END RESTRAINT EFFECTS IN THE TRIAXIAL TEST

EFFETS DES RESTRICTIONS D'EXTREMITES DANS L'ESSAI TRIAXIAL

W.H. PERLOFF, Associate Professor of Soil Mechanics
L.E. POMBO, Graduate Research Assistant

Purdue University, Lafayette, Indiana

SYNOPSIS An analysis of the triaxial compression test is presented. For an assumed elastic-plastic constitutive law, the influence of end restraint on conditions within the test is determined. The significance of end restraint effects is shown to depend upon the constitutive relation for the material.

INTRODUCTION

The Triaxial Compression Test

The triaxial compression test is one means of subjecting soil specimens to loading systems for the purpose of determining the mechanical response of the material. The test is considered useful because it is assumed to represent a simple boundary value problem for which a solution can be found by inspection. If it is assumed that:

1. The right circular cylindrical specimen (Figure 1a) is homogeneous and isotropic,
2. Only normal stresses are applied to the boundary surfaces. The stress applied to the end planes σ_a, is the same at all points in the plane. The stress applied to the lateral surface σ_R, is the same at all points of the surface,
3. The constitutive law is a single-valued function of stress, strain, and loading history (among other factors),

then the state of stress and strain within the specimen will be homogeneous. That is, at any point within the specimen (Figure 1b):

$$\begin{aligned} \sigma_z &= \sigma_a \\ \sigma_r &= \sigma_\theta = \sigma_R \\ \tau_{rz} &= \tau_{z\theta} = \tau_{r\theta} = 0 \end{aligned} \tag{1}$$

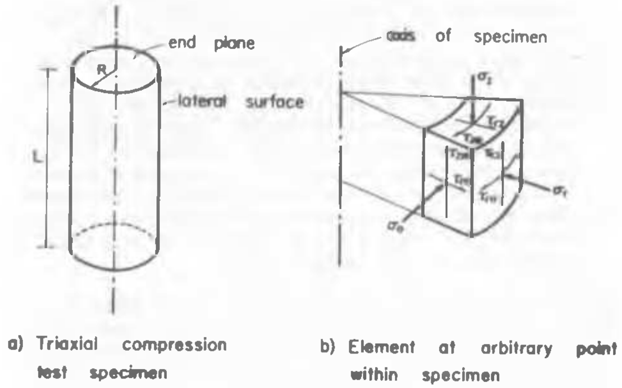
and, if strains can be considered small,

$$\begin{aligned} \epsilon_z &= \frac{\Delta L}{L} \\ \epsilon_r &= \epsilon_\theta = \frac{\Delta R}{R} \end{aligned} \tag{2}$$

in which ϵ_z, ϵ_r, ϵ_θ are normal strains in the z-, r-, θ-directions respectively, ΔL and ΔR are changes in the length L of the specimen and its radius R. Thus the specimen has "point-like" attributes and the material response, which pertains to a material point, can be determined from measurements of the response of the gross specimen.

Discrepancies Between Assumptions and Reality

The assumptions which make the triaxial compression test readily interpretable are generally not satisfied. The discrepancies between these assumptions and actual conditions derive principally from two sources:



a) Triaxial compression test specimen

b) Element at arbitrary point within specimen

Figure 1. Conditions Within Triaxial Compression Test Specimen

1. Inhomogeneity of the specimen.
2. The boundary conditions at the ends of the specimen differ from those assumed. Although the end planes are displaced approximately parallel to each other, restraint against radial expansion of the ends introduces inhomogeneous conditions. The degree to which this restraint is a factor varies, but it exists in all experiments.

The first of these discrepancies is not considered herein, except insofar as material inhomogeneity is created by inhomogeneous stress conditions. The second, termed end restraint, is the primary concern of this paper. This problem has been studied both analytically and experimentally.

Experimental results indicate that end restraint influences triaxial compression tests on cohesionless materials in the following ways:

1. End restraint leads to non-uniform total stress (Shockley and Ahlvin, 1960) and strain (Shockley and Ahlvin, 1960; Roscoe et al, 1963;

Kirkpatrick and Belshaw, 1968) conditions within the specimen.

