THE MECHANICAL BEHAVIOR OF CROSS ANISOTROPIC CLAYS

LE COMPORTEMENT MECANIQUE DES ARGILES ANISOTROPES

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SYNOPSIS Stress-strain relations of an incremental nature are presented and their validity is experimentally checked on kaolinite clay test specimens with four different degrees of cross anisotropy. Four groups of tests, each with fourteen different stress paths and seven different inclinations of the principal stresses on the axis of rotational symmetry of the clay were conducted: They point to the validity of the proposed stress-strain relations for the kaolinite tested and to the possibility of obtaining them from a potential function. The equivalent stress and the equivalent strain at failure as well as Henkel's pore pressure coefficient are found to depend on the inclination of the principal stresses on the axis of symmetry and on the stress path. The analysis of the test results in terms of effective stresses points to the necessity of developing failure criteria which take into account the directional properties of the material.

INTRODUCTION

Due to the process of sedimentation followed by one dimensional consolidation, naturally deposited clays have their platelets oriented such that the material on the macroscopic level has the property of cross-anisotropy. The direction of the axis of rotational symmetry is the direction of consolidation. In the laboratory, clay specimens prepared from clay water mixtures one dimensionally consolidated have this property of cross anisotropy and can be advantageously used to obtain an understanding of the behavior of naturally deposited clays under various stress fields. This study is composed of 3 parts. The first part consists of the experimental investigation and is a joint effort of both authors. In the second part, a theory adapted to cross-anisotropic materials is presented and used to analyze the test results in terms of total stresses. In the third part the test results are examined in terms of effective stresses. The senior author alone is responsible for the second and third parts.

THE EXPERIMENTAL PROGRAM

Testing Apparatus and Test Specimens

The directional properties of cross anisotropic materials are studied by varying the inclination of the principal stresses with respect to the axis of symmetry. This can be done either by using test specimens cut at various inclinations to this axis or by using vertical specimens and rotating the principal stresses. To insure that circular cross sections remain circular under stress, nearly all the specimens used in this investigation were hollow vertical cylinders i.e. cylinders having their axis parallel to the axis of symmetry of the material. The systems of stresses were symmetrical with respect to the axis. The rotation of the principal stresses was obtained through a combination of axial and torsional stresses.

All the tests were conducted in a specially designed triaxial cell. The various stress combinations were linearly applied to the specimens by means of a pneumatic analog computer programmed in advance for their size, the rate of increase of the stresses, and the inclination of the principal stresses to be kept constant during each test. The testing machine has been described in detail by Saada (1967,1968).

The Kaolinite clay used in this investigation has a liquid limit of 62.5% and a plasticity index of 23.5%. The clay fraction is 73%, and the specific gravity 2.61. For the preparation of the cross anisotropic hollow cylinders of clay, the dry powder was placed in an 8 in. diameter plexiglass cylinder and deaired distilled water was drawn into the cylinder under vacuum to form a slurry. The consolidation pressure was linearly increased to the desired final pressure. To obtain 4 different degrees of anisotropy, the final one dimensional consolidation pressure was given 4 different values. Solid clay cylinders of 2.85 in. diameter were obtained from the 8 in. diameter blocks. A 2 in. diameter core was carefully removed from the inside of these cylinders as described by Saada and Baah (1967). Four groups of identical specimens were made, each group with a different degree of anisotropy.

For testing, the specimens were placed in the cell under a hydrostatic stress of 78 Psi, and allowed to consolidate for 3 days. For the first three groups of specimens, the triaxial consolidation was allowed to proceed with a back pressure of 18 Psi. For the fourth group, it was allowed to proceed with a back pressure of 38.5 Psi. Before the application of the deviator, the drainage lines were connected to a transducer for pore water pressures measurements. Groups 1, 2 and 3 are normally consolidated. Group 4 is overconsolidated.

Testing Procedure

Fourteen tests were conducted in each group of specimens: One test on a specimen whose axis is normal to the direction of consolidation, herein referred to as horizontal, and 13 tests on specimens whose axis is parallel to the direction of consolidation, herein referred to as vertical. The 13 vertical specimens were tested as follows:

a.--6 Tests were conducted at constant cell pressure. Both the axial and torsional stresses were linearly applied in a way such that their ratio was constant during the whole test. To each ratio there corresponds an inclination of the major and minor principal stresses on the vertical as shown in Fig. 1.

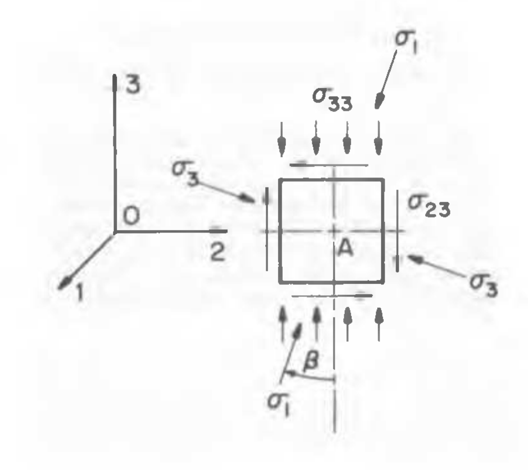

Fig. 1 Reference Axes and Stress System

The intermediate principal stress is radial, constant, and equal to the pressure in the cell. The tests in this series will be called "Direct Tests".

