PREDICTION OF SETTLEMENTS OF FOUNDATIONS ON SAND

PREDICTION DES TASSEMENTS DES FONDATIONS SUR SABLE

ПРОГНОЗ ОСАДОК ФУНДАМЕНТОВ НА ПЕСКЕ

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SYNOPSIS. The subject studied hereafter is the settlement of superficial foundations on sand. The traditional empirical formula of Terzaghi-Peck (1948), based on the bearing plate test of 1 ft square, is inquired into. It is demonstrated that its applicability is restricted and unsatisfactory. On the other hand, the importance and validity of the equation of Housel-Burmister is evidenced. Dealing with the latter, the results of settlement measurements (collected by Bjerrum-Eggested, 1963) described by several authors are analyzed. It is shown how to determine the characteristics of deformability of sandy soils subjected to loaded plates and footings of varied dimensions.

INTRODUCTION

In this paper, the Author shows that experimental and field results (collected by BJERRUM EGGESTAD, 1963) of settlements of plates, footings and fills on the surface of sandy grounds, coincide quite adequately with the results obtained by the application of the theoretical-experimental method of HOUSEL-BURMISTER. Besides, it is demonstrated that the traditional formula of TERZAGHI-PECK (1948) has restricted applicability, while the expression of HOUSEL-BURMISTER presents a much more general character.

Also, the Author contests some hypothesis of Bjerrum-Eggestad (op.cit.) related to the influence of sand density over the settlement ratio of different foundation's size.

THE EXPRESSION OF TERZACHI-PECK

It is sufficiently known and largely divulged the empirical expression of TERZAGHI-PECK (1948):

$$\delta = \delta_o \cdot \left(\frac{2 \cdot B}{B + B_o}\right)^2 \qquad (1)$$

where:

δ = the settlement of a square plate (B x B) subjected to pressure p;

δ_o = the settlement of a "standard" square plate (B_o = 1 ft ≅ 30 cm) subjected to the same pressure p.

The expression (1) may written:

$$\frac{\delta}{\delta_o} = \frac{4}{\left(1 + \frac{B_o}{B}\right)^2} = \frac{4}{\left(1 + \frac{1}{\frac{B}{B_o}}\right)^2} \qquad (2)$$

BJERRUM and EGGESTAD (1963) worked out the curve corresponding to (2) in a log-log graph, correlating δ/δ_o with B/B_o (see Fig. nº 1).

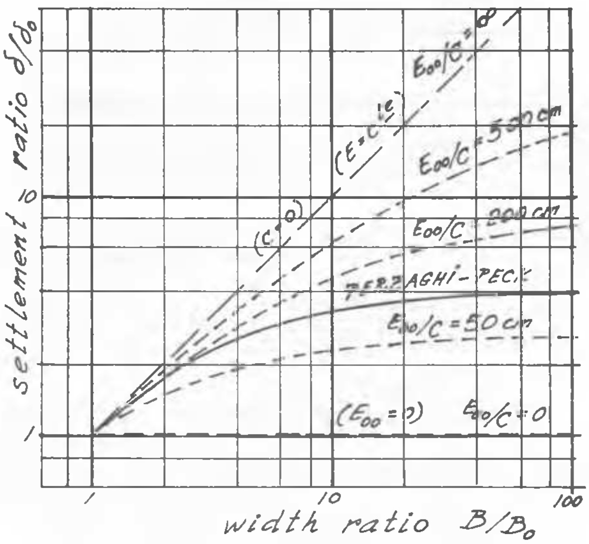

Fig. 1 - Curves correlating δ/δ_o to B/B_o in a log-log graph.

In Fig. nº 2, in a natural scale graph, the same correlation is presented, with the advantage of more precision and detail in the range of usual dimensions of foundations (up to ratio B/B_o = 30, i.e., $B \cong$ 900 cm = 9 m).

