CREEP POTENCIAL AND CREEP RUPTURE OF SOILS

POTENTIEL DE FLUAGE ET RUPTURE PAR FLUAGE DANS LES SOLS

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SYNOPSIS A generalized stress-strain-time function has been used for the study of creep potential and creep rupture in soils. Soils can be classified into three categories: those that lose strength with time, those that gain strength with time and those whose strength is essentially independent of time. A scale is proposed to quantify the creep potential of soils. For those soils that lose strength with time a method has been proposed for determination of the time to achieve a specified strain or creep rupture under a sustained load. The technique has been justified mathematically and by laboratory tests.

INTRODUCTION

Considerable attention has been directed in recent years to the study of stress-strain-time effects in soils including creep, creep rupture, strength after creep, strain hardening, strain softening and long-term strength.

Some investigators [e.g. Murayama and Shibata (1964); Schiffman (1959)] have suggested rheological and/or mathematical models for representation of creep behavior; others [e.g. Skempton (1964); Paduana (1966)] have studied the effect of sustained load on the shear strength of soils, while still others [e.g. Mitchell, Campanella and Singh (1968)] have used the Theory of Rate Processes to better understand mechanisms of creep deformation of soils.

Creep strain rates generally decrease with time under sub-failure stress conditions. After long times the strain rates may almost cease, continue at ever decreasing rates or, in some cases, start increasing, eventually resulting in failure (creep rupture). For any given situation where creep movements have been detected, it is desirable to be able to predict whether future creep movements are likely to cease, continue to decrease in rate, or accelerate, eventually leading to failure.

Casagrande and Wilson (1951) found for some shales that the strength was reduced by long-term loading. They also indicated that some partially saturated soils may in fact show an increase in strength with time. Numerous other investigators have since obtained similar data and have shown that under long-term loading conditions, the strength of saturated soils may range from as low as 50% to more than 100% of the strength obtained using normal testing procedures. However, even within this range of sub-failure stress it is not always clear what potential the soil possesses for future creep movements; i.e., whether they cease, decrease or will accelerate and lead to failure.

In this paper the potential of a soil creep is quantified and guide lines are presented to assist in the prediction of future creep movements from past performance data for a particular site or soil specimen.

CREEP POTENTIAL

Singh and Mitchell (1968) proposed the following three-parameter, phenomenological general function for soils which express the strain rate, $\dot{\epsilon}$, at any time t, after application of sustained deviator stress, D

$$\dot{\epsilon} = A\, e^{\alpha D} \left(\frac{t_1}{t}\right)^m \tag{1}$$

where

- A = strain rate at time t_1 and D = 0 (Projected Value)
- α = value of the slope of the mid-range linear portion of a plot of logarithmic strain rate versus deviator stress, all points corresponding to the same time after load application.
- t_1 = unit time
- m = slope of a logarithmic strain rate versus logarithmic time straight line.

Equation (1) was derived both from experimental data obtained by the authors and data in the literature. It appears valid irrespective of whether the clays are undisturbed or remolded, wet or dry, normally consolidated or overconsolidated, or tested drained or undrained.

Taking t_1 as unity, equation (1) can be rewritten in the form

$$\dot{\epsilon} = A\, e^{\overline{\alpha}\overline{D}} \left(\frac{1}{t}\right)^{m} \qquad (2)$$

where $\overline{D}$ is a normalized stress level, defined as the ratio of the deviator stress to the deviator stress at failure, or it may be the stress ratio q/p where q is the deviator stress and p, the mean normal stress, and $\overline{\alpha}$ is the dimensionless parameter defined as the value of the slope of the mid-range linear portion of the logarithmic strain rate versus stress level or stress ratio plot, all points corresponding to the same time after load application.

Equations (1) and (2) have been found to be particularly suited for study of the creep potential of soils. The parameter 'm' in these equations is a key factor in defining the creep potential of a soil; the smaller the value of 'm' the higher is the creep potential of the soil; i.e., the more rapid are the creep movements and the shorter is the time to a given strain under a given stress level. This is illustrated by creep curves shown in Fig. 1. Creep behavior of Redwood City clay (m = 0.75) is shown in Fig. 1a; that of Osaka clay (m = 1.0) is shown in Fig. 1b; and of Sault Ste. Marie clay (m = 1.25) is presented in Fig. 1c. It may be observed that in all cases the strain accumulates with time at an ever decreasing rate. However, the rate of decrease is less, for small values of m, than for large values of m.

