SEQUENTIAL FAILURE IN STRAIN-SOFTENING SOILS

DEFAILLANCE SEQUENTIELLE RELATIVE A LA DIMINUTION DE LA DEFORMATION DES SOLS

ПРОЦЕСС РАЗРУШЕНИЯ ГРУНТОВ, УМЕНЬШАЮЩИХ ПРОЧНОСТЬ ПРИ ДЕФОРМАЦИИ

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SYNOPSIS - A hypothesis is developed which indicates that failure in simple slopes and embankments involving strain-softening materials necessitates only line ruptures to produce a kinematically admissible mechanism. A classification of such failures in terms of the number of line ruptures is proposed. The failure of a slope at Aberfan in South Wales is examined in the light of the hypothesis and, using the author's clastic analysis it is shown that a factor of safety of less than one is predicted for a third order mechanism described as sequential. The case of a granular embankment on a strain-softening foundation is examined and a simple design procedure to deal with a second order mechanism involving a spreading type failure is presented.

THE LINE-RUPTURE HYPOTHESIS

Consider the equilibrium of the body shown in Figure 1 which is subjected to a general external traction T and also undergoes a kinematically admissible virtual displacement Δ along a discontinuity. It is implied in the first instance that there is no gross geometric anomaly in the shape of the discontinuity due to inhomogeneity in the material. It should also be noted that there is no limitation on Δ other than that the original geometry of the body is not significantly altered.

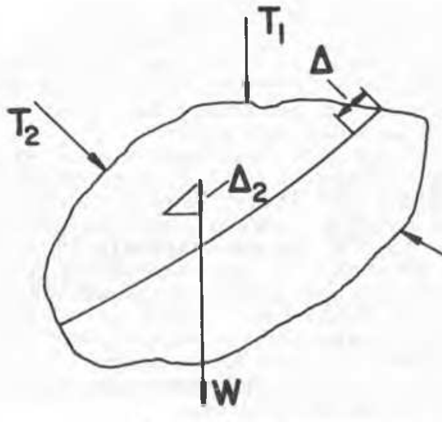


FIGURE 1 - THE GENERAL BODY

As a result of this virtual displacement the external work done is

$$\int_A T|\Delta_1|dA + W|\Delta_2| \tag{1}$$

where Δ_1 represents the surface displacements associated with T and Δ_2 is the vertical displacement of the centroid of the body above the discontinuity. (It is assumed that the centroid of the lower part does not change position.)

The internal work dissipated is

$$\int_V dW\, dV + \Sigma \int_{A_d} \tau|\Delta|dA_d \tag{2}$$

where dW is the work done in a volume element dV on the general stress system (σ^c) and τ is the shear stress on the discontinuity of area A_d. (The summation sign indicates there may be more than one discontinuity required to produce a kinematically admissible mechanism.) Thus, for equilibrium

$$\int_A T|\Delta_1|dA + W|\Delta_2| = \int_V dW\, dV + \Sigma \int_{A_d} \tau|\Delta|dA_d \tag{3}$$

In order to test for stability of the body it is necessary to minimise the right hand side of equation (3) and to assign limiting values to the parameters involved (generally a limiting shear strength). Where surface tractions are included, as for example in retaining wall problems, the determination of the various terms in equation (3) becomes quite complex.

The unloaded slope however, is a case of considerable practical interest and is much simpler to handle. Putting T = 0 in equation (3) we get

$$W|\Delta_2| = \int_V dW\, dV + \Sigma \int_{A_d} \tau|\Delta|dA_d \tag{4}$$

Now if (σ^c) represents the set of generalised stresses these may be subdivided into the hydrostatic or spherical components $(\sigma_s{}^c)$ and the deviatoric component $(\sigma_d{}^c)$, so that we may rewrite

$$\int dW\,dV = \int dW_s\,dV + \int dW_d\,dV \qquad (5)$$

where dW_s is that part of the internal work contributed by the spherical components of stress and dW_d that contributed by the deviatoric components. Thus equation (4) becomes

$$W|\Delta_2| = \int_V dW_s\,dV + \int_V dW_d\,dV + \Sigma \int_{A_d} \tau|\Delta|dA_d \qquad (6)$$

