MODEL FOR SLAB FOUNDATIONS ON EXPANSIVE CLAYS

UN MODELE DES FONDATIONS A BETON FLEXIBLE OU NON-FLEXIBLE SUR DES ARGILES DILATEES

МОДЕЛЬ ПЛИТЫ ИА НАБУХАЮЩИХ ГЛИНАХ

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SYNOPSIS. The behaviour of unsaturated expansive clay, particularly in relation to soil-structure interaction, is not properly understood either at the theoretical or the practical level and current published work on this subject has little relevance to practical problems.

In this paper a deterministic, mathematical model is described which was developed previously by the author for unsaturated and saturated expansive clay with non-linear properties. It is an extension of Biot's consolidation theory to include unsaturated soils and has been checked satisfactorily against available mathematical solutions for saturated soils. Extrapolation to unsaturated soils still requires field verification.

A mathematical model for soil-structure interaction has now been set up by coupling the above soil model to the structural model for various structures, such as reinforced slab-on-ground foundations. Some preliminary results obtained from this application are described. A hypothetical but realistic example was chosen to show centre-heave behaviour of a slab-on-ground foundation, a phenomenon commonly observed in Australia. Several interesting and possibly important conclusions emerged:

1) thin concrete slabs exhibit sufficient rigidity to reduce differential displacement significantly;
2) the behaviour of slabs is very sensitive to the non-linear and cracking criteria used for the soil;
3) for a rough slab-soil interface, the behaviour of the slab may depart considerably from simple slab theory.

While this model is not proposed as a design procedure or an accurate method of analysis of practical structures, it is presented to show that the whole problem can be approached in a rational manner. It gives a realistic framework within which relevant and important research areas can be defined to lead to a more profitable and efficient application of research effort.

INTRODUCTION

The engineering problems associated with structures built of, on or retaining expansive clays have defied most theoretical and practical attempts made for obtaining a solution. This is particularly true for many areas in Australia where negative pore pressures in clay under sealed road pavements are invariably within the range of 1 Bar to 100 Bars at least (Aitchison and Richards, 1965).

Current published research work bears little relationship to practical problems. On the other hand, many good practical soil engineering techniques, presently employed with expansive clays may, in some circumstances, actually be the cause of serious failures; for example, the use of filter sand or gravel behind retaining walls and gravel under slab-on-ground structures. For the design of these structures as well as large embankments, better physical concepts of the behaviour of unsaturated expansive clay (Richards, 1967, and Richards and Gordon, 1972) and suitable instrumentation and techniques for the measurement of the relevant parameters (Richards, 1968), particularly the psychrometric techniques for soil moisture measurement, make it possible to move away from current empirical *ad hoc* practices.

Since saturated (i.e. pore pressures are positive) clay soils have been extensively investigated (e.g. London Clay), adequate techniques are now available for design procedures. However, when such soils are drained and the pore pressures decrease and become negative, they do not undergo any sudden magical transformation at zero pore pressure.

Their behaviour is similar to that documented for expansive clays and it is reasonable to assume that it is continuous into the unsaturated state. While some more complex boundary conditions and modified soil parameters might have to be taken into account, the extension of existing consolidation theories into unsaturated soils should be possible.

In this paper a deterministic mathematical model is discussed which has previously been developed for the instantaneous and time-dependent deformation of compressible material (either saturated or unsaturated) due to applied loads and changes in pore pressures.* It is an extension of Biot's consolidation theory to permit analysis of unsaturated soils with non-linear properties. When checked against available classical solutions for saturated and unsaturated soils, it gave very satisfactory results, and the Mandel-Cryer effect could be produced accurately. The extension of the model into field problems encountered with unsaturated soils still requires a large number of field trials before it can be used with any confidence. While the details of this model are the subject of a separate publication, a brief summary is given later on.

This model will also be extended to include reinforced concrete structures to enable preliminary analyses of simple soil-structure interaction problems. Its application to reinforced concrete slab-on-ground structures is discussed later on and some simple, typical analyses are shown. This application is extremely important as the stiffness of the slab must obviously have an effect in reducing the often large potential differential displacements that occur in expansive clays. This effect should be the real basis of slab design for houses and other similar structures.

