BEARING CAPACITY OF DEEP FOOTINGS IN SENSITIVE CLAYS

CAPACITE PORTANTE DES FONDATIONS PROFONDES DANS LES ARGILES SENSIBLES
НЕСУЩАЯ СПОСОБНОСТЬ ГЛУБОКИХ ОПОР В ЧУВСТВИТЕЛЬНЫХ ГЛИНАХ

B. LADANYI, Professor, Ecole Polytechnique, Montreal (Canada)

SYNOPSIS. It has been known for some time that the value of the bearing capacity factor N_c in the Terzaghi's formula for the evaluation of undrained bearing capacity of clays may be smaller than its original Prandtl's value if the clay is either very deformable or very sensitive. In the former case, a reduction by one third of the N_c factor, proposed by Terzaghi, is commonly used. For sensitive clays, in turn, a formula for calculating the reduced N_c factor for deep circular footings was proposed by the author in an earlier paper. A similar type of solution has been used in this paper for finding the corresponding expressions for the N_c factor, valid for rectangular footings of any aspect ratio and at any depth. The reduced values of N_c factor are intended to be used in connection with reasonably undisturbed values of the peak undrained strength of sensitive clays, obtained by modern sampling and testing methods.

INTRODUCTION

The short-term ultimate bearing capacity of saturated clays beneath shallow footings is commonly estimated by using the total stress concept and $\phi_u = 0$ analysis, for which the original Terzaghi's (1943) bearing capacity formula reduces to

$$q_{ult} = \gamma D + c_u N_c \qquad (1)$$

where q_{ult} is the ultimate bearing pressure, γ is the average total unit weight of overlying soil, D is the depth of footing, c_u and ϕ_u are undrained shear strength parameters of the clay within the failure zone, and N_c is the bearing capacity factor. For the latter, the Prandtl's value $N_c = \pi + 2 = 5.14$ (Terzaghi, 1943), valid for a vertically and centrally loaded infinite strip footing, resting on the surface of a rigid - ideally plastic frictionless material, is commonly used. For estimating the bearing capacity of rectangular and circular footings located below the surface of the bearing layer, an approximate method, proposed by Skempton (1951), Brinch Hansen (1961) and Meyerhof (1963), consists in multiplying the basic value of the N_c factor by convenient shape and depth factors. In Brinch Hansen's notation, Eq.(1) becomes then

$$q_{ult} = \gamma D + c_u N_c s_c d_c \qquad (2)$$

where N_c is defined as before, while s_c and d_c are the shape and the depth factor, respectively, the values of which will be discussed later.

It is commonly considered (Terzaghi, 1943) that the Prandtl's value of N_c factor, which is based on a rigid - plastic assumption of material behavior, is well suited for estimating the bearing capacity of stiff clays of low sensitivity. On the other hand, for either weak clays, which fail at large strains, or sensitive clays, which show a strain-softening post-peak behavior, it is considered that the value of the N_c factor should be generally smaller than 5.14 given by Prandtl.

For weak clays, Terzaghi (1943) has proposed to reduce by one-third the value of c_u in Eq.(1). This is, obviously, the same as reducing the N_c factor by the same amount and using peak c_u value in Eq. (1). For sensitive clays, a reduction in N_c of the same order was anticipated by Osler and Peck (1963) and Brown and Paterson (1964). A theoretical value of the reduced N_c factor for such clays was obtained by the author (Ladanyi, 1967a) for the case of a deep circular footing, by using an approximate analysis based on the mathematical model of an expanding spherical cavity.

The purpose of this paper is to extend the same type of analysis to cover also rectangular footings of any aspect ratio and at any depth. It is obvious that, for any particular case and type of clay, a complete numerical solution of such a problem can be obtained by a convenient numerical method. Recently, the finite element analysis was used by Höeg (1972) to obtain complete solutions for two cases of circular foundations located at and below the free surface of a saturated clay with strain-softening characteristics. In the two cases a reduction in bearing capacity of approximately 40%, with respect to the perfectly plastic soil, was found. The reduction is of the same order as that calculated by the author for a sensitive clay (Ladanyi, 1967a).

