ELASTIC ANALYSIS OF PILE GROUP IN GRANULAR SOIL

ANALYSE ELASTIQUE DU COMPORTEMENT DU GROUPE DE PIEUX DANS LE SOL GRANULEUX

УПРУГИЙ АНАЛИЗ РАБОТЫ КУСТА СВАЙ В СЫПУЧЕМ ГРУНТЕ

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SYNOPSIS. A pile group subjected to vertical and lateral loads is analysed by developing a stiffness matrix relation between external forces and displacement vectors. The pile constants involved in the matrix are determined using the Mindlin's solution for forces acting in side the elastic half space. The effect of spacing on the efficiency of pile group is analytically evaluated. Field tests one on noninstrumented 4 pile group and other on instrumented 9 pile group were conducted. It was found that the elastic theory gives good results provided (a) the soil parameters are properly assessed and (b) the magnitude of loads is about one third to half of ultimate loads.

INTRODUCTION

A pile group subjected to vertical and lateral forces acts as a structural framework embedded in a semiinfinite elastic space. It can be analysed by developing a stiffness matrix relation between the external forces and the displacement vectors. The matrix involves certain quantities called the pile constants, which are defined as the forces exerted by pile on pile cap per unit deflection or rotation of the pile head. They can be correlated with soil properties using the Mindlin's solution (Mindlin, 1936) for forces acting inside the elastic space.

ANALYSIS

Assumptions - (1) The piles are embedded in elastic, homogenous and isotropic soil. The modulus of elasticity, E_s, increases linearly with depth in granular soil and is constant in overconsolidated clay. (2) The soil properties do not change due to presence of pile in soil. (3) The piles in a group are connected by a rigid pile cap. (4) The problem is two dimensional i.e. the forces are applied only in vertical and lateral directions. (5) The load carried by each pile is proportional to the displacement of the pile head.

The first and second assumptions are not strictly valid, as the soil does not behave as an elastic material. However, it was found (Shrivastava, 1971) that they do not introduce significant error in the range of working loads which are generally half to one third of the ultimate loads.

Definitions - Based on the last assumption, the pile constants are defined as the forces exerted by a pile on the footing or the pile cap when the head is given a unit displacement or rotation. Thus we have pile constant k_a corresponding to axial displacement; k_t and m_t due to it translation and m_θ and k_θ for unit rotation as shown in Fig.1. By reciprocal theorem, it can be shown that $m_t = k_t$. Hence, there are only four independent constants namely k_a, k_t, m_t, and m_θ.

The axial forces acting downward, the lateral forces towards the right and moment in clockwise direction are considered as positive.

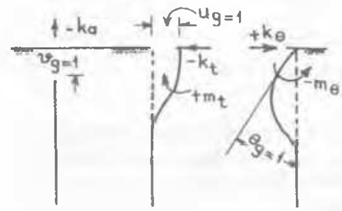

Fig.1 PILE CONSTANTS

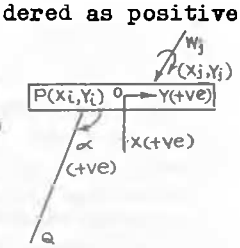

Fig.2 COORDINATES

Coordinate System - Let ABCD be the footing and P the point where pile PQ meets the footing at an angle α with horizontal as in Fig.2. Let us choose a coordinate system X-Y with origin at O. The coordinates of P are (X_i, Y_i). Similarly let the coordinates of an external load vector (W_j) be (X_j, Y_j).

If there are n number of piles in a group and m number of load vectors, the displacement vector of a pile group in terms of external forces are given as

$$\{\Delta\} = \sum_{i=1}^{n} \left[[YY_i]\right]^{-1} \sum_{i=1}^{m} [T_w] \{W_j\} \quad (1)$$

where

$\{\Delta\}$ = Displacement vector of pile group;

$[YY_i]$ = Stiffness matrix of soil-pile system;

$[T_w]$ = Transfer matrix of external forces;

$\{W_j\}$ = External force vector.

The stiffness matrix can be written as

$$[YY_i] = [T][R][K]\left[[T][R]\right]^T \quad (2)$$

where,

$$[T] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ Y_i & -X_i & 1 \end{bmatrix} \qquad [R] = \begin{bmatrix} \sin\alpha & -\cos\alpha & 0 \\ \cos\alpha & \sin\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$[K] = \begin{bmatrix} -k_a & 0 & 0 \\ 0 & -k_t & m_t \\ 0 & m_t & -m_\theta \end{bmatrix} \qquad [T_w] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ Y_j & -X_j & 1 \end{bmatrix}$$

$[A]^T$ = Transpose of $[A]$

Thus knowing the constants and the geometry of pile group, $[YY_i]$ can be computed and the displacements worked out.

