LIMIT LOADS OF PRESTRESSED PLATES AND FRAMES ON ELASTIC FOUNDATION

CHARGES LIMITES DES PLAQUES PRECONTRAINTES ET DES CADRES A FONDATION ELASTIQUE

ПРЕДЕЛЬНЫЕ НАГРУЗКИ ПРЕДВАРИТЕЛЬНО-НАПРЯЖЕННЫХ ПЛИТ, МНОГОПРОЛЕТНЫХ РАМ И ДРУГИХ ЖЕЛЕЗОБЕТОННЫХ КОНСТРУКЦИЙ, РАСПОЛОЖЕННЫХ НА УПРУГОМ ОСНОВАНИИ

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Sinopsis. The prestressed plate placed on elastic half space is a static undermined system, and the preliminary stress creat a selfequilibrium diagram of subgrade reactions.These reactions involve supplementary bending moments in the plate. The proliminary stress cause the increase of the rigidity of the plate. The design of the frames and slabs on soil foundation is solved as a nonlinear. The I partof this paper is written by A.P.Sinitsyn, the II part by I.A.Simvulidi and the III part by V.I. Solomin.

I PRESTRESSED PLATES. The bearing capacity of prestressed plates with consideration of the redistribution of the sub grade reactions are studied. The design method of the multylayered plates with elastic stratum and natural fastened foundation is elaborated too. The prestressed plate placed on elastic halfspace is a static undeterminate system therefore the preliminary stress of the plate create in foundation a selfequilibrium diagram of subgrade reactions. These reactions cause supplementary bending moments in the plate, which depends on the values and excentricity of the preliminary stresses and on the rigidity rate of the plate and foundation. The preliminary stress cause the increase of the rigidity of the plate. The transition beyond the elastic limit for the prestressed plate is connected with a considerable redistribution of stresses in the systm plate-foundation, and as result thd plastic hinge in usual neanihg can not appear. More favorable distribution of bending moments correspond to a certain value of the rate of the plate and elastic foundation rigidities. The elaborated design method permit to find the more advantageous solution. If the rate of foundation and plate rigidities is taken inconvenient then the subgrade reactions should have a great value. The influence surface may be alaborated for the temperature effects but they should have more complicated contours , and the important influence of the rigidity rate on the subgrade reactions are valid too. Now the design of a prestressed plate placed on elastic foundation begin with determination of the reactions of elastic halfspace caused by preliminary stress. The designing scheme is shown in Fig.1

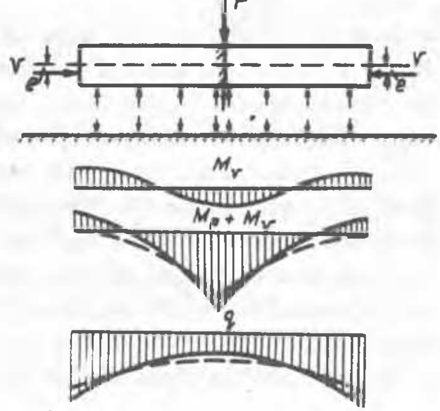

Fig.1 Desægning scheme.

The resultant forces "X_i" were determined by system of linear equations

$$[\delta_{ik}]\cdot[X_i] + y_0 + \Delta_{ip} = 0 \qquad (1)$$

where

$[\delta_{ik}]$ -rigidity matrix;

The item Δ_{ip} may be calculated by multiplication of the bending moment diagrams

$$\Delta_{ip} = -K_i \frac{Ve}{C}\frac{C^3}{6EJ} \qquad (2)$$

The solution of the equations (1) give the values of X_i. The subgrade reactions and the bending moments diagrams are shown in Fig.1. The value of moments caused by selfequilibrium subgrade reactions depend on the value and rigidity rate of the plate and foundation too. The diagrams of subgrade reactions and bending moments due by external loads may be external loads may be constructed independently of selfequilibrium reactions but the rigidity of the plate used in this design must be calculated with respect of influence of the preliminary stress on the value of rigidity. The displacements of the plate due to a unit force are calculated by following formulas:

$$V_{ik} = \frac{C^3}{6EJ}\left(1 - \frac{W_{ik}^{v}}{W_{ik}}\right) W_{ik} \qquad (3)$$