It is interesting to note that, in spite of theoretical predictions of that kind, until very recently, most studies presented in the literature in which predicted and measured undrained bearing capacities of sensitive clays have been compared, have found a satisfactory agreement without having to reduce the N_c factor. Höeg (1972) gives a number of reasons for this apparent inconsistency, the most important of which may be the fact that, in most of the older studies, the reference shear strength of clay was determined either by field vane tests or by conventional unconfined or unconsolidated-undrained triaxial tests performed on samples taken from boreholes by means of various types of thin-walled tube samplers. It is, however, known at present (Eden, 1966, 1970; Conlon and Isaacs, 1970) that such a sampling and testing procedure tends to underestimate the true in-situ undrained strength of sensitive clays sometimes as much as 50%. The apparent agreement between the observed and calculated bearing capacities obtained by using non-reduced N_c values, quoted in some earlier studies (E.g., Bjerrum, 1955; Legget et al, 1961) may, therefore, be due to a compensation of two errors of similar magnitude.

Increased use of more advanced sampling and testing methods in recent years (Raymond et al, 1971) has had as a result that undrained strengths of sensitive clays measured more recently tend to be higher and closer to their true in-situ value than before. It would therefore be unsafe to continue using non-reduced N_c values with these higher strengths when estimating the bearing capacity of sensitive clays. The reduced N_c values calculated in the following are intended to be used with such higher shear strengths obtained by modern sampling and testing methods.

THEORETICAL PREDICTION OF N_c VALUE

To date it has not been possible, by using the principles of the theory of plasticity, to find a rigorous closed-form, solution of the problem of deep punching of saturated clays. Two approximate solutions have, however, been proposed. The first one was obtained by extending to the deep punching problem the original Prandtl's theory valid for punching at the surface (Meyerhof, 1951). Since the problem is based on a rigid-plastic assumption of material behavior, the calculated N_c values are relatively high and correspond well to those associated with the bearing capacity of deep footings in stiff clays of low sensitivity.

The second approach, based on the work by Bishop et al (1945) on the indentation of ductile metals, assumes that the resistance to deep penetration is of the same order of magnitude as that necessary for expanding a small cavity in the medium under the same conditions. In soil mechanics, the approach was first used by Gibson (1950) for estimating the end-bearing capacity of deep foundations in clay, and was further discussed by Skempton (1951) and Meyerhof (1951).

With respect to the first one, the second approach has the advantage of being able to take into account not only the ultimate strength but also the whole stress-strain behavior of the indented material, which makes it suitable for studying the bearing capacity of weak and sensitive clays. The theory of cavity expansion owes its great versatility to the fact that it considers a highly symmetric problem, enabling relatively simple analytical solutions to be obtained even for a rather complex material behavior.

When used in connection with a less symmetric problem, such as the bearing capacity of a deep footing, it necessitates some additional assumptions. In soil mechanics, the transformation from a cavity expansion solution to a deep punching problem has usually been made by assuming, as proposed by Gibson (1950), that during the penetration of the punch a rigid soil cone (for a circular punch) or wedge (for a rectangular punch) is formed at the base of the punch, the lateral surface of which is acted upon by a uniformly distributed soil pressure whose normal component is equal to the ultimate cavity expansion pressure, p_{ult}. Assuming that the semi-angle at the tip of the cone (wedge) is 45° and that, at failure, the shear strength of clay over the whole area of the cone (wedge) has already dropped to its residual value, c_{ur}, one gets from static equilibrium

$$q_{ult} = p_{ult} + c_{ur} \tag{3}$$

In order to get the conventional bearing capacity factor, q_{ult} can be expressed by Eq. (2) in which the term γD is replaced by the average total normal pressure p_o at the footing level. From Eqs (2) and (3) one gets

$$N_c\, s_c\, d_c = c_{ur}/c_u + (p_{ult} - p_o)/c_u \tag{4}$$

In all cases in which clay failure under the footing is of a punching type, i.e. without formation of distinct failure surfaces, the value of p_{ult} in Eq. (4) can be determined from the cavity expansion theory. In reality, in weak clays the punching failure is commonly either the only one observed or it precedes a general shear failure which occurs only after very large displacements. In relatively stiff clays, the punching failure is observed only when the footing depth is greater than about four times its width, i.e. when the effect of the free surface becomes relatively small (Meyerhof, 1963). In all such cases, the value of the bearing capacity factor corresponding to a punching failure can be deduced from Eq. (4) if the ultimate cavity expansion pressure p_{ult} is known.

