THE RESISTANCE CONCEPT APPLIED TO DEFORMATIONS OF SOILS

APPLICATION DU CONCEPT DE RESISTANCE A DEFORMATION DES SOLS

N. JANBU, Professor of Soil Mechanics and Foundation Engineering

The Technical University of Norway, Trondheim, Norway

SYNOPSIS. Several years of research have shown that resistance concepts are powerful and instructive tools in clarifying the stress- and time-dependent behavior of soils under compression, swelling or recompression. The stress-resistance (tangent modulus) has been dealt with previously while the time-resistance is used herein for the first time. It is also demonstrated that the coefficient of consolidation is in reality a relative resistance. Because the resistance concept is rationally defined, in familiar engineering and mathematical language, the corresponding formulae for predicting stress- and time-dependent settlement become simple, and can be expressed in terms of simple dimensionless parameters with well-defined mechanical meanings. Therefore, these parameters are simple to explore by laboratory tests. As important biproducts, the resistance concepts may render several new ways of determining the preconsolidation load.

INTRODUCTION

All media possess a resistance against a forced change of existing equilibrium conditions. The resistance of a medium, or of an isolated part of it, can be determined by measuring the incremental effect of a given incremental cause, whereafter

$$\text{Resistance} = \frac{\text{Incremental cause (given)}}{\text{Incremental effect (measured)}} \qquad (1)$$

Examples: Electric resistance (R or ρ), elastic resistance (E), dynamic resistance (mass), hydraulic resistance (k^{-1}) and heat resistance (C).

For non-linear processes, the resistance is in general defined as the tangent to the cause-effect curve. For linear cause-effect relationships the resistance degenerates to a constant or proportionality, without changing the basic definition.

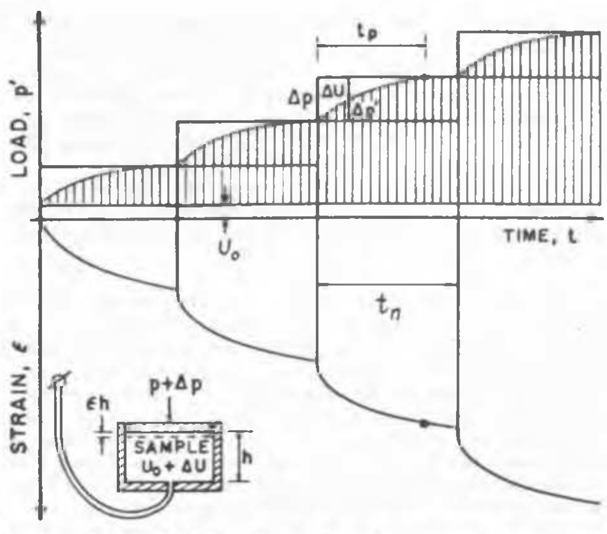

Fig. 1 Obtainable readings in oedometer tests.

Also soils respond to forced changes in existing equilibrium conditions. As schematically illustrated in Fig. 1 the strain ε which develops over an elapsed time t after the application of a stress change Δp, is a function of stress and time, hence

$$\varepsilon = f\ (p't) \qquad (2)$$

The manner in which strain develops with time and stress can best be studied by separate analysis of the soil resistances with respect to time and stress, respectively.

DEFINITIONS

(a) For each load increment (total p = constant) the <u>arithmetic</u> t-ε-plot will render the <u>time resistance</u> R, defined as

$$R = \frac{dt}{d\varepsilon} \qquad (3)$$

Usually R varies with time and stress. For clays in particular it has been found that the <u>long term time resistance</u> varies linearly with time, so that for $t \geq t_p$

$$r = \frac{dR}{dt} = \text{constant} \qquad (4)$$

This dimensionless, long term time resistance is the main parameter required for predicting "secondary" deformations of clays and fine silts.

