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THEORY OF PILES IN SWELLING AND SHRINKING SOILS

THEORIE DES PIEUX DANS DES SOLS SUJETS AU GONFLEMENT OU AU RETRAIT

ТЕОРИЯ СВАЙ В НАБУХАЮЩИХ И УСАДОЧНЫХ ГРУНТАХ

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SYNOPSIS. Piles are frequently used for foundations in swelling and shrinking soils to suppress structural movements. Current methods of analysing such foundations usually only consider the ultimate uplift or downward load capacity of the pile and little or no consideration is given to prediction of the pile movements. In this paper, a method of analysing the movement of a pile in swelling soil is presented. The analysis is based on the simplifying assumption that the soil can be treated as an elastic material but modifications to the analysis are described which allow consideration of such factors as non-homogeneity, slip at the pile-soil interface and crushing or tensile failure of the pile.

In order to illustrate the major factors affecting pile movements, a number of idealized theoretical solutions are presented. Comparisons between the predicted characteristics of behaviour of piles in swelling soils show encouraging agreement with the behaviour observed in a number of full-scale field tests.

INTRODUCTION

Foundations in expansive clays are frequently subjected to severe movements arising from moisture changes within the clay with consequent cracking and damage due to distortion. Piles have been used extensively for foundations in swelling soils in order to anchor the structure down at a depth where changes in moisture content are negligible, so that movements of the structure are minimized. However, considerable uplift forces are then induced in such piles due to the action of the swelling soil. An analogous problem arises with piles in soils undergoing shrinking or consolidation, when downdrag forces are induced in the piles due to negative friction. In this paper, the effect of soil movements on the behaviour of floating piles will be considered. Because of the similarity in approach between a pile in a swelling soil and a pile in a consolidating or shrinking soil, attention will be concentrated on the case of a swelling soil.

In designing piles in swelling soils, there are three requirements -

a) the pile must be able to carry the structural load safely i.e. there must be adequate ultimate load capacity.

b) the pile must have sufficient tensile strength to withstand the tension developed in the pile due to uplift forces.

c) the movement of the piles due to the net effects of uplift forces and the structural load must be less than the prescribed limit.

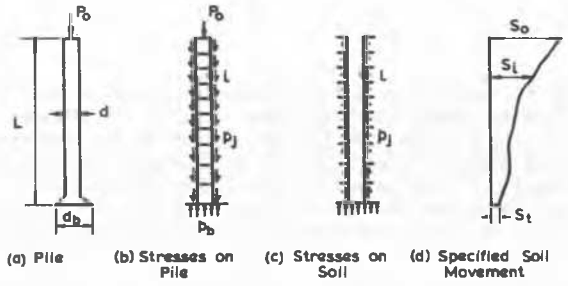
Attention is focussed on the latter two aspects since few problems arise in obtaining adequate load-carrying capacity in expansive soils. An analysis based on elastic theory is described, and typical results are given. Some comparisons between theoretical and observed pile behaviour are also presented.

Existing methods of analysis of piles in swelling and shrinking soils are generally confined to the estimation of the forces induced in a pile by soil movement. Typical of such approaches are those described by Collins (1953), Mariotti and Khalid (1969) and Bozozuk (1972), which in general assume that full slip occurs between pile and soil along the shaft. Sahzin (1968) has obtained expressions for the movement of a pile in swelling soil by considering the work done by friction forces in the upper portion of the pile tending to lift the pile and the applied load and the friction forces in the lower half of the pile tending to resist uplift. A more satisfactory analysis can be carried out by employing elastic theory in a similar fashion to that described by Poulos and Davis (1968) for pile settlements and Poulos and Mattes (1969) for negative friction on end-bearing piles.

THEORETICAL ANALYSIS

Basic Analysis

The problem is illustrated in Fig.1.A circular pile,length L, diameter d and base diameter d_b, is situated in a soil mass in which, away from the pile, a general specified distribution of movement, S (either swelling or shrinking) with depth occurs. The pile is divided into n cylindrical elements each with a uniform shear stress p_j acting on the periphery. In the basic analysis the soil is assumed to be homogeneous and linearly elastic and it is assumed that no slip occurs at the pile-soil interface.