It has been known for quite a long time in metal plasticity (Nádai, 1931) that the problem of expansion of thick-walled cylinders and spheres could be solved approximately for any given stress-strain law of the material if the cylinder or sphere is considered as an assemblage of a great number of thin concentric layers, all of them responding to that common stress-strain law. At a given expansion of the bore, the behavior of any particular layer will be governed by the portion of the common stress-strain curve corresponding to the interval of shear strains to which the layer is submitted. In this type of solution, the differential equation of equilibrium has to be integrated only for one single layer, governed by a simple stress-strain law. The complete solution of the problem is then obtained by numerical integration. It is essentially this method that has been used by the author for a number of years for solving several cavity expansion problems both in clays and in sands (Ladanyi, 1963a,b).

It was shown in an earlier paper (Ladanyi, 1967a) that such a numerical solution for the ultimate spherical cavity expansion pressure, $p_{ult,sph}$ based on an undrained stress-strain curve of any shape (Fig.1), could be written in the form

$$p_{ult,sph} = p_o + \frac{2}{3} q_1 + \frac{2}{3} \sum_{i=0}^{n-2} \Big[q_{i+1} - d_{i,i+1} \; \gamma_{i+1} \Big] \ln(\gamma_i/\gamma_{i+1}) \qquad (5)$$

where, as shown in Fig.1, q_i and γ_i denote the principal normal stress difference and the principal normal strain difference at the point i of the curve, while $d_{i,i+1}$ is the slope of the straight-line segment replacing the curve within the interval i, i+1,

$$d_{i,i+1} = (q_i - q_{i+1})/(\gamma_i - \gamma_{i+1}) \qquad (6)$$

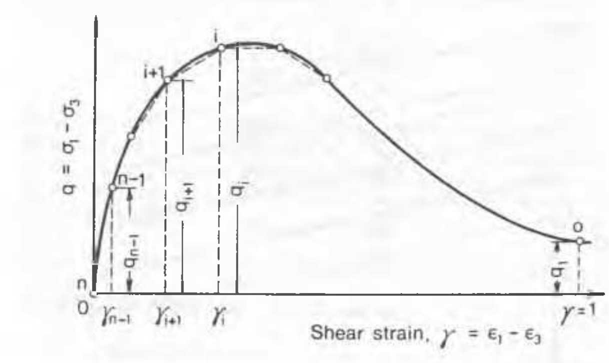

FIG. 1. LINEARIZATION BY SEGMENTS OF UNDRAINED STRESS-STRAIN CURVE OF CLAY.

It can be shown that, when applied to a simplified strain softening stress-strain curve, such as that in Fig. 2, OABC, Eq. (5) can be written as

$$p_{ult,sph} = p_o + \frac{2}{3} q_r \Big[1 - (1 - A)\ln \gamma_r - A \ln \gamma_p \Big] \qquad (7)$$

where A denotes

$$A = \frac{(q_p/q_r)(\gamma_r/\gamma_p) - 1}{(\gamma_r/\gamma_p) - 1} \qquad (8)$$

in which subscripts p and r denote peak and residual, respectively.

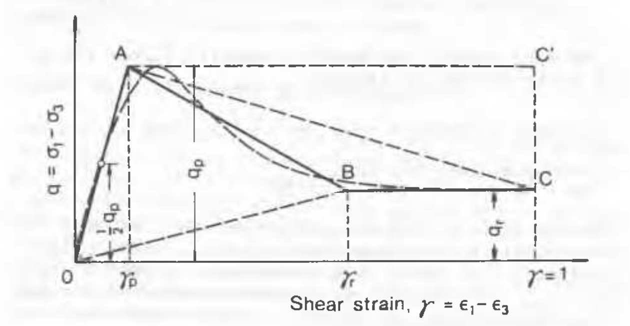

FIG. 2. VARIOUS CONSIDERED ASSUMPTIONS ON THE SHAPE OF THE UNDRAINED STRESS-STRAIN CURVE

Substituting Eq. (7) into Eq. (4) and taking into account that $q = 2 c_u$, one gets the expression for the bearing capacity factor of a deep circular footing

$$N_{c,circle} = N_c \, s_c \, d_c = (c_{ur}/c_{up}) + (4c_{ur}/3c_{up}) \times \Big[1 - (1-A)\ln\gamma_r - A\ln\gamma_p \Big] \qquad (9)$$