It is not the purpose of this paper to present a fully developed model for design purposes; it rather shows that it is possible to formulate a rational model for effective planning of laboratory and field research programmes that will lead to a more efficient and profitable application of research efforts. In fact, design procedures of the future for small structures will probably need to be much simpler than this model and include probability concepts of what could be called 'risk design' (Lytton, 1971).

CONSOLIDATION MODEL FOR UNSATURATED SOILS

The model developed by the author is similar to the model by Christian and Boehmer (1970) and Sandhu and Wilson (1969) for saturated clays. It will therefore not be described in detail here. The flow chart for the computer programme is shown in Fig. 1. The programme is written in Fortran for the CSIRO CDC3600 computer and uses the finite element

*Richards, B.G. "The transient behaviour of saturated and unsaturated clays under load and changing moisture conditions" (in preparation).

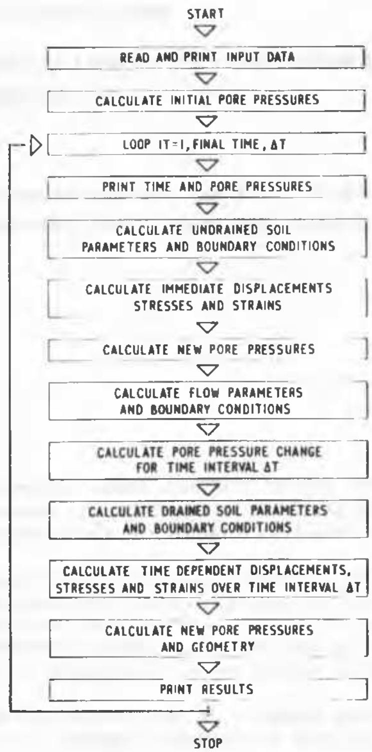

FIG. 1. FLOW CHART FOR COMPUTER PROGRAMME.

method of solution analyses of load-displacements and changes in pore pressures (Zienckiewicz and Cheung, 1967).

For their linear model, Christian and Boehmer (1970) made the assumptions that

a) infinitesimal strains and small velocities exist,
b) Darcy's law is valid for fluid flow,
c) the soil is saturated,
d) the pore fluid is incompressible relative to the compressibility of the soil skeleton,
e) the effective stress principle is valid,
f) there is a linear reversible (i.e. elastic) relation between effective stress and strain.

It proved to be a useful method of solution for Biot's two-dimensional consolidation theory and gave results comparable to published solutions. The model described in this paper was checked against these solutions and gave similar results. An example is shown in Fig. 2 for the MIT plane strain shear testing device (Rixner, 1968).

The assumptions made above can still apply to clays when pore pressures become negative. There are suggestions for example that the intact clay in many of the Australian expansive soils remains practically pore space saturated to negative pore pressures of the

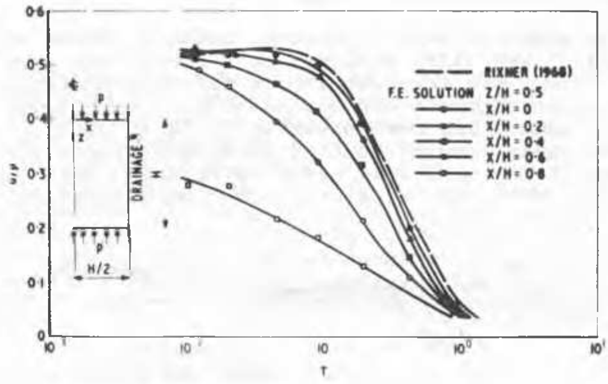

FIG. 2. CONSOLIDATION OF PLANE STRAIN SAMPLE.

order of -100 atmospheres (Holmes, 1956). However, several of these assumptions may not be valid in general and may have to be modified as follow.

A. STRAINS ARE FINITE

Expansive clays can exhibit very large strains (often in excess of 25 per cent). Due to the incremental nature of the solution, the geometry of the problem could be changed for each increment of loading and time interval. This is a step-wise variation of the finite strain theory proposed by Philip and Smiles (1969) where the geometry is related to the soil skeleton and not fixed in space.