(a) Pile (b) Stresses on Pile (c) Stresses on Soil (d) Specified Soil Movement

FIGURE 1 PILE IN SWELLING OR CONSOLIDATING SOIL.

Defining downward soil displacements as positive,and shear stresses positive as shown in Fig.1, the soil displacements along the pile can be expressed as

$$\{S^{p}\} = -\frac{d}{E_s} /I_s/ \{p\} + \{S\} \qquad (1)$$

where $/I_s/$ = the (n+1) by (n+1) matrix of displacement influence factors.

E_s = Young s modulus of soil.

$\{S\}$ = the (n+1) vector of soil movements, negative for swelling.

$\{{}_s\rho\}$ = the (n+1) vector of soil displacements adjacent to pile.

$\{p\}$ = the (n+1) vector of shear stresses at the soil-pile interface and the base pressure, positive directions as in Fig.1.

The elements of $[I_s]$ are obtained by double integration of the Mindlin equations as described by Poulos and Davis (1968). Poisson's ratio of the soil ν_s is not a very important parameter of the values of I_s.

Without slip the pile displacements, ${}_p\rho = {}_s\rho$. For the general case of a compressible pile, the pile displacements must be compatible with the elastic properties of the pile, and the analysis could proceed along the lines given in Mattes and Poulos (1969). Here only the simple case of an incompressible pile is considered and therefore ${}_p\rho = {}_s\rho = \rho$ = constant. It follows that

$$d\,\{p\} = E_s\,[I_s]^{-1}\{\rho - S\} \qquad (2)$$

Also, from equilibrium of the pile,

$$P_o + \sum_{j=1}^{n} p_j \pi d\, L/n + p_b \pi d_b^{\,2}/4 = 0 \qquad (3)$$

where P_o = applied downward load on pile top.

Equations (2) and (3) may be solved to obtain the displacement ρ and the distribution of shear stress p from which the load in the pile P at any depth can be calculated.

The above basic analysis fails to take account of several factors which are likely to be important in real situations. Modifications can be made to allow for these factors.

Local Failure Between Pile and Soil

The effects of pile-soil slip along the shaft can be allowed for by specifying a limiting value of shear stress, τ_a, at each element along the pile. τ_a will usually be expressed in terms of effective stress by the Coulomb expression. In addition, a maximum value of base stress p_b can be specified, corresponding to the ultimate bearing capacity of the base if p_b is negative (see sign definition in Fig.1) or to the ultimate uplift capacity of the base (usually taken as zero unless the base is enlarged) if p_b is positive. The solution is re-cycled until the shear stresses and base stress do not exceed these limiting values (Poulos and Davis,1968).It should be noted that if full slip occurs along a pile, the load distribution in the pile will remain constant and the movement of the pile subsequent to full slip will be the same as that of the soil adjacent to the pile at the point of shear reversal.

Compression Failure of Pile

When the soil is shrinking,allowance can be made for compression failure due to downdrag on the pile by including in the computation the requirement that,when the load in any pile element reaches the compression strength of the pile, it remains constant at this value. Since the load on such a crushed element must be the same at the top as at the bottom of the element, the shear stress between this element and the soil drops to zero. A redistribution of the shear stresses elsewhere at the soil-pile interface therefore occurs and there is an increase in pile displacement. The procedure is explained more fully in Poulos and Davis (1972).

Tension Failure of Pile

When the tensile load reaches the tensile load capacity of the pile, the pile is assumed to fracture and in effect becomes two piles. Two equilibrium equations now apply:

a) for the upper fractured portion

$$P_o + \sum_{j=1}^{m} p_j \frac{\pi d L}{n} = 0 \qquad (4)$$

where m = number of elements in fractured portion.

b) for the lower portion

$$\sum_{j=m+1}^{n} p_j \frac{\pi d L}{n} + p_b \frac{\pi d_b}{4} = 0 \qquad (5)$$

A new variable, the displacement of the fractured portion of the pile, is now introduced so that the n+3 equations may now be solved for the n unknown shear stresses, the base pressure, and the displacements of the upper and lower portions of the pile.

