SOME APPLICATIONS OF THE ELASTICITY THEORY TO DESIGN OF FOUNDATIONS

LES APPLICATIONS DE LA THEORIE D'ELASTICITE AUX CALCULS DES FONDATIONS

О ПРИМЕНЕНИИ ТЕОРИИ УПРУГОСТИ К РАСЧЕТУ ОСНОВАНИЙ СООРУЖЕНИЙ

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SUMMARY. The report deals with some problems of foundation design using the methods of elasticity theory.

K.E.Egorov describes the effect of Poissons's ratio and the type of contait conditions on the solutions of classical problem of the plate on elastic base. V.A.Barvashov presents recurrent method of stress-strain evaluation in the case when a load is distributed over rectangular area in elastic half-space. V.G.Fedorovsky proposes an approximate solution for the problem of point load in the multi-layered half-space

1. The problems of applicability of elasticity theory to foundation design are being widely discussed now. But in practice it is the only theory for relatively simple evaluation of stress-strain state in foundations of structures.

Of course, it is necessary to have the experimental deformation parameters of soil for the design of foundation. These are deformation modulus E and Poisson's ratio ν Poisson's ratio is known to vary from 0 to 0.5 for elastic solids. The effect of this ratio on results of the design is often negligible. But in some cases the determination of Poisson's ratio is necessary especially for the computation of horizontal displacements of soil under the footing. As an example we may give the problem of foundation under strip-footings in the conditions of plane strain. In the case of uniform load p distributed over the interval 2a wide on the boundary of half-plane the components of stresses may be found using Michell's formulae which do not depend on Poisson's ratio ν. At the same time displacements considerably depend on the value of

$$v = \frac{ap(1-\nu^2)}{\pi E}\Big\{(1+\xi)\ln\left[(1+\xi)^2+m^2\right] + (1-\xi)\ln\left[(1-\xi)^2+m^2\right] - \frac{1-2\nu}{1-\nu}\, m \times \left(\operatorname{arctg}\frac{m}{\xi+1} - \operatorname{arctg}\frac{m}{\xi-1}\right)\Big\} + C \qquad (1)$$

$$u = \frac{ap(1-\nu^2)}{\pi E}\Big\{m\ln\frac{(1-\xi)^2+m^2}{(1+\xi)^2+m^2} - \frac{1-2\nu}{1-\nu}\Big[(1+\xi)\times \operatorname{arctg}\frac{m}{1+\xi} - (1-\xi)\operatorname{arctg}\frac{m}{\xi-1} - \pi\Big]\Big\} \qquad (2)$$

Here v and u stand for vertical and horizontal displacements, $\xi = x/a$ and $m = z/a$ are non-dimensional coordinates with the origin in the center of load interval.

In the case of rigid smooth plate (without friction) pressed into half-plane by the load P=2ap the displacements are (Egorov, 1940).

$$v = \frac{P(1-\nu^2)}{\pi E}\left[2\arcsin\sqrt{\frac{1}{2}\left(\sqrt{A}-1+\xi^2-m^2\right)} - \frac{m}{1-\nu}\sqrt{\frac{1}{2A}\left(\sqrt{A}+1-\xi^2+m^2\right)}\right] + C, \qquad (3)$$

$$u = \frac{P(1+\nu)}{\pi E}\left[m\sqrt{\frac{1}{2A}\left(\sqrt{A}-1+\xi^2-m^2\right)} - (1-2\nu)\arccos\sqrt{\frac{1}{2}\left(\sqrt{A}+1-\xi^2+m^2\right)}\right], \qquad (4)$$

where $A = (1-\xi^2+m^2)^2 + 4\xi^2 m^2$

There is an arbitrary constant C in the formulae (1) and (3) for vertical displacemens. To over come this difficulty the compression of soil layer (active zone) has to be considered. Let us consider horizontal displacements along the line passing through the edge of the stripfooting. For that it is necessary to adopt x=a in formulae (1) and (3), that is =1. Then we have

$$u = \frac{2ap}{E}K, \qquad (4')$$

where

$$K_u = \frac{1+\nu}{\pi}\left[(1-\nu)m\ln\frac{m}{\sqrt{m^2+4}} + (1-2\nu)\left(\frac{\pi}{2} - \operatorname{arctg}\frac{m}{2}\right)\right], \qquad (5)$$

$$K_r = \frac{1+\nu}{\pi}\left[\sqrt{\frac{m}{2(m^2+4)}\left(\sqrt{m^2+4}-m\right)} - (1-2\nu)\arccos\sqrt{\frac{m}{2}\left(\sqrt{m^2+4}-m\right)}\right] \qquad (6)$$

Here K_u and K_r are respective for the uniform load and the rigid plate. The formula (4) is valid also in the case of circular footing when a=R-a radius of footing.

