PROBLEMS OF STRESS-STRAIN CONDITIONS IN SUBSOIL

LES PROBLEMES DES RELATIONS CONTRAINTE-DEFORMATION DANS LE SOUS-SOL

ПРОБЛЕМЫ НАПРЯЖЕННО-ДЕФОРМИРОВАННОГО СОСТОЯНИЯ ОСНОВАНИЯ

H.J. KRIEGEL, Research Engineer
H.H. WIESNER, Research Engineer, VEB Baugrund, Berlin (GDR)

SYNOPSIS. The observations of settlement on depth gauges under slab foundations of tall buildings, silos, and chimneys showed, depending on the prevailing subsoil conditions, a decrease of deformations with depth, which differred from the results of conventional computation methods. The influence of non-linear stress-strain behaviour is regarded as being the cause for it.

The paper displays some results of settlement measurement on structures over glacial deposits and demonstrates also the non-linear stress-strain behaviour of these soils by means of triaxial tests. Medium sands and boulder-clay were investigated and, as a comparison, the results of laboratory investigation on a mud are given.

INTRODUCTION

Though settlement observations on structures are in use for decades with regard to verifying and confirming the adapted calculation models, only a few measurements on depth gauges under structures' hitherto have been known. Information about the decrease of deformation with depth is, however, important for an interpretation of structure settlements. We know so far only some Soviet papers about measurements under loading plates and test foundations of little dimension (Konowalow 1964, Svec and Kasakow 1965 et all) and under two chimney foundations as well as under the Moscou television tower (Jegorow 1961, Vikow and Makarow 1964, Nikitin et all, 1970).

The gauge measurements confirm the specification of several national standards, that from certain depths on deformation might be neglected. As for a definition of these depth, different conceptions are still to be found.

Furthermore, gauge measurements show, that the decrease of deformations with depth, when measured, differs from the decrease computed by means of the model of linear-elastic half-space. This is attributed i literature to a number of causes, two of which, appearing superior, may be quoted:

- The strain by the structure has to overcome a soil specific structural strength before resulting in deformations (Steinfeld 1968, Kezdi 1968, Pocaevec 1971)
- The stress-strain relations of the subsoil being non-linear, another stress-deformation condition forms than if being linear-elastic (Vinokurov 1968, Shirokov et all 1971).

The results of settlement measurement

Since 1964 the VEB Baugrund has taken measurements on structures with large foundations. The results of these measurements, however, have not yet been evaluated up to the last detail.

Among others, twelve objects on glacial valley sands and on boulder clay were examined, mainly structures on slab foundations. The dimensions of the foundation slabs are between 2R = 6 m and L x B = 30 x 60 m.

The installation design of the gauges consists of rods with welded base plate, with a pedestal concreted upon, and in a protective pipe. After finishing the foundation slab boreholes were sunk through cutouts, where gauge-rods were forced into soft concrete placed into before. The protective pipes extend down to about 50-80 cm above the foot and shield the gauge rods from the surrounding subsoil. Gauge depths vary between 2.5 m and 30 m under the foundation. As a rule 3 - 4 gauges were provided per object.

In addition to the observation with bench marks by high-precision nivellisation, the relative displacement between gauge and foundation slab was measured. The attained accuracy of measurement for the determination of relative displacements was between ± 0.1 and 0.2 mm.

Figs. 1 and 2 show the gauge settlements of two objects on the sand of the marginal valley passing through Berlin. The sand shows a thickness of 40 - 50 m and is of medium density. The non-uniformity coefficient of the medium sand is $U = d_{60}/d_{10} = 1,5$ to $2,5$.

Figs. 3 and 4 show gauge settlements of 2 objects on boulder clay in the North of the GDR. The layers of glacial till are more than 30 m thick and only interrupted partially by sand layers of little thickness (1 m). The plasticity index of boulder clay is between I_p = 0.06 and 0.10, the consistency index about I_C = 1.0, and the void ratio between e = 0.25 and 0.35.