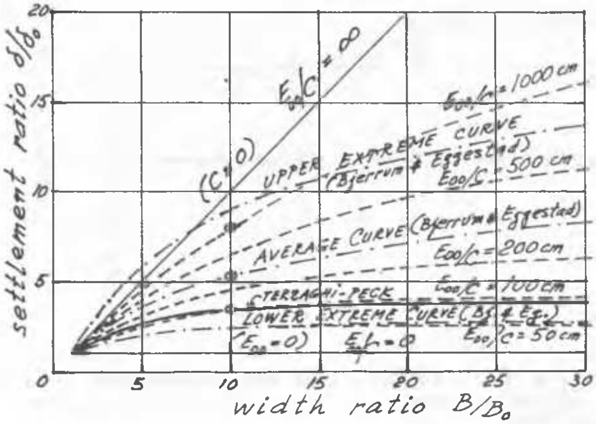

Fig. 2 - Curves correlating δ/δ_o to B/B_o in a natural graph.

THE EXPRESSION OF HOUSEL-BURMISTER

According to HOUSEL (1929), in case of square plates (side B = 2.b) on the surface of the ground, the settlement is produced by the pressure

$$p = n_o + m_o \cdot \frac{P}{A} \qquad (3)$$

where P is the perimeter and A the area of the plate, n_o and m_o characteristic coefficients of the ground.

In another form:

$$p = \frac{C \cdot \delta}{C \cdot (1 - \mu^2)} + \frac{1}{4} \cdot \frac{E_{oo} \cdot \delta}{C \cdot (1 - \mu^2)} \cdot \frac{2}{b} \qquad (4)$$

according to BURMISTER (1947), who adapted the empiric expression (3) of HOUSEL to the Theory of Elasticity, considering that in the soils the Modulus of Deformation E_z may vary (to increase, in general) with the depth.

Comparing (3) and (4) Burmister obtained:

$$n_o = \frac{C \cdot \delta}{C_\delta \cdot (1 - \mu^2)} \qquad (5)$$

$$m_o = \frac{E_{oo} \cdot \delta}{4 \cdot C \cdot (1 - \mu^2)} \qquad (6)$$

being $\frac{2}{b} = \frac{4}{B} = \frac{P}{A}$ and:

E_{oo} = Modulus of Deformation at the surface of the ground;

C = Increment of Modulus with the depth ($E_z = E_{oo} + C.z$)

μ = Coefficient of Poisson

C_δ = Coefficient dependent of the shape and rigidity of the plate.

The expression (4) will be referred to, from here further, as Expression of HOUSEL-BURMISTER (for superficial plates). We do justice to Terzaghi recalling that he had already arrived at an expression similar to (6), concerning m_o, but without setting forth the corresponding expression (5) to n_o - see TERZAGHI (1943), art. 142, page 402.

It is important to bear in mind that the expression (4), in spite of having been originated from the Theory of Elasticity, does not demand for application that the material be of elastic behaviour; it may be applied to soils, when loaded by plates, since there has been proportionality between pressures and settlements - see BARATA (1967).

The Author (BARATA, 1966) demonstrated theoretically that the expression (4) is general and valid for any dimensions of plates, since it deals with plates on the surface of the ground. In case of plates at depth, there will be corrections needed (derived and divulged by the Author in his former papers), which do not concern with the scope of the present paper.

THE GRAPHIC REPRESENTATION OF HOUSEL-BURMISTER'S EXPRESSION

The expression (4) may be written in the classical form:

$$\delta = C_\delta \cdot p \cdot \frac{B}{E_{oo} + C \cdot B} \cdot (1 - \mu^2) \qquad (7)$$

according the modification suggested by BURMISTER (1947).

For a "standard" plate (B_o = 30 cm)

$$\delta = C_\delta \cdot p \cdot \frac{B_o}{E_{oo} + C \cdot B_o} \cdot (1 - \mu^2) \qquad (7\text{-a})$$

Graphs are presented in Figs. nº 3 to 6, in order to solve graphically the expression (7) where the case be, respectively, B = 30 cm, 100 cm, 150 cm and 300 cm, for the conditions:

C_δ = 0,85 (rigid square plate)

μ = 0,30 (average value for soils)

p = 1,00 kg/cm^2

(NOTE: Those graphs may be used for p or C_δ other than the original; it will be necessary, only, to correct proportionally to the other values).