Introducing the generalized rigidity

$$D_v = D \Big/ \left(1 - \frac{W_{ik}^{v}}{W_{ik}}\right) \qquad (4)$$

The value $\left(1 - \frac{W_{ik}^{v}}{W_{ik}}\right)$ is less unity and the generalized rigidity D_v, should be more than real rigidity D.

The value of P_{lim} may be calculated as it is given in paper (1) (Sinitsyn 1964). Relative value of the bearing capacity of the plate when the concentrated load is applied on the edge may be obtained by following formul:

$$P_{lim}/P'_{lim} \approx 0{,}5\left(1 - \frac{0.8\, Ve}{\sigma_e W_{pe}}\right) \qquad (5)$$

P'_{lim} if the load is placed on the edge of the plate. P'_{lim}- if the load placed on the middle of the plates space. σ_e the limit normal stress. W_{pl} the plastic resistant moment. From formul (5) is clear, that the great preliminary stress (V) can reduce the bearing capacity of the plate, if the load is applied on the edge of the plate. Design of the treelayered plate is evaluated with solution of the differential equations of equilibrium with respect of the inertia forces (2) (Sinitsyn 1971).

$$D_i \nabla^4 W_i + D_0(W_i + W_{i+1}) + \left(\mu_i + \frac{h_0\mu_0}{3}\right)\ddot{W}_i + \frac{h_0\mu_0}{6}\ddot{W}_{i+1} = $$
$$= q_i(x,y,t) - \frac{1}{1-\sigma^2}\nabla^2 M_T \qquad i = 1,2 \qquad (6)$$

There is W_i- displacement of the upper plate, W_{i+1}- displacement of the lower plate D_1, D_2- rigidities of upper or lower plates, μ_1, μ_2- masses of a square metre of upper and lower plates; σ- Poisson coefficient D_0- rigidity of elastic stratum; $q_1(x,y,t)$-the load; $q_2(x,y,t)$ -subgrade reactions M_T -bending moment due to temperature

$$M_T = \alpha E \int_{-h/2}^{+h/2} T z\, dz \qquad (7)$$

Where α -coefficient of linear temperature expansion E-modul of elasticity of the plate; h- thickness of the plate; T- temperature distribution function.

The value of M_T-should be calculated by formul

$$M_T = \alpha E \frac{q h^3}{24\lambda}\left(1 - \frac{96}{\pi^4}\sum_{n=1,3\ldots}\frac{1}{n^4} e^{-n^2\pi^2\frac{Kt}{h^2}}\right) \qquad (8)$$

Where is q-heat flux applied to the plate λ -heat conductivity coefficient of the plate material, K-temperature conductivity coefficient.

If the bonds placed between two plates should been nondeformable than $W_1=W_2$ and by addition of equations (6) obtain :

$$(D_1 + D_2)\nabla^4 W + (\mu_1 + \mu_2 + \mu_0 h_0)\ddot{W} = (q_1 - q_2) - \frac{1}{1-\sigma^2}\nabla^2 M_T \qquad (9)$$

Taking $D_1=D_2=D$ and $\mu_1+\mu_2+\mu_0 h_0 = \mu$ and $q_2=KW$ we have :

$$D\nabla^4 W + \mu\ddot{W} + KW = q_1 - \frac{1}{1-\sigma^2}\nabla^2 M_T \qquad (10)$$