Equations (7) and (9) can also be written in terms of deformation moduli if γ_p and γ_r are replaced by $3q_p/2E_p$ and $3q_r/2E_r$, where E_p and E_r denote the slopes of the lines OA and OB in Fig. 2. For example, when applied to a linear-elastic perfectly plastic type of stress-strain law, OAC' in Fig. 2, Eq. (9) reduces to the well known formula (Gibson, 1950),

$$N_{c,circle} = 1 + \frac{4}{3}\Big[1 + \ln(E/3c_u) \Big] \qquad (10)$$

Another simple formula for N_c is obtained from Eq.(9) if the assumed stress-strain relation is given by the line OAC in Fig. 2, which is a typical strain softening behavior. Then, as $\gamma_r = 1$ and $A \approx c_{ur}/c_{up}$, one gets

$$N_{c,circle} = (c_{ur}/c_{up}) + \frac{4}{3}\Big[(c_{ur}/c_{up}) + \ln(E_p/3c_{up}) \Big] \qquad (10a)$$

In order to get a corresponding solution for the deep strip footing, the same analysis should be repeated for the model of an expanding cylindrical cavity, whose longitudinal axis coincides with the centerline of the strip. Assuming temporarily that plain strain information on the undrained stress-strain behavior of the clay is available, (such information can, e.g., be obtained from a pressuremeter test as shown in Ladanyi, 1972), and that it is represented as in Figs. 1 and 2, similar general expressions as in the spherical case can be obtained. For the cylindrical cavity expansion case, one obtains for the curve in Fig. 1,

$$p_{ult,cyl} = p_o + \frac{1}{2} q_1 + \frac{1}{2} \sum_{i=0}^{n-2} \Big[q_{i+1} - d_{i,i+1} \; \gamma_{i+1} \Big] \times \ln(\gamma_i/\gamma_{i+1}) \qquad (11)$$

Equation (6) and (8) remain unchanged, while Eq. (7) becomes

$$p_{ult,cyl} = p_o + \frac{1}{2} q_r \left[1 - (1-A) \ln\gamma_r - A \ln \gamma_p \right] \tag{12}$$

In the same manner, the bearing capacity factor for a deep strip footing is given by

$$N_{c,strip} = N_c s_c d_c = (c_{ur}/c_{up}) + (c_{ur}/c_{up}) \times$$
$$\times [1 - (1-A) \ln\gamma_r - A \ln\gamma_p] \tag{13}$$

Equations (11) to (13) can also be written in terms of stress-strain information obtained in an ordinary undrained triaxial test, i.e., under axial symmetry conditions, if the validity of a common octahedral stress-strain relationship is assumed. In undrained case, the following relationships between the plain strain (subscript ps) and axial symmetry (subscript a) information are obtained (Ladanyi, 1967b),

$$q_{ps} = 2q_a/\sqrt{3} \tag{14}$$

$$\gamma_{ps} = 2\gamma_a/\sqrt{3} \tag{15}$$

$$E_{ps} = 4E_a/3 \tag{16}$$

The plane strain γ values in Eqs (11) and (13) may then be replaced by

$$\gamma_{ps} = 2q_{ps}/E_{ps} = \sqrt{3}\, q_a/E_a \tag{17}$$

In terms of axial symmetry information, Eq. (12) becomes

$$p_{ult,cyl} = p_o + (q_{ar}/\sqrt{3})[1 + (1-A)\ln(E_{ar}/\sqrt{3}\, q_{ar}) +$$
$$+ A \ln(E_{ap}/\sqrt{3}\, q_{ap})] \tag{18}$$

from which for a deep strip footing, and after dropping the subscript a ,

$$N_{c,strip} = \frac{c_{ur}}{c_{up}} \left\{ 1 + \frac{2}{\sqrt{3}} \left[1 + (1-A)\ln\left(\frac{E_r}{2/3\, c_{ur}}\right) + \right.\right.$$
$$\left.\left. + A \ln\left(\frac{E_p}{2/3\, c_{up}}\right) \right] \right\} \tag{19}$$

