3– D. FINITE ELEMENTS FOR FOUNDATIONS IN SOIL

3– D. ELEMENTS FINIS POUR FONDEMENTS EN SOL
МЕТОД КОНЕЧНЫХ ЭЛЕМЕНТОВ ПРИМЕНИТЕЛЬНО К РАСЧЕТУ ФУНДАМЕНТОВ

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SYNOPSIS. A three-dimensional Finite Element technique, originally developed for the solution of rock mechanics problems (Wittke, Rodatz, Wallner 1972) is described. The method based on a simple bilinear stress strain relationship for the subsoil as well as for structural elements enables us to account for a rearrangement of stresses, if the strength, described by the Mohr Coulomb hypothesis, is locally exceeded. The influence of excavations and structures embedded in the subsoil can be considered. Further, a special set up of the Finite Element meshes is briefly described. As an idealized example, the three-dimensional stress and displacement field adjacent to a cable foundation for a suspension bridge is investigated. Further, an excavation adjacent to a tall structure and stabilized by a grouted zone of the subsoil, which is tied back by prestressed anchors, has been examined.

1. INTRODUCTION

Along with the development of modern transportation systems in our large, growing cities, deep excavations adjacent to tall structures, which are not allowed to settle, are to be stabilized by temporary bracing systems. A safe and economic design of these and other foundation engineering structures
653 has to account for complex geometrical and loading boundary conditions. Also, interactions between structures and the very often inhomogeneous and partly even anisotropic soil, with complex stress strain characteristics, have to be considered. Conventional methods very often represent a far too simplified model of the real situation and should therefore at least be partly replaced by modern, more comprehensive design tools such as the Finite Element method, specially developed for electronic computation of problems in continuum mechanics.

2. FUNDAMENTALS

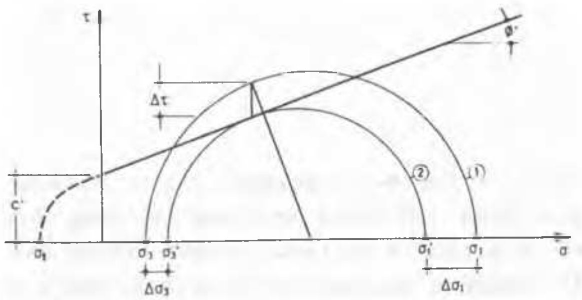
The principle of this method has been outlined elsewhere (Desai 1972, Zienkiewicz, 1971) Therefore the following considerations can be limited to only the most important aspects of this theory.
The calculations begin with a subdivision of the continuum into elements of finite size, usually triangles for two dimensional continua and tetrahedra for three dimensional cases. These elements are assumed to be interconnected only at the corner points. At these nodal points, the internal stresses are assumed to be transmitted from one element to the other by means of substitute nodal forces. Also, volume forces and external loads such as self weight and anchor forces can only be applied to the system by single forces concentrated at the nodes.

Another assumption applied in the standard Finite Element procedure is, that the displacements within one element are linearly dependent on the coordinates. The coefficients of these linear relationships can be expressed by the corresponding nodal coordinates and displacements, the latter ones being the unknowns of the system. By simple partial differentiation the strains, and applying Hooke's law also the stresses, can be determined as functions of the nodal displacements. A relationship between the nodal forces and the stresses and consequently also the nodal displacements can be determined by applying the principle of virtual work to each element. Fulfilling the equilibrium conditions at every node of the system, as well as the boundary conditions, finally results in a system of linear equations for the unknown nodal displacements, from which subsequently the stress field can also be evaluated.

It should be noted, that as a consequence of the above mentioned assumptions, the stresses within each element are constant and that consequently in zones of high stress gradients sufficiently small triangles and tetrahedra respectively have to be selected. Any boundary conditions concerning the geometrical shape, the stresses and displacements as well as different Young's moduli within the subsoil and for the structure can be accounted for by the above described standard Finite Element procedure. The assumption of unlimited linear elasticity however, is an oversimplification with regards to the subsoil and has to be modified by special techniques.

