KINETICS OF STRUCTURAL DEFORMATIONS AND FAILURE OF CLAYS

CINEMATIQUE DES DEFORMATIONS STRUCTURALES ET DE LA RUPTURE DES ARGILES
КИНЕТИКА СТРУКТУРНЫХ ДЕФОРМАЦИЙ И РАЗРУШЕНИЯ ГЛИН

S.S. VIALOV, Dr.Sc. (Eng.) Professor, Laboratory Head
YU.K. ZARETSKY, Dr.Sc. (Eng.) Senior Researcher
R.V. MAXIMYAK, M.S., Researcher
N.K. PEKARSKAYA, M.S., Senior Researcher, Research Institute of Bases and Underground Structures, GOSSTROY USSR, Moscow, USSR

SUMMARY. Tho paper sets forth a theory of the deformation and failure of clayey soils, based on an investigation of the kinetics of changes occurring in the soil microstructure. The investigations consisted of creep tests of soils with simultaneous observation of the changes in their structure. On the basis of these investigations, the mechanism of deformation and failure was revealed, and its laws were established. A generalized equation of deformation and an equation of long-term strength were derived. Results of investigations, carried out in recent years by the authors under the supervision of Prof. S.S.Vyalov, are summarized.

As a rule, soil mechanics equations are of a phenomenological nature and do not take into account features of soil structure and its changes in the deformation process.

A number of works, by Mitchell, et al, Murayama and Shibata, Christensen and Wu, etc., have appeared in recent years which dealt with equations of soil flow based the Eyring theory of rate processes. The application of this theory is a doubtless advancement in soil mechanics. It is necessary, however, to point out that the theory of rate processes deals with the flow process as motion of molecules of a liquid medium when, actually, soil deformation is related mainly to the displacement of structural elements of the soil - mineral particles surrounded by a film of adsorbed water. Therefore, a study of the deformation mechanism should be based primarily on an investigation of the kinetics of structural elements of the soil.

In the flow equation for a perfectly viscous fluid: $d\gamma/dt = \tau/\eta_0$, the coefficient of viscosity η_0, in accordance with molecular kinetic theory, has the following physical meaning:

$$\eta_0 = \frac{1}{A} e^{U/k\Theta} \qquad (1)$$

where U = activation energy, i.e. the energy it is necessary to impart to a particle in order to break its bonds with neighbouring particles and, overcoming the energy barrier, to pass over to the adjacent equilibrium state; k = Boltzmann constant; Θ = absolute temperature in ^{o}K; and A = a certain parameter which, strictly speaking, also depends on the temperature.

Equation (1) was derived for a viscous medium with invariable properties. In soils the viscosity varies in the deformation process. This is due, primarily, to changes in the soil microstructure in the creep process. In their turn, these changes lead to changes in the activation energy since the bonds are altered between the particles.

It shall be shown in the following that the basic changes in soil microstructure consist in the displacement, rearrangement and reorientation of particles on the one hand, and the development of impairments (microfissures and other defects), on the other. Consequently. the activation energy in equation (1) may be written as

$$U^* = U_o^* - U_d^* - U_{or}^* \qquad (2)$$

where $U^* = U/k\Theta$; U_o = initial energy required to break the bonds between the particles at the instant of time t = 0 (it is determined by the interparticle forces of interaction); U_d = change in the initial energy as a result of the breaking of bonds and the development of microfissures and other damages in structure upon stress action; and U_{or} = change in energy due to the displacement, rearrangement and reorientation of particles.

The aim of the conducted investigations was to study the kinetics of structural changes in soils and to develop a theory of the deformation and failure of clay in accordance with equation (2). Experimental investigations consisted in creep tests of a series of specimens of soils with simultaneous studies of the changes that occur in the soil microstructure.

Investigations were carried out on artificially prepared specimens of monomineral (kaolin) and polymineral (Jurassic) clays of semisolid consistency. The water content indices for the kaolin were: w = 38 to 40%, w_L = 58% and w_P = 38%; those of the Jurassic clay were: ŵ = 32%, w_L = 50% and w_P = 26% (with a content of fractions smaller than 0.005 mm equal to 56%).