1. For uniform load (Egorov,1958)

$$K'_u = \frac{1+\nu}{4}\left\{(1-2\nu) - \frac{m}{\pi\sqrt{m^2+4}}\left[(3-2\nu)(m^2+4)(F(k) - E(k)) - 4F(k)\right]\right\}; \quad \left(k^2 = \frac{4}{m^2+4}\right) \qquad (7)$$

where F(k) and E(k) stand for full elliptic integrals of 1st and 2nd type.

2. For rigid plates (Egorov, 1958)

$$K'_r = \frac{1+\nu}{4}\left\{(1-2\nu)\left[1-\sqrt{\frac{m}{2}(\sqrt{m^2+4}-m)}\right] - \frac{1}{2}\sqrt{\frac{m}{2(m^2+4)}}\,(\sqrt{m^2+4}-m)^3\right\} \qquad (8)$$

The values of K evaluated using formulae (5)-(8) are represented in the table I.

1. For strip-footing vertical stresses are

$$\sigma_z = \frac{p}{\pi}\left[(1+A)e^{-\beta\alpha} + e^{\beta\alpha}\right]\frac{1}{\sqrt{m^2+1}}, \qquad (9)$$

where

$$A = \frac{2m(m-2\beta)}{m^2+1}, \quad \alpha = \pi - 2\,\mathrm{arctg}\,m, \quad \beta = \frac{\ln(3-4\nu)}{2\pi}$$

2. For circular footing

$$\sigma'_z = \frac{P}{4\sqrt{3-4\nu}\,(m^2+1)}\left\{\left[1+\frac{4m(m-\beta)}{m^2+1}\right]e^{(\pi-\alpha)\beta} + (3-4\nu)\,e^{(\alpha-\pi)\beta}\right\} \qquad (10)$$

In the case of ν =0.5 formulae (9) and (10) coincide with formulae for smooth footings as follows

$$\sigma_z = \frac{2}{\pi}P\left(1+\frac{m^2}{m^2+1}\right)\frac{1}{\sqrt{m^2+1}} \qquad (11)$$

Table I

m=z/a		0	0.2	0.4	0.6	0.8	1.0	2.0	3.0	4.0
VALUES OF K FOR STRIP FOOTINGS										
K_u	for ν =0.3	-0.260	-0.110	-0.039	-0.005	0.033	0.050	0.071	0.062	0.053
	for ν =0.5	0	0.110	0.156	0.179	0.189	0.192	0.165	0.132	0.107
K_r	for ν =0.3	-0.260	-0.100	-0.043	-0.007	0.018	0.035	0.062	0.059	0.050
	for ν =0.5	0	0.101	0.134	0.153	0.163	0.168	0.154	0.126	0.104
VALUES OF K FOR CIRCULAR FOOTINGS										
K'_u	for ν =0.3	-0.130	-0.014	0.026	0.044	0.050	0.051	0.034	0.021	0.013
	for ν =0.5	0	0.082	0.103	0.108	0.112	0.106	0.046	0.031	0.020
K'_r	for ν =0.3	-0.130	-0.010	0.020	0.034	0.041	0.043	0.042	0.032	0.020
	for ν =0.5	0	0.073	0.087	0.089	0.087	0.082	0.075	0.050	0.030

It is known that in these problems stresses do not depend on Poisson's ratio ν . However we may not consider stresses without displacements which essentially depend on ν (look at the table I). When the load is applied the vertical lines passing through the edge of a circular plate distort as it is shown at fig.1. Analogous distortion takes place in the case of strip-footings (look at the table I). According to the theory when $\nu < 0.5$ soil close to the footing moves to the center of it. That phenomenon contradicts the experimental data. Components of stresses evaluated not tabing into account friction between footing and soil depend on Poisson's ratio not directly i.e. for each value of ν stresses correspond to the certain shape of foundation displacements (fig.1)