3. ADOPTED STRESS-STRAIN CHARACTERISTICS

The Finite Element formulation applied in this paper for stresses below failure is also based on linear elasticity. Isotropic as well as transversely isotropic elasticity can be accounted for, the letter assumption being specially applicable to layered subsoil conditions, if the compressibility normal and parallel to the layer can be determined by different moduli E_2 and E_1 respectively. In addition, two Poisson's numbers ν_1 and ν_2 and one shear modulus G_2 are required, whereas in the isotropic case only two independent elastic constants, the Young's modulus E and Poisson's number ν, determine the stress-strain characteristics of the subsoil (Lekhnitskii 1972). Time dependency of the stress-strain properties is neglected.



φ' = EFFECTIVE ANGLE OF INTERNAL FRICTION ; c' = EFFECTIVE COHESION INTERCEPT
σ_t = TENSILE STRENGTH
① MOHR's CIRCLE FROM 1st STEP OF CALCULATIONS (ELASTIC)
② REDUCTION OF STRESSES TO RESIDUAL STRENGTH

Fig. 1. Mohr-Coulomb-criterion of failure

As a failure criterion, the Mohr Coulomb hypothesis is introduced. The selected envelope, represented in fig. 1 for the two dimensional case, is for stresses $\sigma \geq 0$ a straight line determined by the effective angle of internal friction ϕ' and the effective cohesion intercept c'. The method for cohesive soils also allows for tensile stresses not exceeding the tensile strength σ_t. If the strength is locally exceeded, then it is assumed that strains of any size may occur without an increase of the corresponding stresses. Consequently, the stress strain law adopted in this paper is of the elastic-plastic or the so called bilinear type (fig. 2e, (Desai 1972, Duncan 1972). 653

4. CHARACTERISTICS OF THE DEVELOPED PROCEDURE

The rearrangement of stresses due to local failures is accomplished by an iterative calculation, which shall be briefly outlined by means of the idealized, uniformly loaded, 9 element slab (fig. 2a, b). Element *e* obeys the stress strain law 1-2-3 and the 8 adjacent elements follow the straight line 1 - 4 in the diagram of fig.2e. Element e will be considered overloaded and the excess strain $\Delta\varepsilon_0$ (fig.2c, e) produced by the overstress $\Delta\sigma_0$ is taken up by an internal stress condition. This is done by transforming the excess stress $\Delta\sigma_0$ for the next interation into so called rearrangement forces:

$$\{F^*\}^e = [B]^{Te} \cdot \{\Delta\sigma_0\}^e \cdot V \qquad (1)$$

with: $\{F^*\}^e$ = vector of rearrangement forces

$[B]^{Te}$ = matrix determined from the coordinates of the nodal points

$\{\Delta\sigma_0\}^e$ = vector of excess stresses

V = volume of element e

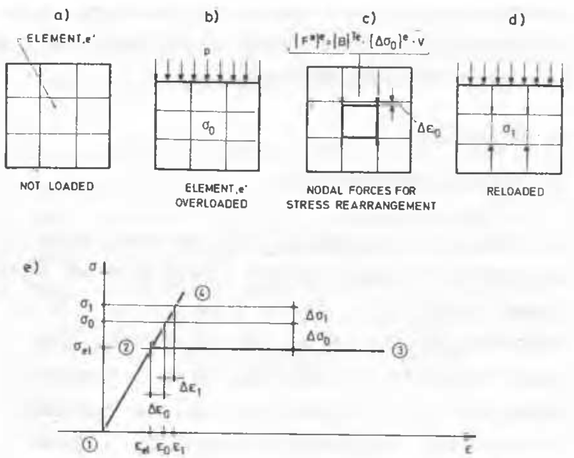


Fig.2. Iterative calculation accounting for rearrangement of stresses due to local failures