The tests were conducted by subjecting hollow cylindrical specimens to torsion with different shear loads, constant for each specimen. The tests were interrupted at various stages of deformation and microsections were taken at several points of the specimens for studying the microstructure and its changes. Structural investigations were carried out with the aid of optical and electron microscopes. In order to record the origination and development of possible impairments in structure (microfissures, etc.), the soil was saturated with a liquid polymer (in a vacuum), followed by polymerization and fixing.

The change in soil structure was quantitatively evaluated by the degree of impairment $\omega = \frac{S_d}{S}$ 100% and the degree of orientation $\rho = \frac{S_{or}}{S}$ 100%, where S_d = area occupied by defects in structure; S_{or} = area occupied by particles oriented in the shear direction; and S = total area of the microsection. These quantities were measured by means of an optical microscope.

In determining index ω, microfissures from tens to hundreds of microns in size were taken into account as well as micropores of the same sizes. The inclusion of pores in the magnitude of ω was due to the impossibility of classifying pores and microfissures. Thus, the obtained value of ω should not be regarded as the absolute value of the degree of impairment, but as a relative value which represents with sufficient reliability the dynamics of microfissure formation.

The results of the creep tests are presented in Fig.1 by curves showing the development of deformation with time t at various shear loads τ. As is evident, the process is of a damped nature at low stresses. Undamped creep, ending the failure, occurs at high τ values. Correspondingly, in the first case the rate of deformation $\dot{\gamma} = d\gamma/dt$ approaches zero and in the second, after reaching a minimum value, it begins to grow (Fig. 1a).

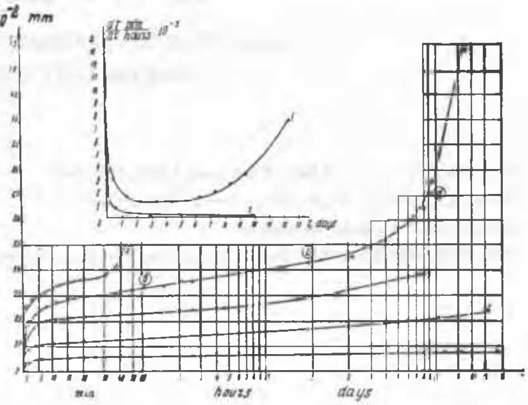

Fig.1.
Creep curves for shear (kaolin, w = 38%):
1. - τ = 83 g/cm^2; 2.- τ = 90 g/cm^2;
3. - τ = 100 g/cm^2; 4.- τ = 133 g/cm^2;
5. - τ = 165 g/cm^2.
(a) curves showing the change in the rate of deformation with time: 1 - τ=100 g/cm^2; 2 - τ = 93 g/cm^2.

The changes in the soil microstructure in the course of the above-mentioned processes can be presented as the scheme in Fig.2.*

The initial structure of the soil was of a block-disordered nature, 60 to 70% of the microsection area being occupied by aggregates consisting of clearly oriented particles. The space between the aggregates was filled with a randomly oriented mass of clayey particles.

To be noted in the first place upon damped creep is the reduction of the size and quantity of defects with time. Thus, the degree of impairment of kaolin equalled ω_0 = 24 to 25% in the initial state and, after deformation for 6 days at τ = 83 g/cm^2, it was reduced to ω = 21%. Subsequent changes in ω occurred with much less intensity and, at t = 22 days, the value of ω was reduced only to 20%. After this, deformation was stabilized. Along with the "healing" of the defects, displacement of the particles occurs and they become more densely packed, thereby strengthening the soil. This is the explanation of the increase in soil strength observed by many investigators following prolonged deformation under conditions of damped creep. Thus, in the experiments of S.S.Vyalov and N.K.Pekarskaya, both the instantaneous and long-term strength of kaolin specimens increased by 18 to 20% following preliminary deformation, in comparison with

---

* Photomicrographs of the changes in structure have been presented previously (Vyalov, Pekarskaya and Maximyak, 1970).

the strength of the initial specimens.