Perfectly Plastic Materials. Consider now the perfectly plastic material which has a limiting shear strength C and is incompressible. In this case the first term on the right hand side of equation (6) vanishes as the spherical components do no work. If we now want to minimise the remaining two terms we note that the third term, by definition, cannot be zero if a slip mechanism is to occur, but that the second term does approach zero if no plastic deformation occurs within the body. This implies that everywhere, except on the discontinuity the internal stresses lie within the yield surface in principal stress space. The important conclusion is reached therefore that, for the ideal perfectly plastic material instability will tend to develop on a single surface discontinuity and that the remainder of the body will act as a rigid entity. It can readily be shown that for the simple two dimensional condition the discontinuity will take the form of a line of constant curvature (line rupture) and it is well known that a circular arc provides the critical shape. This forms the basis of the well established circular arc method of stability analysis for $\phi = 0$ materials.

Strain-Softening Materials. There are many materials which exhibit the phenomenon, of strain-softening. Such material show a characteristic loss of strength when strained past the condition where the peak strength is obtained.

For simplicity it will be assumed that the total strains (volumetric and shear) are negligible in the pre-peak range.

The first class of materials to be considered is where dilation occurs in the strain-softening range. Examples are - dense sand and overconsolidated clays. It is obvious that if such materials are strained past the peak strength condition then the term $\int_V dW_s\,dV$ in equation (6) will not be zero as work will be done against the ambient stresses as dilation occurs. Thus for this term to be zero the volume of material involved must be zero, so that the zone of failure degenerates to one of zero volume, i.e. a line rupture. The requirement of rigid body movement again implies that the line of rupture will be of constant curvature.

The next class of materials is represented by (saturated) collapsing soils and quick clays. Here the internal work is dissipated by the tendency to decrease the void volume against the pore pressure increment generated by the structural rearrangement of the constituent particles. Thus, in the work sense, these soils are similar to those which dilate and similar conclusions with regard to the nature of the initiating failure mechanism are warranted. As such failures develop however the post-initiation deformations are very different from those in dilating materials. The latter tend to be stable outside the first rupture surface whereas the collapsing materials become increasingly unstable and the characteristic liquefaction or flow slide develops.

Finally, there is the class of brittle materials where the loss of surface energy associated with crack formation may be represented as a transient equivalent crack pressure which does work against the ambient stresses in a manner similar to that done by the pore pressure generated in collapsing soils. Space limitations preclude a more detailed examination of this phenomenon here however and such materials will not be considered further.

It will be apparent that for the above materials, the requirement of line rupture only would hold if a single slip surface is considered even if the strength of the material in slip remained constant. When the strength reduces with continued slip the requirement becomes even more definitive.

If more than one slip surface is required to produce a kinematically consistent mechanism it is not immediately obvious whether failure will occur by developing the additional slip surface(s) or whether zone distortion will contribute to a lower work situation and still permit the necessary deformation. Such a possibility appears to be remote in most slope or embankment situations. The exception could be the situation where a gross inhomogeneity occurs and the surrounding material is either quasi-plastic or only slightly strain-softening. For example a thin lens of soft clay may be surrounded by very much stronger material or a weak material may overlie a very much stronger one. In both cases the geometry of the preexisting discontinuity may vary significantly from the the condition of constant curvature. If the contrast in strength is sufficiently great slip will tend to take place along the surface of weakness and this will not then be of constant curvature. In such circumstances there will be complex failure surfaces (or perhaps zones) in the material above the primary discontinuity. Approximate solutions may be obtained by fitting a series of straight lines or arcs of circles to produce the required kinematic viability. In this way account is taken of the work that has to be done in deforming the material above the primary discontinuity.

It should be pointed out that stability analyses which claim to deal with non-circular slip surfaces but do not take into account the necessary internal distortions (Morgenstern and Price (1965) can be seriously in error, although the error is fortunately on the safe side. Similarly the use of these methods by Skempton and Hutchinson (1969) in an attempt to correlate laboratory test results with actual failures is open to question.

