ON THE DISTRIBUTION OF THE VERTICAL AND HORIZONTAL CONTACT PRESSURE COMPONENTS UNDER A RIGID FOUNDATION

SUR LA REPARTITION DES COMPOSANTS VERTICAL ET HORIZONTAL DE LA PRESSION DE CONTACT SOUS UNE FONDATION RIGIDE

О ВЕРТИКАЛЬНОМ И ГОРИЗОНТАЛЬНОМ КОМПОНЕНТАХ КОНТАКТНОГО ДАВЛЕНИЯ ПОД ЖЕСТКИМ ФУНДАМЕНТОМ

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SYNOPSIS. This investigation is a continuation of the work in the laboratory of foundation engineering and soil mechanics at the University of Technology. The object of the present study is to clarify, with the aid of a systematic test series, in the first place the influence of the load magnitude on the contact pressure distribution. Earlier observations already revealed that the most significant effect is exerted by load magnitude (Hartikainen 1970). The effects of foundation depth and of reloading have also been studied. Best efforts were made, in constructing the test foundation and in planning the experimental arrangements, to exclude the errors incurred in earlier equipment and arrangements. The distribution of the horizontal component of contact pressure was also measured. The scattering of results was taken into account in the evaluations.

INTRODUCTION

Foundations are required as a rule to transfer the loads from structures to the soil base, because the strength of building materials is higher than that of soil. Most often this transfer is accomplished by means of spread foundations, of which the footing foundation is the one most commonly employed under piers.

When a foundation is being dimensioned, it is necessary to know the magnitudes of the normal forces, shear forces and moments. These can only be calculated if the loads acting upon the foundation and the reactions caused by them are known regarding magnitudes, points of action and directions. These data are more easily found for the loads than for the reactions. In the present report the reaction due to vertical load imposed on the foundation shall be termed "contact pressure".

When soil is subjected to load, the stresses are distributed in the soil, in which the incremental stress from the loads produces a settlement depending on the compressibility characteristics of the soil. Therefore, the soil settles in and around the area under load. The shape of this settlement, together with the elastic properies of the foundation, determines the distribution of contact pressure. If, as generally is the case, there is friction between foundation and soil, then the contact pressure is not perpendicular to the base of the foundation: it can be divided into a vertical contact pressure component, which is at the same time the normal stress at right angles to the base plane, and a horizontal component, which is at the same time the shear stress in the base plane. In the present study both components have been measured and treated along with each other.

It is a fact that, so far, the contact pressure distribution is adequately understood in two extreme cases only, namely, those of the elastic and failure states; opposite results are produced in these cases by theoretical considerations (Fig. 1 a). Results which are greatly at variance are also obtained by theoretical examination of the horizontal contact pressure component (Schweickert 1964, Gorbunov-Possadov 1965) (Fig. 1 b). The actual behaviour of soil is always intermediate between said two extreme states. Since in the theoretical considerations numerous simplifying assumptions and, under these circumstances, combined treatment of a plurality of unknown factors are involved, experimental clarifications are still needed with regard to those factors which influence the contact pressure distribution.

EQUIPMENT AND EXPERIMENTAL ARRANGEMENTS

The experiments were carried out in a test basin, wherein the soil material, consisting of fine sand, was compacted by means of a vibrating plate in layers of 25 cm each. Fig. 2 shows the grain size curve of the sand. The average moisture content of the soil material during the test series was w = 4,4 %. The average dry unit weight was found to be

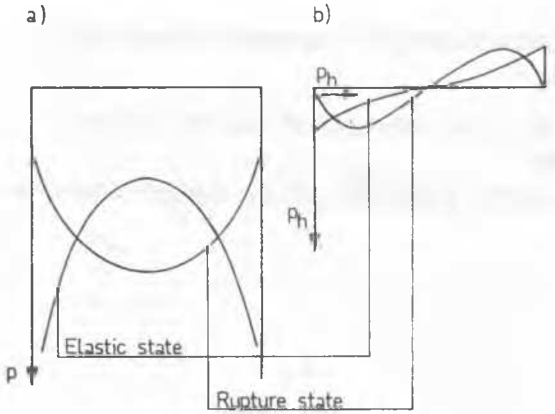

FIG. 1. THE DISTRIBUTION OF (a) VERTICAL AND (b) HORIZONTAL CONTACT PRESSURE COMPONENTS.