b.--6 Tests were conducted with the cell pressure varying linearly. The ratio of the change in the axial and the lateral pressure was always 2 to 1 so that the mean stress was maintained constant and equal to the initial cell pressure. Simultaneously, torques were applied to the test specimens in a way such that the ratio of the torsional stresses to the normal stresses difference was maintained constant during the whole test. To each ratio there corresponds an inclination of the major and minor principal stresses on the vertical. The intermediate principal stress is radial, equal to the pressure in the cell and varied continuously. The tests in this series will be called "Generalized Tests".

c.--One test was conducted in a way such that the specimen was only subjected to torque. It will be called "Torsion Test".

Table I gives the list of the tests conducted in each group, the inclination β of the major principal stress on the axis of rotational symmetry and the system of loading used to obtain this inclination. The testing was entirely automated. The rates of stress were the same for the direct and generalized tests having the same inclination of the principal stresses. The group corresponding to the strongest specimens failed in 7 to 8 hours and the one corresponding to the weakest failed in 5 to 7 hours.

ANALYSIS IN TERMS OF TOTAL STRESSES

The material is nonlinear and nonconservative, and the directions of the principal stresses and the principal strains in general will not coincide. An incremental approach is therefore adopted in the study of the stress-strain relations. In subsequent sections the name "rate of strain" (as in the classical theory of plasticity) is used to indicate that the theory is an incremental one. The theory is adapted to undrained saturated clays. The various components of the stress tensor are either increased or decreased until failure occurs; No reversal in the direction of the loading takes place.

Basic Stress-Strain Rate Relations and Directional Properties

The approach is similar to that used by Hill(1948) and Dorn (1949) in the field of anisotropic metals. It is assumed that the strain increments are linear functions of the stresses. In a system of cartesian coordinates the 6 stress-strain rate relations are written:

$$\dot{\varepsilon}_{ij} = \dot{S}_{ijk\ell}\ \sigma_{k\ell} \qquad (i,j,k,\ell = 1,2,3) \quad .. \quad .. \qquad (1)$$

Table I Tests Conducted on Each Group of Specimens

	Type of Specimen	Angle $\beta°$	Loading System		Designation
Direct Tests	Horizontal	90	Cell Pressure is constant. Mean Stress is Variable.	axial stress increase	D.C. H
	Vertical	0		axial stress increase	D.C. 0
	Vertical	15		axial stress increase + Torque	D.C.R. 15
	Vertical	31.7		axial stress increase + Torque	D.C.R. 31.7
	Vertical	58.2		axial stress decrease + Torque	D.T.R. 58.2
	Vertical	75		axial stress decrease + Torque	D.T.R. 75
	Vertical	90		axial stress decrease	D.T. 90
Generalized Tests	Vertical	0	Cell Pressure is Variable. Mean Stress is Constant	axial stress increase	G.C. 0
	Vertical	15		axial stress increase + Torque	G.C.R. 15
	Vertical	31.7		axial stress increase + Torque	G.C.R. 31.7
	Vertical	58.2		axial stress decrease + Torque	G.T.R. 58.2
	Vertical	75		axial stress decrease + Torque	G.T.R. 75
	Vertical	90		axial stress decrease	G.T. 90
Torsion	Vertical	45		Torque	R. 45

In the system of coordinates shown in Fig. 1, ε_{11} is the strain rate instantaneously in the 01 direction, $\dot{\varepsilon}_{12}$ is the shear-strain instantaneously between the 01 and the 02 direction, σ_{11} is the instantaneous component of the normal stress in the 01 direction, σ_{12} is the instantaneous component of the shear stress, etc. and $\dot{S}_{ijk\ell}$ are variable coefficients having dimensions of a strain rate over a stress. Unlike the coefficients of the elastic stress-strain relations, the $\dot{S}_{ijk\ell}$ are not constant; they depend upon the mechanical history of the material. The number of coefficients in equation (1) can be reduced provided certain symmetries exist. In the case of a cross-anisotropic material with the axis 03 as axis of rotational symmetry, the coefficients remain unchanged in any rotation around this axis; This brings the number of independent coefficients in equation (1) down to 6. For a constant volume material this number is reduced to 4. If in addition, the degree of anisotropy is such that hydrostatic stresses cause negligible shear strains, the number of independent coefficients is further reduced to 3 as shown by Saada and Zamani(1968)

Constants of Anisotropy

When the cross anisotropic material is subjected to large deformations its anisotropic characteristics change. If the changes in the anisotropic characteristics are small, the assumption may be made that the $\dot{S}_{ijk\ell}$ vary in strict proportion, in other words that their ratio remains constant over the deformation. Let

$$\dot{S}_{ijk\ell} = \alpha_{ijk\ell}\left(\frac{\dot{\varepsilon}_{eq}}{\sigma_{eq}}\right) \quad \ldots \qquad (2)$$

where $\alpha_{ijk\ell}$ is a constant of anisotropy, $\dot{\varepsilon}_{eq}$ is an equivalent strain rate and σ_{eq} is an equivalent stress to be subsequently evaluated. Substituting equation (2) into equation (1) and assuming a constant volume and no effect of the hydrostatic stresses on the shear strains, one obtains equation (3).

