INCLINED LOAD TESTS ON SHALLOW STRIP FOOTINGS

ESSAIS DE SEMELLES FILANTES SUPPORTANT UN EFFORT INCLINE

ИСПЫТАНИЯ ЛЕНТОЧНОГО ФУНДАМЕНТА МЕЛКОГО ЗАЛОЖЕНИЯ ПРИ ДЕЙСТВИИ НАКЛОННОЙ НАГРУЗКИ

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SYNOPSIS. The influence of load inclination on the ultimate bearing capacity (failure load) of shallow strip footings was investigated by means of large scale loading tests carried out in non-cohesive soil with a foundation of width 1,0 and length 3,0 m. In five experiments without embedding depth and four experiments with an embedding depth of 0,20 m, loads were centrically applied under an inclination of 10^o, 20^o and 30^o until failure was observed (inclination of applied load parallel to short side of foundation). In agreement with theory, it was established that the ultimate bearing capacity decreases considerably with increasing inclination of the direction of load application. A simple empirical equation is given for this decrease. In addition the experimental results are compared with the results of earlier tests, in which the inclined load acted parallel to the longer side of the foundation, and also with the theoretical inclination factors for the plane state of strain. In this connection it was demonstrated that the influence of the load inclination on the ultimate bearing capacity in the case of inclined application of loads in the direction of the long side of the footing is less than in cases where the inclined load acts in the direction of the short side of the footing. In addition the decrease calculated from the experimental results is generally smaller than that predicted by theoretical inclination factors for identical experimental conditions.

1. INTRODUCTION

As a follow-up on earlier research investigations, in which the ultimate bearing capacity of a 0,5 m wide and 2,0 m long single footing exposed to inclined loads in its longitudinal direction was examined (Muhs and Weiss 1969, 1972), tests were now performed with a 1,0 m wide and 3,0 m long footing with inclined loads applied in the direction of the short side of the footing.

The series of tests with the smaller footing was primarily intended as a means of investigating the failure behaviour of single footings, since these normally are constructed in such a manner that the horizontal load component acts in the direction of the long side of the foundation. In this case, however, the direction of the load does not coincide with the direction in which failure would occur in the case of a vertical load, i. e. in the direction of the short side of the footing. The influence of the load inclination can thus hardly be described adequately in this case (3-dimensional state of strain) by the bearing capacity factors for inclined loads as put forward by Schultze, 1952, and by Sokolovski, 1960, which were derived for the plane state of strain.

The experiments carried out with the larger footing were intended on the other hand to create a genuine basis for comparison between theoretical and experimental inclination factors, since in this case the direction of application of the horizontal load component coincides with the direction in which failure occurs in the case of vertical loads.

The required length of the test footing was derived from the necessity to obtain conditions of approximately plane strain, at least in the central cross-section. The width therefore had to be sufficiently large to ensure that on the one hand gauges for the measurement of contact pressure (normal and tangential stresses, Elmiger and Muhs 1969) could be incorporated in the cross axis of the footing base, and on the other hand load centricity deviations arising inevitably as a result of the experimental arrangements would remain sufficiently small in relation to the overall footing width for the influence of eccentricity of load application to be neglected. As a last condition, the overall dimensions had to remain within the limits set by the test arrangements, the maximal force amounting to 350 t.

All in all a total of nine tests were performed: five tests at the surface of the fill, i. e. at an embedding depth D = 0 and four further tests at an embedding depth D = 0,20 m. The inclination of the centrically applied load was varied between δ = 10^o, 20^o and

30^o and in addition one basic test was carried out with a centrically applied vertical load, both for D = 0 and D = 0,20 m. The tests, their results and the conclusions drawn therefrom are described below.

2. TESTS

2.1 TEST ARRANGEMENTS

The arrangements on the test field of the Degebo available for carrying out large-scale loading tests have already been described in the Proceedings of the 7th International Conference in Mexico City (Muhs and Weiss 1969).

The following quantities were measured in each test:

1. The magnitude of the load, increased stepwise until failure.

2. The movement of the footing, i. e. the vertical settlement of the four corners and the horizontal displacement.

3. The normal and tangential stresses in the cross axis of the footing.

4. The soil movements in the surface of the fill.

2.2 EXPERIMENTAL FILL

The test pit with an area of approximately 150 m^2 was filled with layers of a ununiform sand - gravel - mixture ($d_{50} \approx 0,65$ mm; U $\approx 5,7$) to a depth of approximately 2,5 m. The filling was renewed for the second series of tests carried out at an embedding depth of 0,20 m. The soil data of both fills are compiled in Table I.