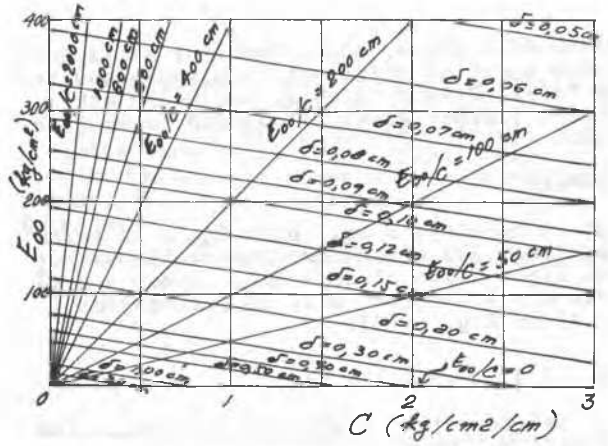

Fig. 3 - Settlements of Rigid Square Superficial Plates (expr. 7) with $B = 30$cm ($\mu = 0,30$ and $p = 1,0$ kg/cm^2).

From the expression (7) and (7-A) we obtain:

$$\frac{\delta}{\delta_o} = \frac{B}{B_o} \cdot \left(\frac{E_{oo}+C.B_o}{E_{oo}+C.B}\right) = \frac{B}{B_o} \cdot \theta \qquad (8)$$

Where

$$\theta = \frac{E_{oo}+C.B_o}{E_{oo}+C.B} = \frac{E_{oo}/C+B_o}{E_{oo}/C+B} \qquad (9)$$

In the particular case of $C = 0$ (soil of Modulus of Deformation constant with depth):

$$\theta = 1 \qquad \frac{\delta}{\delta_o} = \frac{B}{B_o} \qquad (10)$$

equation which in the Fig. nº 2 (and also in Fig. nº 1) is represented by a straight
653 line of 45^o.

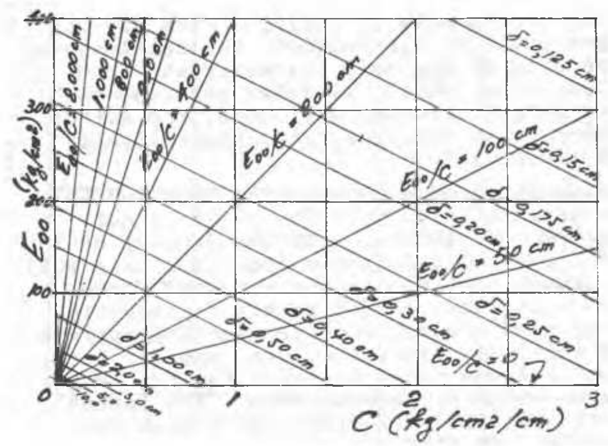

Fig. 4 - Settlements of Rigid Square Superficial Plates (expr. 7) with $B = 100$cm ($\mu = 0,30$ and $p = 1,0$ kg/cm^2).

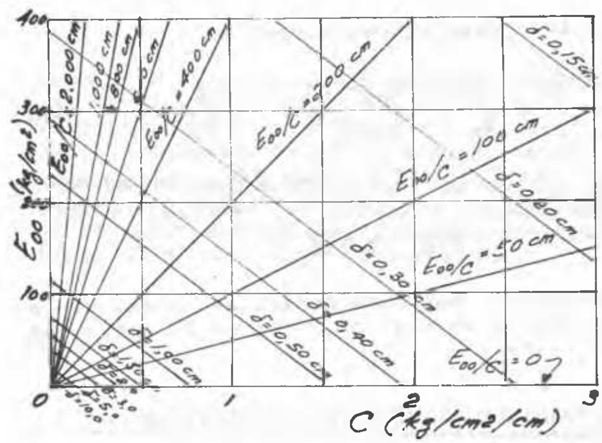

Fig. 5 - Settlements of Rigid Square Superficial Plates (expr. 7) with $B = 150$ cm ($\mu = 0,30$ and $p = 1,0$ kg/cm^2).