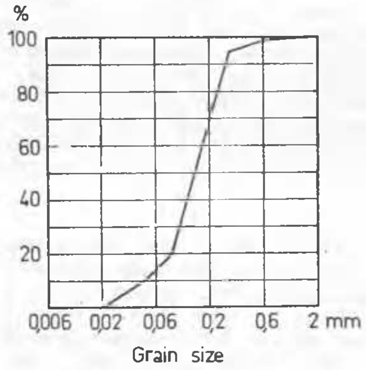

FIG. 2. GRAIN SIZE CURVE OF THE TEST SOIL.

γ_d = 1,53 kp/cm^2, corresponding to relative density D_r = 0,56, determined by normal Proctor tests. The friction angle consistent with the average dry unit weight is φ = 34,4^o and the cohesion corresponding to the average moisture content can be taken as c = 0,04 kp/cm^2.

The base plate of the experimental foundation was square, and of 30 x 30 cm size. Its design is shown in detail by Fig. 3. Endeavours were made in its design to take into account the experience of a previous study (Helenelund 1966, Hartikainen 1970). The upper plate was made more rigid than before, with 50 mm thickness, and the steel rods were shorter. The sections of the lower plate had a square shape, and their number was 36 instead of 20 in the prior tests. For the purpose of studying the horizontal contact pressure component a measuring plate was made of 3 mm aluminium sheet.

The test series concerned here consisted of centric first loading and reloading, performed four times on the surface (D = 0) and three times at greater depth (D = 50 cm).

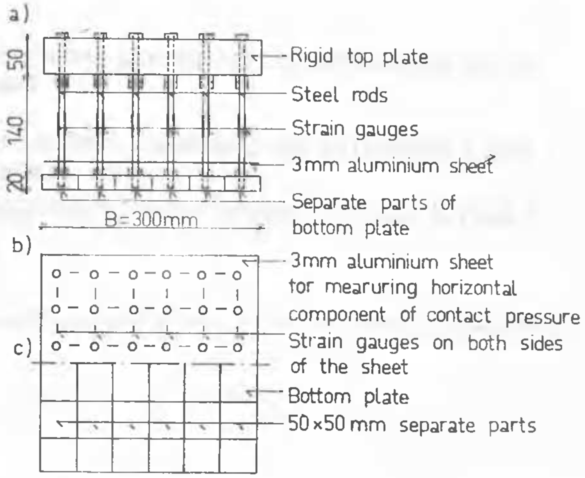

FIG. 3. (a) ELEVATIONAL VIEW OF THE EXPERIMENTAL FOUNDATION. (b) MEASURING PLATE FOR MEASUREMENT OF HORIZONTAL CONTACT PRESSURE COMPONENTS. (c) THE LOWER PLATE OF THE EXPERIMENTAL FOUNDATION.

Each first loading was commenced by first levelling the foundation level. This was done with the aid of a levelling frame of 50 x 50 cm size, made of 10 x 30 mm bar iron. The frame was pushed into the soil to such depth that its upper edge coincided with the foundation level. Levelling was then effected with a metal straight-edge. After the experimental foundation had been mounted, this frame was removed and the groove which had been produced by it was filled in. In the experiments with 50 cm foundation depth, the protective tube was then installed and filling up to surface level was carried out.

The last operation was mounting of the dial gauges. The dial gauges indicating the movements of the slab were placed in the corners of the upper plate of the experimental foundation, and the locations of the dial gauges for soil movements were 25 cm distant from the central points of the sides of the foundation.

