INTERACTION OF FOUNDATION BEAM AND SOIL WITH FRAMES

TRAVAIL COMMUN DES FONDATIONS ET DES CONSTRUCTIONS

ВЗАИМОДЕЙСТВИЕ ГРУНТА И ФУНДАМЕНТНОЙ БАЛКИ С КАРКАСОМ СООРУЖЕНИЯ

K. SEETHARAMULU, Professor, Deptt. of Civil Engineering, I.I.T. Delhi
ANIL KUMAR, Senior Research Fellow, Civil Engineering Deptt. I.I.T. Delhi (India)

SYNOPSIS

The proposed analysis treats foundation beam, soil and super structure as one system. The general formulation presented takes the soil as Winkler's bed. The investigation is limited to the foundation beams and frames only. Wherever possible, if the rafts can be idealised into strips the proposed approach should be applicable. Two independent methods of analysis have been proposed. The foundation beam is taken to be a part of the super structure like any other girder except for the difference that stiffness coefficients of foundation beam are to be derived based on soil interaction. The stiffness matrix for a foundation beam member with soil interaction has been developed. The results of the numerical example presented show the inadequacies of the idealization for treating the components of the system separately.

INTRODUCTION

The foundation beam, soil bed and superstructure constitute one single system and these are not independent components as they interact with each other, contrary to the general design practice where they are treated separately. The idealization which justify the general practice of treating the three components separately are (1) soil pressure is computed assuming the foundation as rigid which gives rise to uniform or uniformly varying pressure distribution. (2) The column bases of the superstructure are assumed to be fixed to the foundation or pinconnected without settlement. (3) The foundation beam is analysed for the reactions obtained from the idealization(2) and the soil pressure based on the idealization (1). Hence, the above idealisations ignore the deformation of the foundation due to its flexibility.

The general formulation presented herein takes the soil as Winkler's bed. The Winkler's idealised bed consists of an infinite number of independent springs which can take both tension and compression and their elastic constant represents the foundation modulus. The most debated part of this hypothesis, that the foundation deforms only along the portion directly under the loading, has been found to be true for many types of soil and geometry of foundation. However, at thee edges it may not be correct because Winkler's idealization suggests a sudden discontinuity. But it does not significantly affect the overall analysis. The value of 'K' (foundation modulus) is not necessarily same as the coefficient of subgrage reaction obtained by standard plate bearing test. Methods have been suggested to find 'K' from this coefficient of subgrade reaction on the basis of the geometry of foundation. But the effect of stiffness of foundation structure is ignored which may be an important factor. Systematic experimental investigation is required for the foundation beams and slabs on classified soils. The analysis of the same set of foundation structures will suggest approximate values of the foundation modulus, K, to be attributed to each type of soil. The other idealizations e.g. elastic half space are comparatively more complicated.

An interaction problem of a simple portal has been solved (Zemochkin and Sintzyp 1962) treating the bed as half space. The computations are quite tedious. It has been tried to co-relate Winkler's approach with that of half space (Biot 1937, Vesic 1961, Barden 1962) by modifying the foundation modulus.

STIFFNESS MATRIX OF TYPICAL MEMBERS:

Stiffness matrices of typical members are obtained as they are used in the proposed method of analysis. The basic elements of a frame are column, beam and foundation beam. The stiffness matrix of column or beam element can be written without difficulty (Livesley 1964). The stiffness matrix of foundation beam is different due to soil interaction.

STIFFNESS MATRIX OF FOUNDATION BEAM

For the foundation beam the equations obtained by Miranda and Nair (1966), which is an initial condition solution of the fourth order, differential equation, can be put in matrix form relating the parametres at the two ends. This matrix will be called as field matrix and in compact form can be written as $(X)_2 = (F)\,(X)_1$ ---------- (1)

where $(X)_2 = (w_2, r_2, v_2, m_2)^t$,

$(X)_1 = (w_1, r_1, v_1, m_1)^t$ and F is the field matrix. The symbols w, r, v and m stand for the vertical deflection, rotation, shear and moment respectively at the ends as indicated by the subscript.