In general it may be hypothesised that for strainsoftening materials-

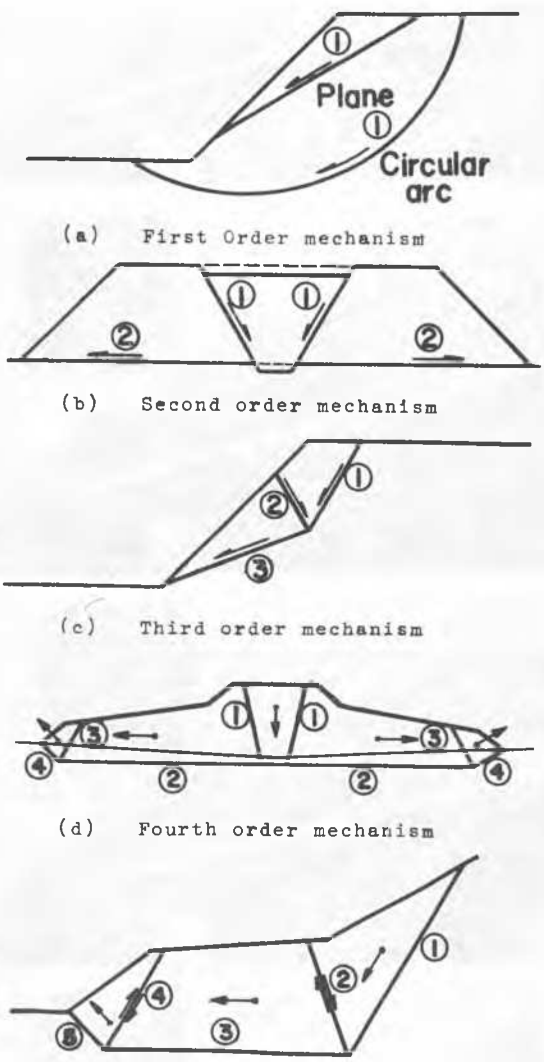
1) collapse will tend to occur on a single rupture surface or on the minimum number of such surfaces as will provide the least resistance in akinematically admissible mechanism.

2) the rupture surfaces will tend to be of constant curvature, i.e. plane (straight lines) or parts of spheres (circles).

MECHANISMS OF SLOPE FAILURE

It is convenient from a practical point of view to classify slope failures in terms of the number of surfaces involved in producing a kinematically admissible mechanism.

A suggested classification scheme is shown in Figure 2.



(a) First Order mechanism

(b) Second order mechanism

(c) Third order mechanism

(d) Fourth order mechanism

(e) Fifth order mechanism

FIGURE 2 - COMPOSITE MECHANISMS FOR SLOPE COLLAPSE

THE ABERFAN DISASTER

An example in which a third order mechanism is considered to have been involved is that of the failure of a colliery waste tip at Aberfan, South Wales, in 1966. Bishop et al (1968) have described the relevant geometry and material properties and the following analysis is based on this information. It is assumed that the general stresses within the slope can be described by the methods of elastic analysis previously given by the author (Trollope, (1968)). Although these methods were derived with the idealising assumption of an infinite wedge, Burman (1972) has shown that when finite wedges are considered and taking into account low shear stiffness at particle contacts, the simplified methods give a good approximation. In particular, the full arching condition appears to be a reasonable upper limit to the likely stress distributions.

The simplified geometry is shown in Figure 3. Assume that the distribution angle θ is 30° and draw BC at this angle to BD. The stresses at C can now be calculated and are -

No arching	σ_x	=	$0.23\ \gamma Z_1$
	σ_z	=	$0.70\ \gamma Z_1$
	τ_{xz}	=	$0.18\ \gamma Z_1$
Full arching	σ_x	=	$0.77\ \gamma Z_1$
	σ_z	=	γZ_1
	τ_{xz}	=	$0.58\ \gamma Z_1$

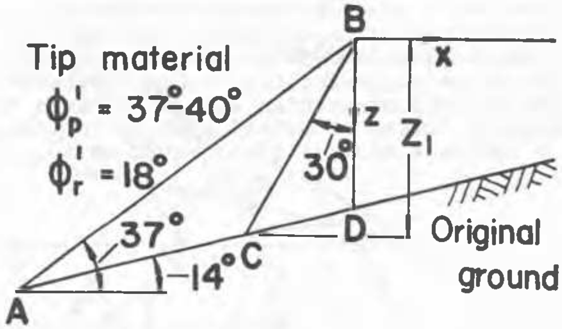


FIGURE 3 - THE ABERFAN SLOPE

If a condition of no-arching obtains in the slope then the required value of the effective friction parameter for stability (ϕ'_{req}) is 39° and for full arching 42°. Under no arching conditions the value of ϕ'_{req} along AC is 24° and for full-arching 38°. Because of the linear nature of the stress solutions a constant value of ϕ'_{req} obtains along AC in each case. The worst case for design is obviously full arching and it should be noted that Bishop et al (loc cit) use the value of $\phi' = 38°$. Thus the factor of safety along AC is 1 even with zero pore pressure.