No separate installation procedures were required for the reloadings. Loading was by means of a 25 ton power jack, rigidly braced against a steel beam. The load was transmitted from the jack to the foundation by a ball joint.

RESULTS AND THEIR TREATMENT

When in connection with an earlier study (Hartikainen 1970) the mode was considered in which the vertical contact pressures under a square slab should be presented, it was decided to present them by surface mapping, which is far more perspicuous than the sectional presentation.

In connection with a computer, the surface maps could be printed either with a diagram unit or with a line printer. Within the scope of the peripheral equipment which was available, printing by line printer was rather more efficient, and this procedure was therefore chosen.

In the computer programme which was set up, the means of the test loading results are first calculated for points which are symmetrically located with reference to the load application point. The results are then corrected on the basis of the sum of the strain readings to correspond to the desired average load p_m, whereby the per cent error incurred in maintaining constant load is eliminated. The grid of observational points (6 x 6) is then converted into a grid of 12 by 12 regular squares by interpolation and extrapolation, whereby the grid extends up to the edges of the slab. Linear interpolation is then applied to calculate the relative contact pressures p/p_m in each square to form a 5 x 3 matrix, whereby for the contact pressures of the entire slab, taking its edge into account, a 61 x 37 matrix is obtained. This is printed with the line printer so that each element only occupies one position; it is thus understood that a given symbol corresponds to each given contact pressure interval. By choosing symbols with suitably alternating blackness values, good contrast between the different stress areas is achieved. The contact pressure maps obtained by this procedure are even in themselves highly perspicuous, and they greatly facilitate the handling of an extensive material of investigation. However, in the contact pressure maps intended for final presentation the constant value lines were entered, following the boundaries between adjacent stress areas, and the values of the corresponding constant pressures were inscribed. Since the contact pressures were calculated as relative values p/p_m, the effect of the magnitude of load could also be made clearly evident in the contact pressure maps.

The treatment of the horizontal contact pressure component followed mainly the same lines as that of the vertical component. This means that the computer programme set up to this purpose is similar in principle to that described above with reference to the vertical component. The sole difference is that in this programme the horizontal contact pressure components are calculated from the projection conditions separately in both directions of measurement and interpolation and extrapolation up to a 61 x 37 matrix is carried out for both directions separately, and the final horizontal contact pressure components are only then calculated. In the maps thus obtained the direction of the horizontal component is outward from the central point. The horizontal components, too, were treated on a relative basis, as p_h/p_m , in order that the effect of load magnitude might be clearly evident.

INTERPRETATION AND COMPARISON OF RESULTS

In experimental studies the scattering of results is a factor greatly impeding the interpretation of results. In order to determine the scatter, three or four test loadings with identical boundary conditions were undertaken. The average standard deviation of the whole test material was 22 % for the vertical and 35 % for the horizontal components of the contact pressure. Considering the accuracy achieved in constructing the test foundation, the observation can be made that the scattering is mainly due to soil inhomogeneity.

All the fourth parts of the contact pressure maps presented in this study (Figs. 4 to 7) represent the mean results of mutually similar tests, whereby the interpretation as regards different factors affecting the results is facilitated.