The definition of $\dot{\varepsilon}_{eq}$ and of σ_{eq} will be completed by assuming that the increase in the deviator stress is a function of the work done per unit volume and that the increment in the specific work dW is always given by

$$dW = \sigma_{eq}d\varepsilon_{eq} = \sigma_{11}d\varepsilon_{11} + \sigma_{22}d\varepsilon_{22} + \sigma_{33}d\varepsilon_{33} + 2\sigma_{12}d\varepsilon_{12} + 2\sigma_{13}d\varepsilon_{13} + 2\sigma_{23}d\varepsilon_{23} \quad \ldots \qquad (4)$$

Equation (4) implies that the relation between σ_{eq} and and $\varepsilon_{eq} = (\int d\varepsilon_{eq})$ will be the same for any stress path. The $\alpha_{ijk\ell}$ are dimensionless ratios and it is appropriate to choose $\alpha_{3333} = 2(\alpha_{1111}+\alpha_{1122}) = 1$ and refer the other constants to it. Substituting equation (3) into equation (4) one obtains

$$\sigma_{eq} = \{\alpha_{1111}(\sigma_{11}^2 + \sigma_{22}^2 + 2\sigma_{12}^2) + \alpha_{1122}(2\sigma_{11}\sigma_{22} - 2\sigma_{12}^2) + (\sigma_{33}^2 - \sigma_{11}\sigma_{33} - \sigma_{22}\sigma_{33}) + \alpha_{1313}(4\sigma_{13}^2 + 4\sigma_{23}^2)\}^{1/2} \qquad (5)$$

and

$$d\varepsilon_{eq} = \frac{dW}{\sigma_{eq}} \quad \ldots \qquad (6)$$

where dW is given by equation (4) and σ_{eq} is given by equation (5).

One notices that the stress-strain rate relations of equation (3) can directly be obtained from a potential function defined by

$$\begin{Bmatrix} d\varepsilon_{11} \\ d\varepsilon_{22} \\ d\varepsilon_{33} \\ d\varepsilon_{12} \\ d\varepsilon_{13} \\ d\varepsilon_{23} \end{Bmatrix} = \frac{d\varepsilon_{eq}}{\sigma_{eq}} \begin{bmatrix} \alpha_{1111} & \alpha_{1122} & -(\alpha_{1111}+\alpha_{1122}) & 0 & 0 & 0 \\ \alpha_{1122} & \alpha_{1111} & -(\alpha_{1111}+\alpha_{1122}) & 0 & 0 & 0 \\ -(\alpha_{1122}+\alpha_{1111}) & -(\alpha_{1122}+\alpha_{1111}) & 2(\alpha_{1111}+\alpha_{1122}) & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2}(\alpha_{1111}-\alpha_{1122}) & 0 & 0 \\ 0 & 0 & 0 & 0 & \alpha_{1313} & 0 \\ 0 & 0 & 0 & 0 & 0 & \alpha_{1313} \end{bmatrix} \begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ 2\sigma_{12} \\ 2\sigma_{13} \\ 2\sigma_{23} \end{Bmatrix} \qquad (3)$$

$$F(\sigma_{ij}) = \frac{1}{2} (\sigma_{eq})^2 \quad .. \; .. \; .. \; .. \; .. \; .. \qquad (7)$$

since

$$d\varepsilon_{ij} = \frac{d\varepsilon_{eq}}{\sigma_{eq}} \frac{\partial F}{\partial \sigma_{ij}} = \frac{1}{2} \frac{d\varepsilon_{eq}}{\sigma_{eq}} \frac{\partial(\sigma_{eq})^2}{\partial \sigma_{ij}} \quad .. \; .. \qquad (8)$$

Summary and Discussion of the Theory

In summary, the incremental theory proposed for experimental verification contains the assumption that the strain increments are linear functions of the stresses; the coefficients of proportionality change such that their ratio remains constant. Hydrostatic stresses cause negligible shear strains and the increment in the specific work during strain is always equal to $\sigma_{eq}\, d\varepsilon_{eq}$. For all the stress paths, the curves σ_{eq} versus ε_{eq} should be identical. This property will be used to test the validity of the theory.

There is no provision in the theory for differences in behavior when the directions of the applied stresses are reversed. Although modifications can be introduced to account for these differences as done by Hsu (1966), it is believed that at this stage they would unduly complicate the equations. For the material under consideration it was noticed that for the same absolute value of the axial stresses, the axial and torsional deformations were quite close in magnitude up to approximately 75% of the failure stress. The validity of the theory will therefore be examined only up to this level.

Determination of the Constants of Anisotropy

The use of cylindrical coordinates is appropriate for the shape of the test specimens and the systems of stresses used. The subscripts 1, 2 and 3 will stand for r, θ and z. Provided the 03 axis is always taken along the axis of rotational symmetry, equations (3) to (8) do not require any modification.