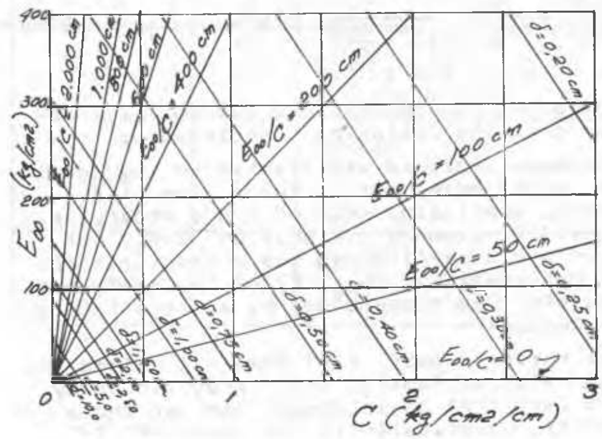

Fig. 6 - Settlements of Rigid Square Superficial Plates (expr. 7) with $B = 300$ cm ($\mu = 0,30$ and $p = 1,0$ kg/cm^2).

In the particular case of $E_{oo} = 0 \quad C > 0$:

$$\theta = \frac{B_o}{B} \qquad \frac{\delta}{\delta_o} = 1 \qquad (11)$$

which corresponds to a straight line parallel with the axis of B/B_o.

The general expr. (8) can be represented, therefore, in Fig. nº 1 or nº 2, by a Family of E_{oo}/C curves, varying between the limits $E_{oo}/C = 0$ (expr. 11) and $E_{oo}/C = \infty$ (expr.10)

There is also the less common case (which is possible, although, to occur, at least to limited depth) which is the one where $C < 0$ (soil of decrescent Modulus):

$$E_z = E_{oo} - C.z$$

In this case it would result:

$$\frac{\delta}{\delta_o} = \frac{B}{B_o} \cdot \left(\frac{E_{oo} - C.B_o}{E_{oo} - C.B}\right) = \frac{B}{B_o} \qquad (9\text{-}A)$$

And, as $C.B_o < C.B$, then $\theta' > 1$ and consequently

$$\delta/\delta_o > B/B_o$$

condition that make possible the existence of points in Fig. nº 2 (or nº 1) above the 45^o line.

COMPARISON BETWEEN THE EXPRESSIONS OF TERZAGHI-PECK AND HOUSEL-BURMISTER

The expr. (1) may be written:

$$\frac{\delta}{\delta_o} = \frac{B}{B_o} \cdot \left[\frac{4 \cdot \frac{B}{B_o}}{\left(1 + \frac{B}{B_o}\right)^2}\right] = \frac{B}{B_o} \cdot \theta_T \qquad (12)$$

Comparing the expressions (8) and (12) we can draw the following conclusions:

The expr. (12) is represented in Fig. nº 1 (or nº 2) by an unique curve, for it depends, explicity, only on B/B_o; otherwise, expr. (8) depends not only on B/B_o, but also and explicity, on the characteristics of deformability (E_{oo}, C) of the soil, and therefore is represented by a large family of curves;

The curve of expr. (12) does not belong to the referred Family; this is evidenced by the fact that it intersect some of those curves, principally in the range $1 < B/B_o < 10$, which is the most important in the study of the usual foundations. The table nº 1 here under demonstrates such fact.

TABLE Nº 1

$\frac{B}{B_o}$	$\left(\frac{\delta}{\delta_o}\right)$ Expr. (12)	$\left(\frac{E_{oo}}{C}\right)$ Expr. (8)
2	1,78	213 cm
3	2,26	153
4	2,56	130
5	2,78	120
7,5	3,11	109
10	3,30	103
20	3,62	96
50	3,85	93
100	3,92	91
∞	4,00*	90 cm

* Asymptotic value

TABLE Nº 1 shows, also, that for soils which ratio E_{oo}/C is different from those covered by Terzaghi-Peck formula, one would make errors (large, eventually) with is application.

