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INFLUENCE OF NEIGHBOURING STRUCTURES ON THE DEEP FOUNDATIONS  
L'INFLUENCE DES CONSTRUCTIONS VOISINES SUR FONDATIONS PROFONDES  
ВЛИЯНИЕ СОСЕДНИХ СООРУЖЕНИЙ НА ГЛУБОКИЕ ФУНДАМЕНТЫ

R. CZARNOTA-BOJARSKI, Prof.

B. RYMSZA, Dr.Sc., Warsaw Technical University (Poland)

**SUMMARY.** The paper discusses hitherto unsolved question pertaining to horizontal pressures caused by the load of neighbouring foundations, transmitted to the structure limiting soil half-space. In analysing methods described in textbooks, the paper points out to great divergencies in approaches and results. Explaining why comparative relations /based on the Boussinesq's solution/ are dependent on the location of the load, the paper also points to a dependence of stresses in soil on physical and mechanical features and the value and type of deformation of the structure. The compilatory ideological schema, resulting from theoretical considerations and model research, approaches the qualitative side of the problem in flat strain.

## 1. INTRODUCTION

In designing structures founded at great depths in over built area, account should be taken of the neighbouring foundations. As the problem of stress distribution in one-sided limited soil half-space so far remains unsolved, various simplified methods are applied in practice. This question is next discussed on the basis of a continuous vertical structure with a linear load  $P$ . Examined are horizontal pressures resulting from load  $P$ , transmitted to the rigid structure, in the form of a unit earth pressure  $\sigma_{xP} = \Delta e_P = \sigma_x$ .

## 2. ANALITICAL METHODS

In analyzing methods of defining horizontal pressures, the paper does not take into account the classical Coulomb method, in which the load  $P$  is included in the weight of the sliding wedge.

### 2.1. Boussinesq's Solution

Lateral pressures (Fig. 1, line 1) are sometimes calculated by applying the formula:

$$\sigma_{xB} = \frac{2P \cos^2 \psi \sin^2 \psi}{\pi z} = \frac{2P x^2 z}{\pi(x^2 + z^2)^2} \quad (1)$$

The Boussinesq's solution does not take account of the properties of soil. Besides, the

basic provisions of the theory discussed are not fulfilled in this case. The paper points out that horizontal soil strain in the surface of the wall  $OM$  (hence stress) are determined by the rigidity of the structure and that radial stresses may occur only in the zone of the cylindrical surface  $MCDE$ .  
2.2. Adjustment of the Boussinesq's Solution  
Tschebotarioff recommends application of the Weiskopf's model (G.P. Tschebotarioff 1951). Assuming that in continuous soil medium at a given point  $M$  of the elastic half-space there would occur lateral displacement  $\Delta(P, E, r, \mu, \psi) > 0$ , the rigidity of the structure counterbalances the impact of a symmetrical-ly located load  $P'$ , with  $\Delta_M = \bar{\Delta}(P) + \bar{\Delta} P' = 0$ . Horizontal pressures represented by the lines 1, 1' are calculated on the basis of the formula:

$$\sigma_{xW} = |\sigma_x| + |\sigma'_x| = 2\sigma_{xB} = \frac{4P x^2 z}{\pi(x^2 + z^2)^2} \quad (2)$$

Though much more reasonable in comparison with Eq. (1), this approach does not constitute a proper solution. Regardless of a failure to meet the boundary conditions (see §2.1), it should be stressed that in

face of the impossibility of stress distribution on the whole cylindrical surface ACE (non-existence of the AM zone), radial stress concentration will occur on the MCE surface, mainly in the MC zone, diminishing in the direction of MDE. It should therefore be assumed that initial stresses are  $\sigma_{x1} > \sigma_{xB}$ , while their proportional differences  $(\sigma_{x1} - \sigma_{xB}) : \sigma_{xB}$ , considered on the surface of the wall, would increase together with a decrease of the angle  $\psi$ .

