CRITICAL LOADS DEPENDING ON LAYER THICKNESS

CHARGE CRITIQUE EN FONCTION DE L'EPAISSEUR DE LA COUCHE

КРИТИЧЕСКИЕ НАГРУЗКИ И ИХ ЗАВИСИМОСТЬ ОТ ТОЛЩИНЫ СЛОЯ

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SYNOPSIS. Bearing capacity of non-cohesive layers of limited thickness subjected to rigid strip loads has been examined by model tests with cylindrical metal rods. Resulting bearing capacity coefficients have been compared with Mandel's coefficients obtained through the theory of limit equilibrium. The influence of non-linear soil behaviour depending on normal and shear octahedral stresses, onto the contact pressures in the foundation base has been investigated by an approximate procedure.

INTRODUCTION

The load applied to a soil layer becomes critical if either it causes (a)displacements of the soil threatening the function of the structure or (b) displacements increasing with constant or accelerated velocity, menacing the stability of the foundation, or (c) stresses overpassing the strength of the foundation material. While this last danger can be avoided by an appropriate design of the foundation, the admissible bearing value related to the criteria (a) and (b) is influenced by the magnitude, shape and depth of the foundation as well as by the static system and stiffness of foundation and structure, and governed by the stratigraphy and the deformability of the soil including the stress-strain-time relationships preceding failure.

The simultaneous solution of this stress-strain problem is hindered by the complexity of the rheological relationships for soils. Therefore, the soil is commonly treated as an elastic medium when considering the criterion (a) and, as a medium obeying Coulomb-Mohr's failure law when applying the criterion (b); in the latter case kinematic conditions are taken into account only in a qualitative way.

In the present paper the bearing capacity obtained, for a layer of limited thickness resting on a rigid base, by the theory of plasticity, will be compared with the results of some model tests, and contact pressures corresponding to the elastic treatment of the layer will be checked with regard to the real soil behaviour.

THEORY OF LIMIT EQUILIBRIUM AND ITS EXPERIMENTAL VERIFICATION

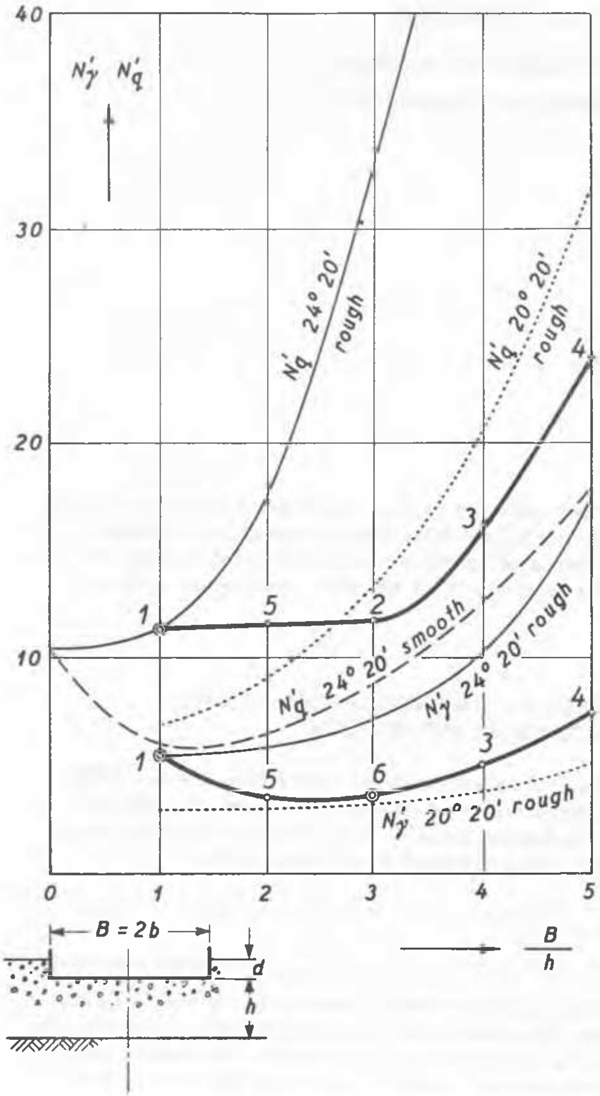
Through the theory of limit equilibrium, Mandel (1972) has found that in plane strain conditions the behaviour of the bearing capacity of a soft ground layer on a rigid base can be expressed in the classical form