The details in Table I are supplemented in each case by the average value q_s of all cone resistances measured in five static penetration tests at depths between 0 and 1,5 m. The average values at a depth of 1,5 m were 220 kp/cm^2 resp. 180 kp/cm^2. In addition the related angles of internal friction are recorded (see Muhs and Weiss 1972, page 3). Both fills must therefore be classified as dense.

In addition in the last column of Table I, the bearing capacity factors N_q and N_γ are indicated as calculated from the experimental results.

Dark-coloured sand cylinders with a diameter of 2cm were inserted in the test fills in the region of the footings parallel to the short foundation axis in five vertical sections at a distance of 50 cm. These were intended to make visible the deformations and in particular the rupture lines in the ground after completion of tests.

2.3 TEST PROCEDURE

All tests were carried out with a water level of 1 - 2 cm above the surface of the fill in order to eliminate the influence of apparent cohesion on the failure load (Fig. 1).

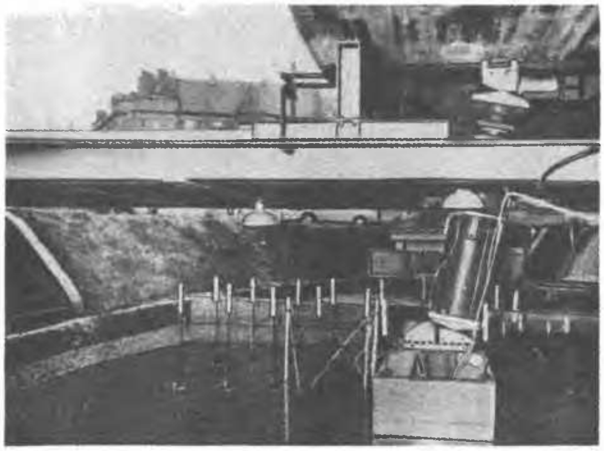


Fig. 1 Set-up of test with centric inclined load ($\delta = 10^o$)
Test No. 3 in fill No. 2

The load was increased stepwise with decreasing increments as the rupture load was approached. This load had been approximately calculated in advance. The measurements quoted in section 2.1 were carried out for each load. On completion of each individual test, the water was lowered several centimeters until it was below the surface of the fill in order to permit measurements of the upheaval and the failure zone at the surface of the fill around the foundation. On completion of all tests in one fill, the water was completely removed from the test pit in order to uncover the dark-coloured sand cylinders in the vertical sections.

Table I
Soil data of the test fills

Fill No. (-)	Unit weight γ_d (t/m^3)	Porosity n (%)	Void ratio e (-)	Uniformity coefficient U (-)	Friction angle ϕ' (^{o})	Cone resistance q_s (kp/cm^2)	Bearing capacity factor N_γ (-)	N_q (-)
1	1,91 $\pm$ 0,01	27,9	0,387	$\approx 5,7$	40	147 $\pm$ 10	55	70
2	1,90 $\pm$ 0,01	28,3	0,395		39	117 $\pm$ 6	44	56

3. TEST RESULTS

3.1 FAILURE ZONES AT THE SURFACE OF THE FILL AND RUPTURE LINES IN THE FILL

These measurements provide a clear picture of the influence of the load inclination on the ultimate bearing capacity.

Figures 2 - 4 show typical failure zones in tests with various load inclinations. Whereas in the case of a centric vertical load, these failures normally occur or can occur in front of both longitudinal sides of the footing, they only arise, in the case of inclined application of the load, in front of one longitudinal side of the footing, i. e. in front of the side lying ahead the direction of the load. The length of the failure zone becomes smaller with increasing load inclination. The ultimate bearing capacity is directly dependent on the size of this failure zone. A small failure zone at the surface of the fill implies a short plane of sliding in the soil and thus a low failure load.

The lines of rupture in the soil can be reconstructed in the five vertical sections by means of the displacements of the darkcoloured sand cylinders. The piercing point of these rupture lines at the surface of the fill or the line of penetration of the entire plane of sliding forms the limit of the upheaval and the failure zone visible at the surface.

