THE RANKINE PRESSURE THEORY IN THE LIGHT OF KINEMATICS OF GRANULAR MEDIUM

THEORIE DE POUSSEE DE RANKINE VUE DU POINT DE LA CINEMATIQUE DU MILIEU SANS COHESION

ТЕОРИЯ ДАВЛЕНИЯ РЭНКИНА С ТОЧКИ ЗРЕНИЯ КИНЕМАТИКИ СЫПУЧЕЙ СРЕДЫ

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SYNOPSIS

The paper is aiming at answering the question whether there exists a possibility of constructing a kinematic solution corresponding exactly to that of Static by Rankine, at an assumption

a) negleible volume changes,

b) of motion of the retaining wall.

After the general formal aspect of the problem have been discussed, a detail analysis of velocity field obtained on model researches was carried out. On a basis of these a velocity field was constructed in accordance with the earlier theoretical ascertainments. Then, using an assumption of the coaxiality of principal directions of stress and the rate of strain, a distribution of horizontal pressure on the retaining wall was found. A diagram of the pressure obtained in this way is similar to the resultes of model researches carried out on a large scale but different from the Rankine anticipations.

INTRODUCTION

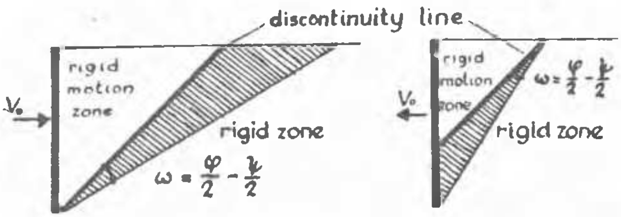
The Rankine pressure theory is one of the classical examples of basic static solutions (in stress) of the plasticity theory applied to the soil mechanics.

Two cases are discussed in the theory: passive earth pressure and active earth pressure; both at an assumption of constant direction of principal stress within the whole region of the solution. The solutions are based on a tacid assumption that there exists a scheme of motion composed of a single slide coextensive with the boundary line of the region of plastic stress.

The tasks could be put alternatively in form of a kinematic solution (see E.H. Davis 1963). The work of gravity forces and the work done along the slide line are calculated and noth are compared to the work of the reaction force on the retaining wall on a given displacement of the wall. When constructing that kind of solution a flow rule should be assumed. For an obvious reason of phisical character (see T. Jeske 1968) the noncoaxial flow is rejected of the coaxial rules the most convincing seem to be the rules of non-associated type (see E.H. Davis 1968) of zero or minimum dilatancy. The associated rule can give a kinematic estimate that differs from static solutions.

The non-associated rule of zero dilatancy gives an estimate which is in accordance with that of static, but in case of an active pressure the scheme of kinematic solution that is in accordance with static Rankine solution does not seem to be kinematic real;

this is shown in Fig. 1. The rigid zone as shown on fig. 1b can not take place in reality. Hence in case of the (problem) on an active earth pressure, even at an admiting of Rankine assumptions, the problem of complete, admissible kinematic as well as static solution still stands open. In the paper there has been suggested a solution of the problem based on an analysis of real velocity field obtained by way of experiment.



a) Passive pressure case b) Active pressure case

Fig. 1. Velocity fields based on Rankine solution

The analysis is preceded by a discussion of some general properties of the velocity field and the sliding line - regarded as the lines of strong discontinuity in the velocity field. The final part of the paper is devoted to the construction of the solution in stresses. Kinematical, geometrical and dynamical relations of the velocity field and their consequences.

The problem is being examined in the plane flow condition. The only thing possible to define in the experiment is the displacement of the particles, that is elements of their path. In order to construct a theoretical solution there ought to be the characteristics of the velocity field to disposal and an envelope curve of those - that is discontinuity line. So, to use fully and interpret correctly experimental data it is necessary to remember some general properties concerning these concepts and the relation between them. It is assumed, that the motion is of steady character. It was admitted, morover, that the displacement will be so small in comparison with initial positions of the particles, that the problem can be considered in its initial geometry.