Non-Uniform Soil

Approximate allowance for the effect of variation of the modulus of elasticity of the soil with depth can be made by substituting the matrix $[I_s/E_s]$ for $[I_s]/E_s$ in Eq.(1). By this procedure, the soil displacement at a particular depth will be calculated as if the modulus at that particular depth was also the modulus at all other depths. This approximation will clearly be unsatisfactory when the variation of modulus from top to bottom is very large or there are sudden major changes.

Variation with Time

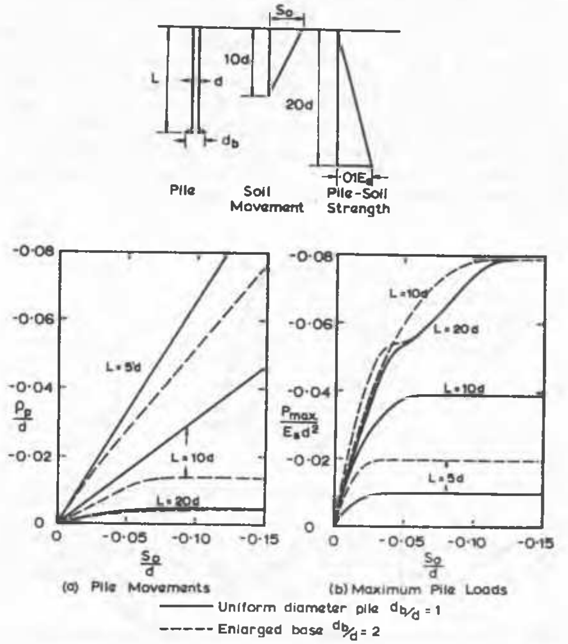
For cases in which the soil movements are time-dependent and the variation of pile loads and displacements with time is required a consolidation analysis may be combined with the above analysis (Poulos and Davis, 1972). Alternatively, appropriate values of soil displacement S may be input at each time considered, and the solution carried out as before.

THEORETICAL TRENDS IN PILE BEHAVIOUR

To illustrate the influence of various factors on pile behaviour, a number of solutions for relatively idealized cases are examined. In all cases, the pile is assumed to be incompressible, ten elements have been used to divide the pile, and the soil modulus along the pile is uniform, ν_s being 0.3. Attention is concentrated on piles in swelling soils. As long as elastic conditions are preserved the solutions also holds for shrinking soils except for a change in sign but, if failure occurs between soil and pile or within the pile, the solution for shrinking soil may be somewhat or even significantly different. In all the cases examined slip between soil and pile starts to occur at the top of the pile and requires a very small swelling movement to initiate it.

The Effect of Pile Length and Base Diameter

For a given soil profile with a linear distribution of soil swelling from S_o at the surface to zero at a depth of 10d, the variation of pile movement and maximum pile force with increasing soil movement is shown in Fig.2 for three different pile lengths. For each length, both a uniform diameter pile and a pile with a base diameter twice the shaft diameter is considered. The pile-soil interface shear strength τ_a varies linearly from zero at the surface to $0.01E_s$ at a depth of 20d, and the base bearing capacity is assumed to vary from $0.36E_s$ for L = 5d to $0.64E_s$ at L = 20d; these values correspond approximately to a soil having $\phi^1 = 30°$.



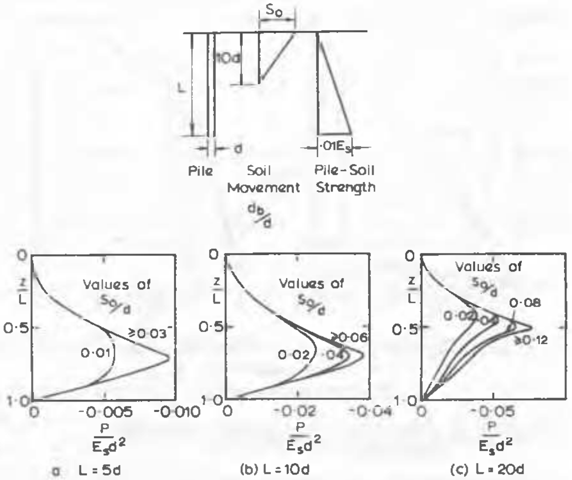
(a) Pile Movements (b) Maximum Pile Loads

FIGURE 2 EFFECT OF PILE LENGTH AND BASE DIAMETER.