Figures 5 - 7 show that for tests with embedding the rupture lines extend only to smaller depths and become shorter with increasing load inclination. The reason for this lies in the fact that the direction of principal stresses for the forces acting in the footing base changes with increasing load inclination.

3.2 FAILURE LOADS

The failure loads measured in the individual tests (see e. g. Figs. 8 and 9) are grouped together in Tables II and III. The influence of the load inclination on the ultimate bearing capacity can, from the point of view of magnitude, at first be demonstrated by comparison of the vertical component of the failure loads in the case of an inclined load $P_{f, \delta \neq 0}$ with the failure load for the related vertical load case $P_{f, \delta = 0}$. The resulting decrease in the bearing capacity in the case of an inclined load is recorded in the last column.

The quotient calculated in accordance with Tables II and III

$$\epsilon_\delta = \frac{P_{f, \delta \neq 0}}{P_{f, \delta = 0}} \qquad (-) \qquad (1)$$

for the case D = 0 for load inclinations of $\delta = 10^o$, 20^o and 30^o demonstrates a decrease in the failure load to 71 %, 43 % and 17 % of those values obtained in the case of a vertical centric load. For tests with D = 0,20 m, the failure load drops to 81 %, 49 % and 17 % of $P_{f, \delta = 0}$. The earlier series of experiments

Fig. 2 Rupture line in front of one side of the footing ($\delta = 10^o$)
Test No. 3 in fill No. 2

Fig. 3 Rupture line in front of one side of the footing ($\delta = 20^o$)
Test No. 3 in fill No. 1

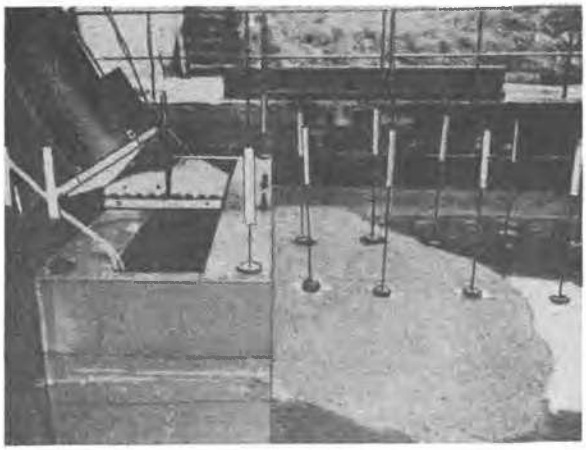


Fig. 4 Rupture line in front of one side of the footing ($\delta = 30^o$)
Test No. 1 in fill No. 2

Fig. 5 Cross section showing lines of rupture beneath footing ($\delta = 10^{\circ}$)
Test No. 3 in fill No. 2

Fig. 6 Cross section showing lines of rupture beneath footing ($\delta = 20^{\circ}$)
Test No. 2 in fill No. 2

Fig. 7 Cross section showing lines of rupture beneath footing ($\delta = 30^{\circ}$)
Test No. 1 in fill No. 2

with inclined load parallel to the long side of the footing - but with a ratio embedding depth/footing width D/B = 1 as compared with the present ratio D/B=0,2 - showed a decrease in the failure load to 83 %, 62 % and 39 % for load inclinations of $\delta = 10^{\circ}$, 20° and