B. THE SOIL IS NOT SATURATED

The assumption that the soil is saturated permits two simple equations to be used, viz.

$$\frac{\partial u}{\partial t} = \frac{\partial \left(\frac{\sigma_x + \sigma_z}{2} \right)}{\partial t} \quad \text{in two dimensions} \qquad (1)$$

where u = pore pressure; t = time; σ_x and σ_z = stresses in x and z directions

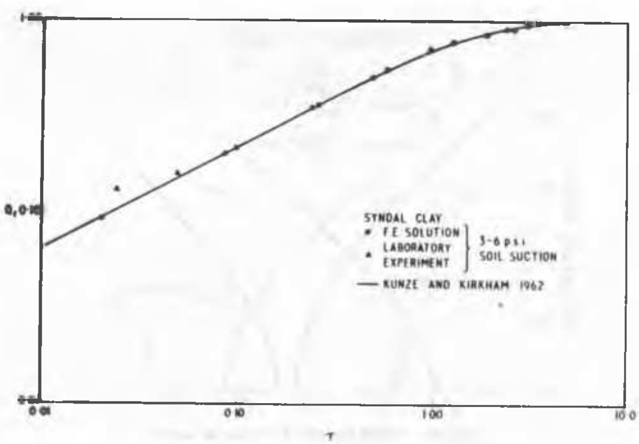

FIG. 3. TRANSIENT FLOW FROM A PRESSURE MEMBRANE SAMPLE.

and
$$\frac{\partial \theta}{\partial t} = \frac{\partial v}{\partial t}\left(\frac{1}{v_0}\right) \qquad (2)$$

where θ = volumetric water content; v = soil volume; v_0 = initial soil volume.

For unsaturated soils, these equations can no longer be assumed to hold and should be replaced by correlations, which can be determined experimentally. In the applications currently being considered by the author, equation (1) has actually been retained as it is approximately correct when the negative pore pressures are of the same order as the applied stresses and the effects of the stresses become insignificant at the lower pore pressures.

The correlations replacing equation (2) can be readily determined experimentally in a number of ways, including the use of the membrane oedometer (Aitchison and Woodburn, 1969). Using such correlations, the model has been applied to unsaturated soils, e.g.to analyse the transient behaviour of a soil sample in the pressure membrane apparatus (Kunze and Kirkham,1962,Richards,1965).Good agree-ment has been obtained as shown in Fig.3.

C. THE EFFECTIVE STRESS LAW IS NOT VALID

While the principles of effective stress still apply, the commonly used effective stress laws may not be valid for unsaturated soils and only total stress concepts have been used. The experimental data must be obtained along the total stress and pore pressure paths similar to those in the field situation. The model itself can be made to closely follow the field paths in an incremental step-wise manner.

D.ELASTIC AND FLOW PARAMETERS ARE NOT CONSTANT

While at any load or time increment, the stress-strain relationships are assumed to be linear and reversible, they will vary with the stress and moisture changes that occur over a period of time. The significant effect moisture has on the moduli of expansive clays has already been investigated (Richards and Gordon, 1972).

Permeability should not be assumed to be a constant since large variations in pore pressures, caused for instance by climatic conditions (Richards and Chan, 1971, Richards, 1972*), result in large variations in permeability (Richards, 1965). The incremental nature of this model permits the moduli, permeability and other parameters to be varied for each load and time increment.

For unsaturated clay, the minor modifications which need to be made to the saturated clay model can be carried out in a rational manner. The changes made to the finite element technique (e.g. finite strains and non-linearity)

*Richards, B.G., "The analysis of road pavements in the Australian environment : Part I. Changes of pore pressure or soil suction" (in preparation).

have already been successfully used in other applications (e.g. Zienckiewicz and Cheung, 1967).