(b) The overall stress-strain result can be studied when all load increments have acted over a specified length of time, say t_n = constant. It is strongly recommended to plot <u>arithmetic</u> p'-ε-curves, from which one obtains the <u>stress resistance</u> (or tangent modulus) M, defined as

$$M = \frac{dp'}{d\varepsilon} \qquad (5)$$

Usually M varies with stress. For stresses above the preconsolidation load of clays, it has been found that the virgin stress resistance M varies linearly with time, so that for $p' \geq p'_p$

$$m = \frac{dM}{dp'} = \text{constant} \tag{6}$$

This dimensionless virgin stress resistance (modulus number), is the main parameter required for predicting the virgin deformation of clays and fine silts.

(c) Immediately after a load application on saturated clays, when $t < t_p$, the strain-time behavior may be governed by a primary consolidation process. If so the <u>coefficient</u> <u>of</u> <u>consolidation</u>, c_v, is a main parameter in defining the early phase of the strain-time-relationship. By definition

$$c_v = \frac{M}{S} = \frac{\text{stress resistance}}{\text{seepage resistance}} \tag{7}$$

in which case

$$S = \frac{\gamma_w}{k} = \frac{\eta}{K} \tag{8}$$

where η = viscosity (g sec/cm^2)
K = permeability (cm^2)
k = coeff. of permeability (cm/sek)
γ_w = unit weight of water (g/cm^3)

It may be of some interest to note that the seepage resistance itself is a product of a fluid resistance (viscosity) and a grain structure resistance (K^{-1}). The most practical dimension of c_v is square meters per year (or square feet per year).

For stresses below preconsolidation both M and c_v are fairly constant for clays, hence for $p' < p'_p$

$$S = \frac{M}{c_v} \tag{9}$$

For stresses well above the preconsolidation load, $p' \gg p_p'$, one often observes that c_v increases linearly with effective stress p', and so does M. Therefore, by differentiating Eq. (7), in the form $M = mp' = Sc_v$, one obtains S from the slope of the $c_v - p'$ curve.

$$S = m\frac{dp'}{dc_v} \tag{10}$$

When the seepage resistance is thus known, the coefficient of permeability becomes,

$$k = \frac{\gamma_w}{S} \tag{11}$$

The principles of determination and application of the resistances defined above will be demonstrated below. Due to space limitations only clays will be considered.

STRESS RESISTANCE AND STRESS-STRAIN FORMULAE

The upper part of Fig. 2 illustrates the arithmetic stress-strain-curves for two clay samples tested in an oedometer. The present effective overburden is denoted p_o'. For zero stress the extrapolated curve may show some strain $\Delta\varepsilon_o$, which is indicative of sample disturbance and "contact settlement", eg because of insufficient fitting of the sample in the test apparatus.