This analysis predicts therefore that failure is likely to be initiated along AC with the slightest increase in pore pressure in this region. It has been established that a spring existed close to point C and it is therefore postulated that the flow of water along AC is sufficient to tend to generate arching stresses and to reduce the strength to the failure condition.

If the material along AC has plastic characteristics and does not show strain-softening effects then total failure of the slope will not necessarily occur. Indeed one can imagine a condition where the strength along AC could be reduced to zero and the slope would still stand if the overlying material were strong enough. This is because failure must develop along at least one other surface to meet the kinematic requirements of a failure mechanism.

In the Aberfan situation however it is obvious that failure will develop due to the strain-softening effect. To maintain equilibrium a ϕ'_{req} of 24° is needed whereas the residual value of ϕ'_r given by Bishop et al (loc cit) is 18°. Hence in this case the very simple calculations outlined above would predict failure. For the more general case the simple graphical 'wedge' method may be used to assess the overall stability. In Figure 4 the polygon of forces is constructed with the assumption that full strain-softening is developed along AC so that the resulting reaction R_1 is inclined at the residual value of ϕ'_r to the normal. Similarly R_2 is assumed to be inclined at the peak value of ϕ'_p to the normal BC. The value of ϕ'_{req} along EC is then determined and the ratio $\frac{\tan \phi'_{req}}{\tan \phi'_p}$ for the material along EC gives the required factor of safety.

For the Aberfan case the calculated factor of safety is 0.8 again indicating instability even if the most favourable assumptions regarding peak strength mobilization on BC and EC are used. Before this latter assumption can be used with confidence however much more information on stress-deformation characteristics is required than is presently available. For the present it is suggested that design in cases similar to that at Aberfan should be based on an adequate factor of safety against initiation of failure along surfaces such as AC.

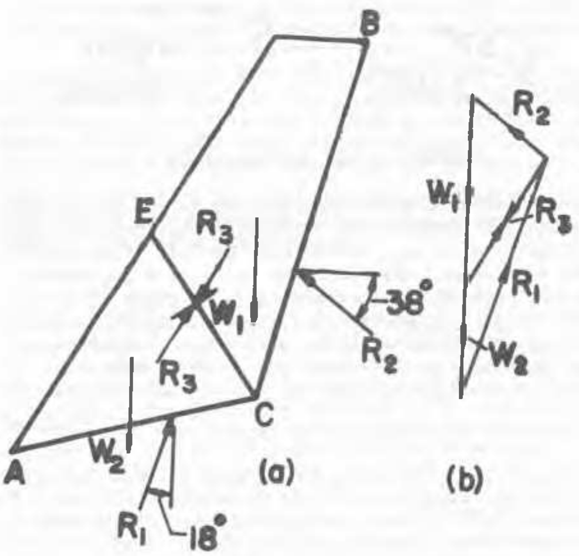


FIGURE 4 - EQUILIBRIUM OF 3rd-ORDER FAILURE MECHANISM

Experimental confirmation of the mechanism described above is given in Figure 5. These photographs have been reproduced from Bishop et al (loc cit) and show clearly the sequence of failure developing along surface similar to AC, followed by one similar to EC in Figure 5. The reasons for describing such a mechanism as sequential failure are obvious from these illustrations.

Figure 5(d) is of particular importance. It will be seen that well after the primary sequential failure is completed the pattern of dark sand lines shows a quasi circular arc configuration.