A new calculation is then made with the forces $\{F^*\}^e$ acting at the corner points of element e, which consequently will be more severely strained but carries a smaller proportion of the system load p. Therefore the adjacent elements are stressed and also deformed more than in the step before, resulting in an additional overstress $\Delta\sigma_1$ of element e (fig.2d,e). Consequently the same procedure is repeated until the stress increase in element e approaches the value of zero. When evaluating the stresses for element e at the end of the iterative procedure, the internal stress condition ($\Delta\sigma_0$ + $\Delta\sigma_1$+...) is not considered. Since the calculation of the irreversible strains $\Delta\varepsilon_0$+ $\Delta\varepsilon_1$+ ...(fig.2c) is based on linear elasticity, the method does not account for ideally plastic stress-strain characteristics with zero volume change beyond failure.

In a manner similar to the example, every overloaded element is investigated at every iterative step. In case of a shear failure, the fictitious nodal forces are evaluated with the assumption, that the stress component normal to the direction of shear remains constant (fig.1). From this special procedure the curved, dotted part of the failure envelope in fig.1 also results (Wittke, Rodatz, Wallner 1972).

Usually, in a first calculative step starting from the unloaded Finite Element mesh, the existing state of stress or the so-called primary case (σ_p, δ_p) is evaluated.

In the secondary case, which again starts from the unloaded system the structural specifications are additionally accounted for. Elements are removed from the areas to be excavated and eventual external loads are added to the system. The resulting displacements δ_s have to be reduced by the primary displacements δ_p, whereas the secondary stresses can be evaluated directly from the total displacements δ_s. Concrete and other structures, e.g. retaining walls, bracing systems or foundations, which may not exist in the primary case, can be considered by special rows of elements, which before evaluating the secondary case are altered e.g. into concrete. Additionally fictitious nodal forces are applied to these elements which exactly compensate for the primary displacements.

Special problems of soil structure interaction, as for example shown in fig.3 (see also section 5.1) can also be solved by the developed method. It is evident, that no tensile stresses can be transmitted from the backfill to the concrete block (fig.3), when

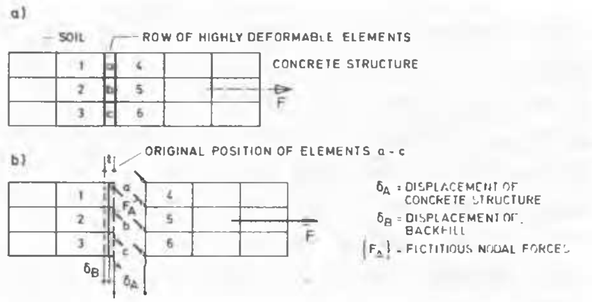


Fig.3. Accounting for special soil-structure interaction

the latter is horizontally displaced due to a load F. Further if the slot between concrete and soil tends to open, the backfill slides and transmits the reduced so called active earth pressure to the foundation.
In order to simulate this behaviour, the interface between backfill and concrete is represented by a thin row of highly deformable elements (a-c, fig.3a).If the force F is applied, the first iterative step of the calculation for the secondary case reveals displacements δ_A and δ_B for the concrete block and the backfill respectively and practically no stresses within element a-c (fig.3b). In order to fulfill the condition that the thickness (t) of the elements a-c has to be constant, fictitious nodal forces (F_A) are applied before the next iterative step is calculated (fig. 3b):

$$\left\{F_A\right\} = \left\{F_A^*\right\} \quad 1-\kappa(\delta_A - \delta_B) \qquad (2)$$

In this equation $\left\{F_A^*\right\}$ are nodal forces equivalent to the shear and horizontal normal stresses applied to the interface by elements 1-3 from the last iterative step and κ is a specially selected convergency factor. If $\delta_A - \delta_B$ 0, then the final result has been achieved. The method allows for shear displacements along the interface.
The set up of a Finite Element mesh usually begins with a subdivision of the continuum into three-dimensional, eight cornered elements (fig.4).These can again be subdivided into tetrahedra in two different ways, which for practical examples usually lead to different stresses at the same point. In this method a special partitioning into two tetrahedra piercing one another has been selected, taking the average stress resulting for the two tetrahedra.The reduced stiffness of the eight cornered element blocks caused by this procedure is compensated by theoretically increasing the volume of the tetrahedra (Wittke,Kodatz,Wallner 1972).