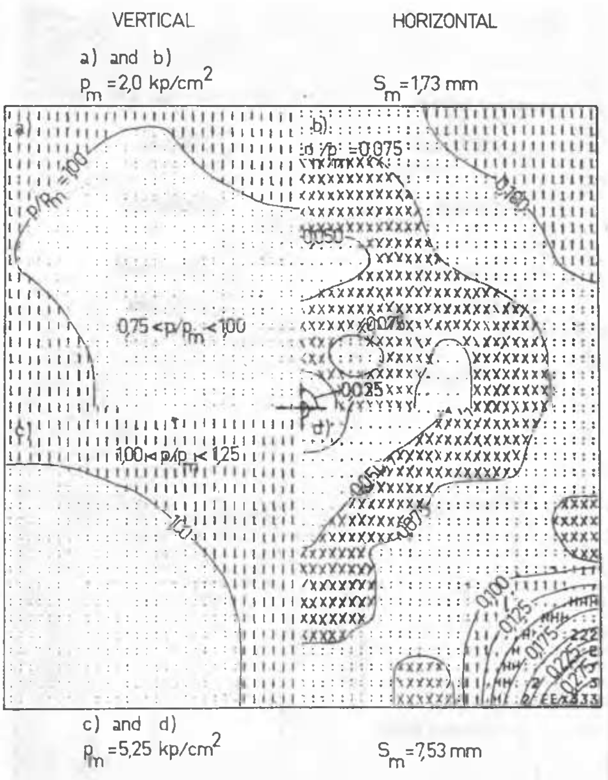

FIG. 4. AVERAGE DISTRIBUTION OF THE VERTICAL AND HORIZONTAL COMPONENTS IN THE CENTRIC FIRST LOADINGS ON THE SURFACE PRESENTED BY THE FOURTH PARTS OF THE CONTACT PRESSURE MAPS.

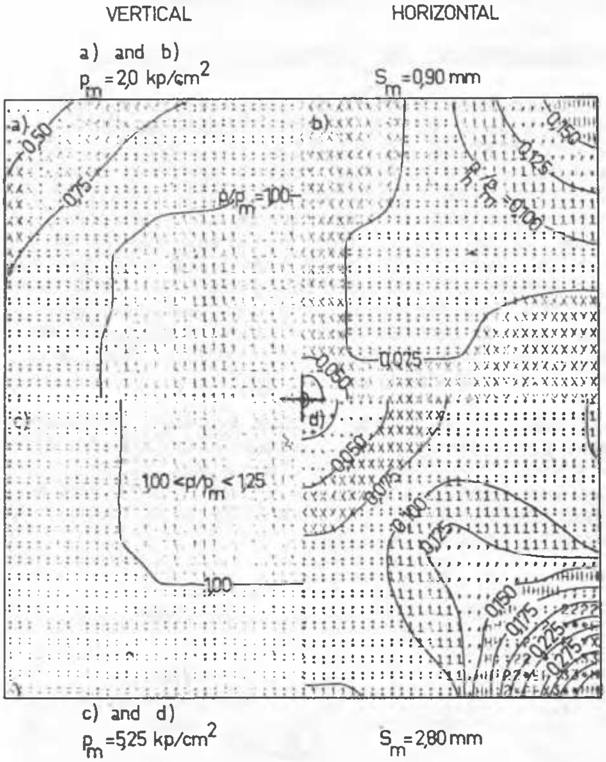

FIG. 5.

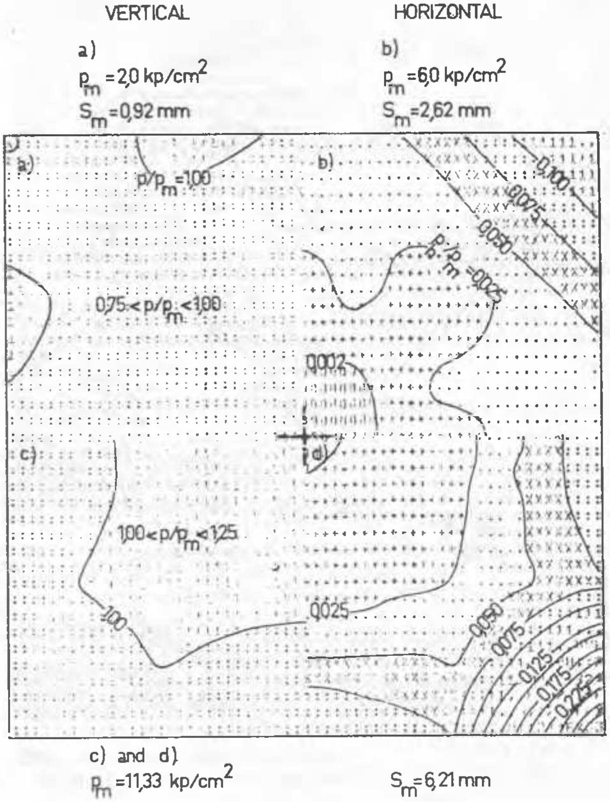

FIG. 6.