Fig.2 shows that as would be expected, the pile movement decreases as the pile length increases. When the pile is entirely situated in the swelling zone (L=5d), movement of the pile continues after full slip has occurred along the shaft. For piles founded below the zone of swelling a limiting pile movement is reached after a certain soil movement occurs. The advantage of founding a pile below the swelling zone is obvious. The maximum tensile load in the pile generally increases markedly as the length increases; relatively small loads are developed when the pile is entirely within the swelling zone.

The presence of an enlarged base leads to a decrease in pile movement although the effect is relatively small, especially for L = 5d and L = 20d i.e. when the pile is entirely in the swelling zone, or anchored well below the swelling zone. In the latter case, the enlarged base has virtually no effect. The corresponding maximum loads are considerably greater for the enlarged base piles except for the L = 20d pile. It is therefore apparent that the enlarged base has the greatest influence when the pile is situated at or near the bottom of the swelling zone, and that the most efficient means of reducing pile movements is either to use a uniform diameter pile founded well below the swelling zone (of length about twice the depth of this zone) or to use an underreamed pile founded at or just below the bottom of the swelling zone.

For the uniform diameter piles considered in Fig.2, the load distributions are shown in Fig.3 for various values of dimensionless soil movement S_o/d. As the pile length increases, the load in the pile increases and the distribution of load also changes; the relative position of the maximum load moves towards the top of the pile. For the L=5d and 10d piles, slip occurs along the whole length of the pile at relatively small soil movements. For the L=20d pile, no change in load occurs after S_o/d reaches about 0.12 but slip only occurs along the upper half of the pile.



a L = 5d (b) L = 10d (c) L = 20d

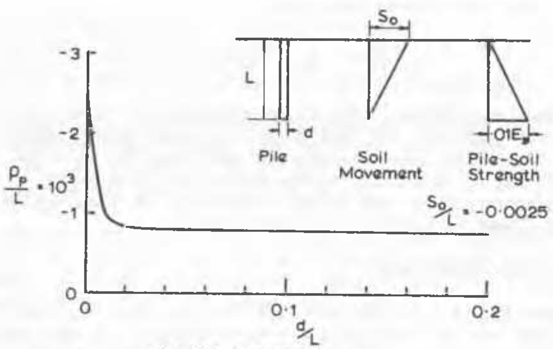
FIGURE 3 TYPICAL LOAD DISTRIBUTIONS

The Effect of Pile Shaft Diameter

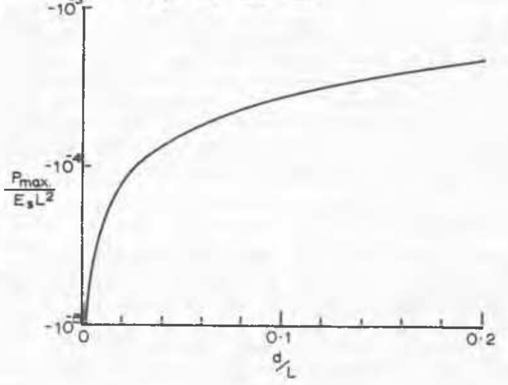
For a given pile length and soil swelling profile, the effect of pile diameter is shown in Fig.4. As the diameter increases, the pile movement decreases but the maximum load increases. However the rate of decrease of pile movement is almost negligible for diameters greater than about 0.03L, and even a slender pile (d=0.01L) moves only about 20% more than a relatively large-diameter pile (d=0.2L). The theory therefore suggests that small-diameter piles founded well below the swelling zone, can satisfactorily suppress upward movements in swelling soils. Donaldson (1967a), has described the successful use of small-diameter piles to support conventional brick buildings on expansive soils in South Africa.