EXAMPLE

Let us consider a soil with $E_{oo} = 48$ kg/cm^2 and C = 0,200 kg/cm^2/cm. The settlement of one rigid plate of $Bo = 30$ cm under a presure $p = 1{,}0$ kg/cm^2 will be (using expr. (7-A) or Fig. nº 3):

$$\delta_o \cong 0{,}43 \text{ cm} = 4{,}3 \text{ mm}$$

The settlement of a plate $B = 5 \times B_o = 150$ cm will be (by expr. (8) or Fig. nº 5).

$$\delta = 1{,}50 \text{ cm} = 15{,}0 \text{ mm}$$

while, by expr. (12) we would obtain (see TABLE Nº 1, directly) for $B/B_o = 5$:

$$\frac{\delta}{\delta_o} = 2{,}78$$

or

$\delta = 2{,}78 \times \delta_o = 2{,}78 \times 0{,}43$ cm $\cong 1{,}20$ cm = 12,0 cm, a value smaller than the one obtained by the expr. (8) of Housel-Burmister. The reason of this difference is because by the Terzaghi-Peck formula, $B/B_o = 5$ corresponds always to $E_{oo}/C = 120$ cm (see TABLE nº 1), which is not the specific case of the example, where

$$\frac{E_{oo}}{C} = \frac{48}{0{,}200} = 240 \text{ cm}$$

(NOTE: For a soil with $E_{oo} = 24$ kg/cm^2 and C = 0,8 kg/cm^2/cm, we would obtain respectively $\delta_o \cong 4{,}2$ mm, $\delta \cong 8{,}5$ mm - by Housel-Burmister - and $\delta \cong 2{,}78 \times 4{,}2 = 11{,}7$ mm - by Terzaghi-Peck, for $B/B_o = 5$, being $E_{oo}/C = 30$ cm in this case).

Thus, the formula of Terzaghi-Peck may underestimate or overestimate the settlements, depending of the soil characteristics. D. J. D'Appolonia, et al, 1968 had verified this risk by experimental way. Here, the Author demonstrates the same fact, theoretically and numerically.

The position of a (B/B_o, δ/δ_o) plotted in the graph of Fig. nº 2 (or nº 1) is not related, absolutely, with the density of the soil. Bjerrum & Eggestad (po.cit., page 202) concluded that "there are, however, some trends indicating that points representing dense sand are situated between the average and the lower extreme curves, whereas the upper extreme curve is valid for the very loose, slightly organic sand". The latest edition of TERZAGHI-PECK (1967) book (see Art. 54) admitted similar hypothesis. However the application of expr. (8). demonstrates that this does not correspond to reality. In fact, TABLES Nº II and Nº III, hereafter, furnish results for plates under pressure p = 1,0 kg/cm^2, which point out (by comparison) that for a given ratio $B/B_o = 10$, for instance, one may obtain in Fig. Nº 2 (or Nº 1) points corresponding to the same ratio δ/δ_o, either above or below the Average Curve of Bjerrum-Eggestad, and representing, indifferently, dense or loose soil.

In other words, it can be staded, also, that the ratio E_{oo}/C does not characterize the density (or deformability) of the soil. For this aim, the knowledge of the values E_{oo} and C, properly, is of paramount importance.

What seems "scattering" (in the opinion of Bjerrum-Eggestad) is merely a consequence of variable and diversified characteristics (E_{oo}, C) of the sandy soils tested.

TABLE Nº II

E_{oo} (kg/cm²)	C (kg/cm²/cm)	$\frac{E_{oo}}{C}$ (cm)	Settlements (cm)		$\frac{\delta}{\delta_o}$	OBSERVATIONS (See Fig. nº 2)
			B = 300 cm	B_o = 30 cm		
			δ	δ_o		
400	0,400	1000	0,450	0,056	8,0	Dense^x soil (point above the average curve)
140	0,500	250	0,780	0,150	5,2	Medium^x soil (point on the average curve of Bjerrum & Eggestad)
25	0,250	100	0,350	0,700	3,3	Loose^x soil (point below the average curve)

x approximated compacity

TABLE Nº III

E_{oo} (kg/cm²)	C (kg/cm²/cm)	$\frac{E_{oo}}{C}$ (cm)	Settlements (cm)		$\frac{\delta}{\delta_o}$	OBSERVATIONS (See Fig. nº 2)
			B = 300 cm	B_o = 30 cm		
			δ	δ_o		
80	0,080	1000	2,200	0,280	8,0	Loose^x soil (point above the average curve)
140	0,500	280	0,780	0,150	5,2	Medium^x soil (point on the average curve of Bjerrum & Eggestad)
200	2,000	100	0,290	0,088	3,3	Dense^x soil (point below the average curve)

x approximated compacity

DEFORMABILITY CHARACTERISTICS OF SOME SANDS

In order to prove and evidence the practical validity of the preconized method, the Author analyzes in the following the results of some cases collected by BJERRUM and EGGESTAD (1963).