FIG. 5. AVERAGE DISTRIBUTION OF THE VERTICAL AND HORIZONTAL COMPONENTS IN THE CENTRIC RELOADINGS ON THE SURFACE PRESENTED BY THE FOURTH PARTS OF THE CONTACT PRESSURE MAPS.

FIG. 6. AVERAGE DISTRIBUTION OF THE VERTICAL AND HORIZONTAL COMPONENTS IN THE CENTRIC FIRST LOADINGS AT 50 CM DEPTH PRESENTED BY THE FOURTH PARTS OF THE CONTACT PRESSURE MAPS.

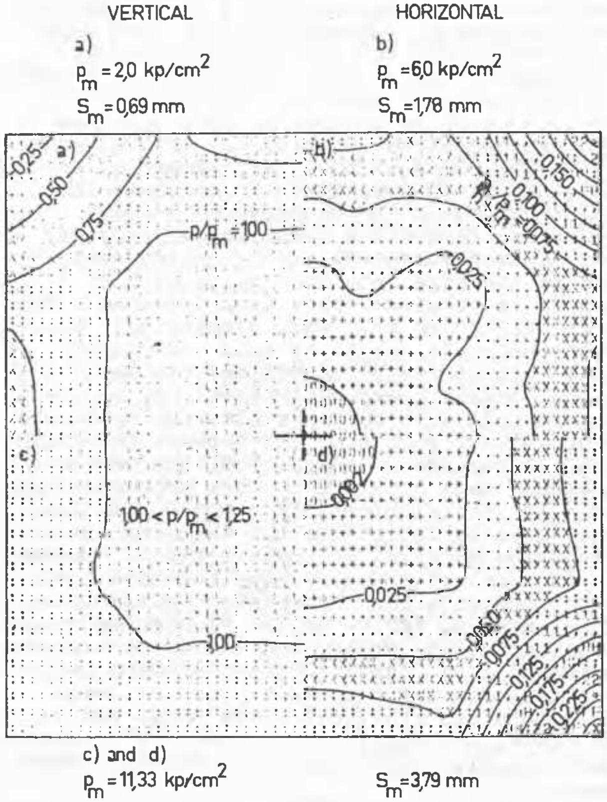

FIG. 7. AVERAGE DISTRIBUTION OF THE VERTICAL AND HORIZONTAL CONTACT PRESSURE COMPONENTS IN THE CENTRIC RELOADINGS AT 50 CM DETH PRESENTED BY THE FOURTH PARTS OF THE CONTACT PRESSURE MAPS.

Increasing of the load causes a shift from the elastic state in the direction towards the rupture state (Fig. 9). This should result in an increase of the relative vertical contact pressure component in the central part of the slab. The phenomenon is indeed clearly observable in the present study, namely, in the results of the centric first loading tests (Figs. 4 and 6).

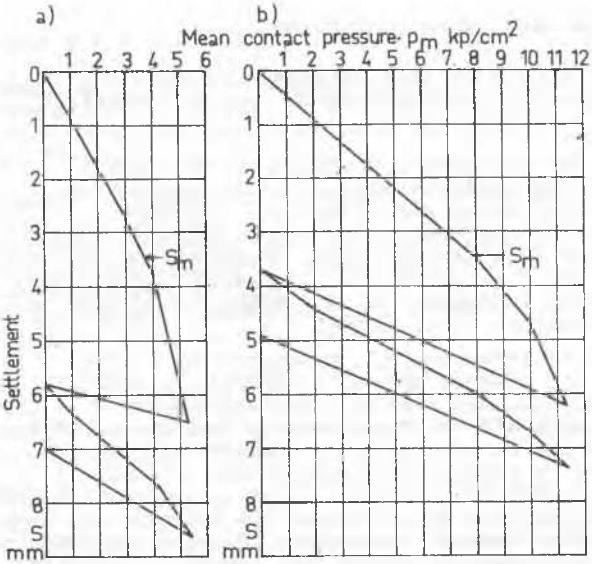

FIG. 8. AVERAGE LOAD-SETTLEMENT CURVES IN THE TESTS (a) ON THE SURFACE AND (b) AT 50 CM DEPTH.

