# ANALYSIS OF THE FOUNDATION SLABS AND UPPER STRUCTURE INTERACTION

## ANALYSE D'ACTION RECIPROQUE DES DALLES ET DES BATIMENTS

## АНАЛИЗ СОВМЕСТНОЙ РАБОТЫ ФУНДАМЕНТНЫХ ПЛИТ И ВЕРХНЕГО СТРОЕНИЯ

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SYNOPSIS. Three methods of continuous slab building's and construction's foundations estimation described in the report. RIVKIN method concerns upper constructions slab foundation and soil basement as a single space multielement system. Deformation properties of the soil basement are being determined by the suggested by the author three-dimensional relation. BOBRITSKY and KLEPIKOV method concerns interaction of the slab foundation with non-uniform linearly deformable soil basement considering stiffness of the upper structure. Deformation properties of the soil basement are being described by the variable stiffness coefficient with the consideration of non-linear relation between settlement and pressure in the foundation bed. MALIKOVA method have been developed for the estimation of large continuity slabs with the consideration of the upper structure stiffness. The method employs Gorbunov-Posadov tables for the estimation of the slabs on the elastic semispace. Recommendations have been given on the choose of the deformation module in accordance with the multistoreyed building's settlement measures data.

The employment of slab foundations is being spread in modern engineering which is due by the increasing of number of floors and enlargement of technical equipment charge.

In connection with this the problem of the improvement of estimation for the economical and reliable projecting of slab foundations becomes of great importance.

Investigations of the last years demonstrated that the estimation of slab foundations without consideration of their common work with upper structure and variability of soils deformation properties cannot present reliable basis for projecting.

RIVKIN method of estimation is based on the next reference positions (Rivkin, 1968).

1. Upper structures, slab foundation and soil present a single space multielement system /Fig.I/
2. Deformation characteristics of this system elements are of considerable variability; their influence on stress-strain state of the system is non-single valued. That creates necessity of consideration such estimation parameter of the system in real variability limits, which permits to

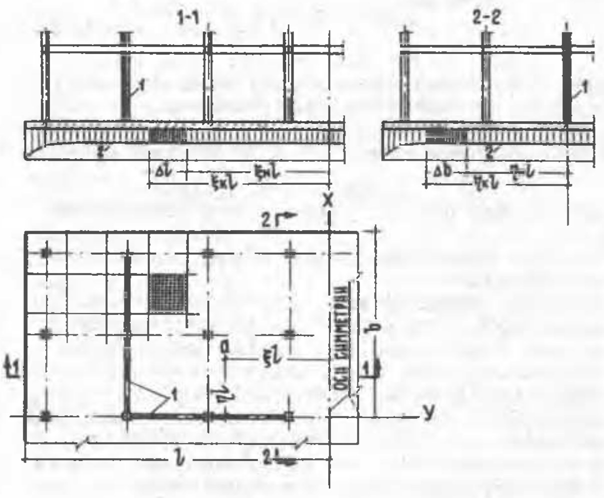


Fig. I.

Frame building on continuous slab foundation; I - rigidity diaphragms; 2 - rigidity epure of soil basement.

indicate the limits of stress changes in construction elements.

3. Estimation model may not consider these characteristics of actual system, effect of which on it's stress-strain state is overlapped by the variability consideration of other characteristics more essential for the problem under consideration.

Concerning these principles as design model of the system, another space rod system equal to the first one by its deformation properties is proposed in which upper building structures are approximated as usually by the frame, slab foundation-by cross bands and soil - by discrete support represented by vertical rods placed in slab band axis cross points.
The legitimation of the rod approximation of system's two-three dimensional elements is proved by comparative estimations.
As a result system with elements of different dimensions, estimation of which caused insurmountable difficulties and forced to refuse from the consideration of common work of slab foundation and upper structure is reduced to one-dimensional system estimated by computer programmed for space rod systems considering bending and longitudinal rod strains.