In each of the four groups of specimens, 3 tests are required to obtain α_{1111} and α_{1313}. These tests are:

a.--A direct compression test on a vertical specimen. In this case

$$\sigma_{eq} = \sigma_{33} \qquad \text{and} \qquad d\varepsilon_{eq} = d\varepsilon_{33} \quad .. \; .. \qquad (9)$$

Thus the curve σ_{33} versus $\int d\varepsilon_{33}$ is equivalent to the curve σ_{eq} versus $\int d\varepsilon_{eq}$. This is curve S in Fig. 2.

b.--A direct compression test on a horizontal specimen. In this case

$$\sigma_{11} = \frac{\sigma_{eq}}{\sqrt{\alpha_{1111}}} \qquad \text{and} \qquad d\varepsilon_{11} = \sqrt{\alpha_{1111}}\; d\varepsilon_{eq} \qquad (10)$$

c.--A torsion test on a vertical specimen. In this case

$$2\sigma_{23} = \frac{\sigma_{eq}}{\sqrt{\alpha_{1313}}} \qquad \text{and} \qquad d\varepsilon_{23} = \sqrt{\alpha_{1313}}\, d\varepsilon_{eq} \quad .. \qquad (11)$$

Since the proposed theory implies that all tests should give the same σ_{eq} versus $\int d\varepsilon_{eq}$ curve, the coefficient α_{1111} for example is determined by plotting the curves obtained from a direct compression test on a vertical specimen and a direct compression test on a horizontal specimen on the same graph: This is shown in Fig. 2. Two equal areas under the curves determine

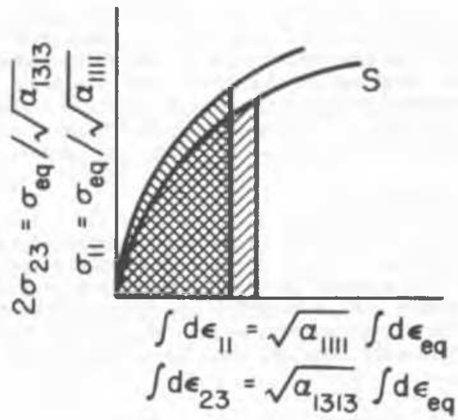

Fig. 2 Determination of Constants of Anisotropy

two points on the abcissa and two on the ordinate. The ratio of the two values on the abcissa must be equal to the inverse of the two corresponding values on the ordinate as shown by equation (10). This ratio gives $\sqrt{\alpha_{1111}}$. In theory, any two equal areas under the curves should give the same coefficient: The values from 4 equal areas were taken and were found to be extremely close. They were averaged to give α_{1111}. The value of α_{1313} is obtained by a similar scheme.

The σ_{eq} versus $\int d\varepsilon_{eq}$ Curves

The parameters needed to check the validity of the proposed theory are σ_{eq}, $\int d\varepsilon_{eq}$ and $\int dW$. To obtain these quantities, the constants of anisotropy $\alpha_{ijk\ell}$ are first computed by the method described in the previous section. The results of these computations for the four groups of tests are shown in Table II. The $\alpha_{ijk\ell}$ are then introduced in equations (5) and (6) to obtain σ_{eq} and $d\varepsilon_{eq}$. All the computations were made on a UNIVAC 1107 computer. The inputs were taken from charts which automatically recorded the normal stresses differences, the total axial deformations, the pore water pressures and the angle of rotation. The curves between σ_{eq} and $\int d\varepsilon_{eq}$ obtained from any stress path should be the same. Figure 3 shows the points obtained from 14 different stress paths in each group. One notices that the points fall within a relatively thin band for which a best curve can be obtained. The curve corresponding to direct compression can be considered as a good average. It thus appears that, for the material under consideration and the degrees of anisotropy tested, there exists a general stress-strain