As before, if Eq. (19) is applied to the linear-elastic perfectly plastic law OAC' in Fig. 2, one gets the known formula (Bishop et al, 1945),

$$N_{c,strip} = 1 + \frac{2}{\sqrt{3}} \left(1 + \ln \frac{E}{2/3\, c_u} \right) \tag{20}$$

Again, for a strain softening behavior such as OAC in Fig. 2, one gets a simple expression

$$N_{c,strip} = \frac{c_{ur}}{c_{up}} + \frac{2}{\sqrt{3}} \left(\frac{c_{ur}}{c_{up}} + \ln \frac{E_p}{2\sqrt{3}\, c_{up}} \right) \tag{20a}$$

In a previous paper (Ladanyi, 1967a) it was found from undrained triaxial tests carried out with Leda clay from Ottawa area with a sensitivity of about 16, that its stress-strain curve could be reduced to the shape shown in Fig. 2, OABC, with the following values of relevant parameters: $c_{ur}/c_{up} = 0.45$; $E_p/c_{up} = 500$; $E_r/c_{ur} = 16$ and $\gamma_r/\gamma_p = 31.25$. Substituting these parameters into Eqs (9) and (13) respectively, one gets the values of generalized bearing capacity factors $N_{c,circle} = 6.72$ for a deep circular footing, and $N_{c,strip} = 5.80$ for a deep strip footing. For comparison, a non-sensitive clay ($c_{ur}/c_{up} = 1$) of the same rigidity ($E_p/c_{up} = 500$) would have according to Eqs (10) and (20), respectively: $N_{c,circle} = 9.15$ and $N_{c,strip} = 7.90$. The reduction in N_c factors due to strain softening character of the clay is seen to be 26.5 per cent, which is close to one-third assumed by Terzaghi for weak clays.

For a linear decrease of strength after the peak, such as shown by the line AC in Fig. 2, the variation of N_c factors with c_{ur}/c_{up} according to Eqs (10a) and (20a) for deep circular and strip footings, is shown in Fig. 3 .

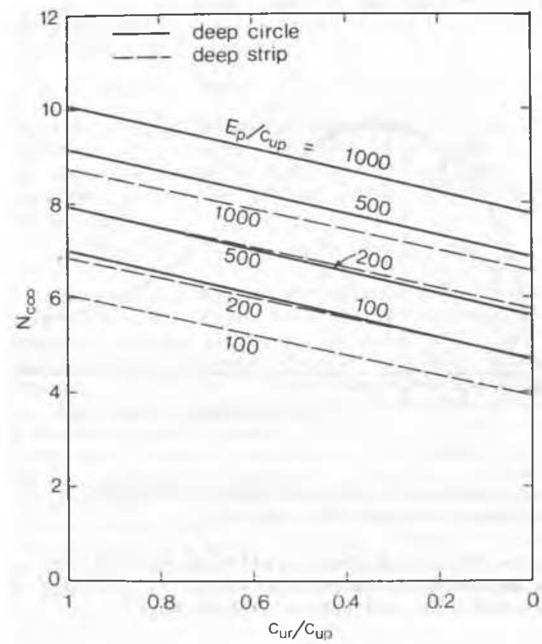

FIG. 3 . VARIATION OF N_c FACTORS FOR DEEP FOOTINGS FOR A LINEAR DECREASE OF UNDRAINED STRENGTH OF CLAY AFTER THE PEAK (AC in FIG. 2).

EFFECT OF FOOTING SHAPE

Once the values of generalized bearing capacity factors for deep strip and circular footings have been determined, the corresponding factors for any rectangular footing with a given aspect ratio, B/L, can be obtained by linear interpolation, as proposed by Skempton (1951). Defining a general shape factor, s_c , by the formula

$$s_c = 1 + \left(\frac{N_{c,circle}}{N_{c,strip}} - 1 \right) \frac{B}{L} \tag{21}$$

the value of bearing capacity factor for a given value of B/L is

$$N_{c(B/L)} = N_{c,strip}\, s_c \qquad (22)$$

It is interesting to note that from the above calculated numerical values of N_c factors for the Leda clay, the value of s_c would be

$$s_c = 1 + 0.16\,\frac{B}{L} \qquad (23)$$

which is close to the formula proposed by Skempton (1951) .