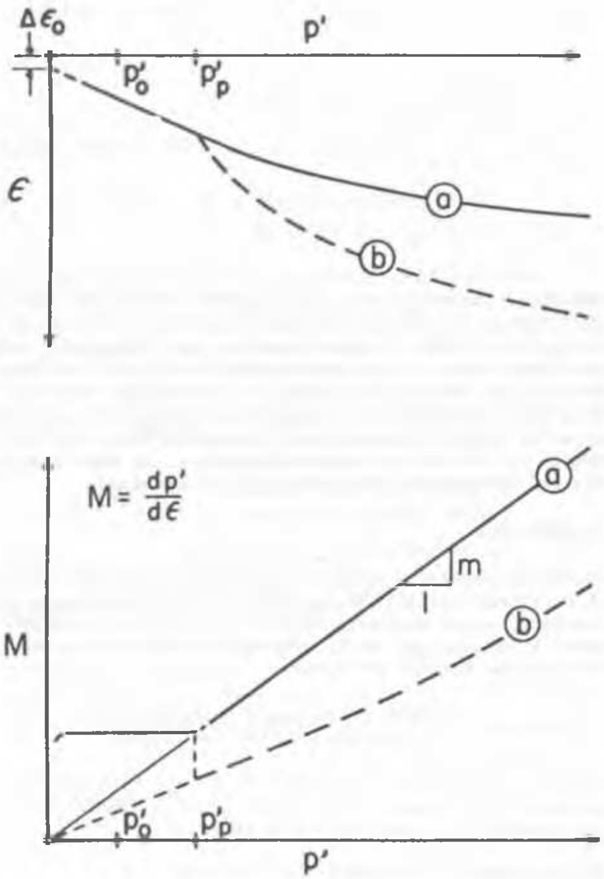

Fig. 2 Illustration of stress-strain and modulus-stress curves for two clay samples.

The behavior of the sample upon loading will not be the same within the preconsolidated ($p' < p_p'$) as within the virgin ($p' > p_p'$) stress range. Therefore, the difference must show up either directly in the stress-strain curve itself, or in the stress-resistance curve, or in both. In order to bring out these differences in behavior and to enable a rational determination of the preconsolidation load, arithmetic plots are absolutely necessary. (For this purpose semi-logarithmic plots may be very deceptive, indeed.)

The lower part of Fig. 2 shows how the stress resistance M (or the tangent modulus) may vary with effective stress. For clays of low sensitivity and/or high preconsolidation ratio the preconsolidation load is best determined from the M-p' curve, marked (a), while for clays of high sensitivity and/or low consolidation ratio the p_p' - value may often show up distinctly in the stress-strain curve itself, such as illustrated by the dashed curve labelled (b). In the latter case a marked drop in M is observed near p_p', signifying a more or less abrupt break-down of the structural resistance of the clay skeleton.

The ideal normally consolidated clay is defined by $p_p' = p_o'$. In addition, for plastic clays the virgin modulus increases linearly with p' when $p' > p_p' = p_o'$ in which case,

$$M = mp' \tag{12}$$

The strain formula can now be derived directly from the basic definitions, since

$$M = \frac{dp'}{d\varepsilon} = mp' \text{ leads to } d\varepsilon = \frac{1}{m}\frac{dp'}{p'}$$

By integration over the stress range from p_o' to any $p' = p_o' + \Delta p$ one gets

$$\varepsilon = \frac{1}{m} \ln \frac{p'}{p_o'} \tag{13}$$

This goes to show that a semi-logarithmic stress-strain relationship is simply explained by a linear stress resistance, or linear tangent modulus.

For overconsolidated clays, where M is independent of stresses below p_p', a stress change from p_o' to $p_o' + \Delta p < p_p'$ will lead to a strain ε equals

$$\varepsilon = \frac{\Delta p}{M} \tag{14}$$

which is equivalent to elasticity in the sense that M = stress independent.