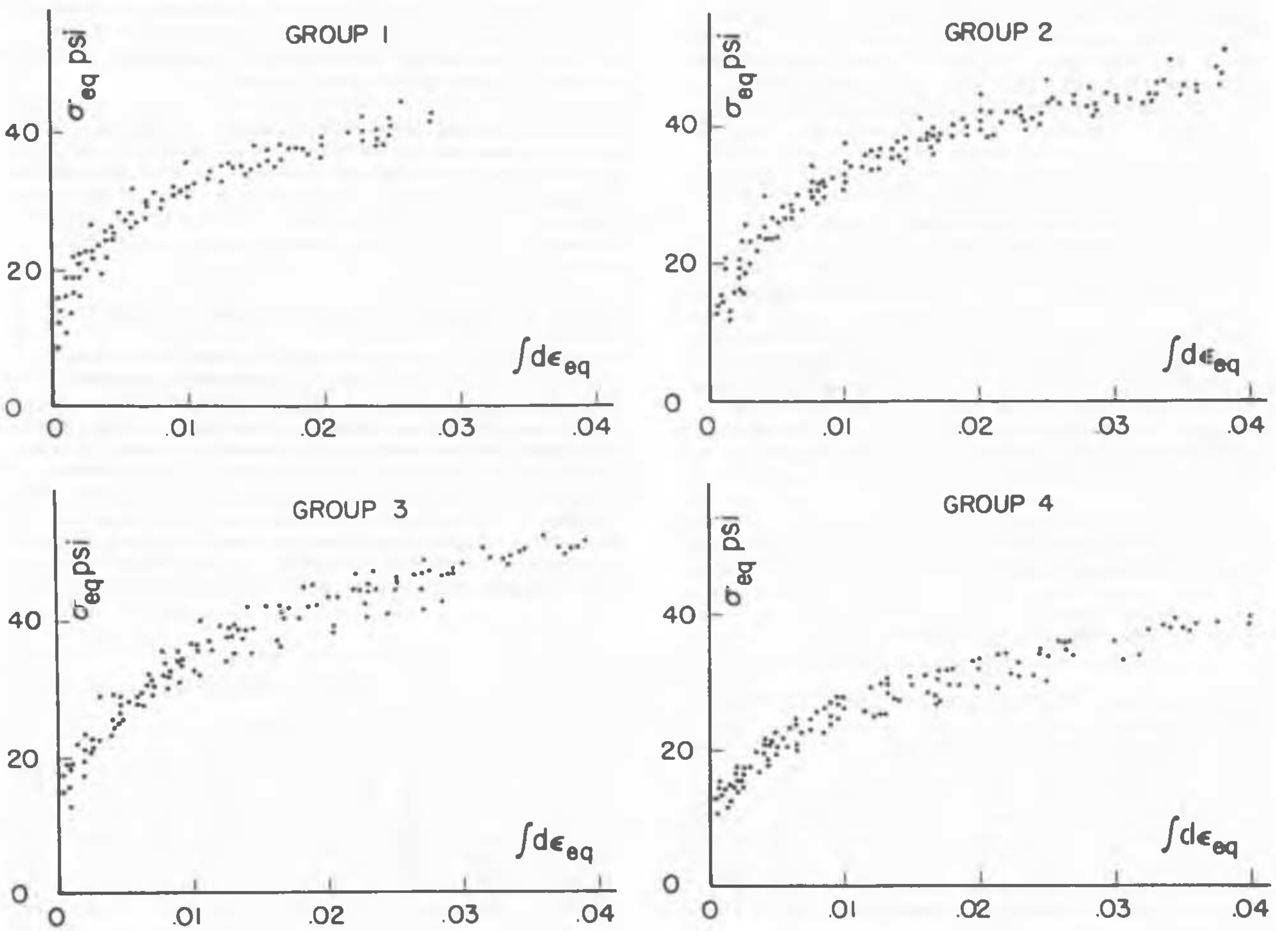

Fig. 3 Equivalent Stress-Equivalent Strain Curves for the Four Groups of Specimens

curve that can be deduced from any type of test. The proposed theory seems to be quite acceptable in describing the stress-strain behavior at least up to 75% of the failure stress. Within this range the stress-strain rate relations are obtainable from a potential function.

Table II: Constants of Anisotropy

	Group 1	Group 2	Group 3	Group 4
α_{1111}	0.871	0.924	0.973	1.130
α_{1313}	0.888	0.845	0.938	0.865
α_{3333}	1.000	1.000	1.000	1.000

Discussion of the Failure Parameters

Traditionally, failure criteria have been expressed in terms of stresses and conveniently represented in space by geometric surfaces. When it comes to anisotropic materials, a complete representation in a three dimensional space is impossible. A previous study on anisotropic specimens made of the same clay by Saada and Baah (1967) showed that a failure criterion independent of the intermediate principal stress would be inadequate. In the same study, because of the small difference between the results in compression and extension, the yield criterion of Hill (1948) was found satisfactory. For the 4 groups of specimens in this investigation, the same conclusions were arrived at regarding the importance of the intermediate principal stresses. On the other hand, the differences in the failure stresses between tests involving compression and those involving extension are too large to allow the use of Hill's criterion. Table III shows, for all the tests conducted, the values at failure of the equivalent stress σ_{eq} in psi, the equivalent strain ϵ_{eq} in in./in. and of the total specific work W in Psi. In Table III one notices that:

a. The value of σ_{eq} changes with the rotation of the major principal stress. The differences between the maximum and minimum values for the four groups of tests are quite substantial and a criterion of the form $\sigma_{eq} = K$, where K is a constant is not acceptable. This criterion is the counterpart of the Von Mises criterion used for isotropic clays.

b. The value of the equivalent strain ϵ_{eq} at failure changes drastically with the rotation of the principal stresses. Although the constants of anisotropy have been determined from the first 75% of the stress-strain curves, ϵ_{eq} is a good representation of the state of strain of the material at failure. Differences as high as 50% in its value indicate that any failure criterion for anisotropic clays should contain parameters representing the strains.

c. The value of the specific work W changes as drastically as that of ϵ_{eq} with the rotation of the principal stresses. W is one of the more logical quantities to appear in a failure criterion. At this stage only a trend is clearly defined and no formulation can be attempted until more tests are conducted on a wide variety of clays.