EFFET OF FOOTING DEPTH

On the basis of observations and theoretical predictions, Brinch Hansen (1961) has proposed the following empirical formula for the depth factor, d_c , to be used in the bearing capacity formula, Eq. (2), which is valid for undrained ($\phi_u = 0$) failure of a footing in clay:

$$d_{c(D/B)} = 1 + \frac{0.35}{B/D + 0.60} \qquad (24)$$

where B and D are the footing width and depth, respectively. Denoting by N_{co} the bearing capacity factor for a footing resting on the surface, the value of $N_{c(D/B)}$ for a footing at any relative depth D/B is given by

$$N_{c(D/B)} = N_{co}\, d_{c(D/B)} \qquad (25)$$

For infinite depth Eq. (25) becomes

$$N_{c\infty} = N_{co}\, d_{c\infty} \qquad (26)$$

Dividing Eq. (25) by Eq. (26) yields

$$N_{c(D/B)} = N_{c\infty}\, d_{c(D/B)}/d_{c\infty} \qquad (27)$$

Since from Eq. (24), $d_{c\infty} = 1.584$, one gets finally

$$N_{c(D/B)} = N_{c\infty} d'_c \qquad (28)$$

where

$$d'_c = d_{c(D/B)} / d_{c\infty} = \frac{0.631 + 0.6\ D/B}{1 \quad + 0.6\ D/B} \qquad (29)$$

is a modified depth factor, enabling to estimate the values of N_c factor for a finite depth from the corresponding values of the factor, valid for infinite depth.

Taking into account simultaneously the shape and the depth of footing, the formula for the bearing capacity factor is

$$N_{c(B/L)(D/B)} = N_{c,strip,\infty}\ s_c\ d'_c \qquad (30)$$

where $N_{c,strip,\infty}$ is given by Eq. (19), s_c by Eq. (21), and d'_c by Eq. (29) .

Taking again the foregoing N_c values for the Leda clay as example, one would get by this method, for a footing on the surface, the following N_c values: $N_{c,strip,o} = 5.80 \times 0.631 = 3.66$, and $N_{c,circle,o} = 6.72 \times 0.631 = 4.25$. For a non-sensitive clay of the same rigidity, one gets $N_{c,strip,o} = 4.99$ and $N_{c,circle,o} = 5.78$, which are close to the values expected for a general shear failure.

SENSITIVITY AND DEGREE OF DISTURBANCE

The foregoing analysis shows that, in order to be able to evaluate the N_c factor for a footing in a sensitive clay, it is necessary to have a knowledge on the complete undrained stress-strain curve of the clay, covering the region of strains from $\gamma = 0$ to $\gamma = 1$. If the curve is subsequently reduced to the shape shown in Fig. 2, the computation of N_c factors can be made by using only four relevant parameters, i.e., the shear strains γ_p and γ_r , and the shear strengths c_{up} and c_{ur} .

It has been shown in an earlier paper (Ladanyi, 1967a) that, if the usual secant modulus method is used for linearizing the ascending part of the curve (Fig. 2), one gets for typical sensitive clays the values of the E_p/c_{up} ratio between the limits of about 250 and 500. As, for an undrained test, $\gamma_p = 3c_{up}/E_p$, these limiting ratios correspond to $0.012 \geq \gamma_p \geq 0.006$. Some more recent field studies, reviewed by D'Appolonia et al (1971), show that E_p/c_p may even be as high as 1200 (or $\gamma_p = 0.0025$) in certain clays.