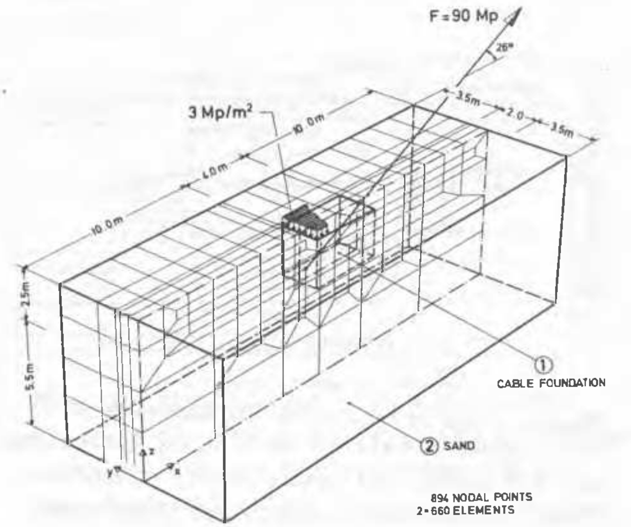
The developed computer program in FORTRAN IV, including all subroutines, requires 100K-bytes and some additional storage dependent on the problem in question. The time and storage required for computation depends on the structure of the matrix of coefficients of the system of linear equations. On an IBM 360/85 a well organized example with approximately 1000 unknowns requires about 40 seconds for the first solution and 2 sec for each iterative step.

5. EXAMPLES 659

5.1 CABLE FOUNDATION

In the first example, the three-dimensional stress and displacement field for an idealized model of a cable foundation for a suspension bridge is investigated. The 2,5m high concrete foundation with rectangular cross section (2,0m x 4,0 m) is embedded in sand and uniformly loaded by 3 Mp/m^2 on the surface (fig.4). The elastic constants (E,), the strength parameters (ϕ',c',σ_t) and the unit weights (γ) of the concrete and the sand are compiled in fig.4. In addition to its own weight, the concrete foundation is loaded by the suspension cable, located in the vertical plane of symmetry and inclined at 26° to the horizontal. The cable force amounts to F=90 Mp.

For the calculation an 8m heigh, 24 m long and 9m wide section of the subsoil was considered. Along the vertical side walls and the lower horizontal boundary of this section the normal displacements are assumed



	E	ν	φ'	c'	γ	σ_t
	[Mp/m²]	[-]	[°]	[Mp/m²]	Mp/m³	Mp/m²
① CONCRETE	2100000	0.18	25	1000	2.4	150
② SAND	4000	0.3	33.5	0	1.8	0

Fig.4. Idealized model of a cable foundation for a suspension bridge

to be zero,whereas along the surface the displacements may have any direction. Because the stresses and displacements will
659 be symmetrical to the x-z-plane of the right hand cartesian coordinate system, the Finite Element mesh was only evaluated for points $y \geq 0$ of the described section (fig.4).