Summary of Analysis in Terms of Total Stresses

An incremental theory relating the strains to the stresses in cross anisotropic undrained saturated clays has been presented. Experiments conducted on a clay with 4 different degrees of anisotropy show that the theory yields good results for both normally consolidated and over consolidated cases. The stress-strain rate relations can be deduced from a potential function. Both failure stresses and strains were found to be highly dependent on the directions of the principal stresses with respect to the axis of rotational symmetry of the material. A failure criterion

Table III: Test Results at Failure

Designation	Group 1									Group 2								
	σ_{eq}	ϵ_{eq}	W	$\bar{\sigma}_1$	$\bar{\sigma}_2$	$\bar{\sigma}_3$	φ'	τ_{oct}	$3a_f$	σ_{eq}	ϵ_{eq}	W	$\bar{\sigma}_1$	$\bar{\sigma}_2$	$\bar{\sigma}_3$	φ'	τ_{oct}	$3a_f$
D.C. H	53.8	.106	4.86	81.8	24.2	24.2	33	27.1	.61	58.9	.100	4.84	86.8	25.5	25.5	33	28.9	.49
D.C. 0	51.5	.093	4.10	75.1	23.6	23.6	31	24.2	.79	57.0	.113	5.50	83.0	26.0	26.0	31	26.9	.56
D.C.R. 15	54.6	.095	4.46	77.3	25.6	21.9	34	25.2	.73	57.3	.129	6.32	80.1	25.5	21.6	35	26.7	.66
D.C.R. 31.7	47.4	.066	2.76	66.7	30.8	17.1	36	20.9	1.04	57.6	.069	3.38	78.0	33.5	16.5	40	25.9	.67
D.T.R. 58.2	48.0	.044	1.81	64.9	51.0	14.6	39	21.2	.78	48.2	.058	2.41	65.2	51.0	13.7	41	21.7	.77
D.T.R. 75	47.6	.064	2.58	57.2	54.0	8.9	47	22.0	.90	48.4	.051	2.08	61.0	57.7	11.6	43	22.5	.73
D.T. 90	42.2	.058	2.21	56.0	56.0	10.7	43	21.3	.89	48.7	.056	2.25	59.7	59.7	11.0	43	22.9	.72
G.C. 0	50.5	.107	4.72	73.2	22.7	22.7	31	23.8	.86	57.3	.115	5.50	82.2	24.9	24.9	32	27.0	.59
G.C.R. 15	51.2	.115	5.14	72.5	24.0	20.5	34	23.7	.88	56.4	.109	4.95	79.0	25.4	21.5	35	26.2	.68
G.C.R. 31.7	47.8	.080	3.36	66.9	30.6	16.8	37	21.1	1.04	56.1	.077	3.62	75.4	32.1	15.5	41	25.3	.75
G.T.R. 58.2	46.7	.041	1.59	65.0	51.5	16.1	37	20.6	.76	49.4	.050	2.00	65.4	50.9	12.7	42	22.2	.74
G.T.R. 75	45.9	.056	2.18	56.1	53.0	9.6	45	21.2	.96	48.7	.057	2.31	60.6	57.3	11.0	44	22.7	.74
G.T. 90	43.5	.051	1.88	55.5	55.5	12.0	40	20.5	.93	50.5	.060	2.52	59.8	59.8	9.3	47	23.8	.71
R. 45	47.5	.062	2.47	64.6	39.4	14.2	40	20.6	1.00	51.5	.059	2.44	70.0	42.0	14.0	42	22.9	.79

Designation	Group 3									Group 4								
	σ_{eq}	ϵ_{eq}	W	$\bar{\sigma}_1$	$\bar{\sigma}_2$	$\bar{\sigma}_3$	φ'	τ_{oct}	$3a_f$	σ_{eq}	ϵ_{eq}	W	$\bar{\sigma}_1$	$\bar{\sigma}_2$	$\bar{\sigma}_3$	φ'	τ_{oct}	$3a_f$
D.C. H	60.8	.086	4.30	87.6	26.0	26.0	33	29.0	.46	49.3	.088	3.53	64.7	18.3	18.3	NOT EVALUATED	21.9	.26
D.C. 0	58.9	.088	4.33	86.4	27.5	27.5	31	27.8	.46	48.5	.079	3.13	68.5	20.0	20.0		22.9	.15
D.C.R. 15	60.2	.095	4.81	86.1	29.4	25.3	33	27.7	.47	48.2	.105	4.24	65.3	19.5	16.2		22.4	.26
D.C.R. 31.7	61.2	.078	4.00	80.5	35.0	17.6	40	26.5	.59	44.3	.078	2.92	58.5	24.5	11.5		19.8	.40
D.T.R. 58.2	54.5	.047	3.39	68.7	53.3	12.8	43	23.6	.64	42.7	.068	2.65	50.5	38.0	5.3		19.0	.42
D.T.R. 75	52.9	.070	3.07	62.1	58.5	8.7	49	24.3	.69	39.9	.064	2.15	43.9	41.2	3.3		18.5	.54
D.T. 90	50.5	.056	2.31	60.1	60.1	9.6	46	23.8	.70	39.8	.082	2.70	43.0	43.0	3.2		18.8	.52
G.C. 0	61.1	.108	5.59	87.7	26.6	26.6	32	28.8	.45	51.0	.116	5.04	72.0	21.0	21.0		24.0	.07
G.C.R. 15	62.0	.088	4.47	85.9	27.7	23.5	35	28.5	.50	48.1	.091	3.57	67.0	21.4	18.1		22.3	.18
G.C.R. 31.7	59.3	.077	3.86	76.6	32.6	15.8	41	25.6	.71	41.7	.096	3.52	56.9	24.9	12.7		18.6	.43
G.T.R. 58.2	54.9	.072	2.21	67.5	51.9	11.1	46	23.7	.69	40.1	.074	2.45	48.6	36.8	6.1		17.9	.50
G.T.R. 75	54.6	.064	2.86	67.3	63.6	12.2	44	25.1	.49	38.9	.067	2.21	44.6	41.9	5.0		18.1	.50
G.T. 90	50.0	.057	2.34	60.9	60.9	10.9	44	23.6	.67	37.0	.058	1.75	41.3	41.3	4.3		17.4	.60
R. 45	57.1	.071	3.34	72.7	43.2	13.7	43	24.1	.70	44.2	.113	4.33	54.1	30.4	6.6		19.4	.47