$$p_{max} = F/B = q\, N'_q + c\, N'_c + 0.5\, \gamma\, B\, N'_\gamma \qquad (1)$$

with the following notation: p_{max} for the average value of the maximum contact pressure, F for the total axial load, B for the width of the foundation, γ for the bulk density of the soil, c for cohesion. The bearing capacity coefficients N'_c and N'_γ are functions of the shear angle of the soil and of the ratio B/h. The coefficient N'_q can be expressed by the formula:

$$N'_q = N'_c \tan\varphi + 1 \qquad (2)$$

In cases of a perfectly rough contact between the foundation base and the layer, the coefficient N'_c has been found to depend on the friction condition between the layer and its base. When a perfectly rough contact is assumed the coefficient N'_c, starting from the classical value, increases steadily with B/h. In the case of the perfectly smooth lower contact of the layer, as B/h increases the bearing capacity decreases from the classical value, reaches a minimum and then, in dealing with wide foundations, it increases, becomig greater than the classical value.



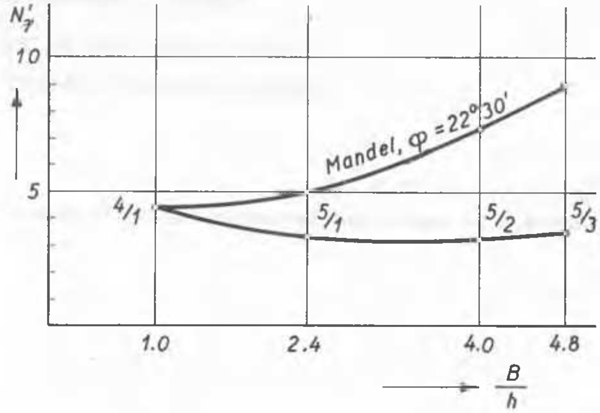


Fig.1. Bearing capacity coefficient versus B/h ratio plots:

(a) Thick full lines represent plots corresponding to our experiments of the series I; Mandel and Salençon's plots for rough contacts are represented by thin full lines for $\varphi = 24^{\circ} 20'$, and by dotted lines for $\varphi = 20^{\circ} 20'$; Mandel and Salençon's plots N'_{γ} for a smooth lower contact at $\varphi = 24^{\circ} 20'$ is given by the dashed line.

(b) The lower line corresponds to our experiments of the series II, the upper line to the Mandel and Salençon charts for $\varphi = 22^{\circ}30'$.

For respective angles $\varphi = 20^{\circ} 20'$ and $24^{\circ} 20'$, the $N'_q = N'_q$ (B/h) plots are presented in Fig. 1 a. As to the coefficient N'_{γ}, Mandel and Salençon have given theoretical values only for the rough lower contact of the layer. For shear angles $\varphi = 20^{\circ} 20'$, $24^{\circ} 20'$ and $22^{\circ} 30'$; the resulting plots $N'_{\gamma} = N'_{\gamma}$ (B/h) are presented in Figs 1 a and 1 b.

Theoretical results of Mandel and Salençon (1972) have been compared, in Fig.1, with the results of two series of model tests made with cylindrical rods l = 6 cm in length (Taylor-Schneebeli method). In the first series (Fig. 1 a), rods of a diameter 3 and 4 mm were combined in the way shown in Fig.2; the initial unit weight of the heap was 2.27 p/cm^3. In the second series (Fig.1 b) the heap was randomly composed with 50% of rods of a diameter of 3 mm and 50% of rods of 5.2 mm; the average unit weight was 2.243 p/cm^3. The base of the wooden loading block was rather smooth. Some data on test conditions and results are given in Table I.