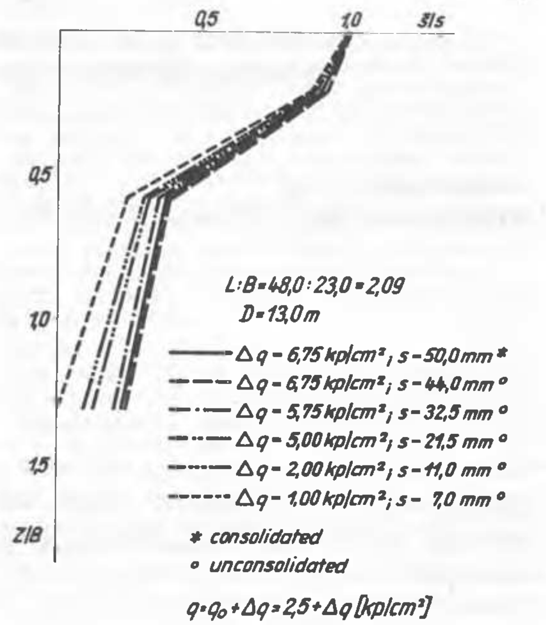

Fig. 1 Gauge settlements under a tall hotel building on sand

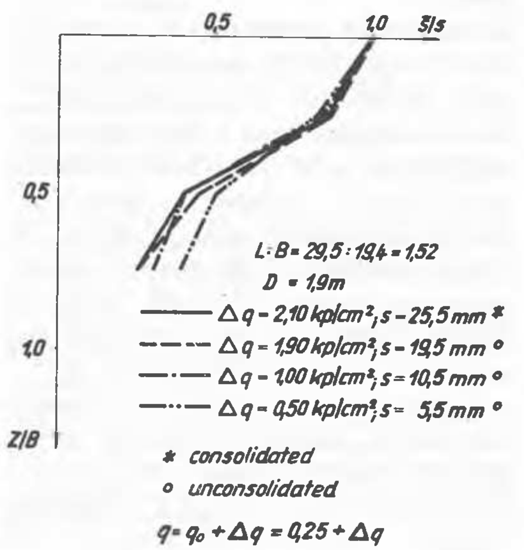

Fig. 2 Gauge settlements under a tall residential house on sand

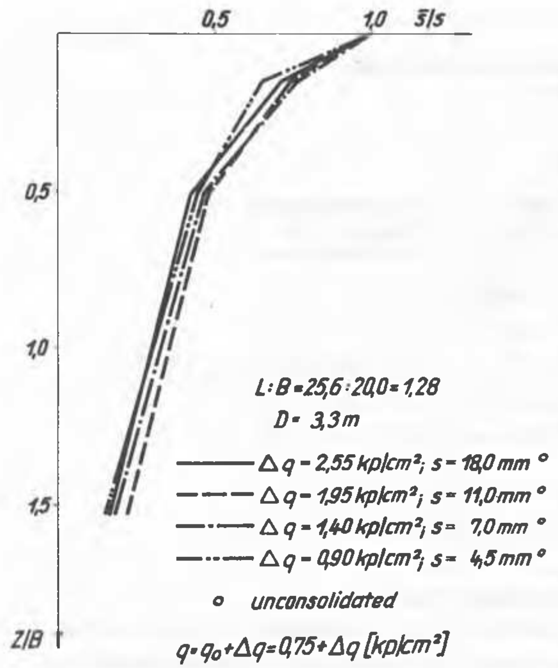

Fig. 3 Gauge settlements under a tall residential house on boulder clay

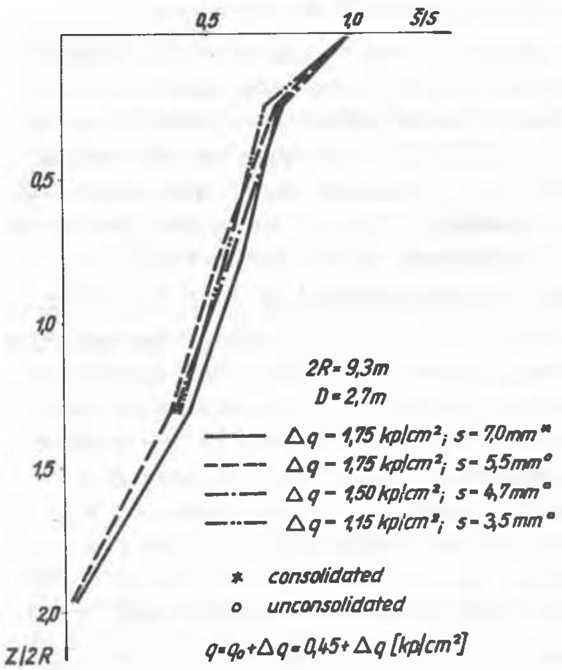

Fig. 4 Gauge settlements under a chimney on boulder clay

653

On the figures the relation between gauge settlement $\bar{s}$ and the corresponding slab settlement s is traced on the abscissa for the range of contact pressure Δq, and the relation of gauge depth z to the foundation diameter 2R or to the least foundation width B on the ordinate. q_o is the contact pressure already existing at the beginning of measurements. The indication "unconsolidated" means that measurement has been performed during the installation, and therefore settlements are still to be expected.