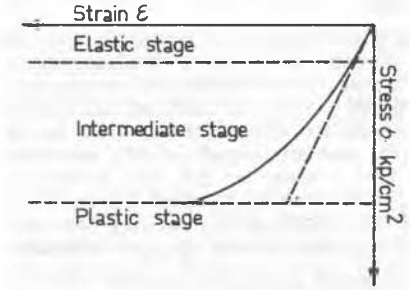

FIG. 9. TYPICAL STRESS-STRAIN CURVE OF SOIL AND ITS VARIOUS STAGES.

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After removal of the load there occurred release of stresses in the edge areas, especially at the corners, with convex transformation of the foundation level. As a result, the relative vertical contact pressure component was at its highest at the initial loads in the reloading tests. When the load was increased, the direction of the contact pressure distribution was reversed, similarly as in the first loadings. Thus the contact pressure distribution at higher loads was similar to that of the first loadings (Figs. 4 to 7). This is due to the fact that the settlements are mainly elastic in the reloadings (Fig. 8).

The influence of the foundation depth on the vertical contact pressure component was minimal. The magnitude of the load has less effect on the distribution of the horizontal contact pressure component than on that of the vertical component, bacause the distribution of the former remains nearly unchanged. In particular, the change of the distribution in the reloadings is minimal. In the first loadings, a more distinct change of the distribution with increasing load is seen than in the reloadings; this is particularly marked at the foundation depth D = 50 cm (Fig. 10). At higher loads, similar distributions of the horizontal contact pressure component were obtained in the first as well as the reloadings (Figs. 4 to 7), as could be expected on the basis of the distribution of the vertical component. The relative horizontal contact pressure components are higher in the centre of the foundation in the tests on the surface than in the test at 50 cm depth.

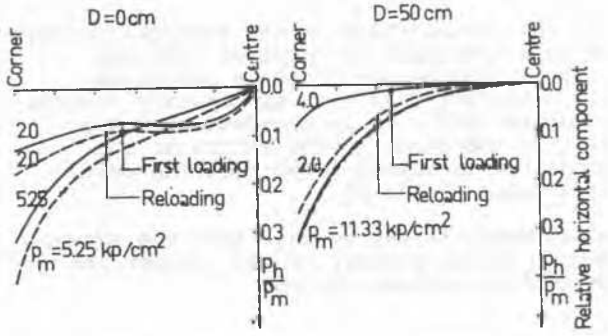

FIG. 10. THE RELATIVE HORIZONTAL COMPONENT OF THE CONTACT PRESSURE ON THE DIAGONAL OF THE FOUNDATION.

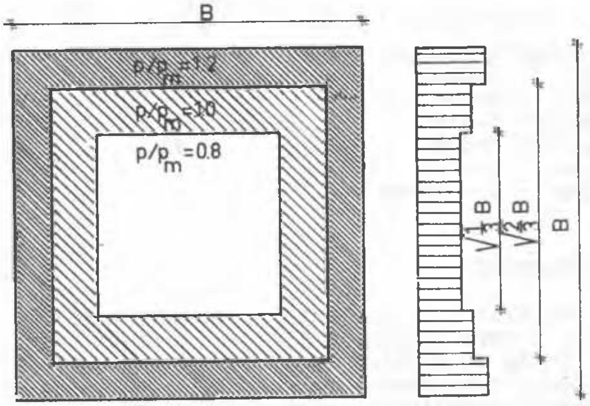

FIG. 11. SIMPLIFIED CONTACT PRESSURE DISTRIBUTION MOST DANGEROUS IN VIEW OF THE DIMENSIONING OF THE FOOTING FOUNDATION AS FOUND IN THE TESTS.