The solution of this equation is

$$W = \sum\sum q_{nm}(t)\sin\frac{n\pi x}{a}\sin\frac{m y\pi}{b} \qquad (11)$$

To determination of the generalized coordinates $q_{nm}(t)$ the following equation must be solved:

$$\ddot{q}_{nm} + \omega_{nm}^2 q_{nm} = \Phi_{nm} \qquad (12)$$

The square values of frequencies ω_{nm}^2 may be calculated by formul:

$$\omega_{nm}^2 = \left\{\left[\left(\frac{n\pi}{a}\right)^2 + \left(\frac{m\pi}{b}\right)^2\right]^2 + \frac{K}{D}\right\}\frac{D}{\mu} \qquad (13)$$

The right side of equation (12) present the items of the expansion series of the external forces and should be calculated as usually by means of self vibration formulas.

Bending moments of the plate were calculated using the displacements values.

The displacements and bending moments are represented by infinite series and they were calculated by means of computer Ural-2

More favorable distribution of the bending moments is connected with a certain value of parameter as a characteristic of the rate of the plate and the foundation rigidities The evaluated method of design allow to determine this more advantageous rate. The subgrade reactions P_{max}/P_0, by which the plate is stable, may be considered by means of influence surface (Fig.2).

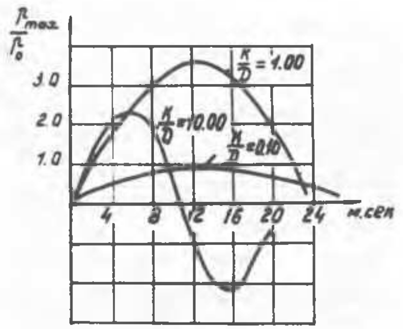

Fig.2 Stabgrade reactions

From Fig.2 is clear that for a unsuccessful selection of the plate and foundation rigidities rate the reactions reach a great value as it is by K/D=1,00.

The results of experimental tests of the prestressed plates made in situ are given in paper /1/ for four types of plates.

II. MULTIPLE-SPAN FRAMES

To derive the principal equation for designing a frames on a soil base, we will consider joint A of a complex frame with all its adjoining members, some of which lie on the soil base.

Assume that members AB, AC, AD and AE lie on the soil base and members Am and An do not. Since the frame under consideration, lying on a soil base, rests at the same time on concentrated nondisplaceable supports (piles) it will be necessary to write additional equations to take into account the influence of the concentrated supports. For this purpose we use the conditions that the reaction pressures, at the points where nondisplaceable supports are arranged, equal zero.

Using these conditions we obtain the equations:

$$\left.\begin{aligned} Y_A &= B_{AB}m_1^{(AB)} - C_{AB}m_2^{(AB)} + m_3^{(AB)}\frac{M_A}{L_{AB}} + m_4^{(AB)}\frac{M_B}{L_{AB}} \\ Y_B &= C_{AB}m_1^{(AB)} - B_{AB}m_2^{(AB)} - m_4^{(AB)}\frac{M_A}{L_{AB}} - m_3^{(AB)}\frac{M_B}{L_{AB}} \end{aligned}\right\} \quad (1)$$

Solving equations (1) and the equations obtained from equations (VI-3) (Simvulidi,1968 Simvulidi,1969) in relation to the joint moments, we obtain after certain transformations: M_{AB}, M_{AC}, M_{AE}, M_{AD}.

$$M_{AB} = \Delta_{AB}\left(\varphi_A b_2^{(AB)} - \varphi_B b_1^{(AB)}\right)\mathcal{L}_{AB}i_{AB} + M'_{AB}$$

where

$$M'_{AB} = \Delta_{AB}\left[L_{AB}\left(E_1^{(AB)}b_2^{(AB)} - E_2^{(AB)}b_1^{(AB)}\right) - \left(\varphi_A^{0(AB)}b_2^{(AB)} - \varphi_B^{0(AB)}b_1^{(AB)}\right)\mathcal{L}_{AB}i_{AB}\right] \quad (3)$$

The joint moments M_{Am} and M_{An} for members Am and An, which do not lie on the soil base, are determined by formulas of structural mechanics.