The Effect of Distribution of Soil Movement

The effect of the shape of the soil swelling profile, for a given depth of movement, is shown in Fig.5. Both the pile movement and the maximum pile load are greatest when the soil movements decrease slowly with depth near the top of the pile (case iii) and least when the soil movements decrease rapidly (case ii).



(a) Pile Movement



(b) Maximum Pile Load

FIGURE 4 INFLUENCE OF PILE DIAMETER

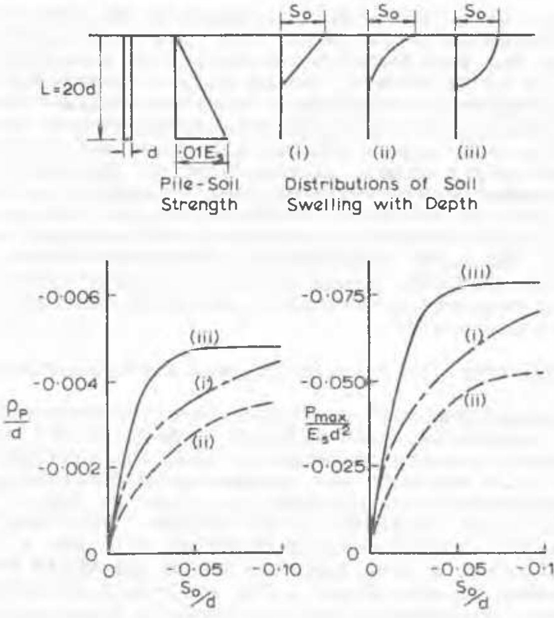
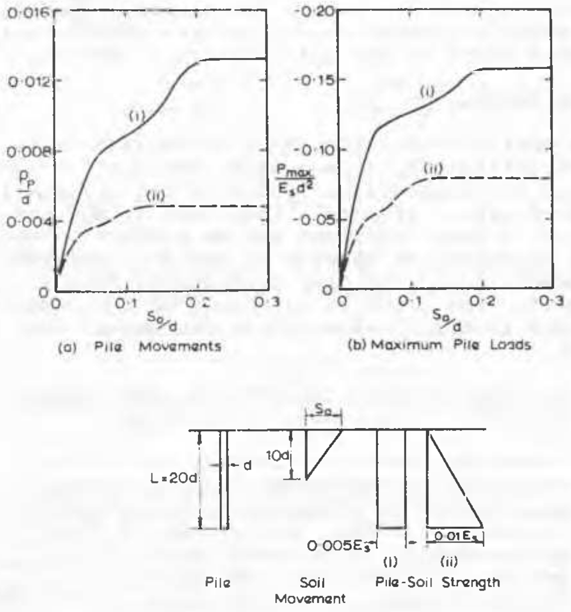


FIGURE 5 INFLUENCE OF SOIL SWELLING PROFILE.

The Effect of Pile-Soil Strength Distribution

In Fig.6 the effect of a uniform distribution of soil-pile strength on the behaviour of a pile with a length to diameter ratio of 20 is compared with that of a strength distribution which increases linearly with depth but has the same average value. For a given soil movement, the pile movement and maximum pile load are considerably greater for the uniform case.



(a) Pile Movements (b) Maximum Pile Loads

FIGURE 6 EFFECT OF PILE-SOIL STRENGTH DISTRIBUTION

The Effect of Axial Load

The effect of axial load on the pile movement is shown in Fig.7. In the case considered, the downward movement increases almost linearly with increasing axial load. Comparison of Figs.7 & 2 shows that the axial load required to prevent upward movement is only about half of the maximum tensile load developed in the uploaded pile. Also shown in Fig.7 are the pile movements calculated on the assumption that the effects of axial load (Poulos and Davis, 1968) and soil swelling can be superposed. Although superposition is not strictly valid in this case because slip occurs along part of the pile shaft, it nevertheless provides an approximate estimate of pile movement. Taking account of pile-soil slip in the solution for axial load would lead to increased movements and closer correlation with the complete solution.