For better correspondance to the experimental data it is necessary to consider a contact problem of elasticity theory taking friction into account. From this point of view the problem when soil is glued to the footing represents much practical interest. There are some solutions of this problem (Ufland,1967) but the formulae obtained for stresses and displacements are expressed in very complicated mathematical form. The simpliest expressions are for stresses along the axis of footing.

$$\sigma'_z = \frac{P}{2}\left(1+\frac{2m^2}{m^2+1}\right)\frac{1}{m^2+1} \qquad (12)$$

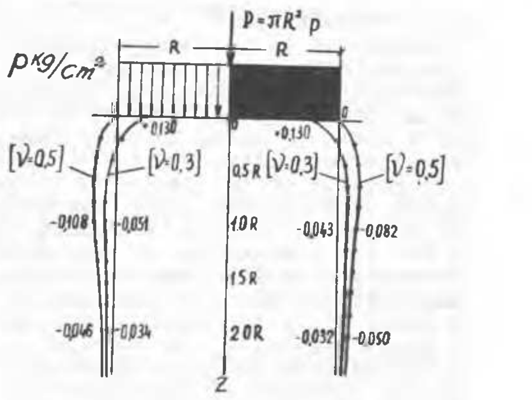

FIG.1

In the case of soil glued to the circular footing the displacement of the footing is

$$W = \frac{(1+\nu)(1-2\nu)P}{2\ln(3-4\nu)ER} \qquad (13)$$

Whence in the case of $\nu = 0.5$ that formula coincides with a formula for dicplacement of circular smooth footing

$$w=\frac{(1-\nu^2)P}{2ER}=\frac{3P}{8ER} \qquad (14)$$

Thus in the foundation design it is necessary to use the formulae which are obtained by solution of contact problems with soil glued to the footing.

2. Let us consider methods of computation of stresses and strains in isotropic and anisotropic half-space with deformation modulus continuously depending on depth. We have obtained analytical recurrent formulae in the case of load distributed inside of half-space over rectangular area according to polinomial law. Let us adopt notification as follows

$$L_{mn}(u,v,w)=\iint u^m v^n \ln(w+R)\,du\,dv;$$

$$J^k_{mn}(u,v,w)=\iint \frac{u^m v^n}{R^k}\,du\,dv;$$

$$T^k_{mn}(u,v,w)=\iint \frac{u^m v^n\,du\,dv}{R^k(v^2+w^2)};\quad J^k_m(u,v,w)=\int\frac{u^m}{R^k}du;$$

$$T^k_m(u,v,w)=\int\frac{u^m}{(u^2+w^2)R^k};\quad S_m(u,w)=\int\frac{u^m}{u^2+w^2}du;$$

where $R=\sqrt{u^2+v^2+w^2}$;
m, n, k-integer number,
k- odd number

Here the sign $\iint$ stands for double indefinite integration. The following recurrent correlations exist:

for $k > 0$

$$J^k_{mn}(u,v,w)=\frac{1}{k-2}\left[u^{m+1}T^{k-2}_n(v,u,w)-(m-k+3)\times T^{k-2}_{mn}(u,v,w)\right] \qquad (15)$$

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$$T^k_{mn}(u,v,w)=T^{k-2}_{m-2,n}(u,v,w)-J^k_{m-2,n}(u,v,w) \qquad (16)$$

$$T^1_{mn}(u,v,w)=J^1_{m,n-2}(u,v,w)-w^2T^1_{m,n-2}(u,v,w) \qquad (17)$$

$$J^1_{mn}(u,v,w)=\frac{1}{m}\left[u^{m-1}(u^2+w^2)J^1_n(v,u,w)+J^1_{n+2}(v,u,w)-J^1_{m-2,n+2}(u,v,w)-w^2J^1_{m-2,n}(u,v,w)\right] \qquad (18)$$

$$J^1_{0n}(u,v,w)=\frac{1}{n+1}\left[v^{n+1}\ln(u+R)-S_{n+2}(v,w)-uT^1_{n+2}(v,u,w)\right] \qquad (19)$$