An important peculiarity of great extension construction's common work with soil that is small changes in nature of distribution of soil deformation characteristics in plan which can cause vital changes of stresses and even of their sign, is taken into consideration within estimation. Mechanical characteristics of the soil under rectangular foundation slab /Fig. I/ are expressed by suggested by author relation

$$p=K\left[1+\beta e^{-\alpha(1-\xi)(m-\eta)}\right]v, \qquad (1)$$

where $p, v$ - correspondingly reactive soil pressure and vertical displacement in any point of slab;
$K, \beta, \alpha$ - parameters of soil design model;
$\xi=\frac{y}{\ell}$, $\eta=\frac{x}{\ell}$ - values of relative coordinates of the point under consideration;
$m=\frac{b}{\ell}$ - relation of the slabs width to its length.
The other symbols are clear from Fig.I.
Parameter $K$, measured in tn/m$^3$ characterised foundation bed rigidity, without considering soil resistance out of the foundation limits. Dimensionless parameters $\alpha$ and $\beta$ consider the soil influence out of the foundation limits on the foundation bed stiffness and nature of contact pressures distributions.

Relation (I) with corresponding values of model parameters permits to accurately describe all vitally important peculiarities of constructions cooperation with soil. In particular cases:
with $\beta=0$ it corresponds to Winkler model;
with $\beta\neq 0, \alpha\to\infty$ it corresponds to Pasternak model;
with $\beta$ =5,5 and $\alpha$ =10 it's solutions are close to elastic semi-space solutions;
with $\beta$ =5,5 and $\alpha > 10$ it corresponds to finite thickness elastic layer model.

Stresses in foundation slab, as a rule, increase with the reducing of soil rigidity.

Consequently, while estimating It's necessary to consider the least probable value of parameter $K$, corresponding to the most forcasted value of the building settlements.

Character of distribution of stresses in foundation slab their value and sign depend on character of the foundation rigidity distribution.

That is why the upper as well as lower confidence lunits of parameter $\beta$ and also the more unfavourable type of the foundation bed rigidity epure, determined by parameter $\alpha$ must be taken into account. It is advisable to take parameter $\alpha$ in all the cases equal to IO and parameter $\beta$ depending on the kind of soil correspondingly to the table below.

Recommended parameter values.

| Soil | Limit upper | Limit lower | Soil | Limit upper | Limit lower |
|---|---|---|---|---|---|
| Sand compact | I.0 | 0 | Clay hard | I.5 | 0.5 |
| average density | 0.5 | -0.25 | half hard | I.0 | 0 |
| loose | 0 | -0.50 | plastic | 0.5 | -0.5 |

With accepted values of $\beta$ and $\alpha$ parameter $K$ is determined depending on the settlement value $S$ and average pressure on soil $p$ in accordance with the relation (I), in formula

$$K=\frac{p}{S}\,\frac{1}{1+\frac{\beta}{\alpha m}A_{3(\alpha m)}}, \qquad (2)$$

where $A_3$ - integral function, which is tabled (Rivkin, 1967).

Variability of the distribution of soil compressibility (Fig. 2) characterised by parameters $\beta$ and $\alpha$ influences considerably stress-strain state; variability of the foundation total compressibility characterised by parameter $K$ has relatively less significance. Rigidity of discrete supports $C$ in tn/m, taken in a form of a rod of I m length, equivalent to this of rectangular foundation area with the length $\Delta\ell=\Delta\xi\ell$ and width $\Delta b=\Delta\eta\ell$ (with relative coordinates of the area origin $\xi_H$ and $\eta_H$ and end $\xi_K$ and $\eta_K$ (Fig.I) is determined by formula

$$C=K\ell^2\left[\Delta\xi\Delta\eta+\frac{\beta}{\alpha}\Sigma A_{3(t)}\right], \qquad (3)$$

where $\sum A_{3(t)} = A_{3[\alpha(1-\xi_H)(m-\eta_H)]} +$

$$+A_{3[\alpha(1-\xi_K)(m-\eta_K)]} - A_{3[\alpha(1-\xi_H)(m-\eta_K)]} - A_{3[\alpha(1-\xi_K)(m-\eta_H)]} \quad (4)$$

So obtained design model have universal properties. By varying rod stiffness it is possible to change deformation characteristics of soil, to take into account the larger rigidity of slab parts, the presence of openings and other constructive peculiarities and also different degree of upper structures participation in the slab work.

