_v \div E_h$ ratio of 1.9. Similarly the B2 and A2 tests give an $E_v \div E_h$ ratio of 1.6, where the lower ratio at the higher initial ambient stress suggests that the application of this symmetric ambient stress increment has reduced the initial degree of cross-anisotropy. A similar finding was reported by Karst *et al.* (1965).

With further increments of stress in the A series tests, the mechanical anisotropy of the samples will retain an 'n' fold axis of symmetry. However, in the B series tests the increase in $\widehat{xx}$ will almost certainly produce an increasing tendency to orthorhombic response with E_x increasing relative to E_y. Under these conditions the cross-anisotropic model becomes an increasingly poor representation of the soil behaviour. It is of interest to note that at all corresponding stages in tests A and B, E_z is greater than E_x reflecting the influence of the initial higher stiffness in the vertical direction.

The non-linearity of the ideal models is shown strikingly for all the tests in Table III. In all cases the modulus values decrease with increase in applied stress while the Poisson's ratios show marked increases. The values of the Poisson's ratios remain at all times within the limits set by the strain energy considerations of the cross-anisotropic model.

CONCLUSIONS

A method is suggested for describing the stress-strain behaviour of a sand. The model accounts for non-linearity by a series of linear increments in each of which the sand is represented by an elastic anisotropic material. The elastic properties of the model in any increment depend upon the initial structure and the subsequent stress history up to the particular increment. Because this dependence is largely taken into account in tests to measure the elastic parameters of the model no great error arises from the fact that no energy is lost in the model while energy is lost in the soil. The examples considered are for monotonic increases in stress. However, a similar approach can be adopted to cases of unloading and repeated loading by dividing the total stress path into several monotonic branches.

For a sand subjected to compressive stress increments the strain energy requirements may be invoked to show that the cross-anisotropic model allows for volume changes of either sign depending on the material parameters. In addition the cross-anisotropic model allows for other observed features in the mechanical response of sand, e.g. $E_h \neq E_v$. The use of such a model should thereby provide closer correspondence with the behaviour of a real soil than an isotropic model.

In triaxial tests with changes in vertical stress and cell pressure, the results may be plotted in ν_h, ν_{vh} space together with the strain energy limitations to define a region within which the elastic parameters must lie. The extent of this region varies greatly and, in some cases, may be relatively small, e.g.

when the volume change is of opposite sign to the applied stress. The fifth parameter ($F_v = 2G_v$) is a shear modulus that can only be found from tests in which a horizontal-vertical shear stress is applied.

Results of tests using a three-axial apparatus in which three principal stresses may be independently varied may be used to obtain some of the parameters directly. However, the type of stress change applied if of different symmetry to the original material symmetry (in its elastic properties) will reduce the order of symmetry so that the results only approximately apply to the original cross-anisotropic model. This occurs, for example, in the application of torsion in the triaxial test and in the application of increments of horizontal stress in the three-axial test.

In all the tests considered, the non-linearity of the models is shown up in the dependence of the moduli and Poisson's ratios on stress level. With the exception of the torsion triaxial tests, all moduli decrease with increase in stress level in both constant stress ratio and constant confining pressure tests, whilst all Poisson's ratios increase. The latter, however, remain within the bounds set by the strain energy requirements for a cross-anisotropic model.

Further developments in the use of cross-anisotropic* models to predict the field behaviour of earthen structures is necessarily coupled with the construction of testing apparatus in which field stress paths can be simulated. This usually requires the application of shear stress as well as three normal stresses. Work is proceeding on the development of such an apparatus which will enable the anisotropic properties for use in the model to be determined using the principles outlined in this paper.

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*or models of lower order of symmetry

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