$$J^1_{1n}(u,v,w)=J^1_{n+2}(v,u,w)+(u^2+w^2)J^1_n(v,u,w) \qquad (20)$$

$$T^1_{m0}(u,v,w)=\frac{v}{m}\left[\frac{u^m}{wv}\,\mathrm{arctg}\,\frac{uv}{wR}-T^1_m(u,v,w)\right] \qquad (21)$$

$$T^1_{m1}(u,v,w)=\frac{1}{m}\left[\frac{u^m}{2}\ln\left(\frac{R-u}{R+u}\right)+J^1_m(u,v,w)\right] \qquad (22)$$

$$J^k_m(u,v,w)=\frac{1}{(k-2)(v^2+w^2)}\left[\frac{u^{m+1}}{R^{k-2}}-(m-k+3)J^{k-2}_m(u,v,w)\right] \qquad (23)$$

$$T^k_m(u,v,w)=\frac{1}{v^2}\left[T^{k-2}_m(u,v,w)-J^k_m(u,v,w)\right] \qquad (24)$$

$$mJ^1_m(u,v,w)+(v^2+w^2)J^1_{m-1}(u,v,w)=u^{m+1}R \qquad (25)$$

$$T^1_m(u,v,w)+w^2T^1_{m-2}(u,v,w)=J^1_{m-2}(u,v,w) \qquad (26)$$

$$S_n(v,w)+w^2S_{n-2}(v,w)=\frac{v^{n-1}}{n-1} \qquad (27)$$

$$S_0(v,w)=\frac{1}{w}\,\mathrm{arctg}\,\frac{v}{w} \qquad (28)$$

$$S_1(v,w)=\frac{1}{2}\ln(v^2+w^2) \qquad (29)$$

$$J^1_0(u,v,w)=\ln(u+R) \qquad (30)$$

$$J^1_1(u,v,w)=R \qquad (31)$$

$$T^1_0(u,v,w)=\frac{1}{uw}\,\mathrm{arctg}\,\frac{uv}{wR} \qquad (32)$$

$$T^1_1(u,v,w)=\frac{1}{2u}\ln\left(\frac{R-u}{R+u}\right) \qquad (33)$$

$$L_{mn}(u,v,w)=\frac{1}{m+1}\left[u^{m+1}J^1_{0n}(u,v,w)-J^1_{m+1,n}(u,v,w)\right] \qquad (34)$$

Using formulae (15)-(34) we can obtaine analytic expressions for integrals

In the w=o (plane problem and the surface of half-space) formulae (15)-(34) may be considerably simplified because of the fact that

$$T^k_{mn}(u,v,0)=J^k_{m,n-2}(u,v,0) \qquad (35)$$

$$T^k_m(u,v,0)=J^k_{m-2}(u,v,0) \qquad (36)$$

$$S_m(u,0)=\frac{u}{m-1} \qquad (37)$$

Derivatives of functions L, I can be easily found. For that purpose we must take into account the following relations:

$$\frac{\partial}{\partial u}J^k_{mn}(u,v,w)=u^mJ^k_n(v,u,w) \qquad (38)$$

$$\frac{\partial}{\partial w}J^k_{mn}(u,v,w)=-kwJ^{k+2}_{mn} \qquad (39)$$

$$\frac{\partial}{\partial u}J^k_m(u,v,w)=\frac{u^m}{R^k} \qquad (40)$$

Analytic expressions for vertical stresses in the elastic half-space caused by the load with intensity $\sum_M\sum_N P_{MN}x^My^N$ distributed over the horizontal area ($a_1 \le x \le a_2$, $b_1 \le y \le b_2$) and applied at the depth C are given below

$$\sigma_z=\frac{1-2\nu}{8\pi(1-\nu)}\sum_M\sum_N P_{MN}\sum_{m=0}^{M}\sum_{n=0}^{N}C^m_M C^n_N x^{M-m}y^{N-n}\times\Big\{(1-2\nu)(z-c)\left[-J^3_{mn}(u,v,z-c)+J^3_{mn}(u,v,z+c)\right]-3(z-c)^3J^5_{mn}(u,v,z-c)-\left[3(3-4\nu)z(z+c)^2-2c(z+c)(5z-c)\right]J^5_{mn}(u,v,z+c)-30cz(z+c)^3\times J^7_{mn}(u,v,z+c)\Big\}\Big|_{x-a_1}^{x-a_2}\Big|_{y-b_1}^{y-b_2} \qquad (41)$$

Formula (41) has been obtained by integraion of Mindlin's solution (1936). In (41) formula of Newton-Leibnitz should be used twice for u and v respectively.

