THE BEARING CAPACITY OF FOOTINGS ON NONHOMOGENOUS CLAYS

LA CAPACITE PORTANTE DES FONDATIONS PLACEES SUR L'ARGILE NONHOMOGENE

НЕСУЩАЯ СПОСОВНОСТЬ ФУНДАМЕНТОВ НА НЕОДНОРОДНЫХ ГЛИНАХ

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SYNOPSIS. An analysis of the stress and velocity characteristics is made for the case in which the undrained shear strength varies with depth. Using the relations along a slip line it is proved that the Hill's mechanism is most suitable, provided the above variation is linear.
Finally, a strip load on the surface of nonhomogeneous soil can be expressed in terms of the usual bearing capacity factor (π+2) multiplied by the cohesion at a depth of 0.4 times the footing width. This theoretical result is compatible with the existing data.

INTRODUCTION

The strength properties of nonhomogeneous materials is a field of considerable engineering importance and warrants further research. An interesting group of nonhomogeneous materials are those with a zero angle of internal friction. The materials, which include saturated clay under undrained conditions, have an engineering property werein plastic deformation takes place without volume changes.

Natural clay deposits exhibit nonhomogeneity
653 to a considerable degree. It is well known that the undrained shear strength of saturated clay increases with depth beyond the zone of desiccation. A solution for the bearing capacity of such a case is given in the literature assuming rupture along a circular surface. (Sreenivasulu V. and Ranganatham B.V., 1971, Button S.J., 1953, Reddy A.S. and Srinivasan R.J., 1967, Reddy A.S. and Srinivasan, 1971). However the solution should be derived from the static and kinematic fields, as shown in the following sections.

THE YIELD FUNCTION

The general form of the yield function in nonhomogeneous medium with a zero angle of internal friction in the case of plane strain is given by Eq. (1):

$$f(j_2;x;y;c_1;c_2...) = 0 \tag{1}$$

where j_2 is the second invariant of stress deviator in the xy plane (see the reference axis in Fig. 1) and c_1, c_2... are constants determined by laboratory tests.

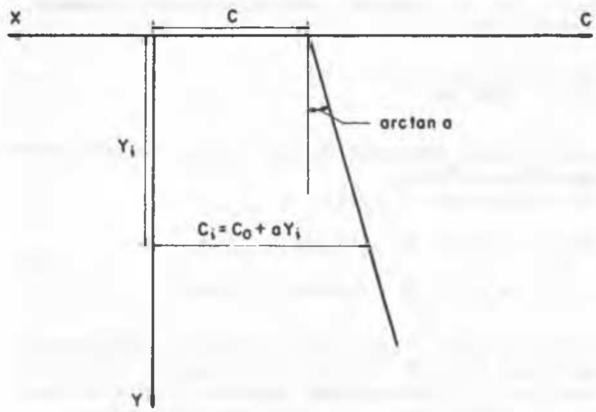

FIG. 1. VARIATION OF COHESION WITH DEPTH

Eq. (1) may also be expressed in the following form:

$$f = \sqrt{j_2} - c = 0$$
$$c = c(x,y,c_1,c_2...) \tag{2}$$

f is sometimes called the plastic potential. Geometrically, c may be regarded as the radius of Mohr's circle during yield. This paper proposes a function c where the cohesion increases linearly with depth (see Fig. 1) i.e.:

$$c = c_0 + ay \tag{3}$$

SLIP LINES

The slip lines in any media (isotropic, anisotropic, homogeneous, etc.) should represent kinematic lines along which relative displacement of the material is possible. Lines along which such phenomena can take place are known as velocity characteristics. To determine these velocity characteristics, the general stress-strain relationships (the flow rule) for perfectly rigid-plastic media are used:

$$\varepsilon_{ij} = \lambda \frac{\partial}{\partial\sigma_{ij}} \text{ , or in the xy-plane} \tag{4}$$

$$\varepsilon_x = \frac{\partial u}{\partial x} = \lambda\frac{\partial f}{\partial\sigma_x} \; ; \; \varepsilon_y = \frac{\partial v}{\partial y} = \lambda\frac{\partial f}{\partial\sigma_y} \; ; \; \gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = \lambda\frac{\partial f}{\partial\tau_{xy}}$$

Where:

- ε_{ij} - is the plastic strain rate tensor.
- u,v - are the velocity components along the x-and y-axes respectively.
- λ - is a positive factor of proportionality, not a material constant.
- f - is the yield function defined by Eq. (1) or Eq. (2).