Dimensionless plotting allows a good comparison of decreasing deformations with depth, independent from size of the foundation and settlement value. Sands and boulder clay show different behaviour. Sands indicate three distinct ranges: up to z/B = 0.25 little relative deformations, from z/B = 0.25 to 0.50 high relative deformations, and from z/B = 0.5 on little relative deformations again. In boulder clay, however, only 2 distinct ranges could be made out: up to z/B = 0.25 high relative deformations, from z/B = 0.25 on nearly uniformly little relative deformations.

The depth of the deformated zone could not be detected clearly, as the gauges were not reaching down to this boundary depth in most cases. From an extrapolation of curve development and from a comparison with the objects, where the boundary depths were measured nearby, we can estimate that from z/B = 2 on no deformations occur. The boundary depth of sands is, as a rule, less than that of boulder clay.

Table I summarizes the dispersion range of the relations between the layer settlements Δs and the total settlement s for all evaluated objects.

Table I

Comparison of gauge values and computation values, obtained by standard processes

z/B	Δs/s Values for sand+	Values for boulder clay*	Computational values for E = constant
0 to 0.25	0.10 to 0.30	0.25 to 0.45	0.15 to 0.20
0 to 0.50	0.50 to 0.65	0.30 to 0.60	0.30 to 0.45
0 to 0.75	0.60 to 0.80	0.40 to 0.70	0.45 to 0.55
0 to 1.00	0.70 to 0.90	0.55 to 0.75	0.55 to 0.65
0 to 1.50	0.80	0.75 to 0.90	0.70 to 0.85

Notes: + Scatter for the measuring range $q = 1.0$ to $6.7\ kp/cm^2$
* Scatter for the measuring range $q = 0.9$ to $3.0\ kp/cm^2$

In the last column of table I some computational values are indicated, which result from standard calculation processes as SNiP II-B.3-62, DIN 4019, and TGL 11464, vol. 1, for foundation dimensions L/B < 2, assuming a constant deformation module. As all objects can be assumed to have a homogenous subsoil for the investigated depth, a comparison is possible.

This comparison shows that the differences of the deformation process are clear for sands from z/B = 0.25 on; for boulder clay, however, computational values range within the scatter of measurement in the depths z/B = 0.25 to 1.5

The results of measurement show, moreover, that the loading value of large foundations has only little influence on the deformation process and the depth of the deformated zone. Only for measurement ranges from $\Delta q = 1.0\ kp/cm^2$ to $\Delta q = 6.75\ kp/cm^2$ (the deformation process on fig. 1) a dependence is to be seen. On the other hand, gauge measurements under loading plates and test foundations with little dimensions indicated much more influence of the loading upon the depth of the deformated zone. Konowalow (1967), for instance, states a linear dependence on contact pressure by an empirical formule developed from his measurements.

Laboratory results

The deformation behaviour of soils with non-linear stress-strain characteristics can be simulated in the laboratory in a limited way only. It seems important that the components of non-dilatational strain and volume strain can be determined separatedly on evaluating these experiments.

The triaxial test on cylindrical samples is suitable for it; first the sample is consolidated under hydrostatic conditions up to $\sigma_m = \frac{\sigma_1+\sigma_2+\sigma_3}{3}$ and then the deviatoric stress is recorded under the conditions σ_m = constant. Fig. 5 shows the results of this experiment on a boulder clay in a plotting, as suggested by Ladany (1969).

In connection to the stress-strain relations of the linear-elastic and the elastic-isotropical half-space, the deformation behaviour is to be described by the modules of compressibility K and the modules of shear deformation G. Now, however, both values do not represent soil specific constants, but are functions of the effective three-dimensional stress-condition.

From the linear and square invariants of the stress condition and strain condition follows