For tests Nos I/1 and 4/1 the coefficients N'_{γ} and N'_q have been ascertained according to Mandel and Salençon's tables; they correspond to respective shear angles $\varphi = 24^{\circ} 30'$ (series I, test I/1, Fig. 1 a) and $\varphi = 22^{\circ} 30'$ (series II, test 4/1, Fig. 1 b). For tests Nos 5/1, 5/2 and 5/3 with q = 0, the coefficients N'_{γ} were computed directly from the test data. Tests Nos I/2 and I/6 permitted to compute, for B/h = 3, both coefficients, N'_{γ} and N'_q. For other tests of the series I, the ratio N'_{γ}/N'_q was taken to be the same as for tests Nos I/2 and I/6.

In all the tests, the contact with the layer base can be considered perfectly rough while the friction resistance between the layer and the foundation was certainly smaller than the shear resistance in the heap. The comparison with Mandel and Salençon's coefficients proves that the experimentally obtained values do not change with increasing B/h ratio in the same manner as theo-

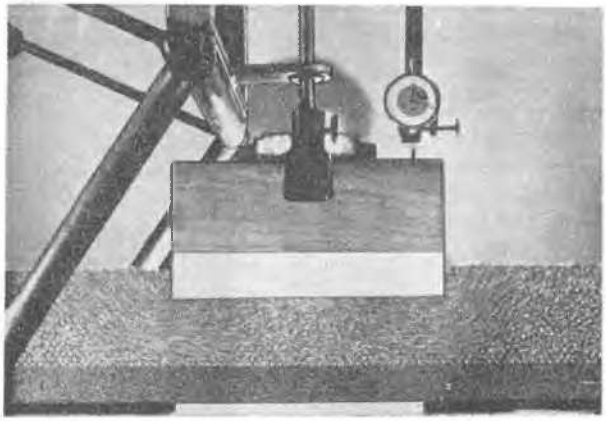


Fig.2. (a) Plastic fields observed during the test I/2.

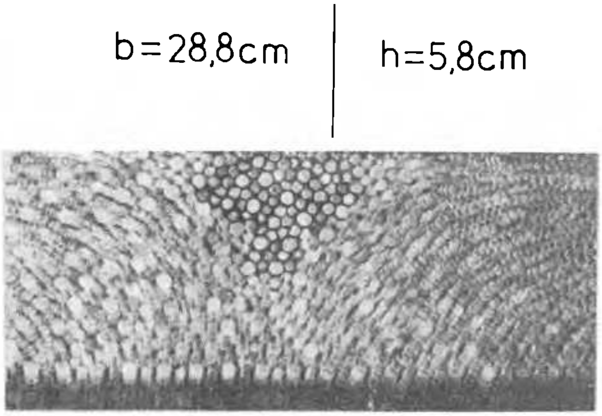


Fig.2. (b) Rigid wedge below the foundation base observed during the test I/4.

Table I

Test series	Test No	d cm	B cm	h cm	B/h -	$p_{max} = \frac{P_{max}}{B\,l}$ kp/cm^2	settlement at P_{max} mm	settlement final mm
I	6	0	28.8	9.6	3	0.117	-	-
	1	3	9.6	9.6	1	0.133	1.70	4.50
	5	3	19.2	9.6	2	0.160	2.00	3.95
	2	3	28.8	9.6	3	0.195	1.75	4.15
	3	3	28.8	7.2	4	0.284	2.07	5.07
	4	3	28.8	5.8	5	0.402	1.55	2.60
II	4/1	4	14.4	14.4	1	0.133	1.70	4.00
	5/1	0	24.0	10.0	2.4	0.097	1.40	3.50
	5/2	0	24.0	6.0	4	0.104	1.10	-
	5/3	0	28.8	6.0	4.8	0.122	0.60	1.27

653 retical values do. They remain constant in the domain $B/h < 3$; further increasing values are greater than theoretical values corresponding to a smooth lower contact, yet by far less than according to the theory related to rough contacts.