This is shown more clearly when the same creep curves are plotted as strain versus logarithm of time as shown in Fig. 2, where the differences in material property become more apparent. Redwood City clay with a small value of 'm' has a creep curve, Fig. 2a, with a positive concave upward curvature; Osaka clay with m=1, Fig. 2b, exhibits a linear relationship; and Sault Ste. Marie clay with still larger m has a creep curve, Fig. 2c, with a concave downward curvature.

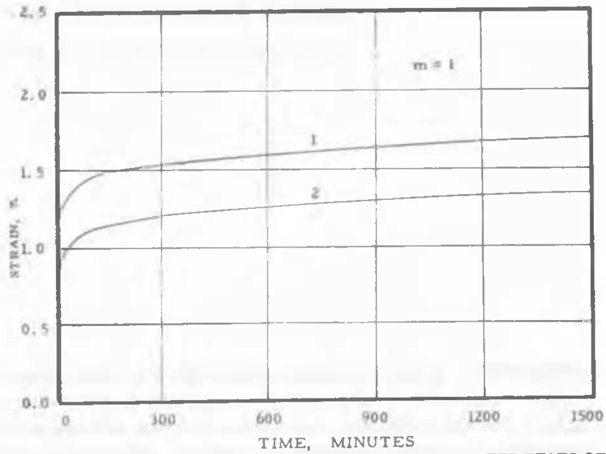

FIG. 1b STRAIN VERSUS TIME: FOR UNDRAINED CREEP TESTS OF UNDISTURBED CLAY. *After S. Murayama and T. Shibata (1964) Cross Plot*

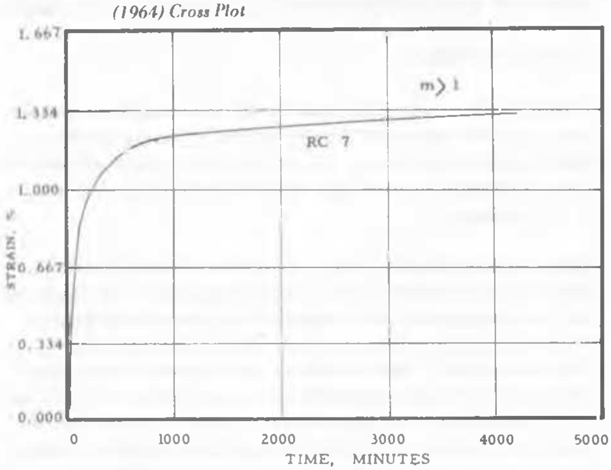

FIG. 1c STRAIN VERSUS TIME: FOR UNDRAINED CREEP TEST OF REMOLDED SAULT STE. MARIE CLAY. *After Christensen and Wu (1964)*

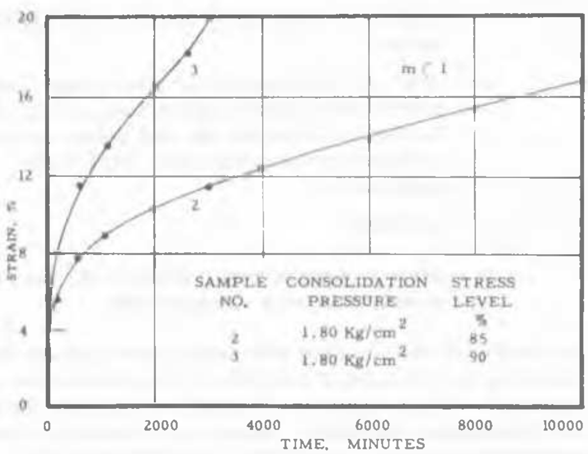

FIG. 1a STRAIN VERSUS TIME: FOR CONSOLIDATED UNDRAINED CREEP TESTS OF UNDISTURBED REDWOOD CITY CLAY (m<1)