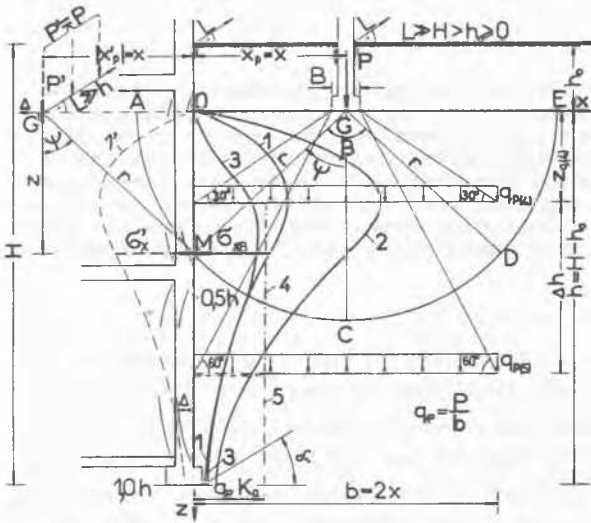


FIG.1. DISTRIBUTION OF HORIZONTAL PRESSURES ACCORDING TO THE FORMULAS OF :  
 1- Boussinesq, 2- Terzaghi, 3- Fröhlich ;  
 4,5- calculation schemata

The obvious fact that transitions of stress increase while the distance  $x_p = x$  of the load P from the structure diminishes is confirmed by a comparative analysis of the dependences (1), (3). The formula (3) defines horizontal pressures when load P acts on the elastic symmetrical wedge GMCD with an apex angle  $\beta$  and  $0 \leq \psi \leq \frac{\beta}{2}$

$$\sigma_{x2} = \frac{2 P \cos^2 \psi \sin^2 \psi}{z(\beta + \sin \beta)} \quad (3)$$

Assuming that stresses occur in the zone of cylindrical surface MD, with  $\widehat{MCD} < \widehat{ACE}$ , we receive  $\sigma_{x2} > \sigma_{xB}$  (when  $\beta \rightarrow \pi$ ,  $\sigma_{x2} \rightarrow \sigma_{xB}$ ). Hence the assumption that a close theoretical relationship, account being taken of

the asymmetric wedge with an angle  $0,5(\beta + \pi)$ , would lead to defining intermediate pressures  $\sigma_{xB} < \sigma_{x1} < \sigma_{x2}$ . The stresses  $\sigma_{x1}$ , defined with taking into account natural lateral deformability of soil medium, would be initial stresses in calculating lateral pressure  $\sigma_{xW1} = 2\sigma_{x1} > \sigma_{xW}$  on the rigid structure  $\Delta_{(x=0)} = 0$ , limiting soil half-space.

2.3. Effect of Linear Load according to K.Terzaghi

On the basis of research by E.Gerber, Terzaghi (K.Terzaghi 1953) presents the following formulas concerning lateral pressures (Fig.1, line 2):

$$\sigma_{xT} = \frac{4 P n m^2}{\pi h (n^2 + m^2)^2}, \text{ for } m > 0,4 \quad (4a)$$

$$\sigma_{xT} = \frac{0,203 P n}{h (0,16 + n^2)^2}, \text{ for } m \leq 0,4 \quad (4b)$$

where:  $n = \frac{z}{h}$  - index of depth,  
 $m = \frac{x}{h}$  - relative distance of the load P from the structure.

The formula (4a) is equivalent to Eq.(2). According to the formula (4b), it should be assumed that with  $x \leq 0,4h$  the value and distribution of pressures  $\sigma_x$  depend to a low degree on the location of the load which is contrary to the findings of other authors. For example, there is cited research by Dubrova, which proves that changes in the value and unit earth pressure distribution increase when the force  $P = \text{const}$  moves closer to the wall (G.A.Dubrova 1963).