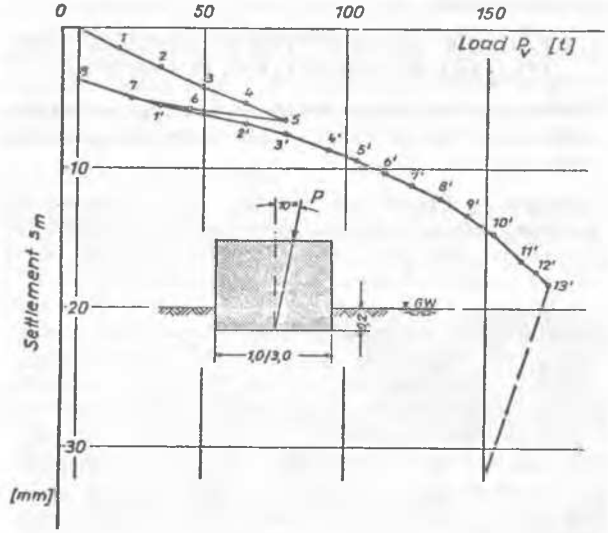


Fig. 8 Load-settlement curve
Test No. 3 in fill No. 2

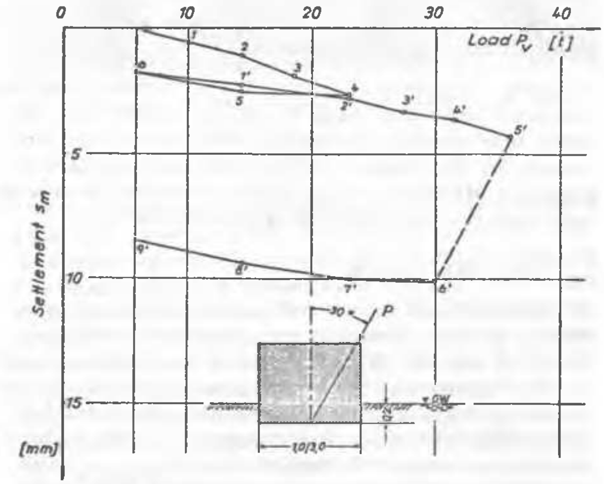
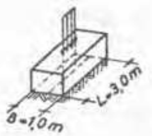
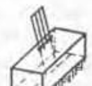
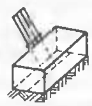


Fig. 9 Load-settlement curve
Test No. 1 in fill No. 2

30° (Muhs and Weiss 1969, 1972). Thus in the case, where the inclined load acts parallel to the short side of the footing, the decrease in the failure load is clearly larger, i. e. the bearing capacity itself is smaller than in the equivalent case with inclined load acting parallel to the long side of the footing, in particular under increasing load inclination.

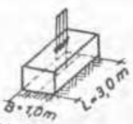
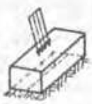
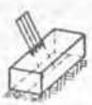
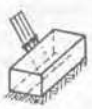
To describe the decrease in the failure load in the case of a load parallel to the long side of the footing, the following equation was derived from earlier experiments:

Table II
Test results, fill No. 1 (D = 0)

Test No.	Arrangement	Fail. Load*) P_f [t]	$\frac{P_{f,\delta\neq0}}{P_{f,\delta=0}}$
1	 $\delta = 0°$	189	1
5		190	1
4	 $\delta = 10°$	134	0,71
3	 $\delta = 20°$	82	0,43
2	 $\delta = 30°$	32	0,17

*) Vertical component

Table III
Test results; fill No. 2 (D = 0,2 m)

Test No.	Arrangement	Fail. Load*) P_f [t]	$\frac{P_{f,\delta\neq0}}{P_{f,\delta=0}}$
4	 $\delta = 0°$	213	1
3	 $\delta = 10°$	173	0,81
2	 $\delta = 20°$	104	0,49
1	 $\delta = 30°$	36	0,17

*) Vertical component

$$P_{f,\delta\neq0} = P_{f,\delta=0} \cdot \epsilon_\delta \qquad (t) \qquad (2)$$

where

$$\epsilon_\delta = (1 - \tan\delta) \qquad (-) \qquad (3)$$

is the reduction factor for the load inclination.

This equation corresponds in its structure to the theoretical equation for inclination factors derived by Brinch Hansen, 1970, which can be described in the general form

$$\epsilon_\delta = (1 - \beta \cdot \tan\delta)^\alpha \qquad (-) \qquad (4)$$

With $\alpha = \beta = 1$, equation (4) coincides in its form with the empirical equation (3) for inclination factors under an inclined load acting parallel to the long side of the footing. For tests with the inclined load acting parallel to the short side of the footing, these factors α and β must be determined from the test results. For obvious reasons it is convenient to make $\beta = 1$ (simpler equation for ϵ_δ, simpler determination of α and better means of comparison with equation (3)).