Next to be considered is the process of undamped creep. In the initial (1st) stage of this process, the above-described phenomenon of strengthening occurs. But, at the same time, micromovements of the particles occur together with the breaking of structural bonds, reorientation of particles, and the origination and development of microfissures. In other words, along with the strengthening, the soil structure is weakened. The predomination of weakening over strengthening leads to the development of irreversible plastic deformations - undamped creep.

The following results were obtained in investigating specimens of polymineral Jurassic clay after deformation (at $\tau = 400$ g/cm^2) for 6.5 hours, 8 and 40 days. Only slight structural changes were observed after 6.5 hours: the cavities were reduced somewhat and reorientation of the particles (in the shear direction) began in small, weakest regions between the aggregates or at points of stress concentration. After 8 days, more pronounced structural changes are observed: the number of oriented particles increases and the aggregates begin to break up and become reorientated. Microfissures are formed along the zones with ordered orientation. Even more noticeable changes occurred after 40 days of deformation. A large part of the aggregates were broken up so that they occupied only 20 to 30% of the area of the microsections. The reorientation process had developed to a degree in which 50% of the area of the microsections was already occupied by oriented particles. The number of microfissures had also increased sharply; the degree of impairment reached 32%.

Similar changes were observed for the monomineral clay (kaolin). This is illustrated by the curves in Fig.2 which show the development of the degrees of impairment and orientation with time.

No reorientation of particles has time to occur upon rapid failure. For example, upon failure of kaolin in two minutes (at $\tau = 180$ g/cm^2), the structure remains practically unchanged. Upon failure in 9 minutes (at $\tau = 160$ g/cm^2) a certain, though not very large, reorientation ($\Omega = 14.3\%$) is observed. Upon long-term failure (after 840 hours at $\tau = 100$ g/cm^2), the degree of reorientation reached 50%. As to the development of microfissures, it occurs both in long-term and rapid failure. For example, upon failure in t = 2 minutes (at $\tau = 166$ g/cm^2), the degree of impairment was $\omega = 36.8\%$ and after t = 1.2 hours (at $\tau = 133$ g/cm^2) it was $\omega = 36.3\%$. The degree of particle-reorientation can be regarded as depending primarily on the length of time for the deformation process, while the degree of impairment depends upon the load and the time during which it acts.

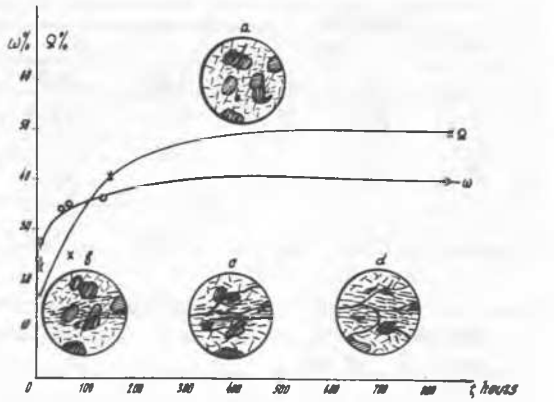

Fig.2.
Curves showing the development of the degree of impairment (ω) and degree of orientation (Ω) with time, and a scheme of the change in soil structure in the process of undamped creep: a - initial structure; b - after 1 hour; c - after 1 day; d - after 9 days.
Soil: kaolin, $\omega = 38\%$, $\tau = 100$ g/cm^2.

As the breaking of bonds between the particles proceeds and the impairment of the structure grows, the rate of deformation increases and the stage of failure begins. Breaking up of the aggregates and particle reorientation continue in this stage. The most characteristic feature of this stage, however, is the development of microfissures which merge, forming a network, and become macrofissures that break up the soil. It is precisely the development of microfissures and the rupture of interparticle bonds that causes soil failure in the creep process. Failure (repture) occurs when the degree of impairment reaches a certain critical value ω_r. This value ω_r depends neither on the load nor on the failure time and is 37.5±2% for kaolin and 40.9±2% for polymineral Jurassic clay (see table).