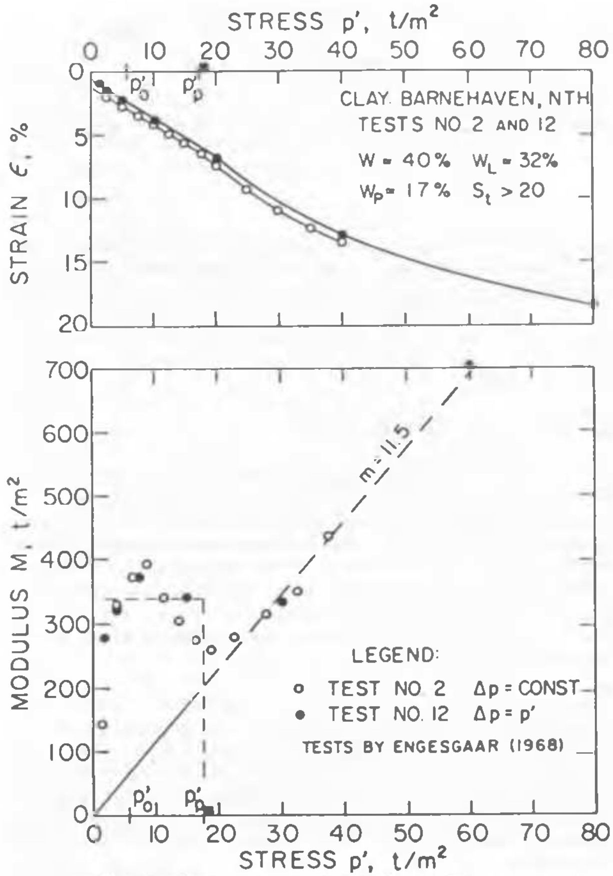

Fig. 3 Observed stress-strain and modulus-stress curves for two overconsolidated, very sensitive clay samples.

Fig. 3 shows the oedometer test results for two overconsolidated, very sensitive clay samples. The present effective overburden is 5.6 - 5.9 t/m², but before the excavation of the site (some 20 years ago) the effective overburden was closer to 20 t/m². For test No. 2 the load increments were constant, while for test No. 12 the load increment equalled the preceding, effective load. Fig. 3 shows that both tests lead to $p_p' = 17 - 19$ t/m², but the test with constant load increments appears to define the preconsolidation load more precisely.

TIME RESISTANCE AND TIME-STRAIN FORMULAE

Fig. 4 illustrates for one given load increment how the time-strain curve, and the time-resistance curve may look in arithmetic plots. The R-t-curve is divided into three zones. In zone A the R-t curve is represented by a second degree parabola, corresponding to the early stage of the primary consolidation. In zone B a transition takes place from the parabolic shape of R in zone A to the linear R-variation of zone C. In zone C the excess pore pressure is zero, corresponding to secondary consolidation. In zone B primary and secondary compression usually overlap.

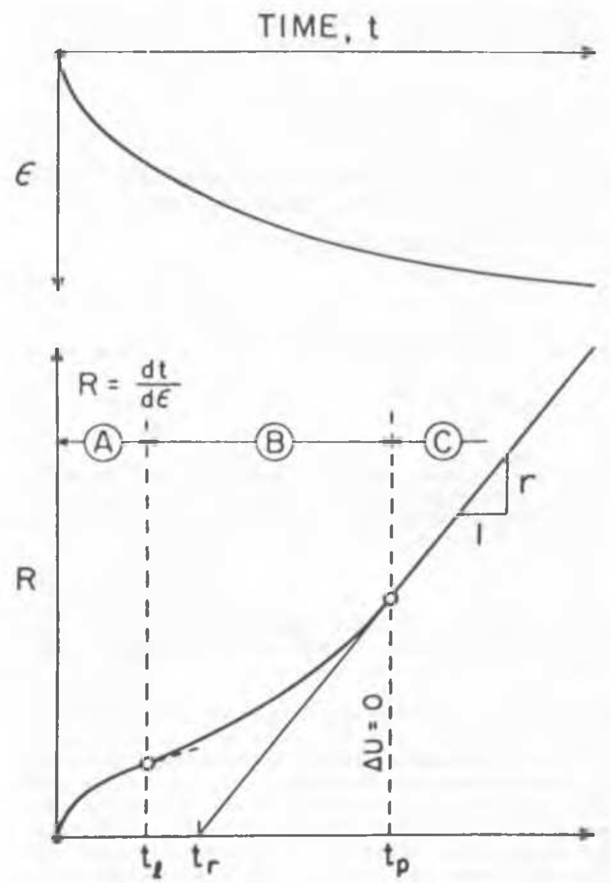

Fig. 4 Strain and time resistance as function of time for one load increment (Δp = p').

The idealized R-t curve shown in Fig. 4 is typical when Δp = p', but for Δp = constant zone B may altogether disappear and in addition t_r may very well be negative.