in terms of total stresses should contain both stresses and strains or stresses and specific work together with the inclinations of the principal stresses.

ANALYSIS IN TERMS OF EFFECTIVE STRESSES

Whether one is studying isotropic or anisotropic clays, an analysis using effective stresses is necessary, to interpret the tests results in terms of the mechanisms now credited for the development of the shear resistance of saturated clays.

Anisotropy and Water Content

In a cross anisotropic clay, a change in water contents induced by one dimensional consolidation is necessarily accompanied by a change in the directional characteristics of the material. This means that the constants of anisotropy in equation (3) will change. On the other hand, if the change in the water content of a cross anisotropic clay is brought about by means of hydrostatic consolidation pressures, the constants of anisotropy in equation (3) "may" remain unchanged. It is to be noticed that these constants are ratios and that they are referred to α_{3333} which is chosen equal to unity: If the directional characteristics change in strict proportion during a change in water content, the $\alpha_{ijk\ell}$ do not change. In equation (3) the only quantity that is modified is the ratio $d\varepsilon_{eq}/\sigma_{eq}$.

Total and Effective Stress Paths

Figure 4 gives the total stress paths followed in three groups of tests. The stress paths are expressed in terms of the octahedral normal and the octahedral shear stresses. The pure torsion test and all the generalized tests have a vertical path through the hydrostatic effective consolidation pressure. The direct tests have an inclined stress path depending on the inclination of the principal stresses on the axis of symmetry. The way the pneumatic analog operates is such that all the total stress paths are straight lines.

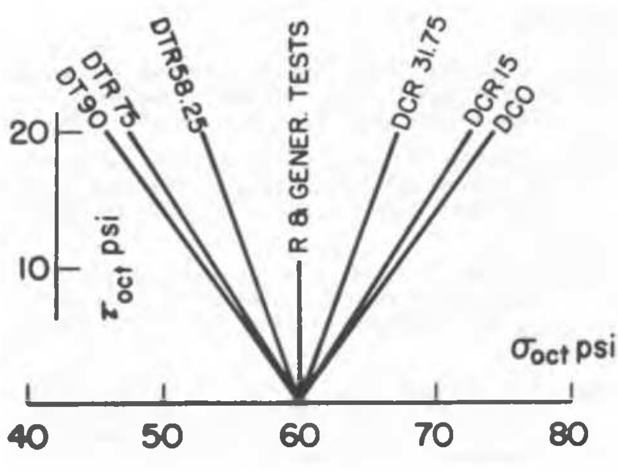

Fig. 4 Total Stress Paths

In each group, every inclination of the principal stresses was found to correspond to a different effective stress path and to a different value of the octahedral shear stress at failure (see Table II). Figure 5 shows the effective stress path for direct compression as well as the zone (between the 2 envelopes) in which all the effective stress paths fall: No two paths coincide.

Discussion of Failure Parameters

For the 4 groups of tests, Table III gives the value at failure of the 3 principal effective stresses $\bar{\sigma}_1$, $\bar{\sigma}_2$, $\bar{\sigma}_3$, in psi, $\phi' = \sin^{-1}(\bar{\sigma}_1-\bar{\sigma}_3/\bar{\sigma}_1+\bar{\sigma}_3)$ in degrees, the octahedral shear stress in psi and $3a_f$ where a_f is Henkel's pore pressure coefficient. There is no ambiguity in the definition of ϕ': Both $\bar{\sigma}_1/\bar{\sigma}_3$ and $(\bar{\sigma}_1-\bar{\sigma}_3)$ reach their maximum value at failure. In Table III one notices that:

a. In all groups a_f depends on the stress path and on the direction of the principal stresses. Different inclinations of the principal stresses on the axis of rotational symmetry of the material result in different principal strain paths. The pore water pressures are a measure of the tendency of the material to change volume. This tendency results from the hydrostatic component of the stress tensor, from the shear strains and from their combined effects. Different strain paths in an anisotropic material necessarily give different pore water pressures.

b. For the 3 first groups of tests (normally consolidated), the angle ϕ' varies with the stress path and with the direction of the principal stresses. If the effects of anisotropy were negligible when the tests results are expressed in terms of effective stresses, and if Coulomb's failure criterion were valid all the values of ϕ' would be equal. This equality would extend not only to all the tests in one group but to all 42 tests in the 3 first groups. The results shown in Table III clearly demonstrate that this is not the case. The differences (as high as 50%) are too large and are repeated in the 3 groups to fall within the experimental error range.