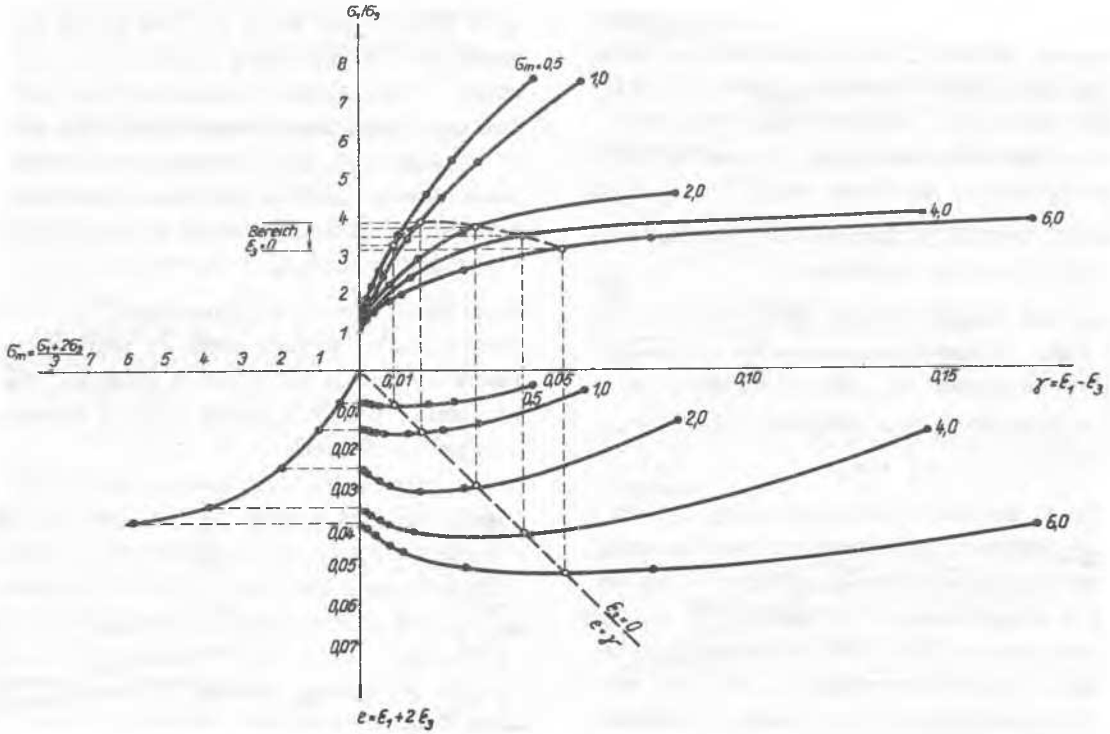

Fig. 5 Deformation behaviour of a boulder clay in a triaxial test

$$\sigma_m = \frac{\sigma_1+\sigma_2+\sigma_3}{3} \qquad (1)$$

$$\sigma_i = \frac{1}{6}\left[(\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2 + (\sigma_3-\sigma_1)^2\right]$$

$$e = \varepsilon_1+\varepsilon_2+\varepsilon_3 \qquad (2)$$

$$\varepsilon_i = \frac{2}{3}\left[(\varepsilon_1-\varepsilon_2)^2 + (\varepsilon_2-\varepsilon_3)^2 + (\varepsilon_3-\varepsilon_1)^2\right]$$

For the triaxial test these relations are reduced to

$$\sigma_m = \frac{\sigma_1+2\sigma_3}{3}$$

$$\sigma_i = \frac{1}{\sqrt{3}}\,(\sigma_1-\sigma_3) \qquad (1')$$

$$e = \varepsilon_1 + 2\varepsilon_3$$

$$\varepsilon_i = \frac{2}{\sqrt{3}}\,(\varepsilon_1-\varepsilon_3) \qquad (2')$$

For linear-elastic material constants result from the following formules

$$K = \frac{\sigma_m}{e} = \text{constant} \qquad (3)$$

$$G = \frac{\sigma_i}{\varepsilon_i} = \text{constant}$$

i thus $E = \frac{9\,KG}{3\,K+G}$ and $m = 2\,\frac{3\,K+G}{3\,K-2G}$.

For soils with non-linear stress-strain characteristic, K and G can be defined analogous to the formules 3 as tangent inclination or secant inclination of the relations

$$\sigma_m = f_1(e\ ;\ \sigma_i) \qquad (4)$$

$$\sigma_i = f_2(\varepsilon_i;\ \sigma_m) \qquad (5)$$

Whether to choose the inclination of tangent or of secant, depends on the process of solving numerically the half-space problem at non-linearity.

With these triaxial tests we intended to obtain these relations. Triaxial tests on the conditions described above comprehend only special cases of the possible existing stress paths and stress conditions in situ, but they give already a good view into the deformation behaviour of the soils during three-dimensional stress and can be regarded as routine inspection useful in practice.

For three soils these relations have been represented: a medium sand of medium density, a boulder clay, both from the subsoil under the observed objects, and a mud having been tested as basis for a causeway over alluvial marine deposits. The mud has been added as a comparison, being an extremely soft soil, sensitive to deformation.