But irrespective of the magnitude of the load increment the R-t relationship will eventually become a straight line for clays. For $t > t_p$ one can hence write

$$R = r(t - t_r) \tag{15}$$

Now, introducing the defining equation $R = dt/d\varepsilon$ one gets

$$d\varepsilon = \frac{1}{r}\frac{dt}{t-t_r} = \frac{1}{r}\frac{d(t-t_r)}{t-t_r} \tag{16}$$

By integrating over the time range from t_p to any $t = t_p + \Delta t$ one obtains the following formula for secondary strain, for $t \geq t_p$

$$\varepsilon = \frac{1}{r}\ln\frac{t-t_r}{t_p-t_r} \tag{17}$$

wherein t_r may be positive, negative, or zero.

In the early stage of the primary consolidation, the time-resistance R is proportional to $\sqrt{t}$, so that for $t \leq t_\ell$

$$R = r_\ell \sqrt{tt_\ell} \tag{18}$$

where r_ℓ is again a dimensionless resistance. Since $R = dt/d\varepsilon$, or $d\varepsilon = dt/R$, integration from $t = 0$ to $t \leq t_\ell$ leads to

$$\varepsilon = \frac{2}{r_\ell}\sqrt{\frac{t}{t_\ell}} \tag{19}$$

For clays that behave in a theoretically ideal way

$$t_p \simeq 1.2\,\frac{d^2}{c_v} \tag{20}$$

where d = drainage path, c_v = coefficient of consolidation. The time limit of the parabolic zone A is given theoretically by

$$t_\ell \simeq 0.2\ t_p \tag{21}$$

and the degree of primary consolidation for $t = t_\ell$ should be about 55%. A systematic analysis of zone B and of t_r is yet to be done.

However, the dimensionless long-term time resistance ratio r has been analyzed extensively in the laboratory by H. Engesgaar (1968), and an example of some of the results obtained is shown in Fig. 5 for the same clay as in Fig. 3. Although the 6 clay samples were tested under widely different conditions, including different sample dimensions, it is clearly seen that for all 6 tests the r-value drops drastically for increasing stress until $p' = p_p'$. For $p' > p_p'$ the r-value remains practically constant, and is roughly equal to 200.

Similar r-variations as shown in Fig. 5 have been observed on all the 5 different types of clay analyzed so far. The main impression is that the long-term resistance is much greater in the preconsolidated

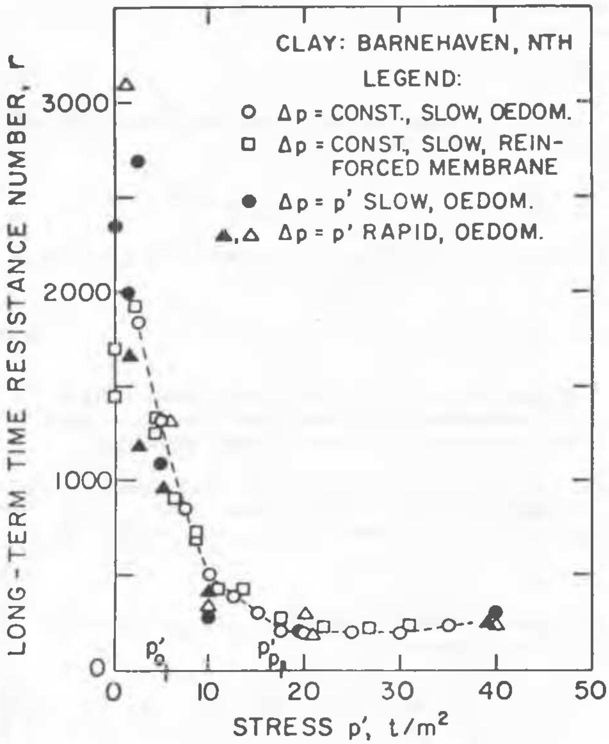

Fig. 5 Observed secondary time-resistance versus effective stress for 5 different test conditions on clay samples from the same depth.

range than in the virgin stress range. Moreover, for all cases studied hitherto the differences have been sufficiently large to enable a separate determination of p_p' directly from the r-p'-plot, as shown by Fig. 5. In the virgin stress range the r-values of the Norwegian clays analyzed so far have varied between 200 and 400.