Non-linear problems connected with the consideration of non-elastic strains of soil and reinforced concrete can be easily solved by step method as each new "step" of estimation needs correction in initial information only of rods rigidity combinations. Clear physical interpretation of model elements permits not only to investigate it with given designing conditions but also to obtain of optimal constructive solution by varying parameters.

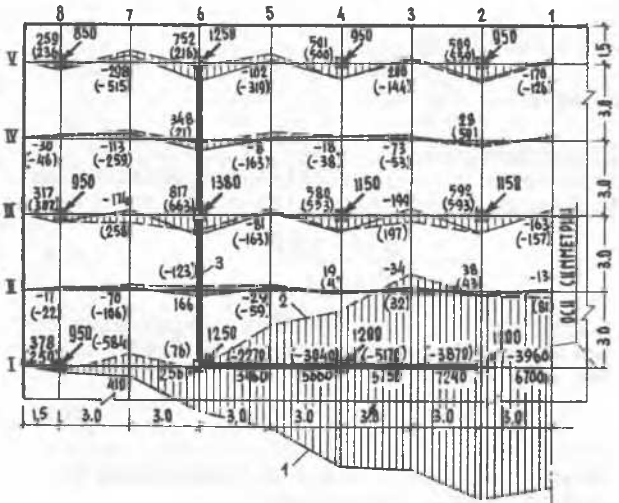

Fig.2.
Bending moments' epure (tn m) in longitudinal bands of slab foundation.

I. parameters of the soil model $\beta$ =I and $\alpha$ =IO;

2. the same with $\beta$ =0.

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On figures 2 and 3 are presented moment epures in longitudinal bands (3m width) of slab foundation of multi-storeyed frame building (Fig. I) solved in connection with flexible rod scheme.

Said design method is used by design institute "Kiev project", "Mosproject" and others.

Solution of the complex problem "soil-slab-upper structure", developed by Bobritsky and Klepikov is devided into two stages first of which proposes the contact problem solution with the determining of interaction stresses between slab and upper structure, and the second, final stage means slab estimation for given out of the interaction area outer loadings and found contact forces.

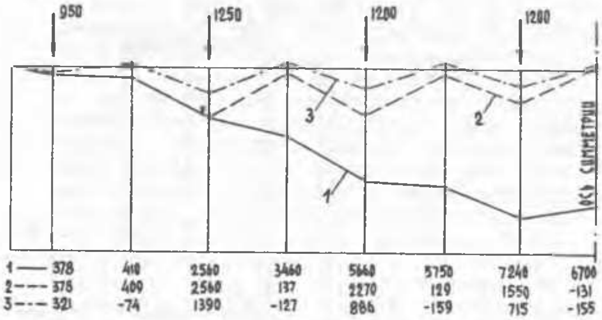


Fig. 3.
Bending moments' epure in the foundations' band along the axis of longitudinal rigidity diaphragm with $\beta$ =I and $\alpha$ =IO.

1. with consideration of upper structures' work;

2 and 3 - without consideration of upper structures' work for the slab with increased rigidity areas in the places, where diaphragms are located and for the smooth slab respectively.

Let us consider the common case of the foundation slab deflection taking the loading through the column system and diaphragm rigidity (Fig. 4a).

The slab lies on non-uniform in plan foundation. For design model let us take single parameter foundation-bed characterised by variable coefficient $K(x,y)$ permitting to effectively consider natural non-uniformity of soils lying in construction base. (Klepikov, 1967).