From the condition of equilibrium of joint A and making certain transformations, we obtain the general equation of angular displacement for joint A:

$$2\left[\sum i_{Am} + \frac{3}{4}\sum i_{An} + K_{AB}i_{AB} + K_{AC}i_{AC} + \frac{3}{4}K_{AE}i_{AE} + \frac{3}{4}K_{AD}i_{AD}\right]\varphi_A + \sum i_{Am}\varphi_{Am} + K_{AB}^{(1)}i_{AB}\varphi_B + K_{AC}^{(1)}i_{AC}\varphi_C =$$
$$= 3\sum\frac{i_{Am}}{L_{Am}}\delta_{Am} + \frac{3}{2}\sum\frac{i_{An}}{L_{An}}\delta_{An} + \frac{1}{2}\left(\sum M'_{Am} + \sum M''_{An} + M'_{AB} + M'_{AC} + M''_{AE} + M''_{AD}\right) \quad (4)$$

In these equations M'_{AB}, M'_{AC}, M''_{AE} and M''_{AD} are joint moments due to the loads applied on rods AB, AC, AE and AD which lie on the soil base: M'_{Am} is the support moment due to the load for a beam not lying on the soil base, and with fixed ends; M''_{An} is the support moment due to the load for a beam not lying on the soil base, and with one end fixed and the other lying free.

To determine the values of quantities $\bar{\varphi}_{3A}^{(A)}$, $\bar{\varphi}_{3B}^{(A)}$, $\bar{\varphi}_{3A}^{(B)}$ and $\bar{\varphi}_{3B}^{(B)}$ we use Table (V-2 a), for those of quantities $\bar{\varphi}_{2A}^{(A)}$, $\bar{\varphi}_{2B}^{(A)}$, $\bar{\varphi}_{2A}^{(B)}$ and $\bar{\varphi}_{2B}^{(B)}$ Table (V-3 a) and the values of $\varphi_A^{0(AB)}$, $\varphi_B^{0(AB)}$, $\varphi_A^{0(AC)}$, $\varphi_C^{0(AC)}$, $\varphi_A^{0(AE)}$ and $\varphi_A^{0(AD)}$ are determined by the formulas (VI-81) (VI-82) (VI-83) and (VI-84) (Simvulidi,1968, Simvulidi,1969). /3,4/.

To determine the values of B_{AB}, C_{AB}, B_{AE}, B_{AD}, C_{AD}, $m_1^{(AB)} \ldots m_4^{(AB)}$, we use formulas that have been previously derived

$$\Delta_{AB} = \frac{1}{a_1^{(AB)}b_2^{(AB)} - b_1^{(AB)}a_2^{(AB)}} \quad (5)$$

$$a_1^{(AB)} = \bar{\varphi}_{2A}^{(A)} + \bar{\varphi}_{3A}^{(A)}m_3^{(AB)} - \bar{\varphi}_{3B}^{(A)}m_4^{(AB)} \quad (6)$$

$$b_1^{(AB)} = \bar{\varphi}_{2B}^{(A)} - \bar{\varphi}_{3A}^{(A)}m_4^{(AB)} - \bar{\varphi}_{3B}^{(A)}m_3^{(AB)} \quad (7)$$

$$E_1^{(AB)} = C_{AB}\left(\bar{\varphi}_{3A}^{(A)}m_2^{(AB)} - \bar{\varphi}_{3B}^{(A)}m_1^{(AB)}\right) - B_{AB}\left(\bar{\varphi}_{3A}^{(A)}m_1^{(AB)} - \bar{\varphi}_{3B}^{(A)}m_2^{(AB)}\right) \quad (8)$$

$$K_{AB} = -\frac{1}{4}\sum\Delta_{AB}b_2^{(AB)}\mathcal{L}_{AB} \quad (9)$$

$$K_{AB}^{(1)} = \frac{1}{2}\sum\Delta_{AB}b_1^{(AB)}\mathcal{L}_{AB} \quad (10)$$

The obtained general equation (4) of angular displacements is sufficient only for designing a symmetric frame structure, to which a symmetric load is applied. Since the frame being considered lies on a soil base and rests, at the same time, on undisplaceable concentrated supports, the relative displacements of the frame joints can be only horizontal ones.