REVISED CONSOLIDATION THEORY

The classical one-dimensional theory of consolidation for saturated soils is based on the assumptions of linear stress-strain and constant coefficient of permeability. Moreover, the differential equation was expressed in terms of pore pressure with the consequence that the immediate pore pressure conditions appeared to play such a predominant role, and no consideration could be given to the stress history of the soil.

Several of these defects of the classical consolidation theory have been criticized independently by Mikasa (1965) and Janbu (1963, 1965, 1967). The common basis for their criticism is the differential equation in terms of strain instead of pore pressure. By deriving the differential equation in terms of strain directly, one will discover the following main features:

It is not necessary to assume a linear stress-strain curve nor a constant k, because the coefficient of consolidation is obtained in full and directly inside one and the same differential operator, in contrast to the classical theory. But most important of all, it is the distribution of remaining strain (and not the pore pressure distribution) that determines the rate of primary consolidation. Consequently, U-T-curves for different final strain conditions had to be developed, see eq. Janbu (1965).

In dimensionless form the differential equation in terms of remaining strain $\bar{\varepsilon}$ may be written as follows (one-dimensional consolidation, constant c_v)

$$\frac{\partial \bar{\varepsilon}}{\partial T} = \frac{\partial^2 \bar{\varepsilon}}{\partial \xi^2} \tag{22}$$

in which remaining strain $\bar{\varepsilon}$ is defined as,

$$\bar{\varepsilon} = \varepsilon_1 - \varepsilon \tag{23}$$

where ε_1 = final primary strain (for $T = \infty$)
ε = actual strain (at depth z and time t)
$T = tc_v/d^2$ = time factor (dimensionless)
$\xi = z/d$ = distance from impervious base/drainage path
$c_v = Mk/\gamma_w = MK/\eta = M/S$, see Eqs. (7) and (8)

The seepage velocity v is given by the equation

$$v = -c_v \frac{\partial \bar{\varepsilon}}{\partial z} \tag{24}$$

The equations above have been solved for different ε_1-distributions, and the corresponding U-T-curves are shown in Fig. 6 for three cases.

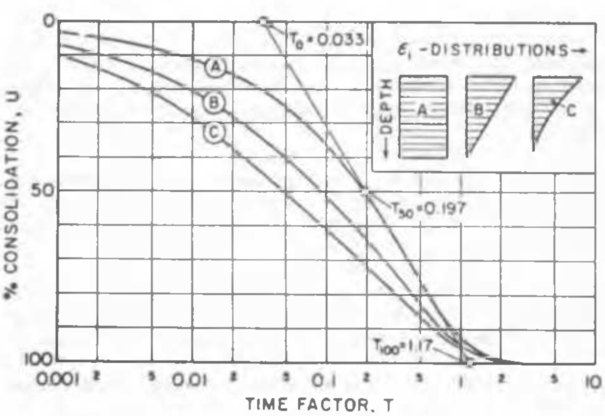

Fig. 6 Degree of consolidation as function of time factor for three different distributions of final strain.

For clay layers of substantial thickness ε_1 decreases with depth even when additional stress is constant over the whole layer because p_o' increases. For such a case Fig. 6 shows that the rate of consolidation on strain basis is more rapid than on stress basis.

DETERMINATION OF c_v

In laboratory tests the sample thicknesses are so small that it appears justified to assume ε_1 = constant, i.e. case A in Fig. 6. Moreover, in those cases when semilogarithmic laboratory (∂-t) curves are S-shaped so that "100% primary" consolidation can be constructed in orthodox manner, one can determine c_v for two or three reference points (one is of course insufficient). The general formula is, by definition of T,

$$c_v = T\frac{d^2}{t} \tag{25}$$

in which the theoretical T-values obtained from Fig. 6

T_0 = 0.033 (0.035)

T_{50} = 0.197 (0.20)

T_{100} = 1.17 (1.2)

must be used together with the test values t_0, t_{50}, and t_{100}, respectively.