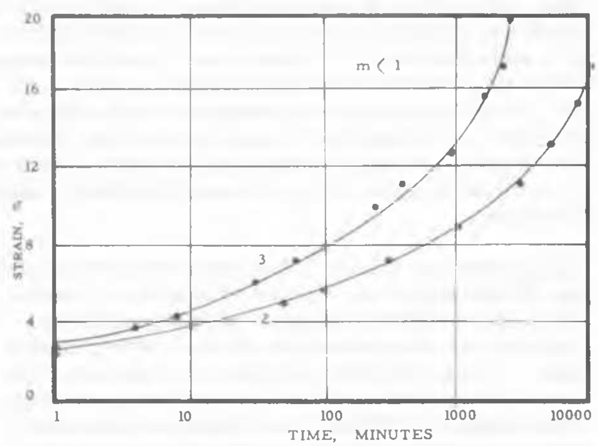

FIG. 2a STRAIN VERSUS TIME: FOR CONSOLIDATED UNDRAINED CREEP TESTS OF UNDISTURBED REDWOOD CITY CLAY (m<1)

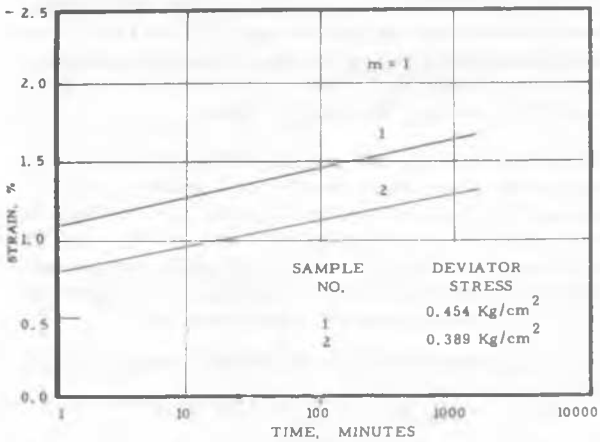

FIG. 2b STRAIN VERSUS TIME: FOR UNDRAINED CREEP TESTS OF UNDISTURBED CLAY. *After S. Murayama and T. Shibata (1964) Cross Plot*

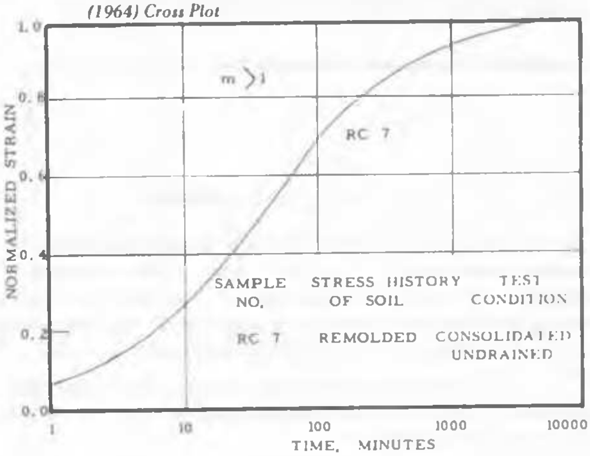

FIG. 2c STRAIN VERSUS TIME: FOR UNDRAINED CREEP TESTS OF REMOLDED SAULT STE. MARIE CLAY. *After Christensen and Wu (1964)*

Figs. 3a through 3c show the relationships between logarithmic strain rate and logarithmic time for the creep curves shown in Figs. 1 and 2. The plots are linear in all cases and the value of 'm' is given by the slope in each case. Results of numerous tests performed in the authors' laboratories and by others have shown that the parameter 'm' is a material property which is only slightly influenced by the test conditions and the stress load imposed. The values of 'm' vary from one soil to another from a low value of about 0.4 to a high value of about 1.3. Table I shows specific values of 'm' for some soils investigated.

The following deductions may be made from Figs. 1, 2 and 3; and equations (1) and (2).

1) Under sub-failure sustained loads, strain accumulates with time at an ever decreasing rate. At a given time, the rate of decrease of strain rate (second derivate of strain with respect to time) may vary from soil to soil.