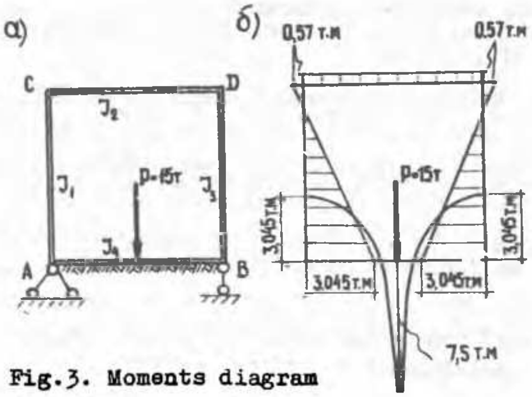

Fig.3. Moments diagram

Example 1. A moment diagram (Fig.3 b) has been plotted for a closed rectangular frame (Fig.3a) lying on an elastic soil base and, at the same time, having its ends resting on concentrated supports (Fig.3).

III. NON-LINEAR PROBLEMS OF FOUNDATION SLAB

The non-linear problem is solved as a sequence of linear ones. When cracks appear in flextarnal reinforced concrete elements, stiffness decreases sharply.
For solving this problem, the variation-differential method is rather effective. The potential energy of this system equals:

$$\Pi = V + V_0 + U \qquad (1)$$

Where V is a slab deformation energy, V_0-is a base deformation energy, U-is a load work. The formulas adopted for moments occuring in the slab are as follows:

$$M_x = -(A\varkappa + B\chi + 2E\omega)$$
$$M_y = -(B\varkappa + C\chi + 2F\omega) \qquad (2)$$
$$M_{xy} = -(E\varkappa + F\chi + 2D\omega)$$

where $\varkappa, \chi, \omega$ are curvatures: A,B,C,D,E,F are stiffnesses. Accepting these notations it is possible to write:

$$V = \frac{1}{2}\iint_\Omega [A\varkappa^2 + C\chi^2 + 2B\varkappa\chi + 4D\omega^2 + 4(E\varkappa + F\chi)\omega]\, d\Omega \qquad (3)$$

$$V_0 = -\frac{1}{2}\iint_\Omega p w \, d\Omega \,, \qquad (4)$$

$$U = -\iint q w \, d\Omega \qquad (5)$$

where p is a reaction pressure; q is an external load; w is a vertical displacement; Ω is a slab area.
An irregular grid is plotted on the slab and equations (3) - (5) are rewritten as follows:

$$V = \frac{1}{2}\sum_\varsigma \sum_\xi [A\varkappa^2 + C\chi^2 + 2B\varkappa\chi + 4D\omega^2 + 4(E\varkappa + F\chi)\omega]_{\varsigma\xi} \Delta\Omega_{\varsigma\xi} \qquad (6)$$

Here the curvatures in any point $\varsigma\xi$ are expressed by means of differential ratics, hence $V = V(W_{\varsigma\xi})$. When (6) is written, boundary conditions should be taken into account.

$$V_0 = \frac{1}{2}\sum_\varsigma \sum_\xi P_{\varsigma\xi} W_{\varsigma\xi} \Delta\Omega_{\varsigma\xi} \qquad (7)$$

If the base model has a distribution capacity (a layer, a half space):

$$P_{\varsigma\xi} = \sum_\rho \sum_\tau \mathcal{B}_{\varsigma\xi,\rho\tau} W_{\rho\tau} \qquad (8)$$