Loading transmitted to slab by diaphragm is reduced to resultant force $P_z$ and to bending moments $M_x$ and $M_y$ relating to axes $x$ and $y$ correspondingly. Taking rigidity of diaphragm in it is plane incomparably greater than foundation slab bending rigidity, we can conclude that the contact zone does not deflect with the slab bending but shifts as absolutely hard body. Diaphragm shifting in common case is determined by vertical translational displacement $y_0$ and rotation angles $\varphi_x$ and $\varphi_y$ around axes $x$ and $y$ correspondingly ( Fig. 4b). Following Zhemochkin (Zhemochkin, Sinitsyn, 1962) let us imagine the link between the slab and diaphragm as a number of absolutely rigid vertical rods with hinged supports (Fig. 4c). Further, let us choose the main system removing all superfluous links and

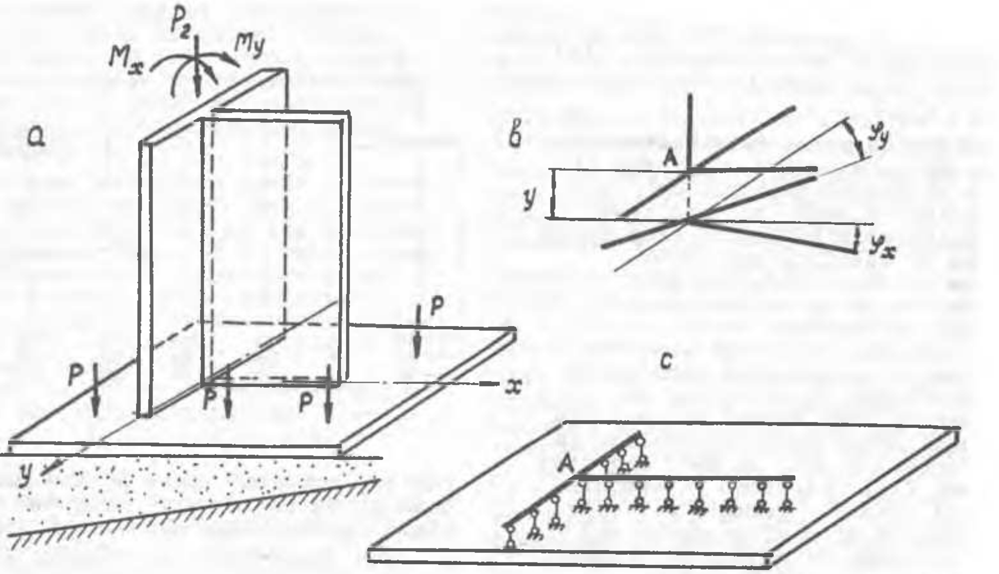


Fig. 4.
Foundation slab with rigidity diaphragm.

a - fragment of building;
b - diaphragm displacement scheme;
c - design scheme

changing them by unknown interaction forces $X_i$. With it diaphragm is conditionally fixed in the point of conjugation $A$ from the turns relatively to principal axes.

The unknown contact forces are determined by means of solution of the canonical equations system of structural mechanic and the equilibrium equation for diaphragm. After estimation of "basis" forces in links the slab is estimated for bending from outer loadings given out of the diaphragm limits and from found contact forces; further slab strain state, components of inner forces in slab $M_x, M_y, M_{xy}, Q_x, Q_y$ and reaction of soil $p$ are determined. This voluminous work can be done with the help of developed in NIISK, GOSSTROY, USSR, programme of calculation of foundation slabs RAFUP-2, permitting to simultaneously solve slab bending problems with 25 possible outer loadings' combinations (Bobritsky, 1970). The programme is based on the solution of the fundamental differential equation by a method of finite differences permitting to consider various boundary conditions and variability of a number of system's estimation parameters.
The consideration of non-linear character of work of foundation is realised with the help of elastic solutions method based on the successive approximation process; in each cycle foundation rigidity is corrected by the results of previous estimation cycle. The procedure continues till given convergence criterion is realised while comparing two adjacent cycles. It needs usually 3-4 approximations. As there is non-linear dependence of soil settlement-pressure the following hyperbolic relation is accepted: 653

$$w = \frac{\bar{w}\left(1-\frac{\bar{p}}{R}\right)p}{\bar{p}\left(1-\frac{p}{R}\right)} \qquad (5)$$

Basing on (5), the following expression of non-linear foundation rigidity coefficient has been obtained:

$$K = \frac{R}{w + \bar{w}\left(\frac{R}{p}-1\right)} \qquad (6)$$

where $R$ - ultimate load corresponding to loss of bearing capacity;
$w$ - foundation settlement;
$\bar{w}$ - settlement for a given (normative, exploitation) pressure;
$p$ - pressure on the foundation bed.