In TABLE Nº IV are reproduced only the cases of Bjerrum-Eggestad paper in which the measurement of settlements were effectuated on superficial plates and loaded areas of diversified (more than two sizes) dimensions.

The characteristics (E_{oo}, C) of the soils included in TABLE Nº IV were determined by a procedure explained in the followings, taking as an example the Case nº 1 of the Table. Thus:

a) In the graph of Fig. nº 7 were plotted the pairs (p, δ) of TABLE Nº IV;

b) It was assumed that the points (p, δ) plotted, belong to the linearity (proportionality) range of the p - δ curves of every test; consequently, connecting such points with the origin of the graph, one obtains the probable initial part of the different curves;

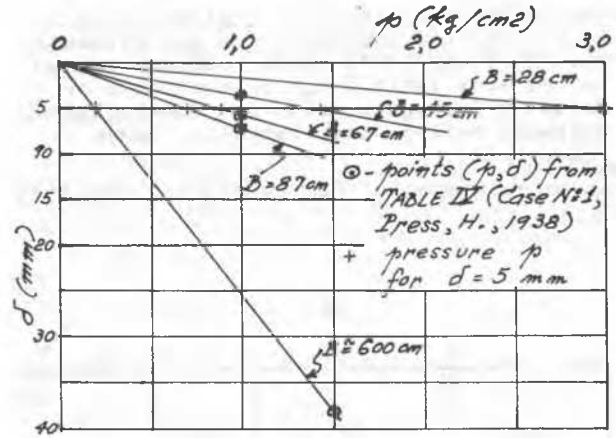

Fig 7 - Curves pressure-settlements of footings (Press, H., 1938).

c) For a given settlement (δ = 5 mm, for instance) one obtains from Fig. Nº 7.

plate 0,28 m x 0,28 m ..... p ≅ 3,000 kg/cm^2
plate 0,45 m x 0,45 m ..... p ≅ 1,420 kg/cm^2
plate 0,67 m x 0,67 m ..... p ≅ 0,910 kg/cm^2
plate 0,87 m x 0,87 m ..... p ≅ 0,710 kg/cm^2
plate 5,50 m x 6,50 m (≈6,0m x 6,0 m) .... p ≅ 0,200 kg/cm^2

TABLE Nº IV

(from Bjerrum & Eggestad, 1963)

Case Nº	Local	Object	Dimensions (m)	Pressure p (kg/cm^2)	Settlement (cm)	Soil	References
1	Berlin (Germany)	Footing Load Test Load Test Load Test Load Test	5,50 x 6,50 0,87 x 0,87 0,67 x 0,67 0,45 x 0,45 0,28 x 0,28	1,5 1,0 1,0 1,0 3,0	3,75 0,70 0,55 0,35 0,50	coarse medium-loose sand n = 42 %	Press. H. 1938
2	Berlin (Germany)	Footing Load Test Load Test Load Test	3,50 x 3,50 0,60 x 0,60 0,45 x 0,45 0,30 x 0,30	1,9 1,9 1,9 1,9	4,90 1,10 0,80 0,60	fine dense	Press. H. 1938
5	Berlin (Germany)	Footing Load Test Load Test	2,80 x 3,00 1,00 x 1,00 0,30 x 0,30	2,2 2,2 2,2	0,60 0,27 0,12	fine loose	Press. H. 1932
8	Purley-Way Croydon (England)	Footing Footing Footing Load Test	1,50 x 1,50 1,50 x 1,50 1,10 x 1,10 0,30 x 0,30	1,0 0,8 0,8 1,0	0,13 0,16 0,19 0,06	sandy gravel medium-loose	Meigh A.C. and Nixon I.K. 1962
10	Verdalsöra (Norway)	Hydr.Fill Test Fill Load Test Load Test Load Test	150,00x200,00 19,00x19,00 Ø 0,80 Ø 0,36 Ø 0,20	0,60 0,76 0,80 1,40 1,40	8,00 5,00 0,70 0,35 0,21	silty fine Sand some organ. content loose n = 45 %	NGI 0,728

d) In Housel's Graph (Fig. Nº 8) were plotted the pairs $(\frac{P}{A}, p)$ for the different plates, and for $\delta = 5$ mm. We can observe that, save the point of plate B = 0,28 m, the rest form a reasonably straight line