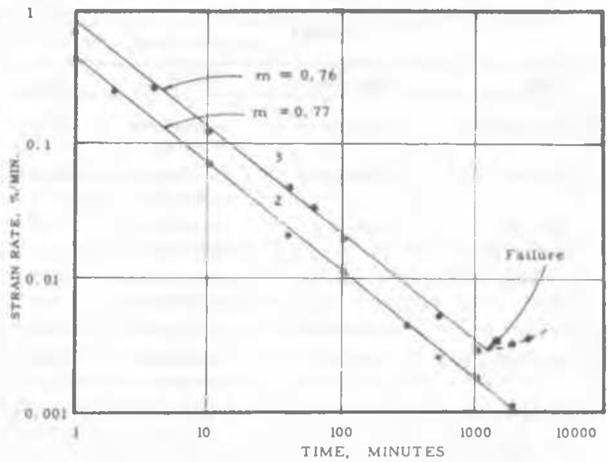

FIG. 3a STRAIN RATE VERSUS TIME: FOR CONSOLIDATED UNDRAINED CREEP TESTS OF UNDISTURBED REDWOOD CITY CLAY (m < 1)

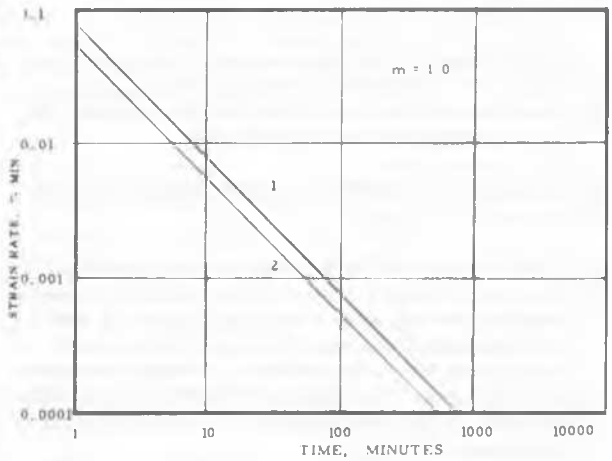

FIG 3b STRAIN RATE VERSUS TIME: FOR UNDRAINED CREEP TESTS OF UNDISTURBED CLAY. *After S. Murayama and T. Shibata (1964) Cross Plot*

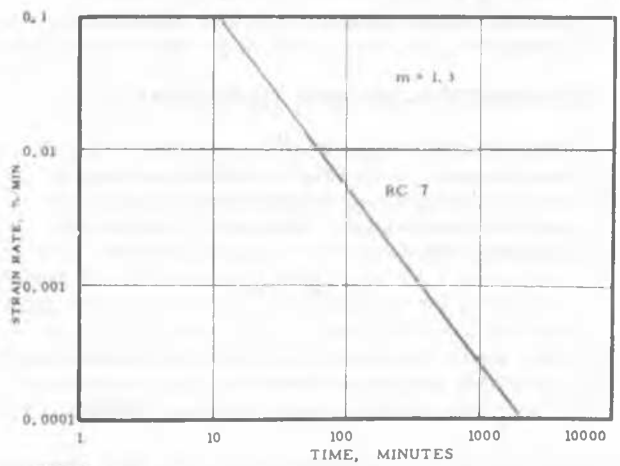

FIG. 3c STRAIN RATE VERSUS TIME: FOR UNDRAINED CREEP TEST OF REMOLDED SAULT STE. MARIE CLAY. *After Christensen and Wu (1964)*

TABLE I

Soil	Stress History	Test Type	'm'
Redwood City Clay	undisturbed	consolidated undrained	0.75
Bangkok Mud	undisturbed	consolidated undrained	0.70
Bentonite Sand Mixture	remolded	consolidated undrained	0.70
Kaolinite Sand Mixture	remolded	consolidated undrained	1.05
Osaka Clay	undisturbed	undrained	1.0
Sault Ste. Marie Clay	remolded	consolidated undrained	1.30
Pacific Palisades Silt	compacted	compressive	1.25

2) For different soils, $d\epsilon/d\log(t)$ may be an increasing function of time; a constant or a decreasing function of time.

3) $d\log(\dot{\epsilon})/d\log(t) = m$ is a constant for a given soil. The value of 'm' varies for different soils. The lower the value of 'm' the greater is the tendency of the soil to accumulate strain with time.

The parameter 'm' provides a useful measure of the creep potential of a soil.