Under these assumptions we obtain the index in equation (4) by simple application of the method of least squares, calculated for the given load inclination angles δ, and the empirically determined reduction factors ϵ_δ in the last columns of Tables II and III. For tests with D = 0, we obtain for this calculation $\alpha = 2,0$; for tests with D = 0,20 m, $\alpha = 1,92$. The numerical discrepancies are not significant; i. e. within the limits of test accuracy both tests, with and without embedding, show the same influence of load inclination on the ultimate bearing capacity. The reason for this result probably lies in the limited number of experiments on the one hand and in the relatively small embedding depth of 0,20 m, i. e. in the low ratio D/B = 0,20 on the other hand. Thus for both test series, a common index α can be determined. We obtain $\alpha = 1,96 \approx 2$ and thus:

$$\epsilon_\delta \approx (1 - \tan\delta)^2 \qquad (-) \qquad (5)$$

Comparison with equation (3), where the index $\alpha = 1$, clearly shows the larger influence of the load inclination on the bearing capacity for cases with the horizontal component acting parallel to the short side of the footing.

From equation (2) it will be clear that the reduction factors ϵ_δ (both in accordance with equation (3) and with equation (5)) only represent a means of calculating the overall influence of the load inclination, i. e. the relation to the total failure load $P_{f,\delta=0}$. In fact - under the actual test conditions under water - $P_{f,\delta=0}$ is composed of the components of footing width and embedding depth. Since in the present case, tests without and with embedding were carried out, whereas earlier experiments with inclined load were performed exclusively with embedding, it is now possible to determine the influence of load inclination on the failure load seperately in the terms of width and depth of the general bearing capacity equation.

In this connection the basic data are supplied by tests No. 1 and 5 for the case of a vertical centric load in the series of tests with D = 0 and the corresponding test No. 4 in the series of tests with $D \neq 0$. If the boundary conditions of these tests are substituted in the general bearing capacity equation, a system of linear equations for the unknown ultimate bearing capacity factors N_γ and N_q is obtained. The evaluation in the present case is however made more complicated by the fact that the soil data of the two fills as provided by the measured cone resistances differ somewhat from one another (see Table I), so that here in fact four values should be determined, namely N_γ and N_q for each fill. For this determination, the empirical relation that the bearing capacity factors are proportional to the cone resistance (Muhs and Weiss 1971, page 27), can be utilized additionally. The bearing capacity factors determined in this manner for the two fills are recorded in Table I.
If the empirically determined bearing capacity factors N_q are compared with the theoretical factor N_q proposed by Prandtl which is generally accepted as valid (N_q = 64 for $\phi' = 40^o$, N_q = 56 for $\phi' = 39^o$), a difference of the empirical from the theoretical value amounting to 9 % is obtained in the first fill, whereas in the second the two values concide.

For the bearing capacity factor N_γ, the empirical relationship

$$N_\gamma = 1,0\ (N_q - 1)\ .\ \tan \phi' \qquad (-) \qquad (6)$$

was derived from the results of large scale loading tests (Muhs 1971 page 15). This equation gives a theoretical bearing capacity factor N_γ = 53 (for $\phi' = 40^o$) and N_γ = 45 (for $\phi' = 39^o$). These values agree with the empirical bearing capacity factors N_γ recorded in Table I within the limits of the experimental accuracy. The empirical relationship between N_γ and N_q is thus confirmed also by these tests.

The equations for determining the reduction factors $\epsilon_{\delta,B}$ in the width term and $\epsilon_{\delta,D}$ in the depth term are now:

$$p_{f,\delta \neq 0} = p_{f,\delta \neq 0}\ .\ F =$$

$$= (p_{fB} \cdot \epsilon_{\delta,B} + p_{fD} \cdot \epsilon_{\delta,D})\ .\ F \qquad (t) \qquad (7)$$

The left side of this equation contains the experimentally determined failure loads for load application inclined at an angle of δ to the vertical direction. By substitution of the test results, we obtain a total of 3 x 2 = 6 equations for the total of six unknowns ($\epsilon_{\delta,B}$ and $\epsilon_{\delta,D}$ in each case for three different load inclinations).

The numerical discrepancies between the reduction factors determined separately for the width term and the depth term are again not significant, so that in this manner the result of equation (5) is also confirmed. The given test conditions thus lead to a common reduction factor $\epsilon_\delta = f(\delta)$.