2. The observed average stress-strain behavior of the gross specimen is different for conventional specimens with end restraint than for specimens with reduced end restraint (Taylor, 1941; Rowe and Barden, 1964) although the degree of difference is not entirely clear (Bishop and Green, 1965; Duncan and Dunlop, 1968).
3. Reduction of end restraint leads to deformations which approach an homogeneous condition (Rowe and Barden, 1964; Duncan and Dunlop, 1968; Kirkpatrick and Belshaw, 1968).

Triaxial compression test results for clay soils indicate that:

1. End restraint produces non-uniform pore water pressures, accompanied by moisture migration when testing time permits (Crawford, 1963; Blight, 1964; Barden and McDermott, 1965), and non-uniform strains (Kraft, 1965).
2. The average stress-strain behavior of the gross specimen is modified by end restraint (Barden and McDermott, 1965; Duncan and Dunlop, 1968). For length to diameter (L/D) ratios of 2, the stresses in specimens with constrained ends are slightly higher than stresses at comparable average strains in specimens with lubricated ends.
3. Suitably reduced end restraint leads to approximately homogeneous conditions as measured by external deformation, pore water pressure distribution, and moisture migration. There is some doubt however, whether unconstrained conditions can be achieved by lubricated end platens in long-term tests (Duncan and Dunlop, 1968).

Previous analytical investigations of end restraint effects (Filon, 1902; Pickett, 1944; D'Appolonia and Newmark, 1951; Balla, 1960) have been restricted to consideration of linear elastic materials. Although these studies have provided insight into the problem, the limitations of a linear elastic constitutive law, in addition to difficulties in satisfying the mixed boundary conditions, have reduced the utility of such results.

It has been concluded (Bishop and Green, 1965; Duncan and Dunlop, 1968) that for specimens with a sufficiently large length to diameter ratio, end restraint effects will be unimportant in triaxial compression test results, except in the measurement of volume changes at large strains for tests on sands. It appears, however, that these conclusions are based primarily on the results of tests on specimens which did not exhibit a sharp peak point in the stress-strain curve. The significance of this point is illustrated by tests on a medium dense sand at low and high confining pressures reported by Lee and Seed (1964). In the case of the lower confining pressure, the stress-strain curve exhibited a sharp peak point, followed by a reduction in stress as the strain increased. The influence of lubricated end platens on the test results was pronounced. For the higher confining pressure, the stress-strain curve did not have a sharp peak, and the effect of lubricated end platens was nominal.

It seems likely that the significance of end restraint effects depend markedly upon the mechanical behavior of the specific material considered, and that a generalization for all soils cannot be drawn.

Objective

It is the objective of this paper to:

1. Delineate the effect of end restraint on local conditions within a triaxial compression test specimen by a numerical analysis, assuming a suitable nonlinear constitutive law.
2. Determine the relation between the observed gross response of the specimen and that which would be observed in the absence of end restraint effects.

ANALYSIS

Constitutive Relation Assumed

Determination of a constitutive relation appropriate for soils over a wide range of stress and strain has been the objective of many experimental and theoretical investigations. Among these are the work of Rowe (1962), Drucker (1964), Ko and Scott (1969). The most complete consistent hypothesis has been developed at Cambridge University (Schofield and Wroth, 1968; Roscoe and Burland, 1968). The complexity of these results, however, has limited their application in the solution of boundary value problems. Previous nonlinear solutions have made use of simpler assumed constitutive laws (Clough and Woodward, 1964; Girijavallabhan and Reese, 1968; Hoeg et al, 1968).