PILE CONSTANTS

The problem is solved if the pile constants are properly assessed. To correlate these constants with soil properties, the Mindlin's solution for stresses and displacements in semi-infinite elastic space due to point loads in it is used.

Pile Constant k_a - Let us consider a pile subjected to vertical load P. The displacement v_g of the pile head at ground level and the load P can be correlated by the following relation -

$$v_g = \frac{P}{E_s \cdot L} \cdot I_v \quad (3)$$

where

E_s = Modulus of elasticity of soil;

L = Length of pile;

I_v = Influence factor for vertical displacement of pile head in a semi-infinite mass which can be determined using Mindlin's solution (Poulos & Davis 1968)

$$\text{Hence, } k_a = \frac{P}{v_g} = \frac{E_s \cdot L}{I_v} = A_1 . E_s . L \quad (4)$$

Pile Constants k_t, m_t and m_θ - Let us consider a free head pile subjected to a lateral force Q and moment M at ground level. The lateral displacement u_g and rotation θ_g at ground level can be expressed as -

$$u_g = I_{uq} \frac{Q}{E_s . L} + I_{um} \frac{M}{E_s . L} \quad (5)$$

$$\theta_g = I_{\theta q} . \frac{Q}{E_s . L^2} + I_{\theta m} . \frac{M}{E_s . L^3} \quad (6)$$

In which, I_{uq}, I_{um} are the influence factors for displacement due to lateral load Q and moment M respectively. Similarly, $I_{\theta q}$, $I_{\theta m}$ are the factors for rotation. They can be determined using Mindlin's solution (Poulos 1968b).

Putting $\theta_g = 0$ and $u_g = 1$ and solving Eqs. (5) and (6) we get

$$k_t = E_s . L . \frac{I_{\theta m}}{I_{uq} . I_{\theta m} - I_{um} . I_{\theta q}} = A_2 . E_s . L \quad (7)$$

$$m_t = E_s . L^2 . \frac{-I_{\theta q}}{I_{uq} . I_{\theta m} - I_{um} . I_{\theta q}} = A_3 . E_s . L^2 \quad (8)$$

Similarly putting $\theta_g = 1$ and $u_g = 0$, we get

$$M_\theta = E_s . L^3 . \frac{I_{uq}}{I_{uq} . I_{\theta m} - I_{\theta q} . I_{um}} = A_4 . E_s . L^3 \quad (9)$$

In the above analysis E_s is assumed as constant with depth. When E_s increases linearly with depth, the influence factors can be determined by the method suggested by Lenci et al (Lenci et al, 1968).

The values of constants, A_2, A_3 and A_4 are tabulated elsewhere (Shrivastava 1971) for different values of stiffness factor $E_p . I_p / E_s . L^4$. Similarly A_1 for different values of E_p / E_s are also tabulated.

EFFECT OF SPACING OF PILES

Poulos (Poulos 1968a, 69) has discussed in detail the effect of spacing on the deflections of piles in a group. Let us consider a group of n piles which is subjected to vertical loads only. The axial displacement v_g of any pile k in the group by superposition is

$$v_g = \sum_{j=1}^{n} \frac{P . I_v}{E_s . L} . \alpha_{vkj} \quad (10)$$

where

α_{vkj} = Interaction factor for v_g;

$$= \frac{\text{Vertical displacement of pile k due to load P on pile j}}{\text{Vertical displacement of pile k due to load P on pile k}}$$

Similarly,

$$u_g = \sum_{j=1}^{n} \frac{Q . I_{uq}}{E_s . L} \alpha_{uqkj} + \sum_{j=1}^{n} \frac{M . I_{um}}{E_s . L^3} \alpha_{umkj} \quad (11)$$

$$\theta_g = \sum_{j=1}^{n} \frac{Q\, I_{\theta q}}{E_s . L^2} \alpha_{\theta qjk} + \sum_{j=1}^{n} \frac{M . I_{\theta m}}{E_s . L^3} \alpha_{\theta mkj} \quad (12)$$

where α's are the interaction factors which depend on the spacing of piles and can be evaluated from Mindlin's equation (Poulos 1968 a, 1969). Since

Eqs. (11) and (12) are similar to (3), (5) and (6) **the constants for piles in group can be evaluated by the procedure discussed earlier.**