On scrutiny of all individual results, the approximate observation can be made that the contact pressure under centric load was never more dangerous, as regards the dimensioning of a footing foundation, than that shown in Fig. 11.

Comparison of the distribution of the vertical contact pressure component with the modulus of compressibility method (Kany 1959) reveals that none of the factors affecting the test results has any substantial effect in Kany's method, owing to the fact that the foundation is highly rigid. It may also be noted that the vertical contact pressure component calculated by Kany's method is clearly higher than that found by tests (Table 1). When a method combining the modulus of subgrade reaction and the modulus of compressibility (Repnikov 1967, Schultze 1969) is applied, the results obtained in the tests are very closely approximated (Table 1), while otherwise the method was altogether unpredictable.

When the distribution of the vertical contact pressure component is compared with the contact pressure distribution consistent with Balla's (1962) bearing capacity formula (Schultze 1962), it is observed that the relative component in the centre of the foundation is always lower that the calculated value (Table 1).

Accordingly, it may be said that the experimental results clearly remain between the theoretical extreme states.

TABLE I. COMPARISON OF EXPERIMENTAL AND THEORETICAL VALUES OF THE VERTICAL CONTACT PRESSURE COMPONENT.

	Point of measurement		
p/p_m	Corners	Edges	Centre
Max. in tests	1,1	1,1	1,2
Min. in tests	0,7	0,9	0,9
Subgrade reaction	1,0	1,0	1,0
Kany 3-dim.	1,82		
Kany 2-dim.		1,35	
Repnikov 3-dim.	1,26		
Repnikov 2-dim.		1,11	
Balla D = 0			1,44
Balla D = 50 cm			1,33

CONCLUSIONS

The distribution of contact pressure constitutes a very extensive field of problems, in which the foundation and the soil and other boundary conditions are essential determinants. Accordingly, of the different boundary conditions only those consisting of the magnitude of load, foundation depth and reloading were chosen as such upon the effects of which the present study was centred, while throughout the study one and the same, square, rigid experimental foundation and one and the same soil material, fine sand, were considered.

The most important observations made in connection with the study can be summarized as follows.

1. The standard deviations of the test results could be approached by performing several tests with identical boundary conditions. These standard deviations were found to be rather high and mainly caused by the soil inhomogeneity; accordingly they hardly instigate any optimism with a view to situations in practice.

2. The influence of the magnitude of load on the contact pressure distribution is distinct, but even so this distribution remains all the time between the theoretical extreme values. This is thought to indicate that as a rule the applications of soil mechanics move within a wide range in the intermediate area between the elastic and rupture states. The effect of the foundation depth proved to be minimal, which is primarily due to the soil, which was loose, compared to that in the earlier studies (Hartikainen 1970).

3. The effect of reloading was found to be highly interesting in that load increase had an effect on the changes in contact pressure distribution opposite to that observed at first loading.

4. With the soil material used in the tests, the test results were close to those consistent with the modulus of subgrade reaction theory. This is not attributable to appropriateness of the theory, but to the fact that its assumptions, which are very crude from the viewpoint of the theory of elasticity, act in the proper direction in the case of the soil material in the present tests by an approximately proper amount.

5. It was clearly noticeable that procedures based on the classical theory of elasticity are poorly suited, at least with the present soil material, resulting in too high edge pressures, among other things.

6. Simultaneous consideration of the vertical and horizontal contact pressure components, in particular, showed clearly that although the distribution of the horizontal component resembles the distribution according to the theory of elasticity, it is derived from plastification. Accordingly, its consideration by the theory of elasticity leads in the wrong direction.

ACKNOWLEDGEMENTS

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