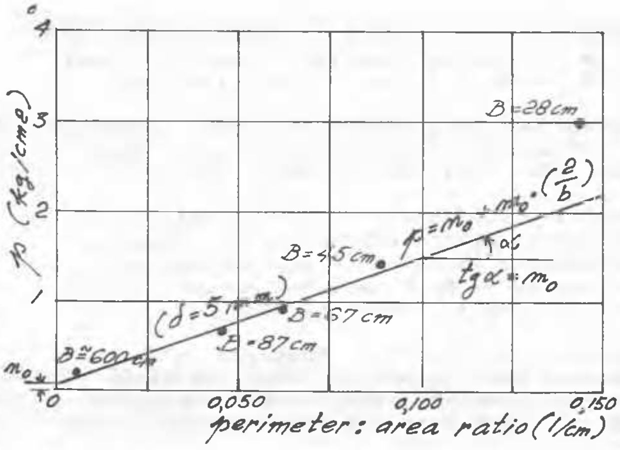

Fig. 8 - Housel's graph corresponding to Fig. 7 case (δ = 5 mm).

(NOTE: The author has established a method to adjust the straight line so obtained, in function of the results of the Duch Cone Test - see BARATA, 1962 and BARATA et al, 1970.b);

e) The straight line drawn permits us obtain (Fig. Nº 8) in the case:

$$n_o = 0{,}075 \text{ kg/cm}^2 \quad ; \quad m_o = 14{,}250 \text{ kg/cm}$$

f) Using the expressions (5) and (6) we get

$$E_{oo} = \frac{4.C\delta \,.\, (1 - \mu^2)}{\delta} \,.\, m_o \cong 90 \text{ kg/cm}^2$$

$$C = \frac{C\delta \,.\, (1 - \mu^2)}{\delta} \,.\, n_o \cong 0{,}116 \text{ kg/cm}^2\text{/cm}$$

Proceeding in the same way the results of the different cases of TABLE Nº IV, we arrive at the values in TABLE Nº V, in which are presented, also (for comparison), the probable settlements of plates B_o = 30 cm and B = 300 cm, under p = 1,0 kg/cm^2, for the soils listed in TABLE Nº IV.

TABLE Nº V

CASE	E_{oo} (kg/cm^2)	C (kg/cm^2/cm)	$\frac{E_{oo}}{C}$ (cm)	δ (cm) (p = 1,0 kg/cm^2)	
				B_o = 30 cm	B = 300 cm
Verdalsora (10)	92	0,072	1280	0,240	2,040
H. Press (2)	75	0,093	800	0,290	2,250
H. Press (1)	90	0,116	775	0,250	1,850
H. Press (5)	407	1,550	263	0,055	0,265
Croydon (8)	319	2,320	138	0,060	0,230

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It will be interesting to compare the settlements of TABLE Nº V (calculated by the theory) with those actually measured in the field (TABLE Nº IV), taking in account the necessary corrections of pressure, shape and dimensions of the plates used.

CONCLUSIONS

Reffering to superficial plates on sandy soils we can conclude:

1) The empirical expression of Terzaghi-Peck has its field of application restricted to certain sands. Should it be put to use, in many cases the results obtained would be smaller than in reality;

2) The expression of Housel-Burmister is of a much more general applicability, since it takes into account, explicitly, the deformation characteristics of the soil, as well as its variation with the depth;

3) In order to foresee the deformability of a given soil in relation with loaded areas of different dimensions, the knowledge of the variation of deformation modulus is indispensable. When dealing with linear

variation we have

$$E_z = E_{oo} + C \cdot z$$

The modulus E_z can be determined (see earlier papers of the Author) from plate bearing tests of Housel type and from correlations with the Dutch Cone Test.

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