For the problem considered herein, the following relations are assumed to define the applicable constitutive law:

$$\varepsilon_{oct} = \frac{1}{K}\sigma'_{oct} \tag{3}$$

$$(\tau_{oct})_y = c_o + c_1\,\sigma'_{oct} \tag{4}$$

$$\frac{d\gamma_{oct}}{d\tau_{oct}} = \frac{1}{G_o+G_1\sigma'_{oct}},\quad \tau_{oct} < (\tau_{oct})_y \tag{5a}$$

$$\frac{d\gamma_{oct}}{d\tau_{oct}} = \frac{1}{G_2},\quad \tau_{oct} > (\tau_{oct})_y \tag{5b}$$

in which

σ'_{oct} is the effective octahedral normal stress,

τ_{oct} is the octahedral shear stress,

$(\tau_{oct})_y$ is the "yield" stress corresponding to σ'_{oct}

ε_{oct} is the octahedral normal strain, i.e. the volumetric strain,

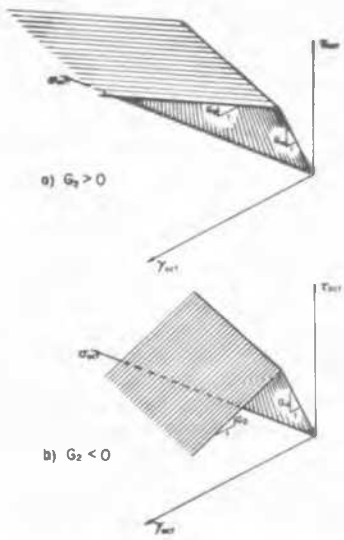
γ_{oct} is the octahedral shear strain,

K, c_o, c_1, G_o, G_1 and G_2 are material parameters.

These relations are illustrated in Figure 2a for $G_2>0$ and Figure 2b for $G_2<0$.

Several features of these relations require comment:

1. Equation 4 is equivalent to an extended von Mises yield criterion. Bishop (1966) concludes that the Mohr-Coulomb hypothesis is more



a) $G_1 > 0$

b) $G_2 < 0$

Figure 2. Constitutive Law Assumed for Analysis

applicable to the behavior of sands. But this is disputed by Ko and Scott (1968). Wu et al (1963) present results which support the Mohr-Coulomb hypothesis as do Roscoe and Burland (1968). However, this issue is not resolved, and for purposes of this discussion Equation 4 has the advantage of simplicity.

2. The yield strength and the shear stiffness prior to yield depend upon the magnitude of the effective confining pressure. These features are common to all soils.
3. The yield function describes an ideal plastic material. That is, yielding does not lead to development of a new yield surface in stress space (Fung, 1965). However, the associated flow rule of the classical theory of plasticity (Drucker, 1964) is not assumed here, because a sharp peak in the stress-strain curve does not satisfy the usual definition of stable plastic materials (Drucker, 1959), and it was deemed desirable to consider the response of such materials, i.e., $G_2<0$.
4. The linear form of Equations 3-5 was selected to permit clearer comparison between constrained and unconstrained conditions. The material is not linear, however, even in the elastic range, because of the dependence of the shear stiffness on the mean stress (Equation 5a).

Method of Analysis

The formidable analytical difficulties associated with the solution of a nonlinear problem with mixed boundary conditions recommend use of a numerical solution. In this case, the finite element method (Clough, 1965; Zienkiewicz and Cheung, 1967) was used. The problem was assumed axisymmetric so that a wedge of the specimen could be considered, with displacements in the θ-direction (Figure 1b) equal to zero, and $\tau_{z\theta}=\tau_{r\theta}=0$. Due to symmetry about the midplane of the specimen, only one-quarter of the specimen, in vertical section, was analyzed. The boundary conditions imposed were:

1. At the midplane of the specimen - all vertical displacements equal, no horizontal restraint.
2. At the outer lateral surface - constant applied normal stress, no restriction on displacements.
3. Along the centerline - no radial displacement, no restriction on axial displacement.
4. At the end of the specimen - no axial displacement, and
   a) Unconstrained case: no restriction on radial displacement, or,
   b) Constrained case: no radial displacement of any node points.

The specimen was loaded, assuming drained conditions, by applying increments of axial displacement to the midplane. Because of the nonlinear nature of the constitutive law assumed, the state of the specimen is a function of loading history. Hence, increments of displacement were applied to produce increments of average axial strain, $\varepsilon_a = \Delta L/L$, from 0.05 to 0.7 per cent depending upon the position on the "stress-strain" curve. Material parameters determined after a given increment, were used as equivalent elastic moduli for the succeeding increment.