The classification parameters of the soils can be seen from the captions.

Examining the volume change behaviour we assumed that volume change $e = \frac{\Delta V}{V}$ = consists of a component e_c, which depends on σ_m, and a component e_d, dependent particularly on σ_i.

$$e = e_c + e_d \qquad (6)$$

e_c was found on the condition $\sigma_1 = \sigma_2 = \sigma_3$. An isotropic deformation behaviour was assumed, which was confirmed approximately on evaluation by a comparison of ε_1 and $e_c/3$. e_d has been determined on the condition of σ_m=constant, i. e. with increasing σ_1, σ_3 was reduced correspondingly. These ways of stress are not identical with those in situ under load change. When choosing other test conditions, as for instance deviatoric stress at σ_3 = constant, the components e_c and e_d cannot be divided anymore clearly.

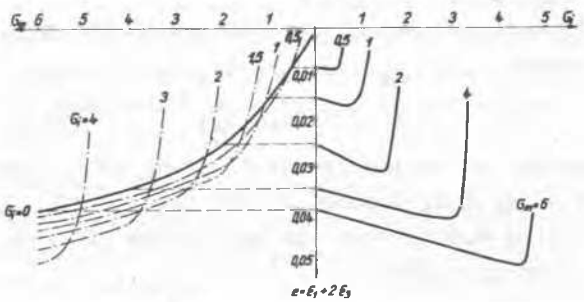

Fig. 6 $e = \frac{\Delta V}{V} = f(\sigma_m, \sigma_i)$ for a boulder clay

Coefficient of non-uniformity	$U = d_{60}/d_{10} = 60$
Plasticity index	$I_p = 0.08$
Consistency index	$I_C = 1.0$
Void ratio at test beginning	$e = 0.30$

Fig. 6 shows the dependance of volume change e on σ_m and σ_i for a boulder clay. The left side of the figure shows the relation $e_c = f(\sigma_m)$ for $\sigma_i = 0$, the right side shows the relations $e = f(\sigma_i)$ for σ_m = constant. The dashed curves on the left side for σ_i = constant demonstrate the effect of constant σ_i at different σ_m on the volume changes. Fig. 7a shows the change of volume for a medium sand of medium density, Fig. 7b for a mud.

From the figures follows that the boulder clay and the medium sand of medium density react by loosening from a special limit of σ_i, while the mud shows only a slowing decrease in volume.

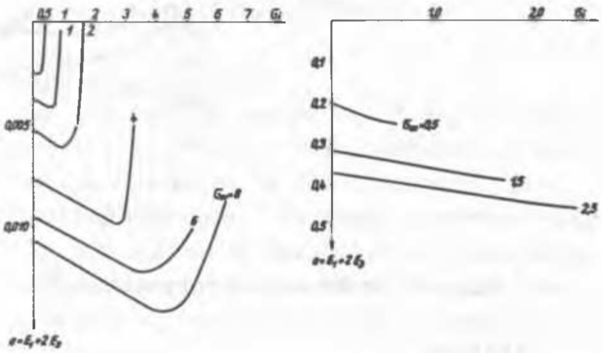

Fig. 7a b 653

a) $e = \frac{\Delta V}{V} = f(\sigma_m, \sigma_i)$ for a medium sand

Coefficient of non-uniformity	U	$= d_{60}/d_{10} = 2.2$
Density index at test beginning	I	=0.373
Void ratio at test beginning	e_a	=0.640
Void ratio in loosest state	e_{max}	=0.744
Void ratio in densest state	e_{min}	=0.465

b) $e = \frac{\Delta V}{V} = f(\sigma_m, \sigma_i)$ for a mud

Plasticity index	I_p	=1.06
Consistency index	I_C	=0.185
Void ratio at test beginning	e_a	=4.0-4.5
Water content at test beginning	w_a	=1.57
Lime content		87 %
Content of organic constituents		1.6%

The transition to loosening is σ_i/σ_m = 0.7 to 0.8 for boulder clay, σ_i/σ_m = 0.5 to 0.6

for sand of medium density, and the transition to the ending of volume decrease for mud $\sigma_i/\sigma_m = 0.8$.