Analoguously we obtained solutions for other components of displacements and stresses in cases corresponding to isotropic and aelotropic half-spaces and in the case of deformation modulus increasing with depth according to polinomial law. For that purpose we integrated well-known respective integral cores.

As numerical illustration we give here the results of computation of displacements under rigid square plate loaded centrally. An approximation of intensity of subgrade reactions in this case has been obtained by M.I.Gorbunov-Posadov for uniform elastic half-space (1953). Taking into account the symmetry we give displacements only for 1/4 of the plate (fig.2).

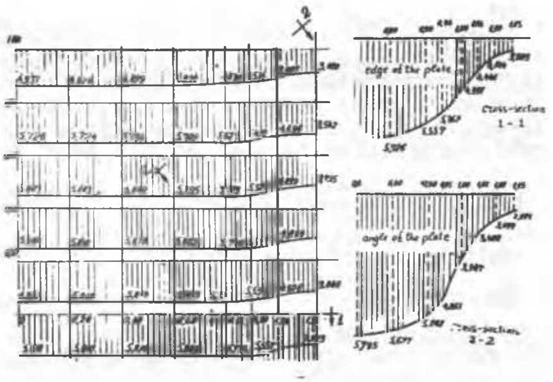

FIG.2

Our results show that approximation by Gorbunov-Posadov for displacements is sufficiently accurate everywhere except zones in the vicinity of angles of the plate.

3. It is impossible to represent exactly stresses and displacements of multi-layered half-space using elementary functions. Here we give a method for computation of stresses and strains in any point of multi-layered half-space using rapidly converging series.

Let us consider a problem of point vertical load P in an elastic half-space $z \geq 0$, consisting of n strata $h_{i-1} \leq z < h_i$ supported by half-space $z \geq h_n$. Elastic parameters of the strata are G_i, ν_i. We find the solution in the i-th stratum in form of biharmonic Love's function. Its representation by Hankel's transform is

$$\Phi_i = \int_0^\infty \{[A_i(\alpha) + \alpha(z-h_{i-1})B_i(\alpha)]e^{-\alpha(z-h_{i-1})} + [C_i(\alpha) + \alpha(z-h_i)D_i(\alpha)]e^{\alpha(z-h_i)}\} J_0(\alpha r)\, d\alpha,$$

$$i = 1, \ldots n \qquad (42)$$

$$\Phi_{n+1} = \int_0^\infty [A_{n+1}(\alpha) + \alpha(z-h_n)B_{n+1}(\alpha)] e^{-\alpha(z-h_n)} J_0(\alpha r)\, d\alpha$$

Here

$$\Phi_i = \tilde{\Phi}_i + \Phi_i^* \qquad (43)$$

where $\tilde{\Phi}_i$ is stress function corresponding to Kelvin's solution for point load in entire space and using Φ_i^* we satisfy boundary conditions between layers and on free surface. Taking into account (42) we may obtain from boundary conditions a system of 4n+2 linear equations with unknown values $A_1(\alpha), \ldots, B_{n+1}(\alpha)$. Expressing C_i, D_i, A_{i+1} and B_{i+1} from equations corresponding to the boundary conditions between i-th and i+1-th layers we come to linear system as follows

$$Z^*(\alpha) = S(\alpha)[Z^*(\alpha) + \tilde{Z}(\alpha)] \qquad (44)$$

Here $Z^*(\alpha)$ and $\tilde{Z}(\alpha)$ represent 2 parts of (4n+2)- dimensional vector $A_1, \ldots B_{n+1}$, corresponding to the Φ_i^* and $\tilde{\Phi}_i$ respectively. Let us write components of the vector Z

$$\tilde{A}_i = -H_i \frac{1-\alpha(c-h_{i-1})}{\alpha^3} e^{\alpha(c-h_{i-1})}$$