Rearranging the elements of Eq. (1) following relation can be established

$$(p) = A\,(S)\,(d) \quad \text{-----------}(2)$$

where $(p) = (v_1, m_1, v_2, m_2)^t$ and

$(d) = (w_1, r_1, w_2, r_2)^t$, $A = 4B^3EI/2y_1^2 - y_2y_3$,

$B^4 = \frac{K}{4EI}$

$S_{11} = S_{33} = y_2\,y_4 + y_1\,y_3$, $S_{12} = y_2^2 + y_3^2/4B$,

$S_{13} = -y_2$, $S_{14} = y_1/B$, $S_{23} = -y_1/B$, $S_{34} = -S_{12}$

$S_{22} = S_{44} = (y_2\,y_1 - y_3\,y_4)/2B^2$, $S_{24} = y_3/2B^2$

y_1 = Sinh Bl Sin Bl, y_2 = Cosh Bl Sin Bl+ Sinh Bl Cos Bl, y_3 = Cosh Bl Sin Bl-Sinh Bl Cos Bl, y_4 = Cosh Bl Cos Bl

The positive directions are in accordance with right hand rule.

METHODS OF ANALYSIS

A. Direct stiffness method.

Using the stiffness matrices of typical beam, column and foundation beam, the stiffness matrix of a complex structure can be assembled by using suitable logical techniques (Tezcan 1963, 1966) in the programming. The next step would be the solution of these equations. The stiffness matrix even for a moderately tall frame is too large to store in the computer in the case of direct inversion. If the effect of axial deformation in columns is considered the size of the stiffness matrix is further increased.

Two methods (a) and (b) are generally applied to overcome this difficulty.

(a) Tridiagonalisation, elimination and back substitution. This standard procedure has been used successfully by Clough etal (1964) for the analysis of tall buildings. The inversion of the storey stiffnesses is the main objection of the method.

(b) Iterative method

The Gauss-Siedal method has been found to converge when displacement method is used. The Kani's method is essentially the Gauss-Siedal method. But, if the axial deformations of columns is thought to be significant for moments and shears (as it is true in tall buildings) Kani's algorithm becomes tedious. The additional unknown will be the vertical deflection and to evaluate them, vertical force equilibrium has to be used.

The Gauss-Siedal method has been chosen for computer program with an improved algorithm which saves computer storage significantly.

In the analysis there are three pivotal equations obtained from horizontal force equilibrium of a floor line, vertical force and moment equilibrium for the corresponding deformation of a node. From the three pivotal equations the equations of equilibrium, equal to the number of unknown deformations, can be generated. The deformation, forces and members are given number designations and in this process the zero coefficients are not stored. The numbers are sequenced in the following way.

(i) The joint rotations and moment loads at joints are numbers from left to right and top to bottom.

(ii) The storeys are labelled from top for horizontal forces and sways.

(iii) The vertical deflections and forces at joints are labelled in the same fashion as the joint rotations after the horizontal sways are numbered.

(iv) Column numbers are independent of the above, and they are designated from top to bottom and left to right.

(v) Similarly girders are also numbered.

If there is foundation beam with overhangs, for systematic programming, fictitious joints and members are assumed as a part of the super-structure frame. Therefore while numbering the structure, these

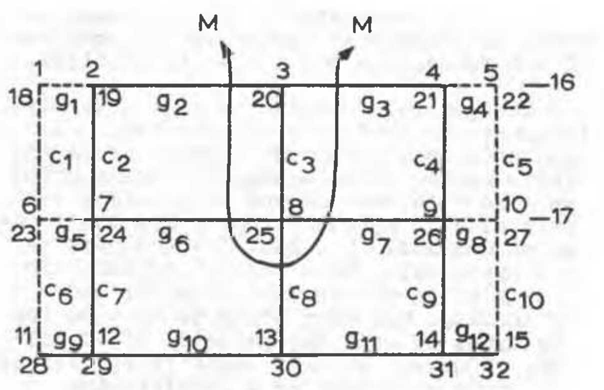

FIG. 1

fictitious joints and members alongwith the foundation beam are included, Fig.1.