The solution was obtained using a computer program which is a modification of that described by Wilson (1963). Material parameters considered herein (Equations 3-5) are:

$K = 1333$ psi

$c_o = 15$ psi, $c_1 = 0.268$

$G_o = 1000$ psi, $G_1 = 0.176$

$G_2 = 100$ psi <u>or</u> $G_2 = -200$ psi

RESULTS

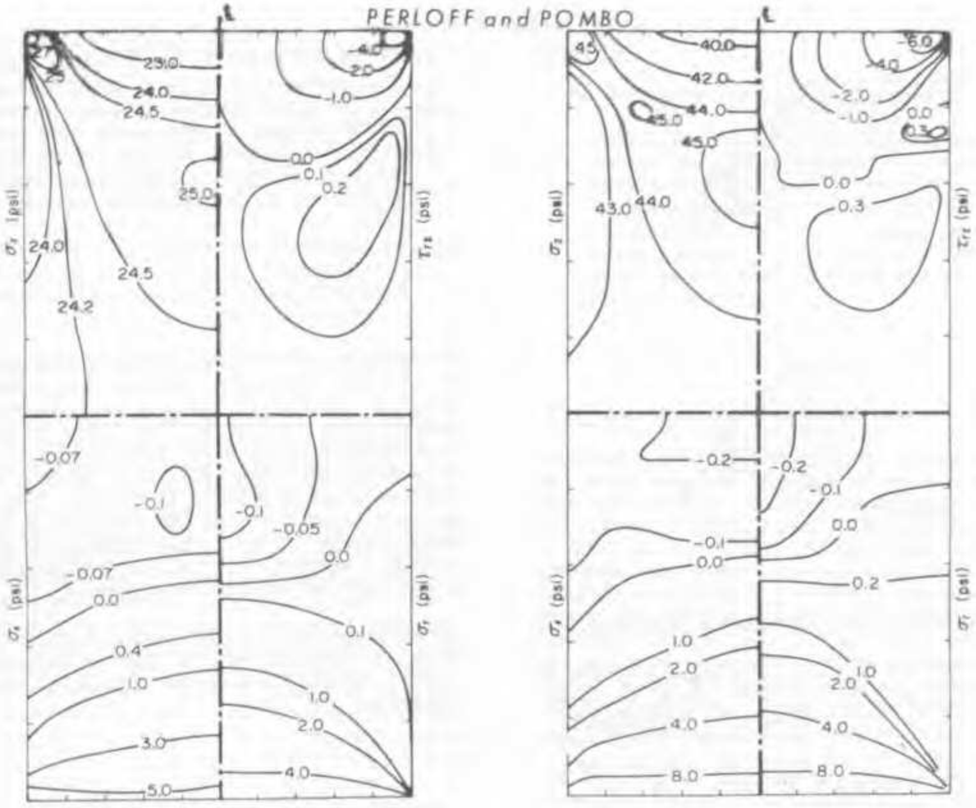
Stress Distribution

The distribution of stresses throughout a constrained unconfined specimen for which L/D = 2, is shown in Figure 3. Contours of σ_z, σ_r, σ_θ, τ_{rz} are shown in respective quadrants of the schematic cross-sections. Figure 3a illustrates results for an average axial strain $\varepsilon_a = 1.0$ per cent. The stresses are given directly rather than in dimensionless form because of the nonlinear behavior. Compressive normal stresses are denoted positive. At this strain magnitude, the material is still entirely elastic. The average axial stress, $\sigma_a = 24.3$ psi. The inhomogeneity of stress conditions is apparent from these results.