In cases when the pore pressure is measured throughout the consolidation test, one can obtain more directly the time of 100% consolidation, t_p, and from Eq. (25) the c_{vp} is found for T_p = 1.2.

Finally, if a $\varepsilon - \sqrt{t}$ plot is used one can determine the time limit of the parabolic curve, $t = t_\ell$, which theoretically corresponds to T_ℓ = 0.24 and yields $c_{v\ell}$.

These 5 different methods of determining c_v have been used on the extensive data furnished by Engesgaar's (1968) experiments, and the results are shown in Fig. 7, which indicates:

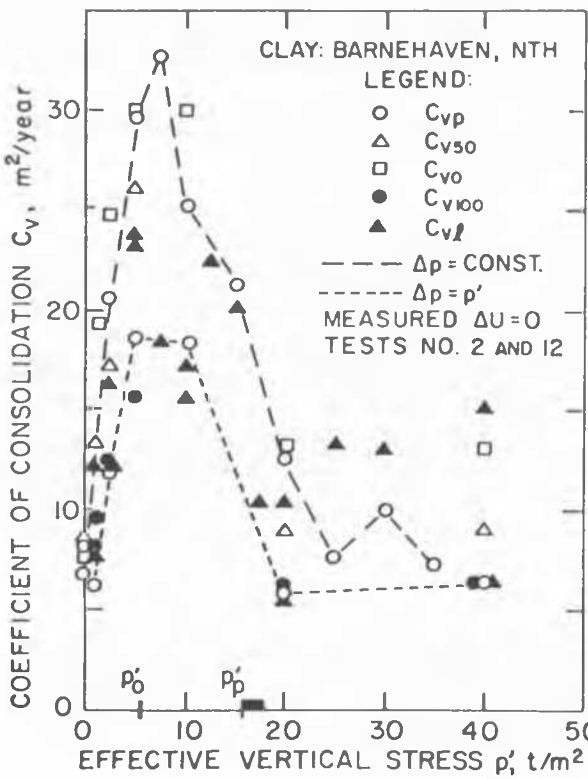

Fig. 7 Coefficient of consolidation determined by five different methods for two clay samples from the same depth.

Constant load increments yield larger c_v-values than $\Delta p = p'$. The c_{v0} is substantially larger than c_{v100} for this clay. This is due to the fact that the ratio t_{100}/t_0 ranges between 50 and 70 instead of the theoretical value of 35-36. This again indicates that the "primary" consolidation is slowed down by overlapping processes of different nature. Most important, the c_v-values are substantially higher in the preconsolidated than in the virgin stress range.

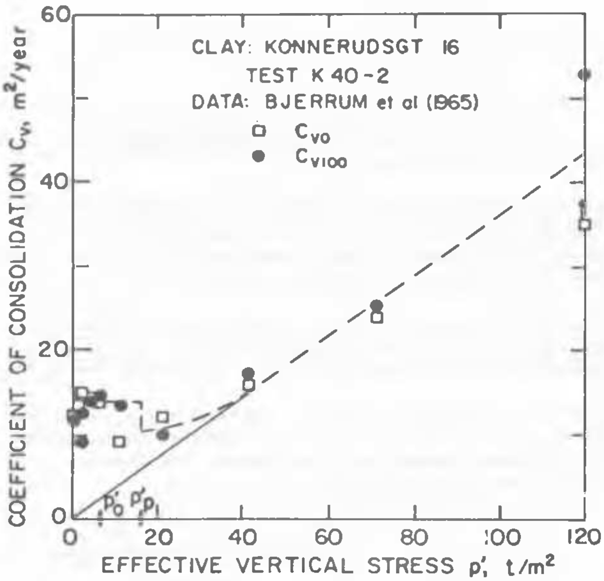

Fig. 8 Coefficient of consolidation as function of stress.