FIELD TESTS

Description of Site - The soil was mainly silty sand with a layer of clay at a depth of about 3.15 m. The water table was at a depth of 2.5 m. The bore-log and penetration resistances are shown in Fig. 3. The modulus of elasticity and poisson's ratio were determined by the triaxial test, horizontal and vertical plate load tests, penetration tests using empirical relations suggested by Schultze et al (Schultze & Melzer 1965) and vibration tests. It was found that the values as determined by different tests varied widely depending on type of test, magnitude of strain etc. and it was difficult to choose a single value of E_s. However, based on the data of single pile, triaxial test and plate load tests, the values of E_s equal to 60 kg/cm^2 in dry con-condition and 20 kg/cm^2 when the site is kept flooded for 7 days, were found to be most probable values

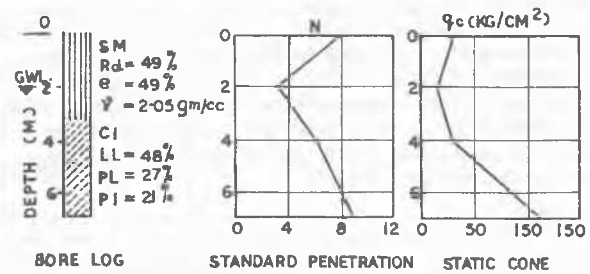

Fig.3 SOIL EXPLORATION DATA

Test Details - A 4 pile group of noninstrumented piles and a 9 pile group of instrumented piles were tested under vertical and lateral loads and moment.

4 Pile Group - The group consisted of 4 steel piles of 10 cm diameter, $E_p I_p$ of 6.20×10^8 $kgcm^2$ and length of 500 cm. The piles were spaced at 4 times the diameter and were connected by a rigid pile cap of concrete. The group was subjected to vertical load initially. The load was applied in gradual steps. In the next test, the vertical load was kept constant and lateral load was gradually increased. The vertical load was applied by a girder whose one end rested on a roller fixed at the centre of the pile cap and the other end on a non-yielding type of support. Thus, the load on the pile cap was equal to half the vertical load applied at the centre of girder. The lateral load above ground level was applied by means of dead weights suspended by a wire cable and pulley arrangement. The test was done in dry condition of soil.

9 Pile Group - The group consisted of instrumented piles of 10 cm diameter, $E_p I_p$ of 3.2×10^8 $kgcm^2$ and embedded length of 500 cm. Fifteen electric resistance wire strain gauges of type of SR-108 with gauge length equal to 8 mm. were fixed on each half of the pile at an interval of 30 cm centres, except for the last which was fixed at a distance of 60 cm. The piles were calibrated for axial load and bending moment.

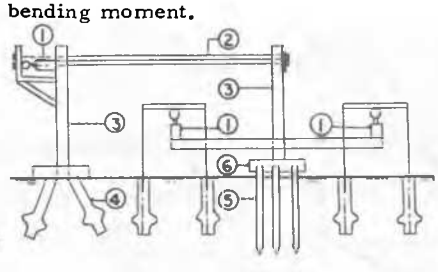

Fig. 4 TEST SET UP - 9 PILE GROUP

The vertical load was applied in gradual increments. Subsequently, the vertical load was kept constant and lateral loads was applied by high tensile wire as shown in Fig. 4. In this case, the area was flooded for 7 days to avoid the effect of dessication.

TEST RESULTS

The vertical load settlement curves and the lateral load deflection curves are shown in Fig.5(a)and 5(b)respectively.The moment,deflection and pressure distribution along depth for one of the piles in 9 pile group are shown in Fig. 5 (c). The elastic analysis is used to predict the load-deformation curves.The moment,deflection and pressure distribution along the depth for 9 pile group are also computed.Value of E_s is chosen as 60 kg/cm^2 for 4 pile group and 20 kg/cm^2 for other group.Also since the penetration resistance increases with depth the following variation of E_s based on N value is assumed for 9 pile group -$E_s = 5 + 15 \frac{X}{L}$.