Where $\mathcal{B}_{\varsigma\xi,\rho\tau}$ are the base pliabilities.
If the base is discrete:

$$P_{\varsigma\xi} = K_{\varsigma\xi} W_{\varsigma\xi} \qquad (9)$$

$$U = -\sum_\varsigma \sum_\xi q_{\varsigma\xi} W_{\varsigma\xi} \Delta\Omega_{\varsigma\xi} \qquad (10)$$

Now Π may be considered as a function of the vertical displacement of the grid points plotted on the slab and a system of linear equations to determine $W_{\varsigma\xi}$ is written.
The behaviour of a slab sample is divided into a number of stages;

1. $M_{max} \leq 0{,}5\, M_{cr}$, where M_{cr} is a moment of crack formation. During this stage the sample acts as a linear-deformable and isotropic one: $A=C=D_0$, $B=\mu D_0$, $D=(1-\mu)D_0$, $E=F=0$ where is a cylindrical stiffness.

2. $0{,}5\, M_{cr} < M_{max} \leq M_{cr}$. The sample is still isotropic, but there appears a physical non-linearity: $A=C=\tilde{D}_0$, $B=\mu\tilde{D}_0$, $D=(1-\mu)D_0$. $E=F=0$, $\tilde{D}=\tilde{D}_0(1{,}2-0{,}4\, M_{max}/M_{cr})$.

3. $M_{cr} < M_{max} < M_e$ where M_e is a moment when stresses in the reinforcement attain the yield limit. A crack runs through the sample considered, and it becomes anisotropic, the physical non-linearity manifesting on a larger scale than in the previous stage.

In this case:
$A=A(M_x, M_y, M_{xy}, \measuredangle)$, $B=B(M_x, M_y, M_{xy}\ \measuredangle)$ .....
...$E=E(M_x, M_y, M_{xy}, \measuredangle)$, $F=F(M_x, M_y, M_{xy}, \measuredangle)$
where is the angle characterizing the direction of the crack.
If there are two cracks, then $M_{xy}=0$ and $A=A(M_x)$, $C=C(M_y)$, $B=E=F=0$

Two examples of calculation of foundation by means of the above equations are considered. They have been calculated by the electronic computer "Minsk-22" using the programmes made by Turinin V.F.and Shishov I.I.

Example I. A reinforced concrete beam of T-section (Fig.4) rests on a linear deformable half space with characteristics E_0=1500 t/m^2, μ_0=0.35. The beam has the following parameters an average pressure on the base ((with 200 ton) is 1.25 kg/cm^2, reinforcement percentages:

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$\mu = 0.72$ $\mu' = 0.36$ (with bottom tensioned reinforcement) ; $\mu = 0.35$, $\mu' = 0.72$ (with top tensioned reinforcement),
As can be seen from the bending moment diagrams the linear calculation (dotted lines) shows that with $P = 80$ ton , a maximum moment, occuring in the middle of the beam, equals M_e .
According to the calculation with non-linearity of behaviour of reinforced concrete (solid lines) it is possible to find that this moment is considerably smaller and attains only the moment of crack formation M_{cr} . With the non-linearity taken into account, the greatest moment becomes equal to M_e only with $P = 200$ ton . At the same time vertical displacements and ments and reaction pressures differ imperceptibly from those obtained bt the linear calculation.

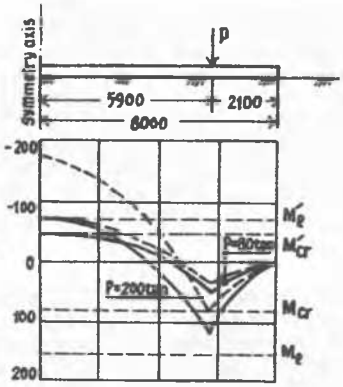

Fig.2. Diagram of bending moments in a beam

Fig 4 Diagram of bending moments

This example along side sith a number of others, considered (5), (6) show that the difference between the strains in foundation beams and slabs, found as a result of the linear and non-linear calculations may appear rather great.

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