2.4. Fröhlich's Formula

In designing underground railway in Munich, with  $\alpha < \frac{1}{2}(45^\circ + \frac{3}{2}\phi)$ , Fröhlich's formula was applied (H.Joas, J.Weber, E.Heine 1971):

$$\sigma_{xF} = \frac{3 P x^2 z^2}{4(x^2 + z^2)^{2,5}} \quad (5)$$

Pressure distribution is presented in Fig.1 by line 3 which practically is the same as the curve 1 at the depth  $z \geq 0,5h$ . In the upper part  $\sigma_{xF} < \sigma_{xB}$ , which cannot be justified in the light of remarks made in §§ 2.1, 2.2. It is stressed that the results of research by various authors analyzed in § 2.3

\*) Provided that  $\phi = 30^\circ + 40^\circ$ , the limiting condition of the application of Eq.(5) is  $x \leq (0,8 + 1,0)h$ .

unanimously shows that with  $x < (0,7+0,8)h$  real resultant earth pressure  $\Delta E_{xP} = \int_0^h \sigma_x dz = E_x > E_{xB} > E_{xP}$  considerably exceeds calculation values.

### 2.5. Replacement Schemata

Pressures (Fig.1, line 4) resulting from the calculation schema Hütte 1970 are defined by the following equation:

$$\sigma_{xB} = q_P \tan^2(45^\circ - \frac{\phi}{2}) = \frac{P}{2x} K_a \quad (6)$$

This schema is characterized by too great freedom of choice (ungrounded reference to Coulomb's theory of wedges). In the case of a rigid structure, rather  $\sigma_{x0} = q_P K_0$  should be assumed, with account taken of the coefficient of earth pressure at rest  $K_0 > K_a$ .

Regardless of considerable divergencies in values and pressures distribution, that is in the value and location of resultant pressure, the reservation arises that with  $x \leq 0,6h$  the result is  $E_x = q_P K \Delta h \approx 0,6 P K_{a,0} = \text{const.}$  Taking into account the principle of superposition, this schema does not meet boundary conditions. Generalizing the calculation schema applied in determining the influence of railway loading (Civil Engineering Handbook, 1940), a constant value of pressures  $\sigma_{xB}$  defined by Eq. (6) should be assumed, below the level of acting of replacement load  $q_P$  (5) (Fig.1, line 5). In this method are no limitations as to the value  $B_{\min}$  and  $\frac{B}{h}$ . With  $B \ll h$ , this method leads to erroneous results. The schema presented does not meet the provisions and boundary conditions of the Coulomb theory, to which it makes a reference. With  $x \rightarrow 0$ , the zone of load  $q$  cause such earth pressure as one resulting from surcharge of the whole area of sliding wedge. Thus this method leads to theoretical contradictions. It is presented in order to illustrate divergencies in approaches to the problem being discussed.

### 3. COMPARATIVE ANALYSIS

If an evaluation of the methods presented is based on the degree of correlation of calculation ( $\sigma_{xi}, E_{xi}$ ) and real ( $\sigma_x, E_x$ ) values, then such an evaluation is difficult since the scope of experimental research in this

domain is narrow. It is impossible to define general comparative relations because parameters determining properties of soil does not appear in dependences based on Boussinesq's solution (1)+(5), while Eq. (6) introduces dependence of pressures  $\sigma_{xB}$  on the angle  $\phi$ . It is emphasized that quantitative divergencies in pressure distribution  $\sigma_{xi}$  represented in Fig.1 by lines 1-5, which in their extremes exceed 300-400%, are variable because they depend on the distance  $x$  of a load  $P$  from structures. As the limited size of the paper make it impossible to present a fuller analysis of this problem, only certain general regularities are discussed in the further part, with reference to the results of research conducted by Dubrova (G.A. Dubrova 1963) and Spangler (G.P. Tschebotarioff 1951), and to the formula (1). It should be assumed that for small distances  $0 < x \leq 0,3h$  pressure distribution is similar to the distribution defined by Eq. (1), with real values  $\sigma_x$  being 2,5+3 times bigger than calculation values. Taking Weiskopf's model, the additional difference (0,5+1,0)  $\sigma_{xB}$  can be explained by the fact of assuming an incorrect comparative value  $\sigma_{xB} < \sigma_{x1}$  (see § 2.2). In principle bigger divergencies, 4-5 times, should be expected. That they are smaller is explained by the fact that bigger values  $\sigma_{xB} > \sigma_x$  are obtained from the Boussinesq's formula with continuous soil half-space. With  $x \approx 0,5h$ , we obtain  $\xi = \sigma_x / \sigma_{xB} \approx 2$ ; as the distance  $x$  increases, this ratio diminishes. Analysing the results of research by Spangler. Tschebotarioff points out that with  $x \approx 0,8h$  experimentally determined pressure distribution is practically the same as the distribution defined by Eq. (1), ( $\xi \approx 1$ ). The results of research by Dubrova also indicate that with the increase of the distance  $x$  horizontal pressures diminish much faster than it would result from formulas based on Boussinesq's theory. Moreover, experimentally defined distribution of pressures for  $x > 0,75h$  essentially differs from the theoretical distribution resulting from Eq. (1), with  $E_x < E_{xB}$ .