Stress contours for the $G_2 = 100$ psi material when $\varepsilon_a = 2$ per cent are given in Figure 3b. The average axial stress, $\sigma_a = 43.4$ psi. These results may be compared to the homogeneous conditions for unconstrained ends:

$\varepsilon_a = 1.0\%$, $\sigma_z = \sigma_a = 24.0$ psi, $\sigma_r=\sigma_\theta=\tau_{rz} = 0.0$

$\varepsilon_a = 2.0\%$, $\sigma_z = \sigma_a = 43.6$ psi, $\sigma_r=\sigma_\theta=\tau_{rz} = 0.0$



(a) - ε_a = 1 %

(b) - ε_a = 2 %

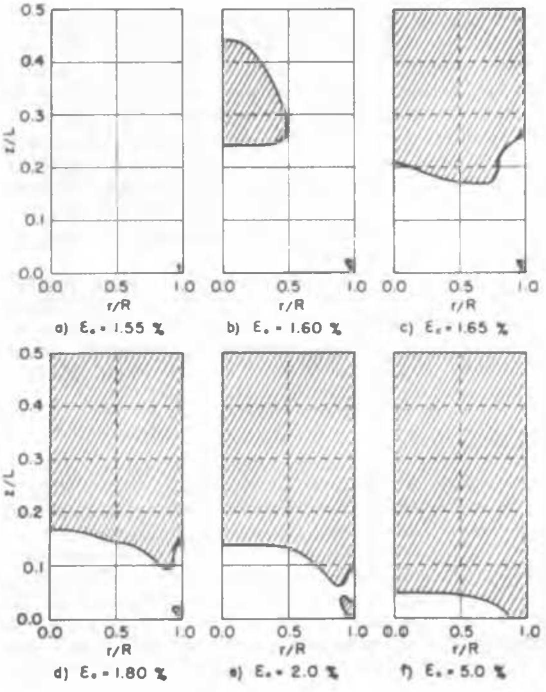
Figure 3. Stress Contours Within Constrained Unconfined Compression Specimen for Which L/D = 2, G_2 = 100 psi.

The appearance of the contours in Figure 3b is modified from that shown in Figure 3a because parts of the specimen have yielded plastically. This effect can be related to the progress of yielding throughout the specimen as it is strained, as illustrated in Figure 4. The shaded areas in this figure show the zones which have undergone plastic yielding at various average axial strain levels. The results are similar for both G_2 = 100 psi and G_2 = -200 psi.

Deformations

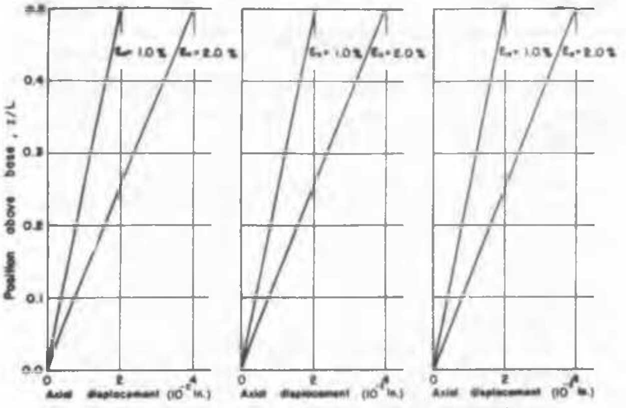
Axial and radial deformations within a constrained specimen are indicated in Figures 5 and 6. Axial deformations are shown as displacements relative to the base of a 2-inch diameter by 4-inch high specimen along various vertical lines. Radial deformations are shown as displacements relative to the centerline at various horizontal sections. The deformations are given for two strain magnitudes, one prior to yield, the other subsequent to initial yielding.

Axial displacements, shown in Figure 5, indicate relatively uniform strains except near the ends of the specimen at both strain magnitudes. The reduction in axial strain for a given elevation, at radial positions away from the centerline is qualitatively similar to that found by Shockley and Ahlvin (1960).