Thus, the degree of impairment upon failure is a constant for the given soil (ω_r = const) and it can be regarded as a strength criterion. Indeed, if the inimpaired area of the specimen cross section is denoted by $S^* = 1 - \omega$, then the actual stress will be $\tau^* = \tau/(1-\omega)$. Evidently, τ^* increases with the impairment ω and failure occurs when this stress reaches the limiting value equal to $\tau^*_r = \tau/(1-\omega_r)$. The lower the stress, the longer the time required to reach the critical degree of impairment ω_r. Thus, for Jurassic clay, the value $\omega_r = 40.9\%$ was reached at $\tau = 550$ g/cm^2 in 30 seconds; at $\tau = 460$ g/cm^2 in 1.9 hours; at $\tau = 425$ g/cm^2 in 6.6 hours; etc. These processes, along the

Table

Values of the degree of impairment ω_r (%) at failure

Kaolin					
Load τ, g/cm^2					
100	133	160	166	180	200
36,9	39,1	36,8	37,0	38,4	38,0
37,3	37,1	36,8	37,2		36,0
39,7	36,3	37,4	38,5		37,5
40,5	37,0	35,9			37,3
Mean					
38,6	37,3	36,7	37,6	38,4	37,2
ω_r(mean) = 37,5					

Jurassic clay				
Load τ, g/cm^2				
	416	425	460	550
	40,3	43,0	39,9	41,0
	41,1	41,6	41,4	
		39,8		
Mean	40,7	41,4	40,6	41,0
ω_r(mean) = 40,9				

breaking of the interparticle bonds and the rearrangement of the structure, are the cause of the reduction in soil strength upon prolonged action of the load.

The kinetics of structural changes can be represented by the following relationships which are approximations of the curve in Fig. 2.

The degree of orientation Ω, varying within the limits $\Omega_0 < \Omega < 1$, is expressed by the equation

$$1 - \Omega = (1 - \Omega_0)(1 + t)^{-\lambda} \tag{3}$$

The degree of impairment ω, varying within the limits $\omega_0 < \omega < \omega_r$, can be expressed by one of two relationships, the difference between which is to be explained further on:

$$1 - \omega = (1-\omega_0)e^{-f_1(\tau)t} \quad \text{or} \tag{4}$$

$$1 - \omega = (1 - \omega_0)(1 + t)^{-f_2(\tau)}$$

where $f(\tau)$ is a certain function of the stress.

Let us return to the initial equation (2). The energy U_0^* is a constant of the soil depending on its composition and state. The energies U_d^* and U_{or}^* can be determined by conditions that follow from a consideration of the process of structural changes as a stochastic one. The change in the energy U_d^* turns out to be proportional to the increment of impairment with respect to a certain unimpaired cross section, and the reduction in the energy U_{or}^* is proportional to the degree of reorientation referred to the area of the unoriented particles, i.e.

$$\Delta U_d^* = \varkappa_2 \frac{\Delta\omega}{1-\omega} \quad \text{and} \quad \Delta U_{or}^* = -\varkappa_3 \frac{\Delta\Omega}{1-\Omega} \tag{5}$$

Let us consider the first of these equations.
It follows from equation (4) that

$$\frac{\Delta\omega}{1-\omega} = \varkappa \Delta t \tag{6}$$

where coefficient $\varkappa$ is a function of only the stress $\varkappa = f_1(\tau)$ on the basis of the first of the equations (4) and a function of both stress and time $\varkappa = f_2(\tau)/(1+t)$ on the basis of the second.

Integrating equation (6), the following is obtained:

$$\int_0^{t_r} \varkappa dt = \int_{\omega_0}^{\omega_r} \frac{d\omega}{1-\omega} = \ln\frac{1-\omega_0}{1-\omega_r} = \text{const} \tag{7}$$

This equation can be regarded as the condition for long-term failure of the soil; it is equivalent to the condition U_d = const.