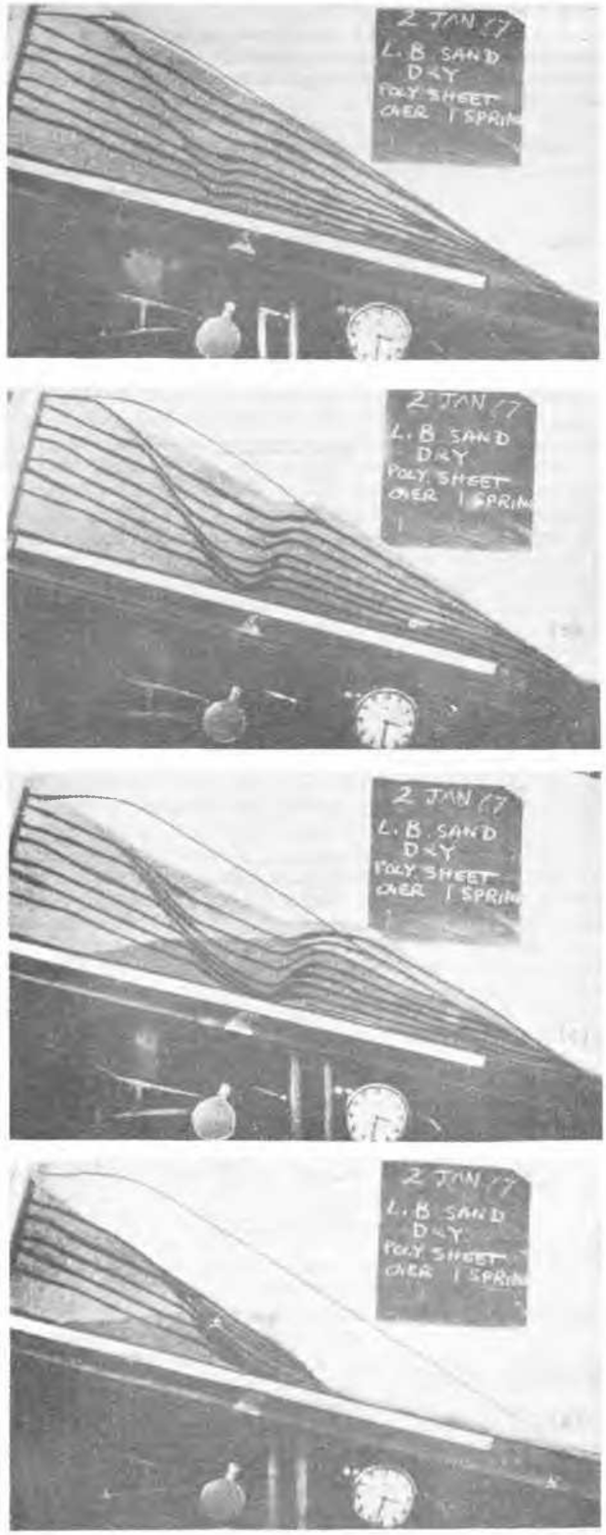


FIGURE 5 - THIRD ORDER FAILURE MECHANISM IN SAND MODEL (FROM BISHOP ET AL, 1968)

This evidence suggests that post failure examination of a slope might reveal disturbed surfaces which would lead to the assumption that failure had occurred on a circular arc surface. Such an assumption would clearly be in error.

GRANULAR EMBANKMENT ON A STRAIN-SOFTENING FOUNDATION

The building of granular fill embankments on soft clay foundations frequently gives rise to serious stability problems. Inevitably the building up of the fill causes consolidation of the clay and the resulting base deformation is of the type that generates arching stresses in the fill.

Consider the embankment shown in Figure 6.

$\tan\theta = \frac{1}{\sqrt{3}}$ then the minimum value of i is given by $\cot_i = \frac{7}{\sqrt{3}}$ whence $i = 14°$.

For slopes flatter than 14° in this material account must be taken of the fact that the critical stress at C may be reached before a condition of full-arching develops ($k > 0$) and the magnitude of the shear stress that can be developed along the base is consequently limited.

The value of the above analysis lies in that it permits an estimate of the maximum shear stress that can be developed at the base of the embankment. Its application in practice is relatively simple. Once the position of D is determined, then from statics, the total shear force along AD is equal to a