Schultze, 1952, and Sokolovski, 1960, page 76, have derived bearing capacity factors for the width and depth terms in the plane state of strain. They can be related with the bearing capacity factors under vertical load and furnish another method for the calculation of the inclination factors. If a slight additional influence of the angle of internal friction on the inclination factors is neglected, these results correspond approximately with the inclination factors given in the German Standard DIN 4017, part 2:

$$\epsilon_{\delta,B} = (1 - \tan \delta)^3 \qquad (-) \qquad (8)$$

$$\epsilon_{\delta,D} = (1 - 0,7 \tan \delta)^3 \qquad (-) \qquad (9)$$

The reduction factor in the width term (equation 8) leads to a greater decrease in the failure load than obtained by the tests. The reduction factor in the depth term on the other hand (equation 9) agrees within the range tested ($\delta \leq 30^o$) approximately with the empirical reduction factors obtained in accordance with equation (5). The above mentioned inclination factors derived by Brinch Hansen, 1970, give generally lower failure loads for $\delta \leq 30^o$.

3.3 STRESS DISTRIBUTION AND BASE FRICTION

The final analysis of the stress measurements in the base of the footing has not yet finished. As a preliminary result however, the following can be stated. In each test - independent of the load inclination - a parabolic distribution of normal stresses is reached in the centre cross-section during failure, a result which could be expected also on the basis of theoretical considerations.

Of particular interest in this connection, however, is the knowledge of the magnitude of the base friction, since it provides a means of judging whether failure is possible under high load inclinations or whether the footing will move under further loading by sliding in the base joint.

Normal and tangential stresses are known by the measurements at three points in the cross axis of the footing base. So the magnitude of the base friction at these points can be calculated. In the case of a vertical load, the base friction forces are directed towards the longitudinal axis of the footing, opposite to the displacements of the soil particles in the base plane. In the case of inclined loads, the friction forces in the footing base have the same direction as the horizontal load component. With increasing load inclination, the angle of base friction also increases. For load inclinations of $\delta = 10^o$ and $\delta = 20^o$ for example, an average angle of base friction of 10^o and 19^o resp. was measured, whereas at a load inclination of $\delta = 30^o$ an average angle of base friction of approximately 25^o was obtained. However, the maximum measured angle of base friction in this case amounted to approximately 38^o.

In view of the limited number of stress gauges, these results can not be considered as representative for the whole base area. All in all however, the measured angles of base friction imply that in the whole base area no sliding can have taken place and that the angle of friction necessary to prevent sliding must have been present outside the measuring points too. This agrees also with the observations made on the rupture lines in the vertical planes in the centre cross-section of the footing (see Figs. 5 - 7), since in the case of pure sliding of the footing on the soil (relative displacement between base and soil) the observed pronounced rupture lines could not have occurred in the soil.

In connection with the question whether, in the case of footings exposed to inclined loads, sliding can occur at all or whether the ultimate bearing capacity is overcome as a result of failure even before the base frictional resistance is fully mobilized, comment was made in a discussion contribution at the 3rd Danube-European Conference in Budapest 1971 (Weiss 1972). On this particular question some additional large scale loading tests were carried out. The results already obtained suggest that the base frictional resistance μ - even in the case of prefabricated footings with a relatively smooth base - in the case of load inclinations used in practice ($\delta < 25^{\circ}$) is always large enough to prevent sliding. Furthermore the permissible horizontal component according to the required safety factor against failure is always lower than this component would be computed with the required safety factor against sliding (e. g. 1,5 in the German Standard DIN 1054).

4. CONCLUSIONS

The investigation of the influence of an inclined load acting parallel to the short side of a footing led to the result that the decrease of the vertical component of the failure load resulting from load inclination is larger than in cases where the inclined load is applied parallel to the long side of the footing.

To describe the failure load $P_{f,\,\delta\neq 0}$ (always lower than the failure load $P_{f,\,\delta=0}$ for the vertical load case), the following equation was found in the case of a load acting parallel to the short side of the footing:

$$P_{f,\delta\neq 0} = P_{f,\delta=0} \cdot \epsilon_{\delta} \qquad (t)$$

with the reduction factor:

$$\epsilon_{\delta} = (1 - \tan\delta)^{2} \qquad (-)$$

The decrease is - considering the accuracy of the test procedure - of equal magnitude in the width and depth terms of the general bearing capacity equation, i. e.:

$$\epsilon_{\delta,B} = \epsilon_{\delta,D} = \epsilon_{\delta} \qquad (-)$$

In contrast to this, on the basis of the results of earlier tests, the reduction factor ϵ'_{δ} for an inclined load acting parallel to the long side of the footing amounts to only:

$$\epsilon'_{\delta} = (1 - \tan\delta) \qquad (-)$$

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