On the basis of the probabilistic nature of impairment accumulation, it can be shown that the stress function can be taken in the form

$$f_1(\tau) = \varkappa_1 \frac{\tau - \tau_\infty}{\tau_0 - \tau} \tag{8}$$

where τ_0 = conditionally instantaneous strength corresponding to the resistance to rapid failure; and τ_∞ = long-term strength.

Substituting $\varkappa = f_1(\tau)$ from equation (8) into equation (7), we obtain at τ = const

$$T\frac{\tau_0 - \tau}{\tau - \tau_\infty} = t_r$$

where $T = \frac{1}{\varkappa_1} \ln \frac{1-\omega_0}{1-\omega_r}$. Hence parameter T characterizes the ratio of the degrees of impairment in the initial state and at the instant of failure. Equation (9) is the long-term strength equation relating the stress and the time after which failure occurs. Equation (9) is based on the assumption that failure occurs only at loads exceeding the limit τ_∞ which demarcates between damped ($\tau < \tau_\infty$) and undamped ($\tau > \tau_\infty$) deformation.

If the assumption is made that failure occurs at any stress acting during an unlimited length of time (corresponding to the law of "secular" creep), then the stress function should be of the form $f_2(\tau)$ =

$= \varkappa_1 \tau / (\tau_0 - \tau)$ and coefficient $\varkappa$ of the form $\varkappa = \varkappa_1 \tau/(\tau_0 - \tau)(1 + t)$. In this case the long-term strength equation will be of the form

$$T \frac{\tau_0 - \tau}{\tau} = \ln (1 + t_r) \qquad (10)$$

For engineering problems, both equations (9) and (10) provide sufficiently close results, coinciding with experiments (which is a consequence of a certain arbitrariness of the conception of τ_∞). These equations and their comparison with experimental data have been dealt with (Vyalov and Meschyan, 1969; Zarrtsky and Vyalov, 1971).

After considering the condition for long-term soil failure, the law of deformation will now be dealt with. Comparing the first of equations (5) with equation (6) and assuming that $\varkappa = f_1(\tau)$, where $f_1(\tau)$ is determined by equation (8), the energy of fissure formation is obtained after integrating: $U_d = \delta_1 \bar{\tau}\, t/T$, where

$\delta_1 = \varkappa_2 \ln [(1-\omega_0)/(1-\omega_r)]$ and

$\bar{\tau} = (\tau - \tau_\infty)/(\tau_0 - \tau)$.

The energy U^*_{or} is determined by integrating the second equation (5), taking into account equation (3), as $U^*_{or} = -\delta_2 \ln (1+t)$, where $\delta_2 = \varkappa_3 \lambda$.

Substituting these equations into equation (2), the creep rate is found to be (Zaretsky and Vyalov, 1971)

$$\frac{\dot{\gamma}}{\tau} = \frac{1}{\eta_0} \cdot \frac{\exp(\frac{1}{T}\ \delta_1 \bar{\tau} t)}{(1 + t)^{\delta_2}} \qquad (11)$$

where $\eta_0 = 1/A \exp (U_0/k\Theta)$ = initial viscosity; $\bar{\tau} = \dfrac{\tau - \tau_\infty}{\tau_0 - \tau}$ = stress level;

and T = parameter from equation (9).

Generally, parameter δ_2 depends on the stress and varies in accordance with whether $\tau > \tau_\infty$ or $\tau < \tau_\infty$. It can be shown, however (Zaretsky and Vyalov, 1971), that under definite conditions

$\delta_1 = \delta_2 = \delta$ = const.

Equation (11) describes both damped and undamped creep, the first case corresponding to the negative sign of the stress level $\bar{\tau}$ (at $\tau < \tau_\infty$) and the second to the positive sign (at $\tau > \tau_\infty$). Equation (11) relates the deformative and strength characteristics. It should be noted that the parameters of equation (11) have a strictly physical meaning. Not excluding the possibility that this equation may subsequently be simplified, it is necessary to emphasize that it was obtained on the basis of a consideration of the physical essence of phenomena, namely, the kinetics of structural changes in soils.