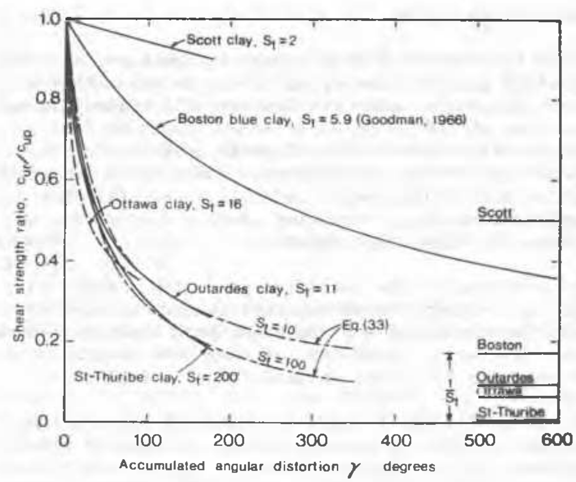

FIG. 4. POST-PEAK LOSS OF UNDRAINED STRENGTH IN CLAYS OF DIFFERENT SENSITIVITIES

The shape of the post-peak descending portion of the stress-strain curve, necessary for determining the parameters γ_r and c_{ur}, has been discussed in two earlier papers (Ladanyi, 1967a; Ladanyi et al, 1968). Experimental evidence on the post-peak behavior of five clays of different sensitivities has been reproduced in Fig. 4. Of the five curves shown in the figure, four (Scott, Ottawa, Outardes and St-Thuribe) have been taken from the above two papers, while the fifth (Boston blue clay) originates from a paper by Goodman (1966). For obtaining the first four curves, the method used was a repeated large strain compression with separate measurement of the disturbed shear strength of clay. The Goodman's curve, in turn, was obtained by measuring the work of distortion in a cyclic, constant volume, shear box test, enabling to deform the clay uniformly up to 40° angular distortion per cycle. The Boston blue clay curve shown in Fig.4 was recalculated from Goodman's Fig.21 by using the following relationship, expressing the degree of disturbance, d_d, in terms of the c_{up}/c_{ur} ratio and the sensitivity, S_t :

$$d_d = \frac{1 - c_{ur}/c_{up}}{1 - 1 / S_t} \tag{31}$$

which gives $0 \leqslant d_d \leqslant 1$ when $1 \geqslant c_{ur}/c_{up} \geqslant 1/S_t$.

The sensitivity of clay is commonly defined as the ratio between undisturbed and completely remolded shear strength of clay. As the testing procedure for sensitivity determination has not yet been standardized, the sensitivities quoted in the literature correspond to various testing methods. At present, most frequently stated are the values of sensitivity obtained from field and laboratory vane tests.

The values of sensitivity according to this definition, while being useful as an index property of clays, have nevertheless very little use in practical foundation design. In fact, the completely remolded strength of clay as a factor in foundation design in natural clay deposits appears only exceptionally and mainly not as a cause but as a result of large localized shear displacements.

If, as considered in this paper, the failure is of a contained plastic flow type, it can be shown that angular distortion under the footing will nowhere exceed the value of 90° . In other words, punching failure of a footing in sensitive clays is governed only by the portion of the stress-strain curve up to 90° of angular distortion, which is still very far from a complete remolding, requiring usually distortions of at least 10 times that value.

The obtaining of the immediate post-peak portion of the stress-strain curve requires, unfortunately, rather time-consuming and unconventional testing methods. An attempt will, therefore, be made here to predict the behavior from the available experimental data.

The curves in Fig. 4 show, in a general manner, that the rate of loss of strength after the peak is proportional to the clay sensitivity. On the other hand, it can be shown that the degree of disturbance, defined as in Eq. (31), follows approximately a hyperbolic law of the form

$$d_d = \gamma^o / (\gamma_a^o + \gamma^o) \tag{32}$$

where γ^o is the angular distortion in degrees and γ_a^o is a constant. Substituting Eq. (32) into Eq. (31) yields

$$\frac{c_{ur}}{c_{up}} = \frac{\gamma_a^o + \gamma^o/S_t}{\gamma_a^o + \gamma^o} \tag{33}$$

It will be seen that, according to Eq. (33), $1 \geqslant c_{ur}/c_{up} \geqslant 1/S_t$ when $0 \leqslant \gamma^o \leqslant \infty$. Equation (33) can be used for approximating the curves shown in Fig. 4. For example, it can be shown that a good approximation for the Scott clay curve is obtained with $\gamma_a^o = 720°$, while for the Boston blue clay one should take $\gamma_a^o = 210°$. For the remaining three very sensitive marine clays, the best approximation is obtained with $\gamma_a^o = 38.6°$. Two calculated curves for sensitivities of 10 and 100 are shown for comparison in Fig. 4 and are seen to agree well with the observed behavior. In other words, it appears that for $10 \leqslant S_t \leqslant 100$, the post-peak behavior of marine clays can be predicted by Eq. (33) with $\gamma_a^o = 38.6°$. When calculating the N_c factors according to the described method, the hyperbolic curve can then be used directly, or replaced by one or two straight lines. It should be noted that γ values in radians should be used in the foregoing formulae for N_c factors.