On account of quasi-stationary character of the flow the paths of particles and the stream lines are given by the equations

$$\frac{v_x}{dx} = \frac{v_z}{dz} \qquad (1)$$

The equations of the velocity field of an assumption of non-associated flow rule are of the form:

$$\frac{\partial v_x}{\partial x} + \operatorname{ctg} 2\theta \frac{\partial v_x}{\partial z} + \operatorname{ctg} 2\theta \frac{\partial v_z}{\partial x} - \frac{\partial v_z}{\partial z} = 0$$
$$\frac{\partial v_x}{\partial x} + \sin\psi \operatorname{cosec} 2\theta \frac{\partial v_x}{\partial z} + \qquad (2)$$
$$+ \sin\psi \operatorname{cosec} 2\theta \frac{\partial v_z}{\partial z} = 0$$

In these equations by θ is denoted the angle between σ_1 and the axis z, and the angle ψ denotes dylatancy measurement. It is assumed that the positive direction of the z axis is oriented down. The equations (2) are hyperbolic and their characteristics are given by the dependencies:

$$\frac{dx}{dz} = \operatorname{tg}\left[\theta \pm \left(\frac{\pi}{4} - \frac{1}{2}\psi\right)\right] \qquad (3)$$

There occur relations along the characteristics

$$\cos\psi\, du^t + (v^t - u^t \sin\psi)\, d\theta = 0$$
$$\cos\psi\, dv^t - (u^t - v^t \sin\psi)\, d\theta = 0 \qquad (4)$$

where u^t denotes the tangential component of the projection of velocity vector on the first characteristical direction and v^t a corresponding projection on the second characteristical direction.

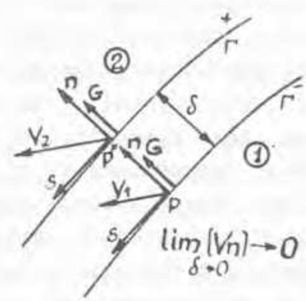
In the light of these relations (1) and (4) it is possible to notice that the stream lines (paths) are coextensive with one of the directions of the characteristics only when $\psi = 0$ and the characteristics are rectilineal or are composed of straight lines and circumscriptions families. In the remaining cases the paths of particles and the directions of the characteristics are not coextensive. The velocity field in which the equations (2) are valid is separated from the rigid zone, and in reality from the region containing elastic-plastic deformations by the discontinuity line. or otherwise by the so called slide line. The line is subject to some dynamic limitations (cf. T.Y. Thomas 1961). The limitations are resulted from the equations of the conservation of momentum and the equation of conservation of mass. According to the denotations in Fig. 2 the equations may be written as follows.

$$[\sigma_{nn}] = \rho (v_{1n} - G)[v_n]$$
$$[\sigma_{sn}] = \rho (v_{1n} - G)[v_s] \qquad (5)$$
$$\rho_1 (v_{1n} - G) = \rho_2 (v_{2n} - G)$$

In the equations (5) the brackets mean a jump of quantity at the passing the discontinuity line. From these equations result some important consequences; as the stress field ought to be continuous, so the left hand sides $(5)_{1,2}$ are equal to zero. On the discontinuity line the quantities $[v_s]$ or $[v_s]$ and $[v_n]$ are not equal to zero, so the quantities in the parathensis ought to be equal to zero. These quantities will be equal to zero when G velocity of propagation of discontinuity line and v_{1n} velocity of particles in the region outside the flow zone are equal to zero. In the analyzed conditions a forced shift of the discontinuity line takes place at the edge of the retaining wall. As a result of this the slope of the discontinuity line to the horizontal line have to increase in comparison to the slope of the stream line. So it is seen that the discontinuity line aught to be convex in the direction to the region (2) Fig. 2.

Independently from the above mentioned conditions the region (1) Fig. 2 in its lower part cannot be rigid in reality. In the analysed boundary conditions only a part of discontinuity line adjacent to the free surface can be motionless and separate the entirely rigid zone from the zone of motion. This element of discontinuity line ought to have a straight-line shape if the stream lines are straight-line. An experimental picture of velocity field.

In order to get a picture of velocity field some model investigations were performed. As model medium some uniform, middle-sized sand of an initial density corresponding to the bulk density $\gamma = 1.55$ G/cm^3 was used. An internal friction angle corresponding to the critical void ratio φ_{cv} was 33°. The experiment was carried out with the displacement rate of the retaining wall Ca 0.25 cm/sec. Pallerns of the velocity field were obtained by comparing displacements as measured with the photogrammetry method at a known time scale. The method used there was that suggested by R. Rutterfield and others (1970).



v_2 and v_1 denote the particle velocities, G denotes velocity of discontinuity line