STRUCTURAL MODEL FOR REINFORCED CONCRETE SLAB

The model used in the following analyses was based on that proposed by Ngo and Scordelis (1967). It applies the basic stress-strain relationships in their exact form and avoids the assumptions made in the theory of plates or slabs. As there are obviously insufficient elements available to accurately represent particularly those parts of the slab, where large stress and strain gradients can be expected, resulting in excessively long constant strain elements, the slab itself was checked without the soil as a simple structural member. Being very sensitive to the mesh and boundary conditions assumed, the model always showed instability in the calculated stresses and strains. The solution obtained by the finite element method however gives nodal displacements as the primary result which were always stable, although significantly less than those predicted by the classical beam theory.

A careful check revealed that, for the same mesh and boundary conditions, the deflections derived at by the finite element method were always a constant percentage of those given by the beam theory, irrespective of the loading conditions, for instance, vertical and horizontal point and distributed loads. Therefore, by decreasing the value of the modulus of the concrete used in the finite element analysis below the value required, an exact load-deflection relationship could be obtained according to the beam theory. A reduction in slab thickness also achieved the same result. Fig. 4 shows the results for a slab used in the following analyses under its own weight and an assumed wall loading, but without foundation support other than at the centre-line. The results discussed below suggest that the small variations in slab stiffness would only have a minor effect on the end result.

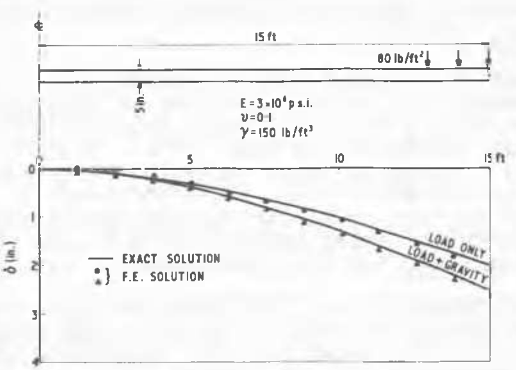

FIG. 4. DISPLACEMENT OF SLAB WITH FIXED SUPPORT AT CENTRE-LINE (TWO-DIMENSIONAL LOADING).

THE SLAB-ON-GROUND MODEL

The complete two-dimensional model as shown in Fig. 5 was first analysed as a non-linear continuum. The adhesion and therefore continuity between concrete and expansive clay when heaving occurs has been observed by Bara (1967). The environmental conditions assumed are typical of the semi-arid areas of southern Australia (Fig. 6). The moisture-flow parameters,

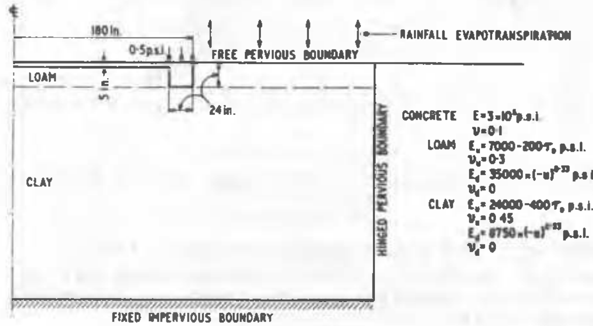

FIG. 5. DETAILS OF SLAB-ON-GROUND FOUNDATION.

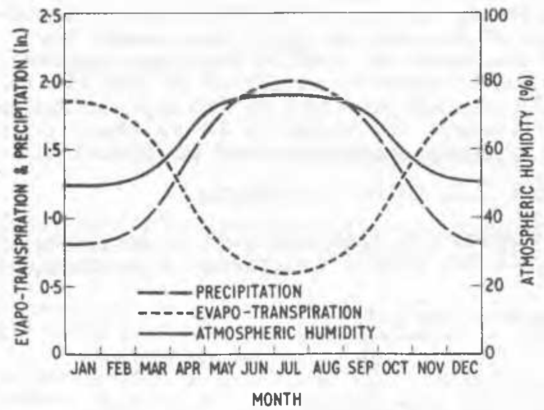

FIG. 6. ENVIRONMENTAL PARAMETERS.