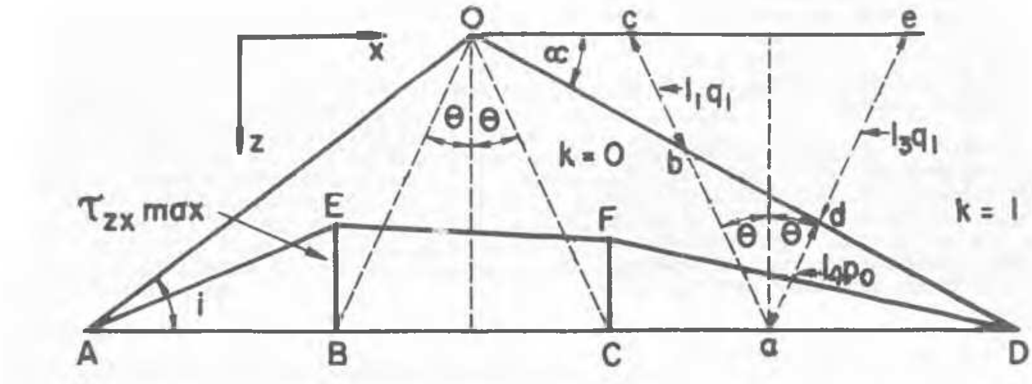


FIGURE 6 - CLASTIC SHEAR STRESSES ON BASE OF EMBRANKMENT

It is assumed that, owing to deflection of the base a state of full arching is established throughout the embankment then as we move inwards from A to D the horizontal stress continues to grow. Obviosly this cannot continue indefinitely as the strength of the fill material will be exceeded. (Trollope and Morgan, (1959)). Suppose the failure condition is just reached at D, then by putting $k = 1$ for the material to the right of D a constant state of stress on the base past D will be obtained. The line OD is the locus of all points such as D. Thus the embankment is divided into two zones - AOD within which a full-arching condition ($k = 0$) is assumed and to the right of OD where no-arching conditions exist.

It can readily be shown, using the method given in Trollope (1968) that

$$\cot\alpha = \left[\frac{1+\sin\phi}{(1-\sin\phi)\tan\theta} - \tan\theta - \cot_i\right] \qquad (7)$$

and the associated shear stress along the base AD is shown in Figure 6. It may now be postulated that if the shear stress along AD is sufficient to cause slip, then a second order failure mechanism may develop with simultaneous slip along AD and OD.

The above analysis applies provided D coincides with or is to the right of C. If $\phi = \theta = 30°$ equation (7) reduces to

$$\cot\alpha = \frac{8}{\sqrt{3}} - \cot_i \qquad (8)$$

and if the limiting value of $\cot\alpha$ is

horizontal force applied to a vertical surface through D and the value of the horizontal force (H) is given by

$$H = \frac{1}{2}\gamma Z^2 \tan^2\left(45 + \frac{\phi}{2}\right) \qquad (9)$$

i.e. the passive thrust within the embankment.

$$\text{Thus } H = \frac{\gamma Z^2}{2}\tan^2\left(45+\frac{\phi}{2}\right) = C.\overline{AD} \qquad (10)$$

where C is the average shear strength along AD and $AD = Z(\cot_i + \cot\alpha)$

If $\cot\alpha \geqslant \tan\theta$ then from (7)

$$AD = Z\left(\cot_i + \frac{\tan^2(45+\frac{\phi}{2})}{\tan\theta} - \tan\theta - \cot_i\right)$$
$$= Z\left(\tan^2(45+\tfrac{\phi}{2})\cot\theta - \tan\theta\right) \qquad (11)$$

so that

$$Z = \frac{2C}{\gamma}\left(\cot\theta - \frac{\tan\theta}{\tan^2(45+\frac{\phi}{2})}\right) \qquad (12)$$

Equation (12) leads to the interesting result that provided the slope is above a critical angle the permissible height of an embankment with respect to the failure mechanism being considered is independent of the actual slope.

When the slope i is less than this critical angle then

$$\frac{\gamma Z^2}{2} \tan^2(45+\frac{\phi}{2}) = CZ\ (\cot_i + \tan\theta$$

and

$$Z = \frac{2C}{\gamma} \frac{(\cot_i + \tan\theta)}{\tan^2(45+\frac{\phi}{2})} \qquad (13)$$

and the required value of i for a given height Z can be determined from equation (13).

In connection with the above analysis it should be noted that the second order mechanism is not the only possible mechanism. The well known first order mechanism - that of a circular slip - must also be considered and may be critical in which case the height of the slope does depend on the slope angle.

The method outlined above is particularly applicable to conditions where the foundation material is sensitive or strain-softening as initiation of failure by spreading along the base will generally be followed sequentially by failure within the embankment behind the initial movement.

If the foundation material has plastic properties however, the arching mechanism can be destroyed by internal stress redistribution following yield along the base. In this case the horizontal thrust drops dramatically to the active value. This mechanism is thought to be responsible for longitudinal cracking in many embankments if the embankment material has a low tensile strength.

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