$$\tilde{B}_i = -H_i \frac{1}{\alpha^3} e^{\alpha(c-h_{i-1})}$$

$$\tilde{C}_i = -H_i \frac{1-\alpha(h_i-c)}{\alpha^3} e^{\alpha(h_i-c)}$$

$$\tilde{D}_i = H_i \frac{1}{\alpha^3} e^{\alpha(h_i-c)} \qquad (45)$$

Here c stands for the depth of load application,

$$H_i = \begin{cases} P/8\pi(1-\nu_i) & \text{if } h_{i-1} < c < h_i \\ 0 & \text{in other cases} \end{cases}$$

Square matrix $S(\alpha)$ is of the form

$$S = \begin{bmatrix} \begin{matrix} 0\,0 & R_1 \\ 0\,0 & \end{matrix} & & \\ M_1 \begin{matrix} 0000 \\ 0000 \\ 0000 \\ 0000 \end{matrix} & R_2 & 0 \\ 0 & & M_n \begin{matrix} 0000 \\ 0000 \\ 0000 \\ 0000 \end{matrix} \end{bmatrix}, \qquad (46)$$

where

$$M_i^{(4,2)} = \begin{bmatrix} (1-4\nu_i)\beta_{i1} & \frac{4}{\varkappa_{i1}\varkappa_{i2}}\{(1-2\nu_{i+1})k_i[1-\nu_i(3-4\nu_i)] - (1-2\nu_i)[1-k_i^2\nu_i(3-4\nu_{i+1})]\} + (1-4\nu_i)\beta_{i1}\, d\alpha_i \\ 2\beta_{i1} & (1-4\nu_i)\beta_{i1} + 2\beta_{i1}\alpha a_i \\ \frac{4(1-\nu_i)}{\varkappa_{i1}} & \frac{4(1-\nu_i)}{\varkappa_{i1}\varkappa_{i2}}\{k_i[1-2\nu_i(3-h\nu_{i+1})-1+2\nu_{i+1}(3-4\nu_i)+\varkappa_{i2}\alpha a_i\} \\ 0 & \frac{4(1-\nu_i)}{\varkappa_{i2}} \end{bmatrix} e^{-\alpha d_i}$$

$$i = 1, \ldots n \qquad (47)$$

$$R_1^{(2,2)} = \begin{bmatrix} -1+4\nu_1 & -4\nu_1(1-2\nu_1)+\alpha a_1(1-4\nu_1) \\ 2 & 4\nu_1 - 1 - 2\alpha a_1 \end{bmatrix} e^{-\alpha a_1}, \quad (48)$$

$$R_{i+1}^{(4,2)} = \begin{bmatrix} \frac{4(1-\nu_{i+1})k_i}{\varkappa_{i2}} & \frac{4(1-\nu_{i+1})k_i}{\varkappa_{i1}\varkappa_{i2}}\{k_i[1-2\nu_i(3-4\nu_{i+1})] - 1 + 2\nu_{i+1}(3-4\nu_i) - \varkappa_{i1}\alpha a_{i+1}\} \\ 0 & \frac{4(1-\nu_{i+1})k_i}{\varkappa_{i1}} \\ -(1-4\nu_{i+1})\beta_{i2} & \frac{4}{\varkappa_{i1}\varkappa_{i2}}\{(1-2\nu_{i+1})[k_i^2 - \nu_{i+1}(3-4\nu_i)] - (1-2\nu_i)k_i \times [1-\nu_{i+1}(3-4\nu_i)]\} + (1-4\nu_{i+1}) \times \beta_{i2}\alpha a_{i+1} \\ 2\beta_{i2} & -(1-4\nu_{i+1})\beta_{i2} - 2\beta_{i2}\alpha a_{i+1} \end{bmatrix} e^{-\alpha a_{i+1}} \quad (49)$$

$$i = 1, \dots, n-1$$

Here we use notifications as follows

$$k_i = G_i / G_{i+1}, \quad \varkappa_{i1} = 3 - 4\nu_i + k_i,$$

$$\varkappa_{i2} = 1 + k_i(3 - 4\nu_{i+1}),$$

$$\beta_{i1} = (1-k_i)/\varkappa_{i1}, \quad \beta_{i2} = (1-k_i)/\varkappa_{i2},$$

$$a_i = h_i - h_{i-1}$$

We can write a solution of the system (44) in the form

$$z^* = \sum_{k=1}^{\infty} S^k \tilde{z} \quad (50)$$

Needed approximate solutions are the partitial sums of above mentioned series. Inserting them into the expressions for displacements and stresses we obtain formulae containing integrals in the form

$$f_1(r,z,c)\int_0^{\infty} \alpha^n e^{-\alpha f_2(z,c)} J_k(\alpha r)\, d\alpha \qquad n = 0,1,2,\dots \quad k = 0,1.$$