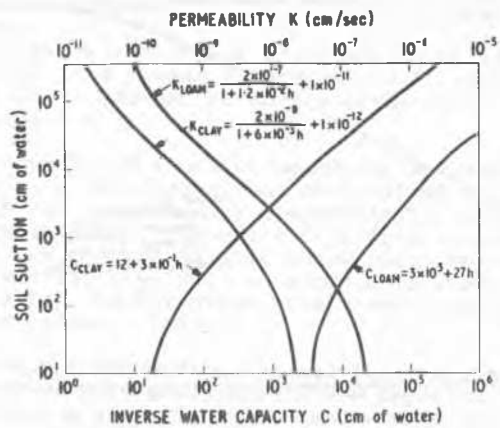

FIG. 7. MOISTURE FLOW PARAMETERS OF SOIL.

presented in Fig. 7, are those of the red-brown earths (one type of expansive clay) found in these semi-arid areas. Fig. 12 shows moisture profiles assumed under natural vegetated surface cover. Starting from the initial conditions (Fig. 8) in July, the results of the pore pressures after 6 months (January) are shown in Fig. 9, and the corresponding surface displacements in Fig. 10, which shows immediate settlements and Fig. 11, which shows the total settlements after 6 months.

An examination of the stresses in the soil revealed very large, obviously unrealistic, tensile stresses, particularly tensile stresses in the vertical direction immediately under the centre of the slab. The second analysis was made with the following specifications for soil behaviour.

a) The soil has tensile strength equal to $E_1/1000$ in the horizontal direction and $E_2/1000$ in the vertical direction.

b) If the tensile strength in the vertical direction was exceeded, E_1 and E_2 were also reduced to 100 p.s.i. and G to zero. This permitted simulation of separation between slab and ground.

c) If the tensile strength in the horizontal direction was exceeded, E_1 was reduced to 100 p.s.i., simulating vertical cracking.

d) If the shear stress exceeded the shear strength (also assumed $E_1/1000$), then E_1 was reduced to 100 p.s.i. and G to zero.

The results of this second analysis are also shown in Fig. 11, indicating that the displacements are considerably reduced. To eliminate a degree of instability observed with time, an additional specification for the soil was introduced into the third analysis. If the specified tensile strength was exceeded, the nodal forces due to volume change were reduced at each increment of time by an amount calculated to reduce the tensile stresses to the tensile strength. This in effect simulates the case, where a soil element can undergo volume decrease until soil fissures open up, after which further volume decrease has little effect on the continuum. This is also one possible mechanism for the χ factor, once commonly used to describe unsaturated soil behaviour.

This third analysis proved to be very stable and gave the displacements shown in Fig. 11.

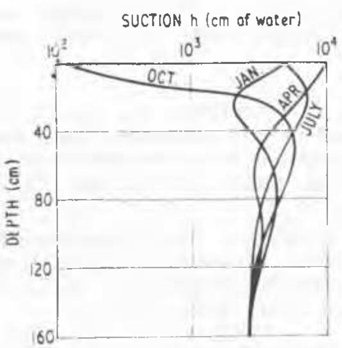

FIG. 8. SEASONAL MOISTURE PROFILES UNDER NATURAL SURFACE COVER.

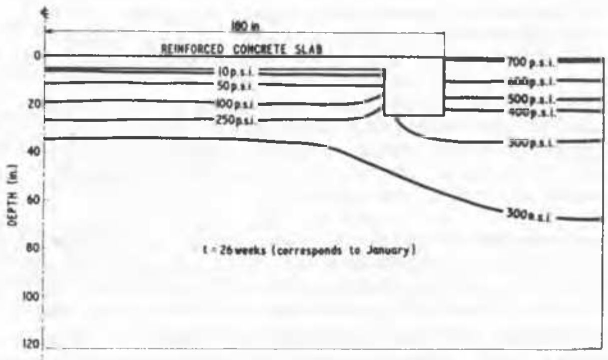

FIG. 9. PORE PRESSURE DISTRIBUTION AFTER SIX MONTHS.