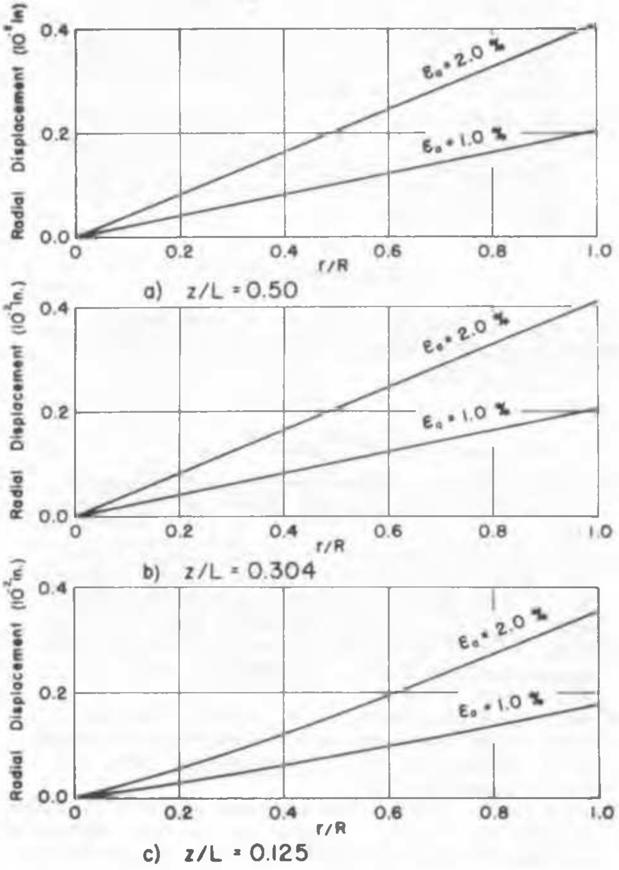
a) E_a = 1.55 %
b) E_a = 1.60 %
c) E_a = 1.65 %
d) E_a = 1.80 %
e) E_a = 2.0 %
f) E_a = 5.0 %

Figure 4. Development of Plastic Zones Within Constrained Unconfined Specimen for Which L/D = 2.



a) r/R = 0.0
b) r/R = 0.5
c) r/R = 1.0

Figure 5. Axial Displacement as a Function of Position Within the Specimen, $L/D = 2$, $G_2 = 100$ psi.



a) z/L = 0.50
b) z/L = 0.304
c) z/L = 0.125

Figure 6. Radial Displacement as a Function of Position Within the Specimen, $L/D = 2$, $G_2 = 100$ psi.

Radial displacements, Figure 6, also correspond to approximately uniform radial strains, except near the ends of the specimen. The results for $z/L = 0.304$ indicate somewhat more uniform deformations than found by Kirkpatrick and Belshaw (1968). The modes of radial deformation for $z/L = 0.50$ and 0.125 correspond well to their data (though no entirely to their interpretation of conditions near the end).

Stress-Strain Relationship

The observed "stress-strain" curves are shown for various L/D ratios for $G_2 = 100$ psi in Figure 7, and for $G_2 = -200$ psi in Figure 8. The average axial stress σ_a, is determined by dividing the total axial force on the specimen by the specimen cross-sectional area. The apparent influence of end restraint on the average relation may be deceptively small, as illustrated in Figure 9.

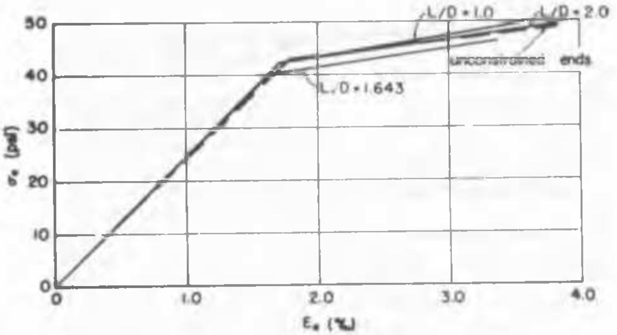


Figure 7. Observed "Stress-Strain" Relation, $G_2 = 100$ psi.