The N'_γ versus B/h plots corresponding to our experiments have a similar concave form as the theoretical plots N'_q for smooth lower contact, yet the minimum appears at greater B/h values (between 2 and 3). The ratio between theoretical and experimental N'_γ values increases with increasing ratio B/h.

Mandel and Salençon's coefficients have been compared also with experimental data of Myslivec (1971) obtained by model tests with sand whose surface has been loaded through a wooden block 4 cm in width. For shear angles of 40° and 35°45' of the model sands, the experimental N'_γ -values have been proved to overpass the theoretical ones in the domain below the respective B/h ratios 2 and 3, and to be much smaller above these ratios.

The disagreement between theoretical and experimental results can partially be due to the imperfection of experimental conditions: deficient homogeneity and isotropy of rod heaps, too narrow wooden blocks in Myslivec's tests. More important seem, however, to be the factors imparing theoretical results:incomplete and non-uniform friction in the foundation base; non-simultaneous mobilization of the same degree of the shearing resistance in the layer; progressive failure at greater displacements accompanied by successive reduction of resistance to residual values; inconsistency of contact pressures preceding failure, obtained by the plastic analysis, with the stress-strain relationships for the medium; and, to some extent, the failure scheme.

The limited influence of the ratio B/h on the increase of bearing capacity coefficients N'_γ and N'_q of non-cohesive soils as depending on the inclination of stress vectors along the upper and lower contact of the layer, has been corroborated also by determining the contact pressures corresponding to statically correct plastic stress fields; they have been computed by using Sokolovsky's numerical method of integrating Kötter's differential equations of stress characteristics.

On the other hand, in a previous paper (Šuklje 1954) proof has been given for a satisfactory agreement between experiments made, at constant volume conditions, with a cohesive layer subjected to surface loading, and the theoretical results obtained by using Meyerhof and Chaplin's (1953) charts of contact pressures. In the case of rough contacts, Mandel and Salençon's flow scheme is just an enlargement of the scheme used by Meyerhof and Chaplin.

CONTACT PRESSURES CORRESPONDING TO THE DEFORMATION CONDITIONS

The influence of the real deformability of a soil onto the pressure distribution in the foundation base will be analysed, in an approximate way, for two concrete cases of rigid foundations resting on a layer of finite thickness. In the first case the foundation is assumed to be an infinite strip of uniform width B = 2 b = 12 m, and in the second, base of the foundation is a circle of a diameter of 12 m. The foundation rests, in a depth of d = 2 m, on a 6 m thick layer of silty sand whose deformability has been ascertained by triaxial testing and the corresponding rheological parameters presented in a previous publication (Šuklje 1969, Fig.17.2). The one-day strains were used. In Fig.3 the plots $\varepsilon^o = \varepsilon^o(\sigma^{o\prime}, \varphi'_m)$ and $\varepsilon_1^d = \varepsilon_1^d(\sigma^{o\prime}, \varphi'_m)$ are reproduced in a square root scale for the octahedral normal stresses $\sigma^{o\prime}$; φ'_m is the slope of the Mohr envelope originating from the point ($\sigma^{o\prime} = 0$, $\varepsilon^o = \varepsilon_1^d = 0$), ε^o is the octahedral strain and ε_1^d the deviatoric strain in the main principal direction. When applying these stress-strain relationships for the stress-states appearing in cases treated in this paper, the slope φ'_m and the stress $\sigma^{o\prime}$ have been chosen parameters of correlation (see Šuklje 1969, p.361).