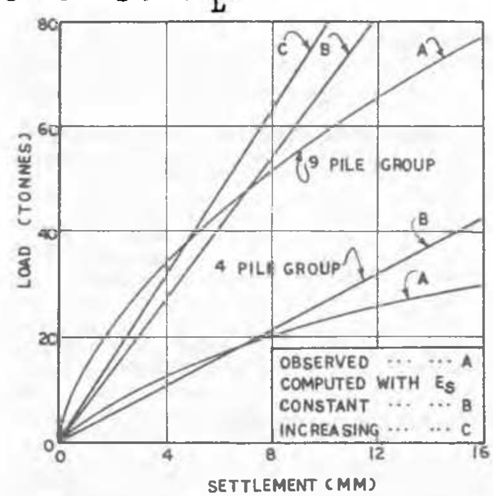

Fig. 5(a) VERTICAL LOAD DEFLECTION CURVES

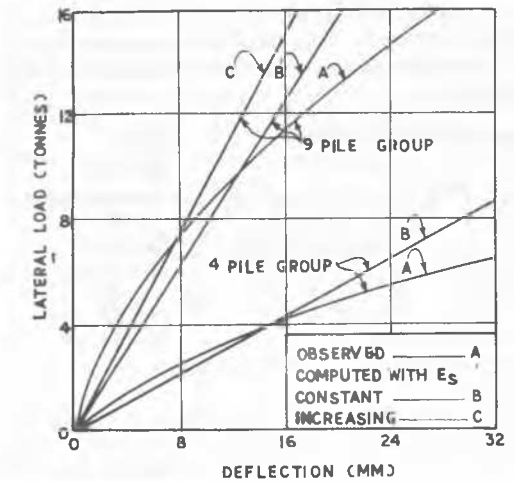

Fig. 5(b) LATERAL LOAD DEFLECTION CURVES

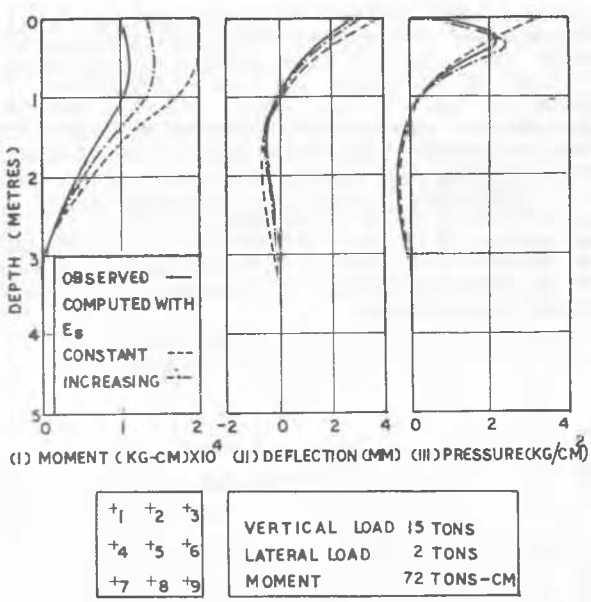

Fig. 5(c) MOMENT, DEFLECTION & SOIL PRESSURE ALONG DEPTH FOR PILE 1

DISCUSSIONS AND CONCLUSIONS

1. The soil tests showed that the soil essentially follows a non-linear stress-strain relation and a single value of E_s cannot be assigned. However, corresponding to working loads a value can be judicially chosen.

2. The observed load deflection curve is curvilinear where the predicted one is a straight line.

3. The observed pressure on pile at the ground level differ considerably as compared to predicted one in case E_s is assumed as constant with depth. However, the agreement between observed and predicted is good if E_s is taken as linearly increases with depth.

4. The error is not significant if the loads are within working range i. e. half to one third of ultimate loads.

It can be concluded that the proposed elastic analysis of pile group is more logical than other existing methods based on subgrade reaction theory because it can take into account the continuity of soil media. However, main difficulty is in chosing a correct value of E_s. The behaviour of pile group in sandy soil is better predicted if E_s is assumed as linearly increasing with depth.

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REFERENCES

LENCI, C., MAURICE, J. and MADIGNIER, F. (1968), "Pieu Vertical Sollicite' Horizontalement", Annales des Ponts et Chausse'es, Vol. VI, Nov. Dec. pp. 337-383.

MINDLIN, R. D. (1936), "Force at a point in the interior of a semi-infinite solid", Physics Vol. 7, p. 195.

POULOS, H. G. (1968 a), "Analysis of the settlement of laterally loaded piles, Part I - Single piles", Civil Engg. Research Report No. R-106, University of Sydney.

POULOS, H. G. (1969), "Analysis of the displacement of laterally loaded piles - Part II - Pile Groups", Civil Engg. Research Report No. R-106, Univ. Sydney.

POULOS, H. F. and DAVIS, E. H. (1968), "The settlement behaviour of single axially loaded pile and piers", Geotechnique, Vol. 18, pp. 351-71.

SCHULTZE, E. and MALEER, K.J. (1965), "Determination of the density and modulus of compressibility of noncohesive soil by sounding", Proc. of the Sixth International Conference on Soil Mechanics and Foundation Engineering, Montreal, Vol. I, pp. 354-358.

SHRIVASTAVA, S. P. (1971), "Elastic analysis of pile groups subjected to vertical and lateral loads," Ph. D. Thesis, University of Roorkee.