The results in Table III show that the clay under consideration, once given the property of cross anisotropy through a process of one dimensional consolidation, will reflect this anisotropy whether one expresses the test results in terms of effective stresses or in terms of total stresses: Strain paths, pore water pressures, effective stresses and failure parameters are all tied to that arrangement of the particles that gives to the clay its cross anisotropic characteristics. The 56 tests conducted in this investigation only represent the behavior of one type of clay and 4 degrees of anisotropy. They do however indicate the serious difference that can result when an anisotropic material is analyzed in terms of formulas established for isotropic ones.

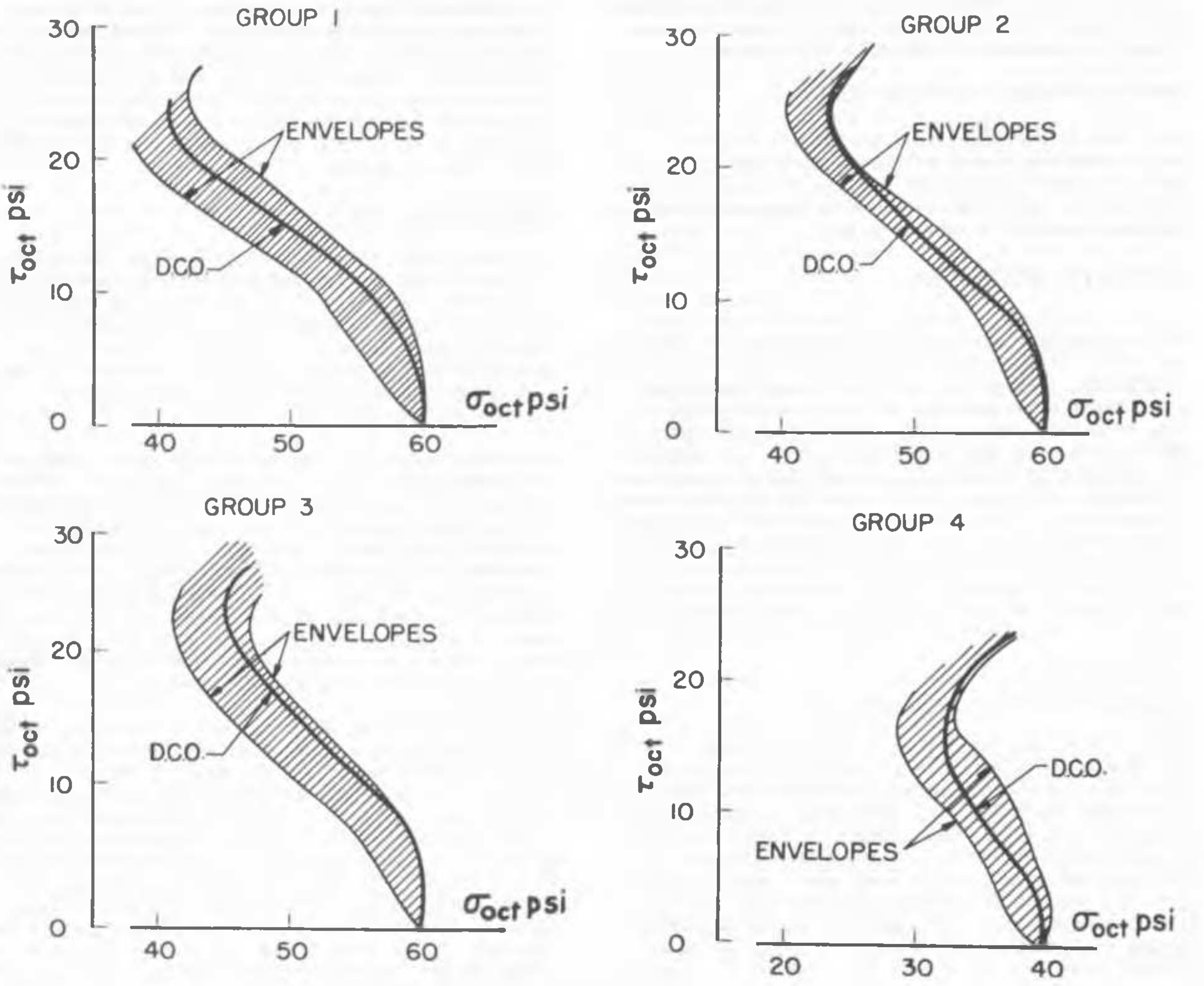

Fig. 5 Effective Stress Paths for the 4 Groups of Specimens

<u>Summary of Analysis in Terms of Effective Stresses</u>

The analysis of the test results in terms of effective stresses shows that the effective stress paths, the pore pressure coefficient a_f and ϕ' are functions of the inclination of the principal stresses. The use of an angle of internal friction obtained from an envelope of Mohr circles is unwarranted. A failure criterion in terms of effective stresses should also contain parameters representing the strains. Expressions for the development of the pore water pressure should contain coefficients describing the directional properties of the clay.

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