From the above relationships it can be shown that a plastic deformation occurs with no volume change and the principal axes of stress and strain rate must coincide. Thus the slope of the two characteristic lines, i and j is:

$$\frac{dy}{dx} = \frac{\sin 2\psi \pm 1}{\cos 2\psi} \equiv \begin{cases} y'_i \\ y'_j \end{cases} \tag{5}$$

Along these perpendicular lines, Geiringer's equation exists:
(H. Geiringer, 1930):

$$\left.\begin{aligned} dv_i - v_j d\gamma &= 0 \quad \text{along i line} \\ dv_j - v_i d\gamma &= 0 \quad \text{along j line} \end{aligned}\right\} \tag{6}$$

where v_i and v_j are the velocity components referred to the i and j slip lines (considered as a right-handed system of curvilinear coordinates) and γ is the anticlockwise angular rotation of the i line from the x axis.

The two Eqs. (6) have three unknowns, v_i, v_j, $d\gamma$. Therefore, in order to determine the velocity characteristics (slip line field), an additional equation must be derived. This equation should express the relationship between the velocity characteristics and the stress characteristics given in the next section.

STRESS FIELD

The substitution of Eq. (2) into the equilibrium equations of a weightless element reduces the number of unknowns to two: p and ψ where p is the isotropic stress and ψ is the angle between the direction of the x axis and the direction of major principal stress (σ_1). The positive direction of ψ is counter-clockwise from x to σ_1.

The two equations that represent a state of equilibrium during yield and that include the above two unknowns are:

$$\begin{aligned} &\frac{\partial p}{\partial x} - 2c\,\sin 2\psi\frac{\partial\psi}{\partial x} + 2c\,\cos 2\psi\frac{\partial\psi}{\partial y} + a\,\sin 2\psi = 0 \\ &\frac{\partial p}{\partial y} + 2c\,\cos 2\psi\frac{\partial\psi}{\partial x} + 2c\,\cos 2\psi\frac{\partial\psi}{\partial y} - a\,\cos 2\psi = 0 \end{aligned} \tag{7}$$

It can be shown that the two equations are hyperbolic and that their characteristics coincide with the velocity characteristics (Eq. 5). The compatibility equations that are valid along the stress characteristics are given by the following equations:

$$\begin{aligned} dp + 2c\,d\psi - a\,dx &= 0 \quad ; \quad \text{along i line} \\ dp - 2c\,d\psi + a\,dx &= 0 \quad ; \quad \text{along j line} \end{aligned} \tag{8}$$

BEARING CAPACITY

The coincidence of the stress characteristics with the velocity characteristics is meaningful. The equilibrium may be determined along the characteristic lines under the given boundary conditions, while at the same time slipping of material may take place along these lines. The critical bearing capacity is determined as the one which gives rise to a slipping mechanism in the nonhomogeneous medium compatible with the boundary conditions. In this study, the bearing capacity for a strip load on the surface of nonhomogeneous soil media is treated. In a weightless, nonhomogeneous medium with a zero angle of internal friction and strength increasing only in the y direction (see Fig. 1) it can be shwon that, when ψ=0, $\pi/2$, ψ and p is constant, there exists a field which contains two families of straight lines intersecting at an angle $\pi/2$ (see fields 1, 2 in Figs. 2, 3). It should be noted that if the cohesion varies in both the x and y directions, fields 1 and 2 are not compatible with the conditions of the problem under discussion. As a first approximation, the zone between fields 1 and 2 is treated as rigid, possessing a circular arc (slip line B C in Figs. 2 and 3).