Such integrals are easily expressed by elementary functions of coordinates. We may have

$$J_{n,k}(r,z) = \int_0^{\infty} \alpha^n e^{-\alpha z} J_k(\alpha r)\, d\alpha =$$

$$= \begin{cases} (-1)^n \frac{\partial^n}{\partial z^n}\left(\frac{1}{R}\right) = \frac{n!}{R^{n+1}} P_n\left(\frac{z}{R}\right) \\ (-1)^n \frac{\partial^n}{\partial z^n}\left[\frac{z}{R(R+z)}\right] = \frac{(n-1)!}{R^{n+1}} P_n^1\left(\frac{z}{R}\right), \end{cases}$$

where $R = \sqrt{r^2+z^2}$, P_n – Legendre's polinomials, P_n^1 – attached Legendre's functions of 1-st kind.

Correction due to the insertion of (k+1)-th member of the series (50) consist in reduction of k-th approximation by "reflection" over layer boundaries. Particularly using the "reflection" over free surface we obtain the solution of R.Mindlin (1936) and analogously in the case of two coherent half-spaces we have the solution of V.P.Plevako (1969), i.e. for that cases exact solutions are already in the 1st approach. For usual cases of soil stratification particularly for soil stratum laying on rigid (rock) foundation 1st and 2nd approximations are sufficiently accurate especially for points in the vicinity of load area. An example we give diagrams of vertical dicplacements W and shear stresses , computed for multi-layered foundation at the distance of 0.5m from the line of action of a point load P-1 ton. At fig.3 continuous curve represents exact solution, and dotted line-1st approximation according the proposed method.

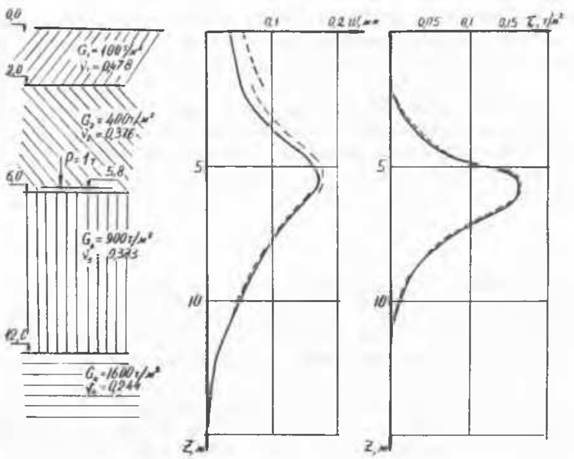

FIG.3.

This method may be spread on the plane and a spacial (non axially simmetric) problems. It is necessary for it to use Furver's transform instead of Hankel's transform.

The above mentioned solutions of problems of loaded elastic half-spaces are easily programmed for computer because obtained formulae are recurrent.

REFERENCES

GORBUNOV-POSADOV M.I. (1953) Design of structures on elastic foundation. Stroiizdat, Moscow.

EGOROV K.E. (1940) Determination of stresses in the foundation of rigid strip footing. Research Institute of Foundation Engineering, Bulletin N 9, Moscow.

EGOROV K.E. (1958). To a question of design of the foundations of ring footings. Research Institute of Foundation Engineering, N 34, Moscow.

•MINDLIN R.D. (1936). Force in the interior of the semi-infinite solid, Physics, 7, N5.

PLEVAKO V.P. (1969). Point load inside of the two coherent half-spaces. "Osnovania, fundamenti i mekhanika gruntov", N 3.

UFLAND Ja.S. (1967). Integral Transforms in problems of elasticity theory. Nauka, Leningrad.

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