In accepted non-linear diagramme (Fig.5) of loading, characteristics are used which can be easily determined by means of soil mechanics methods for given dimensions of foundation embedding depth and physical and mechanical properties of soil. These characteristics permit to describe completely loading diagramm up to the value of pressure $\bar{p}$. In the portion of deformation from known value of pressure $\bar{p}$ to known value of ultimate pressure $R$ loading diagram is described less accurately as there can be presented a great number of curves with different degree of approximation to asymptote $p = R$. While estimating one can employ any

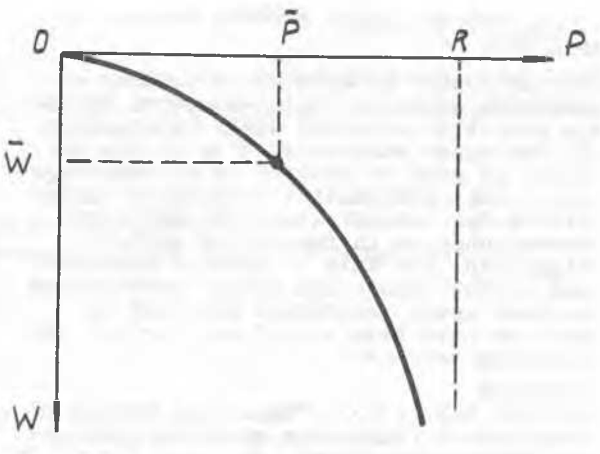


Fig.5.
Settlement foundation diagram.

other kinds of relations $w=f(p)$ including those obtained from experiments.
Said method can be applied for the estimation of a number of constructions.

Malikova considers the peculiarities of work of often met in design practice foundations in a form of rectangular slabs of constant finite stiffness and large extension, with the width $b \geqslant 4L$ where $L$ - elastic characteristic of the slab (Gorbunov-Posadov, 1953) equal

$$L = h\sqrt[3]{\frac{E_1(1-\nu_0^2)}{6E_0(1-\nu_1^2)}} \qquad (7)$$

where $h$ - thickness of the slab; $E_1$, $E_0$ - modules of concrete elasticity and soil deformation;
$\nu_1$, $\nu_0$ - poisson's ratio for concrete and soil.
In connection with large dimensions of foundation slabs in plan it's possible to accept simplificating assumptions concerning slab calculation scheme and soil model.

It's possible while estimating not to consider embedded depth of the slab and to regard it as lying on the soil surface.
Stress state and deformations of rectangular foundation slab under loadings situated near the edge can be accurately determined as in semi-infinite slab on compressible foundation and under the loadings placed far from the edge as in infinite slab. Additional precising of design of large extension foundation slabs is obtained in accordance with indication in the work (Gorbunov-Posadov, 1953) by using elastic semi-space for soil model with high module of deformation determined by the average expected settlement of building.