Considering the frame, Fig. 1, the numbers 1,2, --- 15 designate joint rotations and moments. The numbers 16, 17 represent horizontal forces and sways. The vertical forces and deflections at the joints are designated by the numbers 18, 19,--32. Coding of the equilibrium equations is based on the deformations and members appearing in the equation. This coding is used to generate the equations of equilibrium to solve for different types of deformation (rotation, horizontal sway and vertical deflections).

For faster convergence, the vertical force equilibrium is written by taking column line as a free body above the joint. For joint 8 the free body taken is as shown by section MM, Fig. 1. Similarly for the horizontal force equilibrium, the free body is the frame above the floor line.

Generation of equations of equilibrium:

(1) Moment equilibrium

For writing moment equilibrium at each joint, three numbers (KODJ values) for each joint starting from left only for the top level, are given as input data. The numbers are girder numbers at the left and right of the joint and the sway number for the top level only.

All the code numbers at each joint are generated from the above numbers. From the KODJ values given as data, the KJ values generated in the programme are the required code numbers for generating moment equilibrium. The different KJ values are explained below:

KJ (1), KJ(2), KJ(4), KJ(5) are the rotation numbers at the top, left, right and bottom of the joint respectively. KJ(6) and KJ(7) are the sway numbers of the floor above and the floor to which the joint belongs. The KJ(6), KJ(7) values are generated from the first value of KODJ which is the sway number of top level only. KJ(8),KJ(10) are the numbers of vertical deflections at the left and right of the joint respectively. While KJ(9) is the vertical deflection of the joint itself. KJ(11) and KJ(12) are the girder numbers at the left and right of the joint respectively. These are generated from the second and third value of KODJ for that column line. KJ(13) and KJ(14) are the column numbers at the top and bottom of the joint concerned. The same array KJ(14) is used for all the equations of moment equilibrium.

Suitable provisions are made for the non-existing deformations and members to the left of first column line and to the right of last column line. (For the readers interested in the details, program listing can be made available on request).

So, if it is required to find the effect of other joint rotations on the moment equilibrium of a joint, it is presented in the lib program as below:

```
A1= CR(KJ(13)) * D(KJ(1)) +GR(KJ(11)) *
D(KJ(2))+GR(KJ(12)) * D(KJ(4)) +CR(KJ(14))*
D(KJ(5))
```

where A1 = carry over moment from the joints other than the joint concerned, CR=4EI/L of columns, GR=4EI/L of girders and D= deformations. Similarly the effect of the other deformations can be computed.

2. Horizontal force equilibrium.

For these equations once again three numbers (KODS values) are defined. First two are the numbers for rotations at the top and bottom of the extreme left column for each storey starting from the top and the third is the corresponding number of horizontal sway. Same technique of coding the non-existing deformations and members is used as in the case of moment equilibrium. The array KODS(3 x number of storeys) is used to generate KS(3) which is used for all the horizontal force equilibrium equations. The shears due to rotations of each column are computed as below:

```
    B1, B2, B3 = 0.0
    DO 16J = 1, NCL
    IF (KS(2).EQ.ND+1) J1=1
    IF (KS(2). LT. ND+1) J1=J
    B1=B1+CV(KS(1)+J-1) * D(KS(1)+J-1)
    B2=B2+CV(KS(1)+J-1) * D(KS(2)+J1-1)
    B3=B3+CS(KS(1)+J-1)
16  CONTINUE
```

ND=Total number of deformations, $CS=12\ EI/L^3$ of columns and $CV=6EI/L^2$ of columns.

(3) Vertical force equilibrium.