For comparison, a test performed at NGI has also been evaluated in terms of c_{v0} and c_{v100}, see Fig. 8. For stresses well above the preconsolidation load c_v appears to increase linearly with stress, in the same way as M, indicating that the seepage resistance is less effected by stress increase than the stress resistance. For this Drammen clay the c_{v0} and c_{v100} are roughly equal, indicating negligible overlapping of "secondary" compression into the primary range.

The seepage resistance and the coefficient of permeability can now be determined as outlined by Eqs. (9), (10), and (11). From Figs. 3 and 7 one finds that $k = 2.2 \times 10^{-7}$ cm/sec in the preconsolidated range, and that k drops to half of that value in the virgin stress range.

CONCLUSIONS

In order to obtain a maximum amount of vital information from deformation tests on clay, and in order to establish a more rational basis for practical settlement analysis with respect to stress and time, the following suggestions are made:

(a) Laboratory stress-strain-curves, and strain-time-curves should primarily be studied in arithmetic plots.

(b) The tangent-values of these cause-effect curves are true resistances, by mechanical definition. These resistances should, therefore, be determined and used as the basic deformation characteristics of the soil tested, whether it is a clay, silt, sand, or organic material.

(c) As exemplified again herein, the stress-resistance (tangent modulus) $M = dp'/d\varepsilon$ is in general stress dependent. For clays in particular, this resistance is for logical reasons different in the overconsolidated and the virgin stress ranges. Therefore, preconsolidation loads on clays can be determined most rationally either from M-p'-curves, and/or directly from the arithmetic stress-strain curve. The modulus number $m = dM/dp' \approx$ constant in the virgin stress range, and should replace C_c. (Note: C_c is not a complete compressibility measure, since e_o also must be known.)

(d) For each load increment one should obtain the time-resistance, $R = dt/d\varepsilon$, and present arithmetic R-t-curves. For the long term (secondary) resistance of clays it has been found that the time-resistance number $r = dR/dt \approx$ constant. Moreover, arithmetic r-p'-plots will often determine the preconsolidation load, simply because also the time resistance depends on previous stress history.

(e) Based on a differential equation in terms of strain (instead of pore pressure) the coefficient of consolidation c_v is redefined and re-examined theoretically as well as experimentally. Since c_v is the ratio between the stress resistance [M] and the seepage resistance (S) its dependency on previous stress history is not always so easily detectable, but in general it appears that M plays a more predominant role than S.

REFERENCES

Bjerrum, L., Eide, O. and Foss, I. (1965): "The Settlement of Konnerudsgt.16 and two neibour buildings in Drammen," Norwegian Geotechnical Institute, Reports F. 272, Oslo.

Engesgaar, H. (1968): "Experimental investigation of the time resistance under laboratory consolidation of clay samples," (Diploma Thesis, in Norwegian). The Technical University of Norway, Trondheim.

Janbu, N. (1963): "Soil compressibility as determined by oedometer and triaxial tests," Proc. Europ. Conf., Vol. I, Wiesbaden. (See also discussions, Vol. II.)

Janbu, N. (1965): Consolidation of clay layers based on non-linear stress strain," Proc. 5th Int. Conf. on Soil Mechanics and Foundations Engineering, Montreal.

Janbu, N. (1967): "Settlement calculations based on the tangent modulus concept," Three guest lectures at Moscow State University. Bulletin No. 2 of Soil Mechanics and Foundations Engineering at the Technical University of Norway, Trondheim.

Mikasa, M. (1965): "Consolidation of soft clays," Civil Engineering in Japan, Tokyo. (Synopsis of JSCE prize paper for 1964.)