For particular cases, equation (11) can be reduced to the following modifications. At $\delta/T = 0$, the well-known power law is obtained; at $\delta/T = 0$ and $\delta = 1$, the logarithmic law of "secular" creep; at $\delta = 0$ but $\delta/T \neq 0$, the exponential law; and at $\delta = 0$, the Newton law of linear flow. If the simple relationship $U^*_d = \alpha\tau$ is assumed for U^*_d in the initial equation (2), then the deformation equation (11) will become

$$\dot{\gamma} = \frac{1}{\eta_0} \cdot \frac{\exp(\alpha \tau/\tau_0)}{(1 + t)^{\delta}} \qquad (12)$$

which has been empirically established (Singh and Mitchell, 1968). If in equation (12) it is assumed that $\delta = 0$, the flow equation of the theory of rate processes is obtained (which does not consider changes in the deformation process with time).

An expression for the minimum rate of flow min $\dot{\gamma} = \dot{\gamma}_f$, corresponding to the instant of time $t = t_f$, can be obtained from equation (11):

$$\dot{\gamma}_f = \frac{\tau}{\eta_f} \left(\frac{\tau - \tau_\infty}{\tau_0 - \tau}\right)^{\delta} \qquad (13)$$

where $\eta_f = \eta_0 (T \frac{t_f}{t_r})^{\delta} \exp(-\delta)$ is a parameter that can be regarded as plastic viscosity.

Equations (9), (10), (11) and (13) can be extended to cover a complex stressed state, for which purpose is replaced by the intensity of the rate of shear deformation $\dot{\varepsilon}_i$; by the intensity of shear deformation ε_i and τ by the intensity of shearing stresses σ_i. The ultimate strengths τ_0 and τ_∞ are expressed by intensities as $\sigma_{i(o)} = (H_0 + \sigma_m)\tan \varphi_0$ and $\sigma_{i(\infty)} = (H + \sigma_m)\tan \varphi_\infty$, where

$\sigma_m = 1/3(\sigma_1 + \sigma_2 + \sigma_3)$, and H and φ are strength parameters corresponding to the Mises-Schleicher plasticity condition for the initial and ultimate long-term states.

The deformation equation is obtained by integrating equation (11) which, at τ = const and $\delta = 1$, constitutes

$$\gamma(t) = \tau\left(\frac{1+t}{\eta_0}\right)^{1-\delta} \left[\frac{1}{1-\delta} + \frac{\delta \bar{\tau}}{2-\delta}\left(\frac{1+t}{T}\right) + \frac{(\delta \bar{\tau})^2}{2(3-\delta)} \left(\frac{1+t}{T}\right)^2 + \dots\right] \qquad (14)$$

Using only the first terms of the series, the well-known logarithmic (or power, if $\delta \neq 1$) law of deformation is obtained. If the second terms are also taken into account, an equation of the form

$\gamma = \gamma_{cr} \pm \gamma_f$ will be obtained, where

γ_{cr} = damped creep; γ_f = flow at constant velocity which is added (when $\tau > \tau_\infty$) or subtracted (when $\tau < \tau_\infty$) from the values of the curve for γ_{cr}.

The parameters of the basic equation (11) can be determined directly from experiments. Parameter T and the strength characteristic τ_∞ are determined from the long-term strength (τ vs t_r) curve, and parameters δ and η_0 from the rheological (γ_f vs τ) curve.

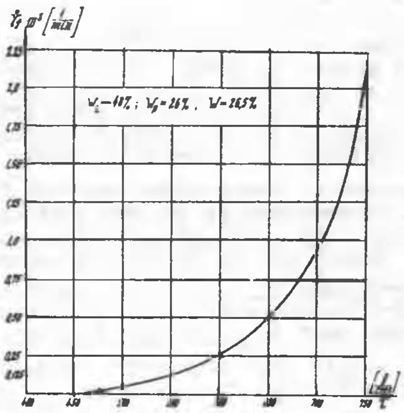

Fig.3.
Rate of flow γ_f vs shear stress τ calculated by equation (13). Experimental data shown by points.