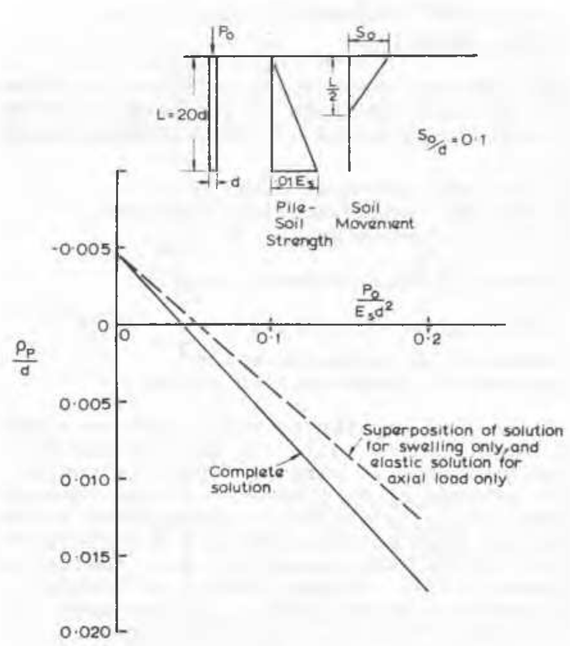
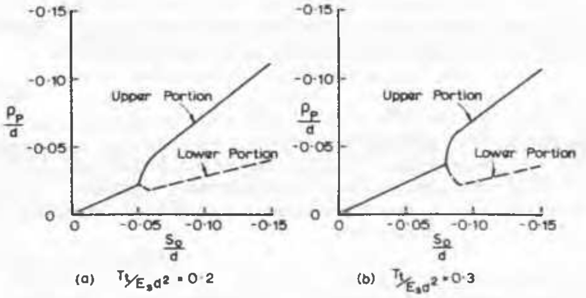


FIGURE 7 EFFECT OF AXIAL LOAD ON PILE MOVEMENT

The Effect of Tensile Failure of the Pile

An example of the effect of tensile failure on pile movement is shown in Fig.8, where pile movement is plotted against soil surface movement for two tensile strengths. When tensile failure occurs, the upper portion moves upward more rapidly than before failure, while the lower part, according to the analysis, suffers a small downward movement at failure and than an upward movement as the soil movement increases. As the tensile strength increases, the soil movement required to cause tensile failure increase also. In this example, the maximum tensile load developed in the pile is 0.393 $E_s d^2$ so that tensile failure will not occur if the tensile strength exceeds this value.



(a) $^{T_t}/_{E_s d^2} = 0{\cdot}2$ (b) $^{T_t}/_{E_s d^2} = 0{\cdot}3$

FIGURE 8 EFFECT OF TENSILE FAILURE OF PILE

It should be noted that the accuracy of solutions in which tensile failure occurs depends to a large extent on the previous history of the pile, since, once failure has occurred and the pile has separated, it remains separated. If too large an increment in soil movement is taken, the point of separation may be inaccurately computed; thus, small increments are desirable for accuracy.

APPLICATION OF THEORETICAL ANALYSIS TO PRACTICAL PROBLEMS

In applying the theoretical analysis described in the previous sections to practical problems, estimates are required of the following quantities:

(i) the soil movement profile
(ii) the pile-soil interface strength
(iii) the soil modulus.

Prediction of Soil Movement Profile

Two cases may be considered:

(a) movement in saturated soils
(b) movements in unsaturated soils.

The first case applies to many problems involving consolidating soils and soil movements may be predicted at various depths by conventional methods of settlement analysis. Methods for predicting movements in unsaturated soils are not as well established, and a variety of approaches has been suggested e.g. Salas and Serratosa (1957), Blight (1965), De Bruijn (1961), Van der Merwe (1964). Approximate methods for estimating variations of pore suction, and hence soil movements, due to moisture changes have been described by Richards (1965) and Blight (1965).