Fig. 2. Geometrical relations on the discontinuity line

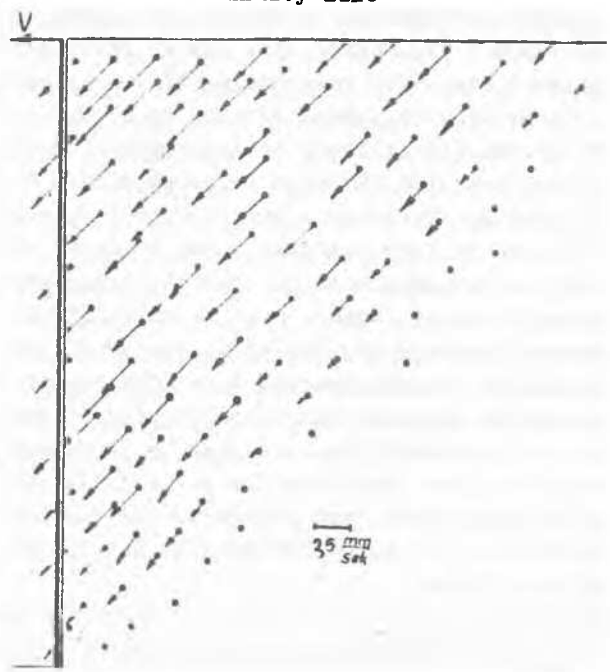


Fig. 3. Experimental velocity field

During an analysis of the obtained results there was taken into account, besides the theoretical premises mentioned above, a approach suggested by W.G. Pariseau (1961). On the basis of the stream line a shape of one of the family characteristics was estimated. As in the experiment some slight changes in the volume were obtained after the phenomenon of the flow occured. There was assumed, when the results were being analysed, that the angle $\psi = 0$.

The achieved in the experiment picture of the velocity field is shown in Fig. 3. In a considerable part of the zone the paths of the particles (stream line) are stright-line and inclined to the horizontal line ca 45°.

Hawing in mind some dynamic dependencies fulfilled along the discontinuity line the shape of it was reproduced; at first it is straight-line and then convect upwards; in its bottom part it is clearly devided into two zones of different velocity displacements in the direction of its tangent. No distinct discontinuities in the velocity field near the point of contact with the retaining wall were stated. This does not contradict the previous statements, as the velocity jump of the tangent to this line has to fade near the wall edge. The determining of the shape of discontinuity line and the knowledge of the stream line allowed to construct a theoretical velocity field; it is shown in Fig. 4. The plastic velocity field got in this way consists of four distinct zones bordered by free surface, wall edge and the discontinuity line. The three zones are in mathematical sense a boundary problem of Cauchy type, one problem of Riemann-Goursat type. The velocity vectors shown in Fig. 4, calculated on the base of characteristic net and a boundary condition, are consistent with the field got in the experiment what indicates the correct admittance of the characteristic net of the velocity field.

Plastic Stress Field

A position of principal directions of a tensor of deformation rate in the flow zone was defined on the ground of earlier established characteristics of the velocity field. They