Disregarding the compatibility conditions for strains, the above stress-strain relationships were applied to the change in the stress field from the initial pure gravitational stage (with γ = 2 t/m^3 and K_o = 0,58) to the end stage corresponding to an additional axially symmetric loading of the rigid foundation by q = 15 t/m^2

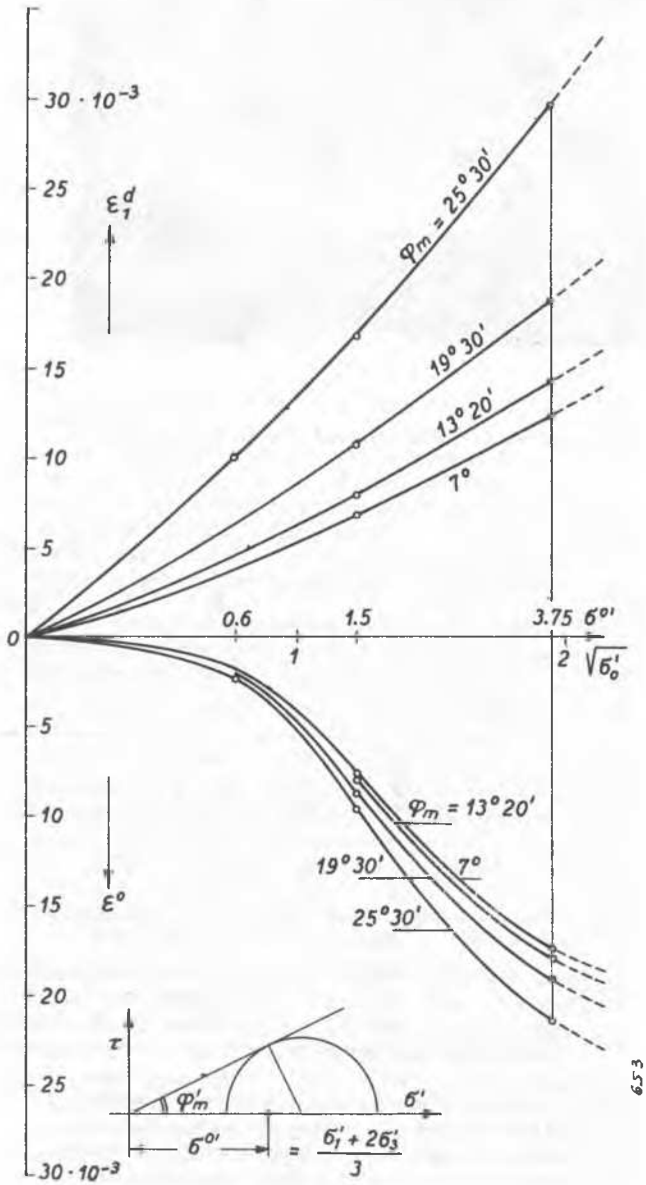


Fig.3. Strain-stress plots of a silty sand as obtained by triaxial testing (according to Šuklje 1969).

on average. Additional stresses have been computed according to formulas resulting for the half space, by using Fröhlich's concentration factor n = 4 in the case of the strip foundation, and by assuming an elastic medium with Poisson's ratio $\nu = 0.50$ in the case of the circular rigid plate. The vertical strains corresponding to the transition from the initial to final stress state was obtained as the difference of the strains obtained for both of them. The approximate values of settlements were found by integrating the diagrams of strain differences.

Governed by the condition of equal settlement in every point of the foundation and combining stresses and strains due to single loading elements of the loaded area, the pressure distribution corresponding to the above data and assumptions was obtained by trial and error. It is presented in Fig 4: thick full line S for the strip foundation and thick dashed line C for the circular foundation. In both cases stress states below the boundaries of the loaded area approach the failure (occurring at $\varphi' = 32^{\circ}$).

These pressure distributions have been compared, in the same figure, with the pressure distributions corresponding to the soil as a pure elastic medium. In the case of the strip foundation, the influencing coefficients a_{ik} expressing the settlement of a point i due to the unit load applied in the area of the element k, have been obtained by using Egorov's solutions for the settlement of the boundary of a uniform strip load applied to the surface of a layer with limitted thickness (Egorov 1958). In the case of the circular foundation, the comparison has been made with the pressure distribution as obtained by Milovič (1971) for the d/h ratio of 1 (the solution for greater d/h ratios is not available).