Pile-Soil Interface Strength

It is commonly assumed that the shear strength between soil and pile increases with depth, approximately in proportion to the overburden pressure. In situations where the soil is normally consolidated and is going to remain saturated (zero air voids but not necessarily positive pore water pressure), the limited experience from measurements of negative friction on piles in soft normally consolidated clays (for example Johannessen and Bjerrum, 1965) may be taken to be reasonably applicable to the case when the pile is affected by shrinking or consolidation of the soil. This experience suggests that, in the equation

$$\tau_a = c_a' + K_s \sigma_v' \tan \phi_a' \qquad (6)$$

where c_a' = adhesion
ϕ_a' = effective pile-soil friction angle
σ_v' = effective overburden stress
K_s = a coefficient of horizontal earth pressure,

the combined term $K_s \tan \phi_a'$ usually lies within the range 0.2 to 0.3 and that c_a' can be neglected. For $\phi_a' \approx 20°$ this corresponds to a range in K_s of 0.5 to 0.8, i.e. somewhat greater than the coefficient of earth pressure at rest K_o.

For swelling situations, especially when swelling occurs from relatively arid conditions, the soil must be in an overconsolidated state and, corresponding to the higher values of K_o for overconsolidated soils, a higher value of K_s may be expected. South African experience (Collins, 1953; Baikoff and Burke, 1965; Donaldson, 1967b) appears to suggest a range in K_s of 1.0 to 1.5 in such situations. The cohesion term is also not necessarily negligible. For soils swelling from an initially unsaturated condition the effective stress concept is of doubtful validity and, as a practical approach it may be better to use the total instead of the effective overburden stress in eq.(6) and appropriately determined values of adhesion and pile-soil friction angle. If the soil is severely cracked before swelling starts, the high values of K_s given above may not be attained in the earlier stages of swelling, but, for design purposes, it would be unwise to rely on the reduction in upward force on the pile this implies.

Soil Modulus

The most satisfactory means of estimation is to backfigure E_s from a pile load test in-situ, using the theoretical solutions for an axially-loaded pile. If such a load test is not possible, a rough estimate may be made by using the correlations between E_s and the undrained cohesion c_u of the clay proposed by Poulos (1971). The value of c_u should be that appropriate to the final moisture content of the soil.

COMPARISONS BETWEEN OBSERVED AND THEORETICAL BEHAVIOUR

An extensive series of observations on the movements of thirty-seven houses in South Africa founded on expansive soils was presented by Collins (1958), twenty-four of these being founded on underreamed piles. The houses were divided into two groups, one in which the average load per pile was 5 tons and the other in which the average load was 10 tons, and each house was founded on piles taken on a specific depth. It is interesting to note that the houses on piles founded at 30 ft. depth showed a greater movement than those at 20 or 25 ft. It was subsequently found that the 30 ft. piles had failed in tension. Considerably smaller movements were experienced with the piles carrying 10 ton loads. Theoretical predictions of the behaviour of the piles at the Leeuhof site were made using the measured soil movement profile, the drained strengths to estimate the distribution of pile-soil strength τ_a and the undrained strength to estimate the distribution of soil modulus E_s. A summary of the measured and predicted pile movements is shown in Fig.9, together with the chosen parameters for the analysis. The agreement is good for the piles with 5 ton loads but for the 10 ton loads, the theory somewhat over-estimates the heave. It is significant to note that the theory predicts that tensile failure occurs for the 30 ft. piles, resulting in larger pile movements. This prediction is substantiated by the observations.