Providing for rigidity of the structure or  $\epsilon = \text{const}$ , it should be assumed that the divergency in comparative relations  $\sigma_x / \sigma_{xB} = \xi(x, z)$  and  $E_x / E_{xB} = \zeta(x)$  results from the fact that Boussinesq's theory incorrectly defines the stress condition in soil medium. This can be proved by comparing the diagrams  $\zeta = f(x)$  and  $E_{xB} = F(x)$  (Fig.2a), where:

$$E_{xB} = \int_0^h \frac{2 P x^2 z dz}{\pi(x^2 + z^2)^2} = \frac{P h^2}{\pi(x^2 + h^2)} \quad (7)$$

The diagram  $\zeta = f(x)$  illustrates the variability of the coefficient of correlation in connection with the value of resultant pressure (divergencies concerning a location of the force  $E_x$  are not taken into account). The line G which defines the variability of the relation  $E_{xG} / E_{xB}$ , determined on the basis of the results of research by Gerber, differs considerably from the curve  $\zeta(x)$  which refers to research by Spangler and Dubrova. It is emphasized that although the formulas (4a, b) constitute approximation of the results of Gerber's tests, they correspond to the value  $\zeta_T \approx 2$ . Analysing conditions of research Terzaghi points out that the values of earth pressure obtained by Gerber are smaller than the real ones. Hence  $E_{xT} > E_{xG}$ . Quantitative

divergencies may also result from different soil conditions; therefor Fig.2a presents a family of curves  $\zeta_1(x)$ , pointing to a dependence of stress on porosity  $n$ , the angle  $\phi$ , etc. The generalized curve  $\zeta(x)$  illustrates the qualitative aspect of the problem. Defining approximatively values we assume that in model research there occurs friction between ground and side walls, which diminishes to a certain degree the pressure on the retaining wall.

4. INFLUENCE OF DEFORMATION OF THE STRUCTURE

It was assumed in the hitherto considerations that pressures are transmitted on the rigid structure ( $\Delta = 0$ ). It is obvious that if there are elastic deformations (or displacements in the case of monolithic structure) - the value and distribution of  $\sigma_x$  will change. A number of authors points to the interdependence between earth pressure and deformations of the structure. Although the resultant pressure  $E_x$  caused by the load  $P$  diminishes with the increase of the displacement  $\Delta$ , there is no correlation between the dependences  $E_x / E_{xB} = \zeta(x, \Delta)$  and  $\sigma_x z_1 / \sigma_{xB}(z_1) = \xi(x, \Delta, z)$  since pressure distribution depends on value and also on the cinematical schema of displacements (G.A. Dubrova 1963).

It results from Boussinesq's theory that in soil half-space natural lateral strain are a diminishing function of the distance  $r$  (with  $z = \text{const}$  - the distance  $x = x_p$ ; see Fig.1). As regards considerations presented in §2.1, it can be assumed that the relative influence of deformation of the structure  $\Delta(\sigma_x, z) = \Delta$  on the pressures  $\sigma_x(P, \Delta, x, z) = \sigma_x$  and  $E_x(P, \Delta, x) = E_x$  is the greater, the greater is the load  $P$  (Fig.2b).