COMPARISON OF PREDICTED AND OBSERVED BEHAVIOR

As mentioned in the introduction, although a number of foundation failure studies, as well as some reports on plate loading tests in sensitive clays can be found in the literature (E.g., Bjerrum, 1955; Legget et al,1961; Brown and Paterson, 1964), the results of these studies are difficult to use as a basis for comparison with the theory, since, due to sampling and testing methods employed, the reported shear strengths were probably closer to an average than to a true peak strength of clay. In other words, if the reported bearing capacities were reanalyzed in terms of undrained shear strengths obtained on block samples, which seem to be the closest to reality, one would obtain for the measured N_c factors values that are lower than those derived from the Prandtl's theory.

That this may, in fact, be the case, has been shown by the results of laboratory and in-situ deep penetration tests in sensitive clays carried out by Ladanyi and Eden, (1969), in which a reduction of close to 40 per cent in the value of bearing capacity factors was found.

On the other hand, the degree of approximation obtained by the proposed method can be estimated by comparing its predictions with the results of computer simulation of bearing capacity problems by modern numerical methods. The reason that they may be expected to be comparable is because both types of solutions consider only a contained plastic flow without general shear failure. From a number of such analyses, that have been published in recent time, only two will be used here for comparison.

Radhakrishnan and Reese (1969) have used the finite element method for analyzing the behavior of a strip footing resting on saturated Wilcox clay and have compared the results with actual model tests. The clay, remolded and recompacted, showed the values of $E = 19\ kg/cm^2$ and $c_u = 0.123\ kg/cm^2$ in unconsolidated undrained triaxial tests. Taking into account that, in the model tests, failure was preceded by a settlement of about 10 per cent, Eqs (20) and (29) give $N_c = 4.47$ and $q_{ult} = 0.55\ kg/cm^2$. The bearing capacity measured in the tests at that settlement was about $0.60\ kg/cm^2$, while $0.65\ kg/cm^2$ was predicted by the finite element method. The slightly higher values in the tests and the calculation are thought to be due to the effect of rigid bottom and walls present in the test and assumed in the calculation.

Höeg (1972) presented a finite element solution for a circular footing resting on and below the free surface of a saturated clay with strain softening characteristics. While insufficient dat do not permit a detailed comparison with his results, it is interesting to note that his calculations show that a circular footing in sensitive clay may not only have its bearing capacity reduced by about 40 per cent, but also that it would fail at much smaller settlements than the same footing in a non-sensitive clay of the same rigidity. He was able to demonstrate the last point by comparing his results with the behavior of a quick clay under a test fill (Höeg and al, 1969).

The available evidence presented shows that the proposed theory leads to a reasonable estimate of the reduced bearing capacity factors for sensitive clays. Additional evidence can only be obtained if future foundation failure studies will be analyzed in terms of the true in-situ peak strength of clay.

CONCLUSIONS

In the past, the bearing capacity failures have most frequently been analyzed by using a general shear failure concept. The concept was usually found satisfactory if used in connection with undrained shear strengths of clays obtained by ordinary sampling and testing methods which, by present standards, are known to furnish shear strengths closer to an average value than to a true peak strength. In recent time, the use of more advanced sampling and testing methods and, in particular, those involving large diameter and block samples, tends to furnish much higher shear strengths of sensitive clays, which, if used with a general shear failure concept, would lead to an overestimate of the bearing capacity of footings. In order to get reasonable bearing capacity predictions with these higher strengths, a use of reduced bearing capacity factors, to compensate for the clay sensitivity, is recommended. A theory enabling to determine the reduced values of the bearing capacity factor N_c for clays of strain softening type and valid for rectangular and circular footings at any depth, is described in this paper. The value of the N_c factor is found to depend considerably on the rate of strength decrease of clay in the post-peak region, which appears to be proportional to the clay sensitivity.

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