For generating these equations eight code numbers were used as defined below: KV(1), KV(3) are the rotations and KV (4), KV(6) are the vertical deflections existing at the left and right respectively of the joints occuring in the chosen free body. KV(2) and KV(5) are the numbers for rotation and deflection of the joints occuring in the same column line. KV(7) and KV(8) are the numbers of girders at the left and right of the joint. The KV values are generated in the program from corresponding eight KODV values for each joint of the top floor only. Say, if the shears, C2, at the end of the girders due to rotation of the joints appearing in the free body are required, it is done as

```
C2=C2+(GV(KV(8)-J2)-GV(KV(7)-J2))*
       D(KV(2)-J1)
```

where $GV=6EI/L^2$ of girders. J2 and J1 are defined in the program to account for all girders and deformations existing in the free body.

The equation with non zero coefficients based on the three basic algorithms for moment equilibrium, sway and vertical force equilibrium, are generated as and when required without having to store them, From the considerations of convergence the sequence of the evaluation of deformations is (i) rotations (ii) sways and (iii) vertical deformations.

B. Force displacement method

The method uses the flexibility of the foundation beam and stiffness of the superstructure while equalizing the deformations at the junctions of columns and foundation beam.

The relations obtained are

$$(d_I) = ((I)-(T_{11}) \times (SS))^{-1} (T_{11}) (p_f)$$

and $(de) = (T_{21}) (p_f) + (SS) (d_I))$

where T_{11}, T_{12} etc. are the submatrices of the flexibility matrix of foundation beam. In this formulation flexibility matrix was obtained by inverting the stiffness matrix of foundation beam using equation (2). d_I is the set of deformation at the base points and d_e at the end points. SS is the matrix obtained by solving the super-structure for unit deformations at the base. (p_f) is the set of reactions obtained by analyzing the frame for given loading with fixed base.

The case of column bases hinged with the foundation is the particular case of the above method.

Example

The problem shown in Fig. 2 was solved by direct stiffness method (iterative technique). The same problem was solved by Blaszkowiak (1966) by relaxation method which is quite tedious. Figure 3 shows the B.M. diagram and the deflected shape.

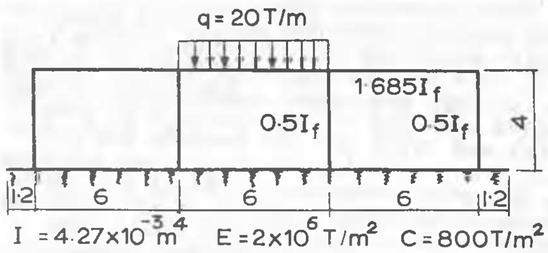

FIG. 2

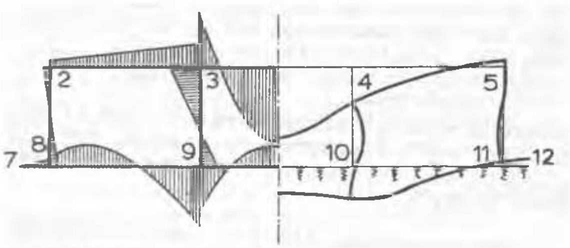

FIG. 3

$m_{8,7}$ =-1.03, $m_{8,9}$ =+ 7.11, $m_{9,8}$=39.8, $m_{9,10}$=-29.23, $m_{2,8}$=-5.98, $m_{8,2}$=-6.07, $m_{3,9}$=-21.6, $m_{9,3}$=-10.8 Tm.

CONCLUSIONS

(i) The numerical example shows the inadequacies of the idealisation used in general practice. The left end settlement for the interaction procedure and idealizations are 0.216 cm and 0.127 cm respectively.

(ii) Direct stiffness method is preferred to Force displacement method.

(iii) The iterative method chosen for computer programming was found to be converging for small frames only.

(iv) The flexibility matrix of the foundation beam on elastic half space can be found by using the method suggested by Cheung (1965) and it may be easily incorporated in the "Force displacement method".

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