Basing on our investigations (Malikova,1968) we conclude that foundation module in a form of elastic semi-space with higher deformation module can be used not only for comparatively homogenous in plan and in depth soil basements but as well in the oase when compressible soils at the depth $H \geqslant 3L$ are underlaying by rocks of low compressibility. Such approach to estimation of the large extension slab permits to reduce substantially volume of calculations by using for determination of stresses and deformations of foundation slabs ready tables (Gorbunov-Posadov,1959) of dimensionless design values, made for semi-infinite and infinite slabs on elastic semi-space. There can be given any number of points of the slab in which design values under vertical concentrated and uniformly distributed loadings must be determined. The number of loadings is not limited as well. To consider common work of foundation slab and vertical rigid elements of the upper structure such as silos bunkers, T.V. towers, rigidity diaphragms of the building mixed method of structural mechanics can be employed.
Between the slab and rigid upper structure elements conditionally absolutely hard rod ties are put in, which imitate their contacts in some points. Forces in these joints are loadings, transmitted to the slab by rigid elements of upper structure. In cases when upper structure's rigid elements rest upon the slab through the flexible ties (columns) deformation, caused by compression of these ties, are taken into consideration. The estimation of large extension foundation slab with consideration of common work with vertical rigid upper structure elements consists of two stages: determination of real loadings transmitted by vertical rigid elements to the slab; determination of foundation slab stresses and strains from found loadings, transmitted by rigid elements and from the loadings, given out of the contact zones with the help of dimensionless design values tables.
According to said method algorithm and programme MRP-3 have been worked out (Malikova, Buhtoyarova, 1972) for computer calculation of rectangular foundation slabs of constant stiffness, large extension with free ends, with or without consideration of interaction of the slab and upper structure rigid elements. The loading on the slab can be vertical concentrated or uniformely distributed over rectangular area.
Maximum time for computer slab estimation is two hours. Algorithm is simple and is suitable for programming and for slab estimation by hand, using dimensionless design values' tables. Estimation of large extension foundation slabs for the loadings taken from the floor loading surfaces, leads as a rule to slab bending by it's convex side in the soil direction; it agrees with the results of slab settlement natural measurements. With it the slab works, chiefly, for positive moments. The consideration of upper structure rigid elements causes super-loading of extreme and unloading of middle areas of the slab. In connection with this the deflection of the slab

decreases approaching to measured in nature values. Positively bending moments also decrease but remain prevailing.

In conclusion we'll mark that determination of average expected settlements of large extension foundation slabs, necessary for calculation of Increased deformation module, is made with consideration of results of settlements observations of slab and box foundations of multistoreyed buildings and tower structures, as well of observations over soil layer deformation (Egorov, 1959; Egorov, Popova, 1971, Malikova, 1972). These observations carried out by Research Institute of Bases and Underground Structures, demonstrated that only a small in comparison with the large extended slab dimensions depth of soil is deformed in the foundation base (compressible layer). That's why, when calculating average settlements of large extension foundation slabs it's possible to use formula for average settlement of loaded surface of elastic layer with introducing of soil deformation module, averaged in the limits of compressible layer (Gorbunov-Posadov, 1953). With it the value of compressible layer thickness is obtained whether taking the actual compressible soil's layer thickness or the conditional design value of the compressible layer thickness $H_{estim}$ if practically non-compressiblo soils are in the large depth. Basing on the analysis of average measured and design settlements of slab and box foundations of multistoreyed and tall buildings, constructed in Moscow, the relation has been established (8) (Malikova, 1972) between conditional design value of the compressible layer thickness $H_{estim}$ and width $b$ of the large extension foundation slab, lying on comparatively uniform in plan clay or sandy soils (non-uniformity coefficient $\alpha = \frac{E_{0\,max}}{E_{0\,min}} \leq 2$ ), mechanical properties of which don't become worse with depth, that's to say soil deformation modules don't decrease and their values determined by deep stamp experiments are within the limits $E_0 = 150 - 600$ kg/cm$^2$.

$$H_{estim} = 4\xi\sqrt[4]{b} \qquad (8)$$

where $b$ - value, measured in m

$\xi$ - coefficient for clay soils

$\xi = 1{,}5$ m$^{3/4}$, sandy soils

$\xi = 1{,}0$ m$^{3/4}$.

The value $H_{estim}$ for the soil, presented by clay and sandy soils' layers, can be calculated by formula

$$H_{estim} = \beta H_{c.estim} + (1-\beta) H_{s.estim} \qquad (9)$$

where $H_{c.estim}$ and $H_{s.estim}$ - design value of compressible layer of clay and sandy soils, calculated by formula (8). $\beta$ - share of clay soil layers in $H_{c.estim}$.

When founding out relation (8) the actual data of layer thickness have been taken into consideration for tower type constructions. (Egorov, 1959; Egorov, Popova, 1971).

CONCLUSION

Given in report methods of estimation of foundation slabs for multi-storeyed buildings permit to consider their interaction with the upper structure. With it the attention is paid to importance of consideration of soil deformation properties variabilities for determination of limits of stresses changes in foundation slab. In estimations the data of natural observations on the foundation slabs' settlements have been used, Programmes realised by computers have been worked out for all the estimation methods.

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