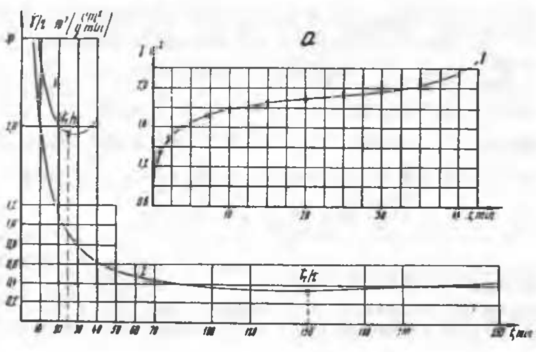

Fig.4.
Variation in the deformation rate with time for constant stress τ and creep curve (a) calculated by equations (12) and (14).
1.- τ = 460 g/cm^2; 2 - τ = 450 g/cm^2.
Experimental data shown by points.

A comparison of experimental and calculated data is given below. Presented in Figs.3 and 4 are data on the torsion tests of Jurassic clay W = 26.5%) as well as a comparison of these data with those obtained in calculations using equations (13) and (14). Similar comparisons have been made (Zaretsky and Vyalov, 1971) for data obtained in shear tests of frozen sandy loam.

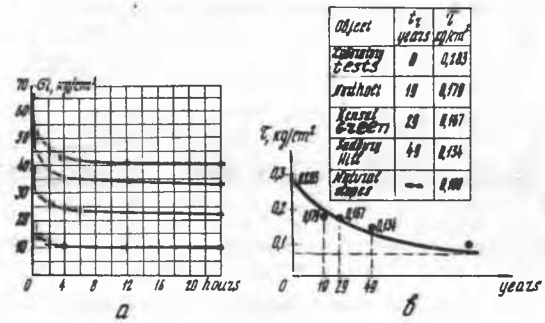

Fig.5.
Long-term soil strength curves calculated by equation (9): a- laboratory confined compression tests of frozen sandy loam at a hydrostatic pressure: 1 - 60 kg/cm^2; 2.- 30 kg/cm^2; 3.- 15 kg/cm^2; 4.-0.
b - field data of L.Suklje on the relationship between the time up to failure and the shear stress in slopes. Experimental data shown by points and in table.

The long-term strength equation (9) is checked against experimental data in Fig.5. Figure 5a refers to data of laboratory confined compression tests (S.E.Gorodetsky), and Fig.5b to data of field observations on the collapse of a series of slopes of London clay processed by L. Suklje. Of interest is the fact that the value of τ = 0.085 g/cm^2, calculated by equation (9), turned out to be close to the value τ_∞ = 0.1 g/cm^2 estimated by Suklje from data on the stability of natural slopes. The applicability of equation (10) has been checked (Vyalov and Meschyan, 1967).

REFERENCES

SINGH, A. and MITCHELL, J.K. (1968), "General Stress-Strain Function for Soils", Journal of Soil Mechanics and Foundations Division, ASCE, Vol. 94, No SMI, January.

VYALOV, S.S. and MESCHYAN, S.R. (1969), "Creep and Long-Term Strength of Soils Subjected to Variable Load", Proceedings of VII International Conference on Soil Mechanics and Foundation Engineering, Mexico City, Vol.I, p.423.

VYALOV, S.S., PEKARSKAYA, N.K. and MAXIMYAK, R.V. (1970), "On the Physical Essence of the Deformation and Failure Processes in Clayey Soils", Journal: Bases, Foundations and Soil Mechanics, No.1, Moscow.

ZARETSKY, Yu.K. and VYALOV, S.S. (1971), "Problems of the Structural Mechanics of Clayey Soils", Journal: Bases, Foundations and Soil Mechanics, No. 3, Moscow.