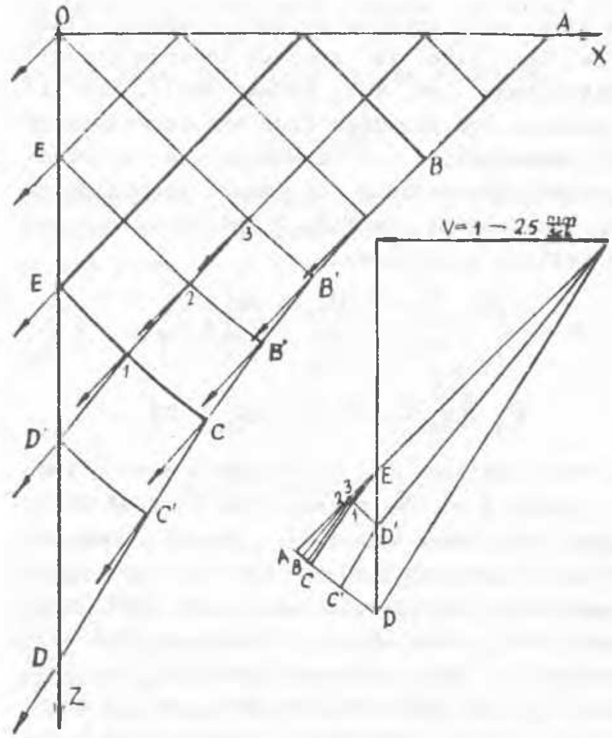


Fig. 4. Theoretical velocity

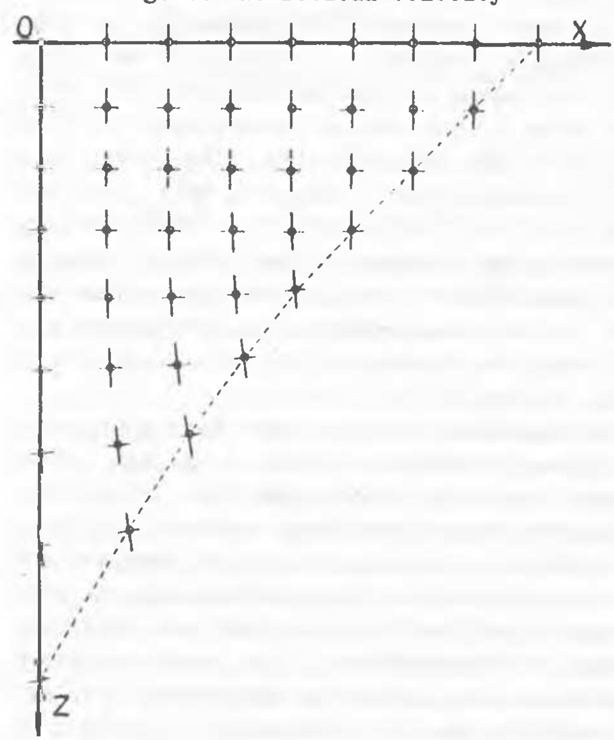


Fig. 5. Principal strain directions

are shown in Fig. 5. Knowledge of principal directions of the deformation rate and the assumption of coaxiality of stresses and the deformation rate made it possible to get the characteristics of the plastic stress field; all of them are given by the equasions

$$\frac{dx}{dz} = tg\left[\Theta \pm \left(\frac{\pi}{4} - \frac{\varphi}{2}\right)\right] \qquad (6)$$

The following differential relations are fulfilled along the characteristics

$$\begin{aligned} &\cos\varphi \, dp + 2 (p \sin\varphi) \, d\Theta = \\ &= -\gamma \sin\left(\Theta - \frac{\pi}{4} + \frac{\varphi}{2}\right) \sec\left(\Theta + \frac{\pi}{4} - \frac{\varphi}{2}\right) dz \\ &\cos\varphi \, dp - 2 (p \sin\varphi) \, d\Theta = \\ &= \gamma \sin\left(\Theta + \frac{\pi}{4} - \frac{\varphi}{2}\right) \sec\left(\Theta - \frac{\pi}{4} + \frac{\varphi}{2}\right) dz \end{aligned} \qquad (7)$$

in which $p = \frac{\sigma_1 + \sigma_3}{2} = \frac{\sigma_x + \sigma_z}{2}$.
Stresses along the retaining wall were defined using the relations (7).

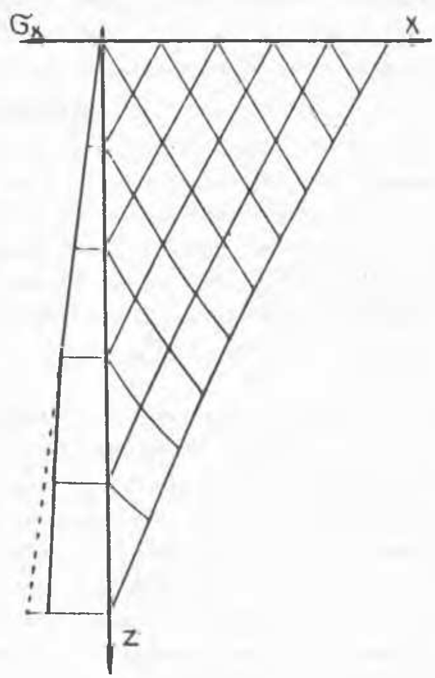


Fig. 6. A part of stress characteristics field and distribution of horizontal pressure

A net of stress characteristics and a diagram of horizontal pressure σ_x are shown on Fig. 6. The diagram is in accordance with the data quoted in bibliography (cf. K. Terzaghi 1943), the data were got from large scale model researches; this confirms the the rightness of the adopted method of solving the problem. It proved to be, independently, that not only elastic solution (see L. Finn 1963) but also a consequent use of plasticity theory gives a curvilineal diagram of pressure on the displacing retaining wall.

CONCLUSIONS

1. In the light of the performed analysis of solution by Rankine for a problem on an active pressure is a static solution and is not real from the kinematical point of view.
2. The presented in the paper construction of the stress field based on experimental picture of the velocity field gives results which are in accordance with some earlier experimental measurements in the range of pressures existing beyond the retaining wall.
3. Because a correct from the kinematic point of view solution provides pressures smaller from the anticipated by Rankine theory, the last gives a safe estimate.
4. Used in the paper method of finding stress field suggested by Pariseau, and founded on the observation and analysis of velocity field, can be affective in comparison with a wholy theoretical solution founded on rules of conservation and the Prandtl-Reuss equations anywhere where a possibility of distinct deformations are admitted. Then there exists certainty of appearing a zone in which stress field as well as field of deformation rate are of hyperbolic character.

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