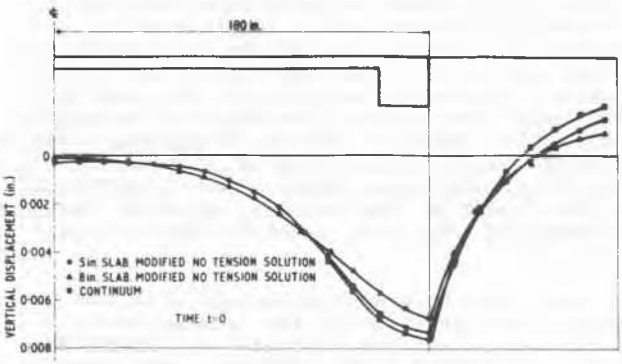

FIG. 10. DISPLACEMENTS AFTER SIX MONTHS.

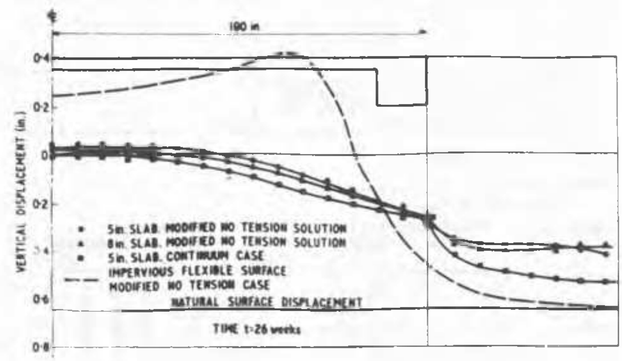

FIG. 11. DISPLACEMENTS AFTER SIX MONTHS WITH SIMPLE AND IMPROVED NO-TENSION CRITERIA.

It was therefore repeated, with the slab elements having the stiffness of the soil but the permeability of the concrete. The fourth analysis was carried out to compare the behaviour of an impervious flexible surface with that of the more rigid slab; the results are also shown in Fig. 11. These results do indicate the significant effect that a concrete slab can have in reducing potential differential displacements in expansive clays. As a point of interest, the same problem was rerun for an 8 in. thick concrete slab. The results are included in Fig. 11 and suggest that increasing the slab thickness from 5 in. to 8 in. would have little effect on the performance of the superstructure or house. This is not entirely unexpected due to the assumed linear relationship between strain and the logarithm of the soil suction and therefore swell pressure.

The results in Fig. 11 also give a maximum differential displacement of 1 in 360 and an average over the half width of the slab of 1 in 600. These figures are satisfactory for the brick veneer construction used in Australia, but probably not for double brick construction (Aitchison, 1970).

Although the model for the slab gave flexure stresses which showed instability and were lower than those obtained from the beam theory, they could be correlated with the theoretical stress data. The estimated stresses for time zero and at six months are shown in Fig.12. With realistic soil stiffnesses, the so-called neutral axis was located below the base of the slab at the centreline, i.e. near the region of maximum tensile stress. Since the maximum tensile stress in the 5 in. and 6 in. slabs after 6 months is of the order of the normally expected tensile strength of concrete, cracking can be expected.

To check the interaction between soil and slab on the position of the neutral axis, several analyses of immediate settlement were made using reduced soil modulus. The results for zero and a low soil modulus, shown on Fig. 12, suggest that at low soil stiffnesses, the beam acts according to the classical beam theory. At higher stiffnesses, usually exhibited by expansive clays in their natural state, the slab and soil begin to act more in a composite manner, with the neutral axis shifting significantly from that determined by the beam theory.

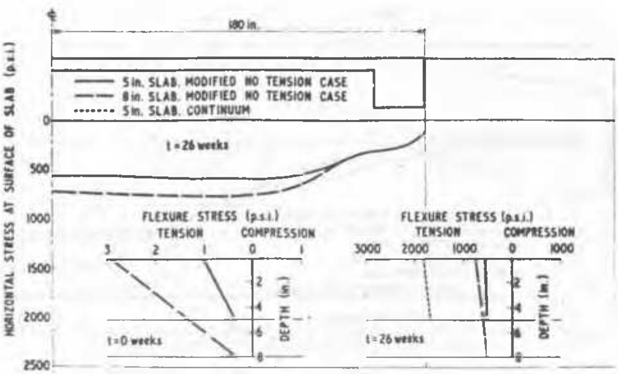

FIG. 12. STRESSES IN SLAB.