Up to these limits linear dependance can be supposed between e_d and σ_i, e_c can be approximated for $\sigma_i = 0$ by a power function of σ_m. Thus for the equation 4 the following approximation results up to the limit mentioned above:

Boulder clay $e = 0.014\ \sigma_m^{0,67} + 0.0030\ \sigma_i$

medium sand of average density (7)

$e = 0.0036\ \sigma_m^{0.5} + 0.0009\ \sigma_i$

mud $e = 0.126\ \sigma_m^{0.5} + 0.06\ \sigma_i$

For linear-elastic material a linearity exists between σ_m and e, the module of compressibility K corresponds to the inclination of this straight line. At non-linearity K can be defined analogous or as an inclination of the tangent for σ_i = constant or as an inclination of the secant for an assumed stress range $\Delta\sigma_m$ and $\Delta\sigma_i$, and thus is a function of σ_m and σ_i. The defined
453 test conditions allow a determination for increasing σ_m and σ_i only.

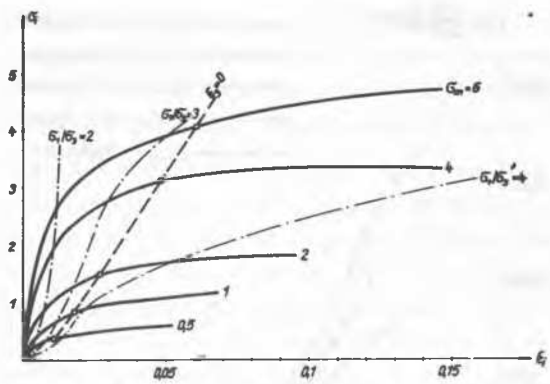

Fig. 8 $\sigma_i = f(\sigma_i, \sigma_m)$ for a boulder clay

The non-dilatational strains adequate to equation 5 is to be seen on the figures 8 to 10. The sets of curves found from the measurements can be approximated mathematically by means of the so-called "spline function" or by predetermining a function of the hyperbolic type. The hyperbolic function $\sigma_i = \frac{A\ \varepsilon_i}{B + \varepsilon_i}$ was assumed. Soviet

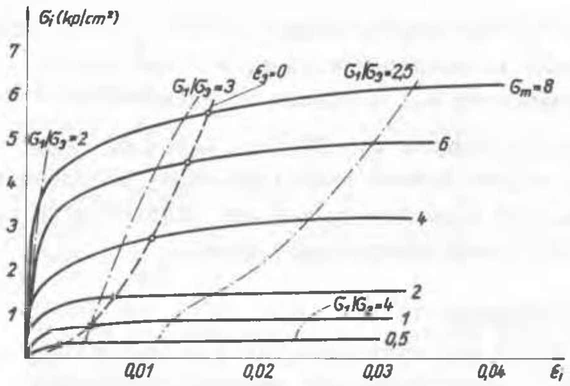

Fig. 9 $\sigma_i = f(\sigma_i, \sigma_m)$ for a medium sand

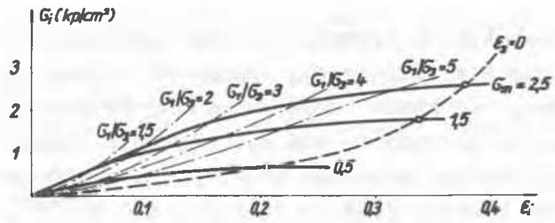

Fig. 10 $\sigma_i = f(\sigma_i, \sigma_m)$ for a mud

publications already applied this function Shirokov et all 1971), thus the results can be compared. The following approximation formules of equation 5 result from this for the investigated soils:

boulder clay $\sigma_i = \frac{(0.75\sigma_m + 0.4)}{0.007 + \varepsilon_i}$

medium sand of average density $\sigma_i = \frac{0.80\sigma_m\varepsilon_i}{0.0008 + \varepsilon_i}$

mud $\sigma_i = \frac{(1.64\sigma_m + 0.1)\varepsilon_i}{(0.085\sigma_m + 0.03) + \varepsilon_i}$ (8)

For linear-elastic material these relations would be straight lines of the form $\sigma_i = G\varepsilon_i$, of which the inclination correspondends to the specific modules of shear deformation. Analogous to this G can be defined in this case of non-linearity as an inclination of the tangent or as an inclination of the secant of an assumed stress interval $\Delta\sigma_m$ and $\Delta\sigma_i$. From the derivation of the function $\sigma_i = \frac{A\ \varepsilon_i}{B + \varepsilon_i}$

$$\frac{d\sigma_i}{d\varepsilon_i} = \frac{AB}{(B+\varepsilon_i)^2} = \frac{A - \sigma_i}{B} \qquad (9)$$

results for $\varepsilon_i = 0$ the modules of shear deformation at the beginning of the shear stress G_o = A/B and with $\varepsilon \to \infty$ at the limit of the acceptable shear stress G_e = 0. This limit is attained with σ_1 = A and is dependent on σ_m, adequate to equation 8.