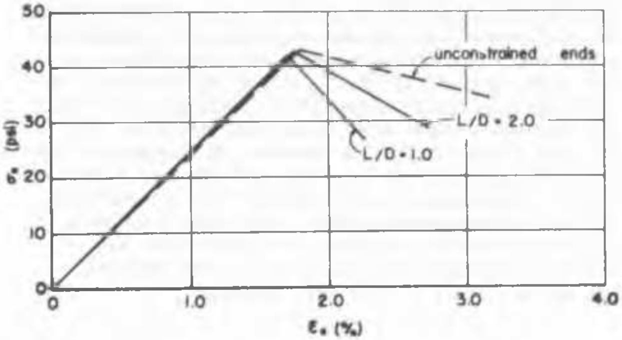
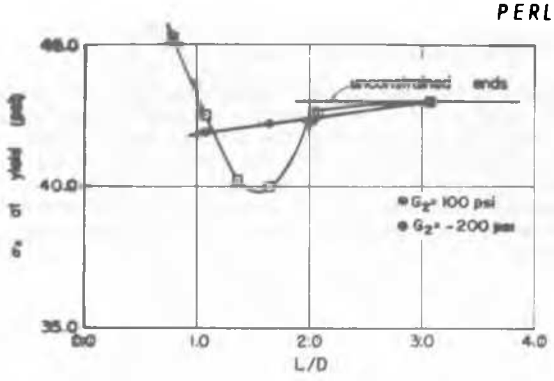
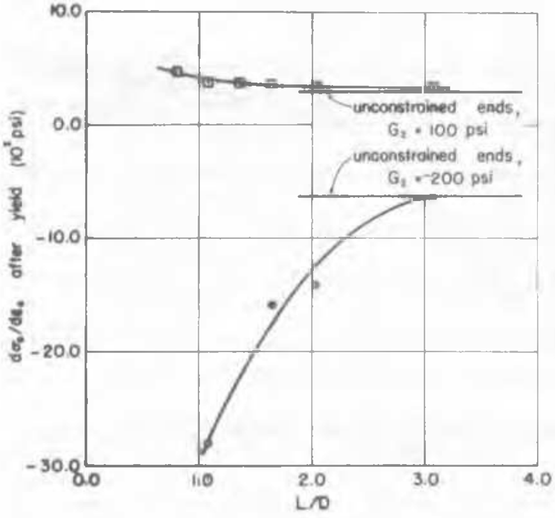


Figure 8. Observed "Stress-Strain" Relation, $G_2 = -200$ psi.



(a) Effect on σ_θ at yield



(b) Effect on slope of "stress-strain" curve after yield

Figure 9. Effect of Specimen Shape on Observed Parameters of Test.

Indicated in this figure is the influence of specimen shape on the apparent yield stress for the gross specimen, and the apparent slope of the stress-strain curve after yield. It is evident from Figure 9a that the confinement and stress concentration introduced by end restraint produce compensating effects on the yield strength of the $G_2 = 100$ psi material, so that yielding may be promoted or inhibited for various specimen shapes. Once yield has occurred, the end conditions produce an apparent stiffening of the $G_2 = 100$ psi material as shown in Figure 9b. Conversely, constrained specimens of the $G_2 = -200$ psi material appear weaker after yield than indicated by the true material response. The data shown in Figure 9 indicate that for L/D = 3, end restraint has little effect on the test results.

CONCLUSIONS

On the basis of the results presented herein for the assumed constitutive law described, the following conclusions have been drawn:

1. Restrained end conditions in the triaxial compression test produce an inhomogeneous state of stress and strain. The importance of these effects is influenced by whether the constitutive relation leads to a positive or negative slope for the stress-strain curve after yielding.
2. Over major portions of the specimen, σ_θ is the smallest normal stress. The degree to which this is important in the interpretation of strength data depends upon the influence of the intermediate principal stress on strength.
3. The effect of end restraint on the observed axial stress-strain curve depends upon the constitutive law for the material tested. The cases investigated herein suggest that the influence of end restraint may be more important than heretofore recognized in the testing of natural "brittle" materials.
4. For those cases in which end restraint cannot be reduced sufficiently, the use of specimens with L/D larger than conventional will assist in eliminating end restraint effects on the observed results.

ACKNOWLEDGMENT

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