There may be two mechanisms of failure: (a) according to Prandtl (L. Prandtl, 1920) (see Fig. 2) and (b) according to Hill (R. Hill, 1950) (see Fig. 3). In the homogeneous case, determination of the bearing capacity (q) by both the above mechanisms is identical. In the nonhomogeneous case, however, determination of q according to Hill's mechanism gives lower values. This result is to be expected since, according to Hill, the material is sheared in a weaker media. This leads to the conclusion that for the case when the undrained shear strength increases with depth, Hill's mechanism should be used for determination of the bearing capacity. Integration of Eq. (8), e.g. along line j (from point A to point D in Fig. 3) yields:

$$q = (\pi+2)\ c_{0.4B} \qquad (9)$$

where $c_{0.4B}$ is the cohesion at a depth of 0.4B.

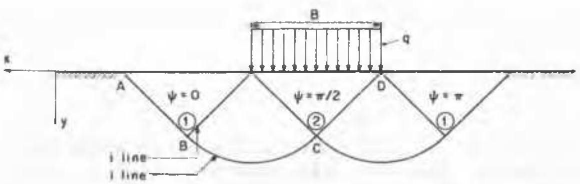

FIG. 2. PRANDTL'S BEARING CAPACITY MECHANISM

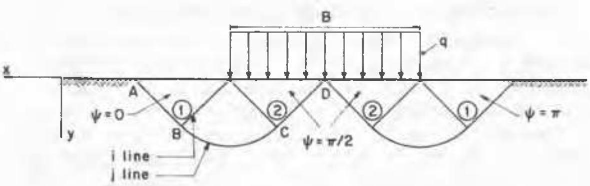

FIG. 3. VARIATION OF COHESION WITH DEPTH

CONCLUSIONS

1. In an incompressible nonhomogeneous medium with a zero angle of internal friction, the slip lines (velocity characteristics) coincide with the stress characteristics and are perpendicular to each other.
2. For the case when the undrained shear strength increases with depth, Hill's mechanism should be used for determination of the bearing capacity.
3. The bearing capacity results, derived mathematically from the theory of plasticity give results comparable with the approximate solution given in the literature (Sreenivasulu, V. and Ranganathan, B.V., 1971; Button, S.J., 1953; Reddy, A.S. and Srinivasan, R.J., 1967; Reddy, A.S. and Srinivasan, R.J., 1971).

REFERENCES

Button, S.J., "The Bearing Capacity of Footing on a Two-layer Cohesive Subsoil", Proc. 3rd Int. Conf. on Soil Mechanics and Foundation Engineering, Vol. 1, (1953) p. 332.

Geiringer, H., Proc. 3rd Int. Conf. App. Mech., Stockholm, 2 (1930) p. 185.

Hill, R., "The Mathematical Theory of Plasticity". The Clarendon Press, Oxford, England, (1950) p. 254.

Prandtl, L., Nachr. Ges. Wiss. Göttingen, (1920) p. 74.

Reddy, A.S. and Srinivasan, R.J., "Bearing Capacity of Footings on Layered Clays", Journal of the Soil Mechanics and Foundation Division, ASCE, Vol. 93, (1967) p. 83.

Reddy, A.S. and Srinivasan, R.J., "Bearing Capacity of Footings on Clays", Japanese Society of Soil Mechanics and Foundation Engineering, Soils and Foundations, Vol. 11, No. 3, Sept. 1971, p. 51.

Sreenivasulu , V. and Ranganatham, B.V., "Bearing Capacity of an Anisotropic Nonhomogeneous Medium under ϕ=0 Condition" Japanese Society of Soil Mechanics and Foundation Engineering, Soils and Foundations, Vol. 11, No. 2, June 1971, p. 17.