In the figures 8 - 10 some lines of equal principal stress relations σ_1/σ_3 = constant and the line for ε_3 = 0 are plotted with a view to additional knowledge.

CONCLUSIONS

K and G are variables, so E is variable, too. K and G are not related invariably, therefore Poisson's coefficient is also not a constant.

The relation between the two specific deformation components, change of volume and change of shape, depends mainly on the relation between σ_1 and σ_m. As this relation has another value at every point in the subsoil under a loading surface, and as it changes during the loading process, the relation between the changes of volume and the changes of shape cannot have a constant value. Therefore the measure of deformation cannot be described by a single value each layer, being constant or only dependent on the hydrostatic stress component.

The figures 11 - 13 show for instance the dependence of a module of deformation $E_S = \Delta\sigma_1/\Delta\varepsilon_1$, defined analogous to the odometer module, on the stress interval $\Delta\sigma_1$ and on the principal stress ratio σ_1/σ_3. The values were defined by conversion from the triaxial tests. The so-called odometer range ε_3 = 0 represents only a special case of the possible stress conditions.

Mathematical solutions for the stress-strain condition, based upon the conventional calculation processes for the linear elastic half-space, are no more possible, when assuming non-linear stress-strain conditions.

Numerical processes, however, like the method of differences and the finit-element-method, allow comprehension of the stresses and the deformations on the basis of the described laboratory results by iteration and graduate

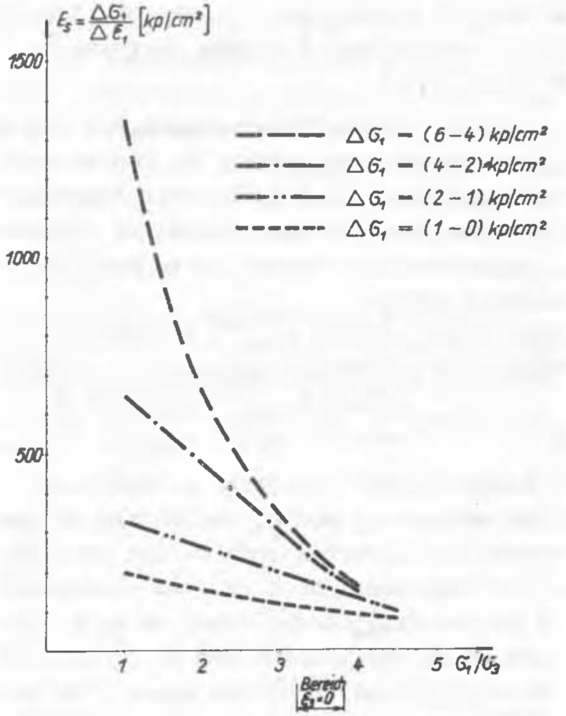

Fig. 11 Dependence of the deformation module $E_S = \frac{\Delta\sigma_1}{\Delta\varepsilon_1}$ on $\Delta\sigma_1$ and σ_1/σ_3 for a boulder clay

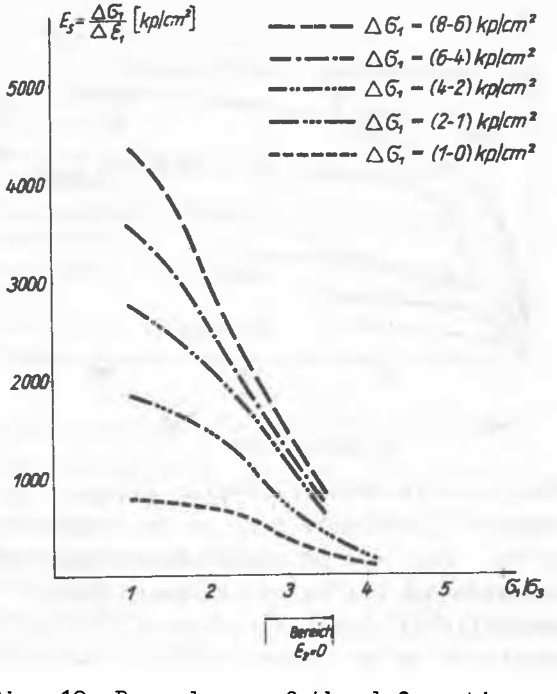

Fig. 12 Dependence of the deformation module $E_S = \frac{\Delta\sigma_1}{\Delta\varepsilon_1}$ on $\Delta\sigma_1$ and σ_1/σ_3 for a medium sand