Donaldson (1967b) has described an instrumented test pile which was also installed at Leeuhof South Africa and in which the development of tensile force with time was measured. The pile was a 9 in. diameter concrete pile, 34 ft. long, with a central steel pipe core containing strain gauges and acting also as tensile re-

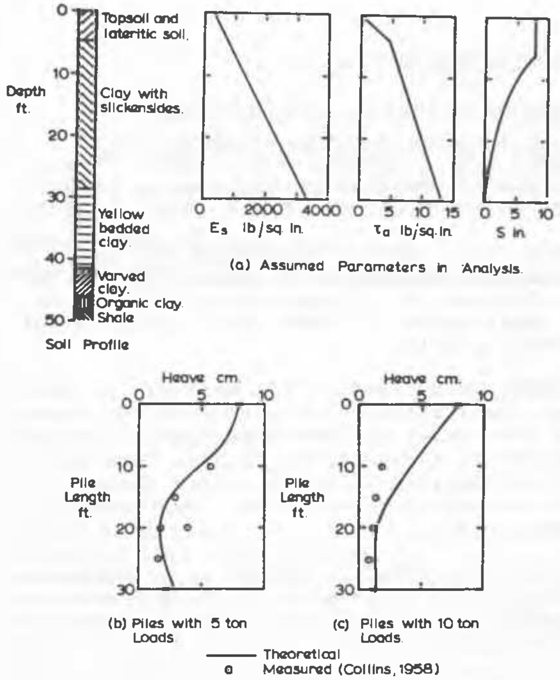


FIGURE 9 COMPARISONS BETWEEN MEASURED AND PREDICTED PILE MOVEMENTS

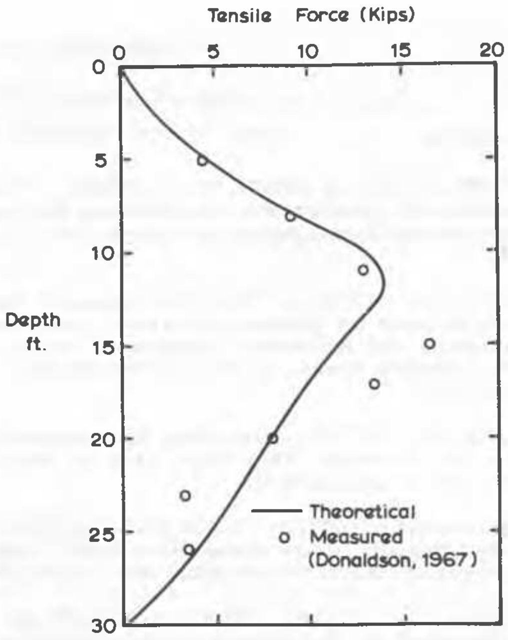


FIGURE 10 COMPARISON BETWEEN MEASURED AND PREDICTED FORCE IN PILE.

inforcement. Heave of the soil around the pile was accelerated by filling, with water, four holes of 4 in. diameter, drilled to depths of 15 ft. at 3 ft. radius from the pile, and hence flooding the site. The tensile force was found to increase significantly as the heave increased. A theoretical calculation was carried out for the latest load distribution above the test pile, using soil data similar to that used for Collins' tests. The soil surface movement remote from the pile was assumed to be 9 mm. A comparison between the theoretical and measured load distributions is shown in Fig.10. Within the obvious limitations imposed by the uncertain soil data, the agreement is satisfactory.

CONCLUSIONS

An analysis has been presented for the movement of, and load distribution in, a pile in a swelling or shrinking soil. The analysis is based on elastic theory, but can be modified to take account of such factors as pile-soil slip and tensile or compressive failure of the pile. From typical solutions obtained for idealized cases, the following main conclusions have been drawn:

(i) in many cases, there is little difference between the behaviour of a pile in a swelling or a shrinking soil, apart from a change of sign in stresses and displacements.

(ii) the most efficient means of suppressing pile movements is either to use a uniform diameter pile founded well below the swelling zone or an underreamed pile founded just below the swelling zone.

(iii) small-diameter piles founded well below the swelling zone are as effective in suppressing upward movements as larger diameter piles.

(iv) an applied axial load of about half the maximum developed tensile load is required to prevent upward movement of the pile.

(v) tensile failure of a pile leads to an increased rate of upward movement if the soil movement continues to increase.

Some suggestions have been made regarding the estimation of the input data required for the analysis of practical cases. Reasonable agreement between theoretical and observed behaviour has been found in some reported field cases.

ACKNOWLEDGMENT

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