In spite of the fact that our computations are of informative character only, they prove that the real stress-strain relationships for soils being non-linear and depending on invariants of the stress tensor, cause considerable deviation of contact stresses in the foundation base from values obtained by elastic treatment of the layer.

CONCLUSION

According to our experiments, the theory of limit equilibrium yields too favorable bearing capacity coefficients as increasing with increasing B/h ratio (width of the foundation/thickness of the layer with rigid base) on condition that rough contacts in the foundation and layer bases were assumed. Contact pressures resulting from stress characteristics which correspond to variable friction mobilization along the contacts, approach better the model test experience.

An improved approach to real contact pressures in the foundation base can be got by applying, to appropriate statically correct stress fields, stress-dependent stress-strain relationships as resulting from triaxial testing. It is expected that, in the future, a computer oriented development of new computation methods (e.g. the finite element method) will allow to take into account real stress-strain relationships for soils expressed in a matrix form.

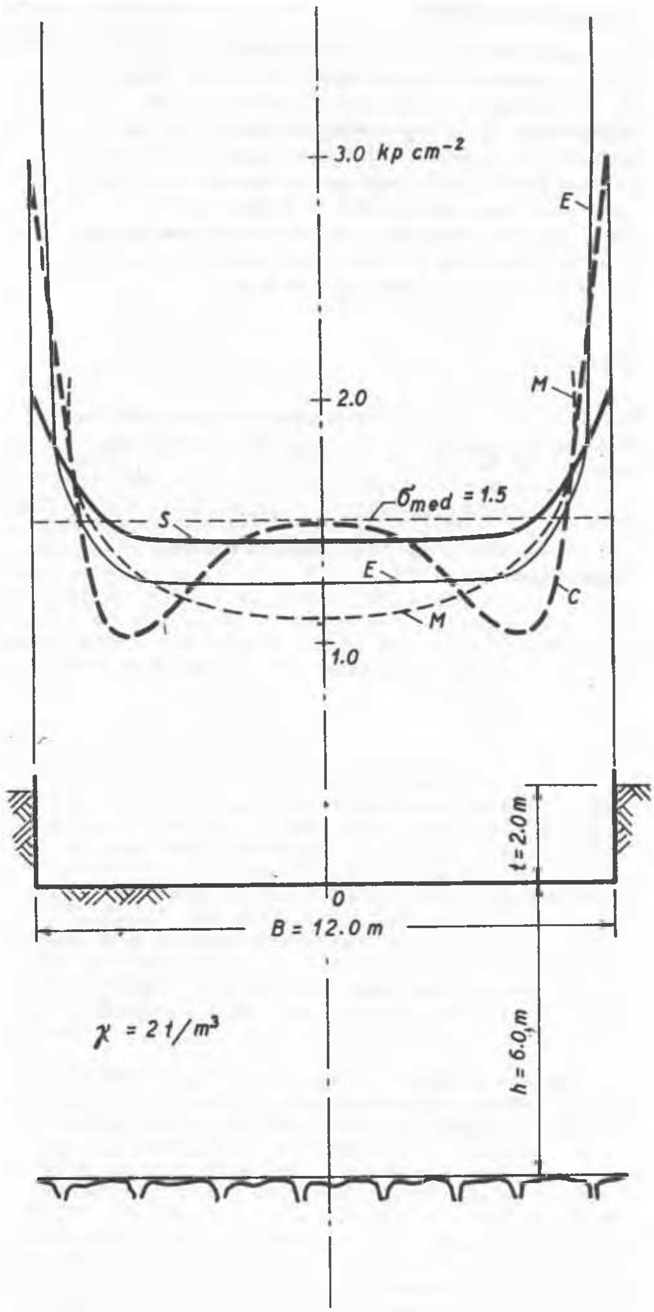


Fig.4. Diagrams of contact pressures in the base of a rigid foundation resting on a layer of limited thickness h : thin lines according to the theory of elasticity, thick lines according to the stress-strain relationships presented in Fig.3 if applied to elastic stress fields: full lines for the strip foundation, dashed lines corresponding to the circular loaded area.

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