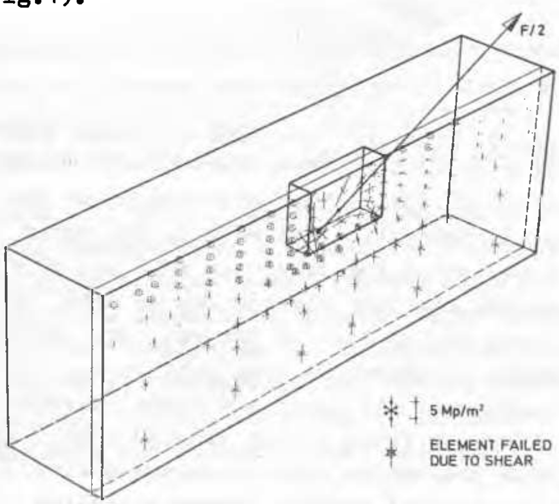


Fig.5.Secondary principal stresses for element layer adjacent to x-z-plane.Three-dimensional perspective plot.

This network consists of 5 vertical slabs of different thickness, each containing 132 eight cornered elements,which as mentioned earlier are again subdivided into two tetrahedra each (fig.4.). Altogether 2 x 660 tetrahedra elements and 894 nodal points result. Based on a handmade quadrangular mesh in the x-z-plane the three-dimensional mesh was semiautomatically evaluated. Within the primary case the stresses and displacements for the vertical loads only were evaluated. Though the method enables a stepwise increase of the suspension force F, in thi example it was applied in one calculational step only.

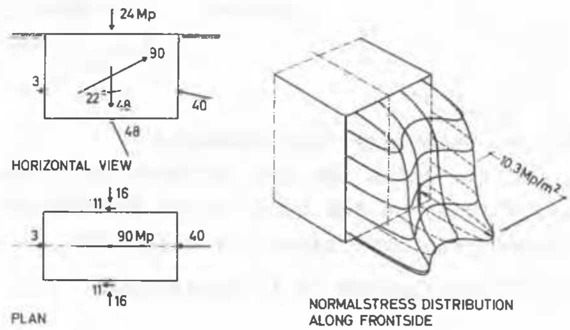


Fig.6. Interactions between soil and cable foundation

Because a representation of the secondary principal stresses for the entire system would be extremely hard to follow up, the results for only the element layer adjacent to the x-z-plane are shown in a three-dimensional perspective plot (fig.5).From the extent of the shear zones in front of the cable foundation (fig.5) as well as from the normal stress distribution along the frontside (fig.6) it can be concluded, that the three-dimensional passive earth pressure has not yet been fully activated. For displacements of 4,7mm (fig.7) this result is in agreement with the investigations performed by Horn 1970. On the other hand,bottom friction and active earth pressure on the backside are fully developed, as can be seen from the corresponding shear zones.

The dislocation of the concrete block

consists of a translation and a superimposed rotation in the x-z-plane. In front and

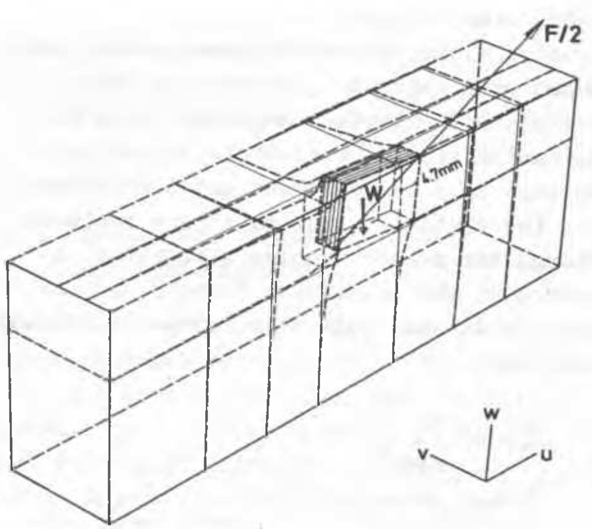


Fig.7. Secondary displacements

along the sides the soil surface is lifted up, whereas in the back of the foundation settlements have taken place (fig.7).