Assuming as a comparative value  $\max E_x$  (earth pressure from the load  $P$  as close as possible to the rigid wall\*), it is probable that the course of the line  $k_1 = f(x, \Delta_1)$

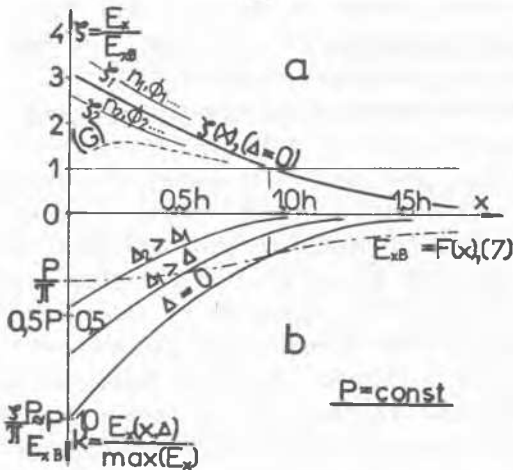


FIG.2. HYPOTHETICAL DIAGRAMS OF:  
 a - Coefficient of correlation  $\zeta = f(x)$  (dash line - relating to Gerber's tests; dash-dot line - Eq.7),  
 b - Influence of displacements of the structure on the  $k$  value

\*) The course of the line G (Fig.2a) indicates  $\max E_x$  with  $x=0, 2h$ , which is in contradiction to Dubrova's tests and Eq.(7). Although Fig.2 assumes  $\max E_x(\Delta = 0)$  with  $x \rightarrow 0$  because then  $\max \sigma_x$  and  $\max \sigma_x = K \sigma_z$ , this problem requires research.

actually depends not only on the size and the type of deformation of the structure but also on soil properties.

Paying attention to this aspect of the problem allows to see difficulties. These difficulties are further complicated in the case of a concentrated load, in the conditions of spacial distribution of pressures.

## 5. CONCLUSIONS

1. The problem of pressures on a structure limiting soil medium caused by the load of neighbouring foundations so far remains theoretically unsolved. Replacement calculation schemata and empirical dependences are differently presented by various authors.

2. Comparative relations in connection with Boussinesq's theory may be divergent because of the dependence of stress on physical and mechanical soil features. There is the need for broad research under various soil conditions. Elastooptical research would also be helpful as it would allow for corrections of the provisions concerning stress distribution in a one-sided limited soil medium. Complex research could provide a basis for defining the general dependence in which there would be included parameters characterising soil properties.

3. Apart from rheological strain, the rigidity of a structure and the static schema of deformation exert considerable influence on the value and pressure distribution on the structure. \* \* \*

This paper, intended by authors to be polemical, does not fully present the problem, pointing to important and hitherto unsolved problems relating to deep foundations in over built area.

## REFERENCES

- (1940), "Civil Engineering Handbook", Mc Graw-Hill Book Company, Inc. New York and London, 1940, pp. 605-606.
- DUBROVA, G. A., (1963), "Vzaymodeystvie Grunta i Sooruzeniya" (in Russian), Hecnoy Transport, Moskva 1963, pp. 24-34.
- (1970), "Hütte Taschenbuch der Bautechnik", Verlag von Wilhelm Ernst, Berlin-München-Düsseldorf 1970, vol. II, p. 214.
- JOAS, H., WEBER, J., HEINE, E. (1971), "Die Sicherung von Gebäuden und sonstigen Anlagen beim Münchner U-Bahn-Bau", Die Bautechnik n 2/1971, p. 37.
- TERZAGHI, K. (1953), "Anchored Bulkheads", Proceedings ASCE, vol. 79, September 1953, Sep. 262, pp. 7-11.
- TSCHEBOTARIOFF, G. P. (1951), "Soil Mechanics, Foundations, and Barth Structures", Mc Graw-Hill Book Company, Inc. New York-Toronto-London 1952, pp. 288-291.