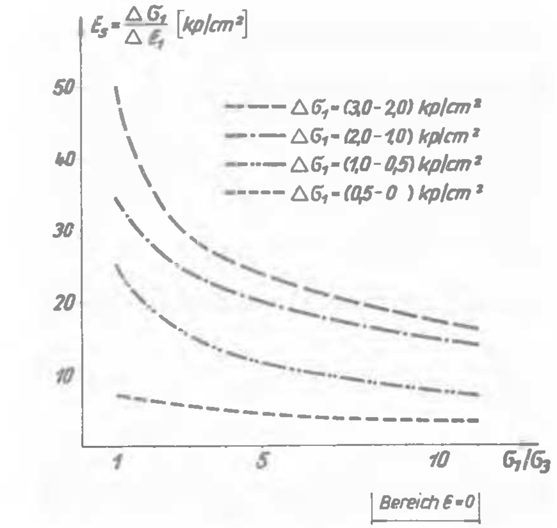

Fig. 13 Dependence of the deformation module $E_s = \frac{\Delta\sigma_1}{\Delta\varepsilon_1}$ on $\Delta\sigma_1$ and σ_1/σ_3 for a mud

approximation. Solutions by the method of differences under limiting assumptions like m = constant or K = constant have been published by Vinodurov (1968) and Shirokov (1971). These results show, that on these assumptions already permit good agreement between calculation and measurement, and that the distribution of contact pressure as well as the settlement near the foundation can also be comprehended realistically.

The calculating operation, however, for these methods is very extensive and can only be managed by electronic calculation technique. For the purpose of research this expenditure can be accounted for; for general practice it is necessary to find better ways, except for special cases.

For pre-calculating the average settlement of structures and the tilting of structures we see a possibility to comprehend the stress-strain conditions at given ranges of loading, given footing dimensions and foundation depths for frequent subsoil conditions by the F-E-method, to compare them with the results of the conventional processes and to adapt these, considering the loading case history and the loading ways. To this effect the above mentioned settlement observations are being evaluated at present.

REFERENCES

EGOROV, K. E. (1961), "Raspredelenie naprjazenij i peremescenij v osnovanii konecnoj tolsciny", Mechanika gruntov, Sb. Nr. 43, Moskva (1961), p. 42 - 63

KEZDI, A. (1968), "Diskussionsbeitrag", Vorträge der Baugrundtagung Hamburg 1968, p. 96 - 102

KONOWALOV, P. A. (1964), "Issledovanie glubiny deformiruemoj zony grunta pod stampami v polevych uslovijach", Osnovanija i fundamenty, Sb. Nr. 54, Moskva (1964), p. 14 - 25

KONOWALOV, P. A. (1967), "Eksperimental'noe issledovanie glubiny szimaemoj tolsci grunta", Osnovanija fundamenty i podzemnye sooruzenija, Moskva (1967), p. 137 - 142

LADANY, B. (1960), "Etude des rotations entre les contraintes et les deformations lovs du cisaillement des pulverulents", Annales Trav. Publ. de Belgique Vol. 1, 1960

NIKITIN, N. V. and MICHAL'CUK, A. I. and TRAVUS, V. I. (1970), "Issledovanie osadok fundamenta televizionnoj basnii v Ostankino" Osnovanija, fundamenty i mechanika gruntov No. 2 (1970), p. 32 - 55

POCAEVEC, A. P. (1971), "Strukturnaja procnost grunta i opredelenie glubiny szimaemoj tolsci", Osnovanija, fundamenty i mechanika, Materialy III vsesojuznogo sovescanija Kiew (1971), p. 220 - 222

SHIROKOV, V. N. et all (1971), "A circular rigid plate on a non-linearly deforming base", Proceedings of the 4th Budapest Conference on Soil Mechanics, Budapest 1971, p. 757 - 764

STEINFELD (1968), "Zur Gründung von 60-geschossigen Hochhäusern in Hamburg", Vorträge der Baugrundtagung Hamburg 1968, p. 35 - 91

SVEC, V. B. and KAZAKOV, P. P. (1965), "Izmerenie deformiruemoj zony v svjaznych gruntach", Osnovanija, fundamenty i mechanika gruntov No. 4, 1965, p. 10 - 12

VILKOV, I. M. and MAKAROV, N. I. (1964), "Issledovanie deformacij osnovanija dymovoj truby", Osnovanija i fundamenty, Sb No. 54, Moskva (1964)

VINOKUROV, E. F. (1968), "Morennye grunty kak osnovanija sooruzenij", Izdatelstvo "Nauka i Technika", Minsk (1968)