5.2 STABILIZATION OF AN EXCAVATION

Adjacent to an existing structure, which transmits a uniform load of 15 Mp/m^2 and a gable load of 25 Mp/m^2 to the subsoil, a 6,5m deep excavation is planned (fig.8). The elastic constants, the shear strength parameters and the different layers of the subsoil are also described in fig.8. The unit weight of $\gamma = 1.9 - 2{,}0$ Mp/m^3 applies for the silty sand as well as for the practically impermeable clay and clay shale layer. For the gravelly sand the unit weight $\gamma' = 1{,}1$ Mp/m^3 for submerged condition is valid.

For stabilization, a limited zone of the subsoil of trapezoidal shape was grouted by chemicals. The resulting increased YOUNG's modulus and cohesion intercept as well as the other constants are also given in fig.8. To prevent tilting, one row of prestressed anchors was installed along with the excavation. The selected Finite Element mesh, not described within

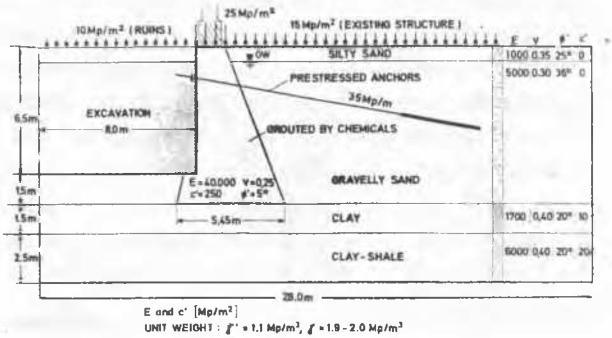


Fig.8. Stabilization of an excavation

this paper, consisted of 2 x 530 tetrahedra and 557 nodes. The nodal points along the lower, horizontal boundary are fixed and for the vertical boundaries the normal displacements are assumed to be zero.

In the calculation of the secondary case also here the excavation and the prestressing effect, represented by equivalent nodal forces along the adhesive stretch and at the anchor heads, were assumed to take place in one step only. Pore pressures in the clay and clay shale were negligible.

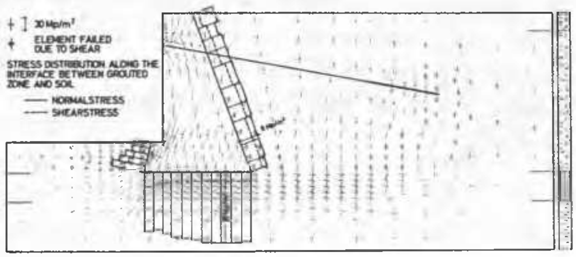


Fig.9. Principal stresses and shear zones

From fig.9 the stress concentrations, which occur at the anchor heads and below the gable foundation, as well as the shear zones below the retaining wall and adjacent to the adhesive stretch can be seen. In front of the foot of the wall the passive earth pressure has been mobilized. A comparison of the shear and normal stresses along the faces of the grouted zone (fig.9) with the active earth pressure derived by conventional methods, shows according to the limited extent of the shear zones developed behind the wall (fig.8) slightly higher values for the Finite Element calculation.

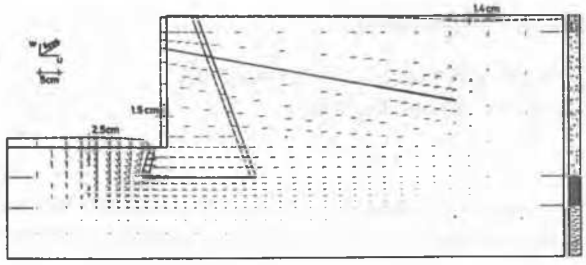


Fig.10. Displacements

From the displacement plot a horizontal movement of the grouted zone and a distinct settlement on the surface above the adhesive stretch of the anchors can be seen (fig.10).Though convergency was reached within the iterative calculations,from the extent of the shear zones (fig.9) it can be concluded that the factor of safety against sliding along a deeply located plane must be low, which could also be verified by standard slip circle analysis.

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