CONCLUSIONS

The model described in this paper is capable of assessing slab-on-ground behaviour of expansive clays. The results generally agree with the author's experience of the behaviour of expansive clays in semi-arid areas. The development of the model for accurate quantitative purposes will only be possible when more relevant input data for material properties and boundary conditions can be obtained and assessed in suitable field trials. Attempts in this direction are currently being undertaken. More accurate and efficient finite element techniques will necessarily have to be developed for quantitative analyses.

REFERENCES

AITCHISON, G.D. (1970), Private communication.

AITCHISON, G.D. and RICHARDS, B.G. (1965)., "A broad-scale study of moisture conditions in pavement subgrades throughout Australia" in "Moisture equilibria and moisture changes in soils beneath covered areas", Butterworths Sydney, pp. 184-232.

AITCHISON,G.D.and WOODBURN , J.A. (1969), "Soil suction in foundation design", Intern. Conf. on Soil Mech. Fdn Engng, Mexico City, Mexico.

BARA, J.P. (1967), Discussion on "Failure mechanism of canal lining in expansive clay" by Kassif, G.Etkin, A.and Zeitlen, J.G., J. of Soil Mech.and Fdns Div., Proc.A.S.C.E 95(SM5), pp. 354-356.

CHRISTIAN, J.T. and BOEHMER, J.W. (1970), "Plane strain consolidation by finite elements", J. Soil Mech. and Fdns Div., Proc. A.S.C.E. 96(SM4), pp. 1435-57.

HOLMES, J.W. (1956), "Water sorption and swelling of clay blocks", J. Soil Sci., 6(2) pp. 200-208.

KUNZE, R.J. and KIRKHAM, D. (1962), "Simplified accounting for membrane impedance in capillary conductivity determinations", Proc. Soil Sci. Soc. Amer. 26(5), pp. 421-426.

LYTTON, R.L. (1971), "Risk design of stiffened slabs", Proc. Conf. on Applications of Statistics and Probability to Soil and Structural Engineering, Hong Kong.

NGO, D. and SCORDELIS, A.C. (1967), "Finite element analysis of reinforced concrete beams" A.C.I. Journal, Proc. Vol.64, No.3, pp.152-163

PHILIP, J.R. and SMILES, D.E. (1969), "Kinetics of sorption and volume change in three-component systems", Aust. J. Soil Research, Vol. 7, No. 1, pp. 1-19.

RICHARDS, B.G. (1965), "Determination of the unsaturated permeability and diffusivity functions from pressure plate outflow data with non-negligible membrane impedance", in "Moisture Equilibria and Moisture Changes in Soils beneath Covered Areas", Butterworths, Sydney, pp. 47-54.

RICHARDS, B.G. (1967), "Moisture flow and equilibria in unsaturated soils for shallow foundations", in "Permeability and Capillarity of Soils", A.S.T.M., STP 417, pp. 4-34.

RICHARDS, B.G. (1968), "Review of measurement of soil water variables and flow parameters", Proc. 4th Conf. A.R.R.B., Vol. 4, part 2, pp. 1843-61.

RICHARDS, B.G. and CHAN, C.Y.L. (1971), "Theoretical analyses of subgrade moisture under Australian environmental conditions and their practical implications", Aust. Road Res., Vol. 4, part 5, pp. 32-49.

RICHARDS, B.G. and GORDON, R. (1972), "Prediction and observation of the performance of a flexible pavement on an expansive clay subgrade", paper accepted for publication Proc. 3rd Intern. Conf. on Structural Design of Asphalt Pavements, London.

RIXNER, J. (1968), Pore pressure dissipation under plane strain conditions, unpublished term paper MIT.

SANDHU, R.S. and WILSON, E.L. (1969), "Finite element analysis of seepage in elastic media", J. Engng Mech. Div., Proc. A.S.C.E. 95(SM3), pp. 641-652.

ZIENCKIEWICZ, O.C. and CHEUNG, Y.K. (1967), "The finite element method in structural and continuum mechanics", McGraw-Hill, London.