It has been further observed that the soils with $m < 1$ eventually fail in creep rupture under sustained loads less than their short term strengths; the soils with $m=1$ seem to exhibit the same strength before and after creep; and the soils with $m > 1$ exhibit cessation of creep with time. Equations (1) and (2) also predict large strains at long times for $m < 1$ and finite strains at infinite time for $m > 1$.

Low values of 'm' are indicative of high creep potential, softening of soils with shear strains, lower strength after creep and creep rupture under sustained loads.

CREEP RUPTURE AND TIME TO FAILURE

In addition to predicting the creep potential of a soil, it is also desirable to be able to predict how long it will take to develop creep rupture or to reach a certain specified deformation. Equation (1) can be rewritten in the form

$$\dot{\epsilon}\, t = A\, t_1^m\, e^{\alpha D}\, t^{1-m} \tag{3}$$

It may be readily seen that the lumped parameter $\dot{\epsilon}t$ is an increasing function of time for $m < 1$; a constant for $m=1$ and a decreasing function of time for $m > 1$.

Since

$$\dot{\epsilon}\, t = \frac{d\epsilon}{d\log_e(t)} = \frac{d\epsilon}{2.73\log_{10}(t)} \tag{4}$$

it may be seen that $\dot{\epsilon}t$ is a measure of the first derivative of strain with respect to logarithm of time. The lumped parameter $\dot{\epsilon}t$ may be considered an instantaneous creep coefficient, and is a measure of the increase of strain per log cycle of time.

A plot of logarithmic (strain rate x time) versus logarithm of time for Redwood City soil with $m<1$ is shown in Fig. 4a; for Osaka clay with $m=1$ in Fig. 4b; and for Sault St. Marie clay with $m > 1$ in Fig. 4c. It may be seen that $\log \dot{\epsilon}t$ versus $\log(t)$ plots are linear in all cases; with a positive slope if $m<1$; a slope of zero if $m=1$, and a negative slope when $m > 1$.

For $m < 1$ and constant D, integration of equation (1) gives

$$\epsilon = \epsilon_1 - \frac{Ae^{\alpha D}\, t_1}{1-m} + \frac{A\, t_1^m\, e^{\alpha D}\, t^{1-m}}{1-m} \tag{5}$$

such that at $t=t_1$, $\epsilon = \epsilon_1$.

By combining equations (3) and (5)

$$\epsilon = c_1 + \frac{\dot{\epsilon}\, t}{1-m} \tag{6}$$

where $c_1 = \epsilon_1 - \dfrac{Ae^{\alpha D}t_1}{1-m}$ = constant.

Thus the strain at any time 't' is a linear function of the creep coefficient $\dot{\epsilon}t$ at that time. The constant in equation (6) is stress dependent. However for $m<1$ computations show its value is approximately constant since an increase of stress increases both ϵ_1 and $Ae^{\alpha D}\, t_1/\overline{1-m}$ and their difference remains relatively unaffected. Fig. 4a also shows equal strain contours. It may be observed that:

a) The $\log(\dot{\epsilon}t)$ versus $\log(t)$ plot consists of two distinct straight lines. The first part has a slope of (1-m) which could have been anticipated from equation (3). The second part is much steeper denoting high strain rates which represents the onset of extremely large deformations and eventual failure. The onset of the second part of $\log(\dot{\epsilon}t)$ versus $\log(t)$ plot has been assumed to denote creep failure or creep rupture. The onset of creep failure can also be observed on a $\log \dot{\epsilon}$ versus $\log(t)$ plot, e.g. Fig. 3a. Strain rate decreases with time, attains a minimum value, and then starts accelerating leading to failure.

b) Equal strain contours are approximately horizontal denoting a unique relationship between $\dot{\epsilon}t$ and ϵ given by equation (6).

The above observations afford a basis for extrapolation to the time required to achieve a specified subfailure $\dot{\epsilon}t$ and therefore specified strain. The time required to achieve a given $\dot{\epsilon}t$ can be determined by observing the creep strain rates over a period of time and plotting $\dot{\epsilon}t$ versus time on double log paper. Simple extrapolation will give the time required to achieve a prespecified instantaneous creep coefficient. For example in Fig. 4a, sample No. 2, the $\dot{\epsilon}t$ at

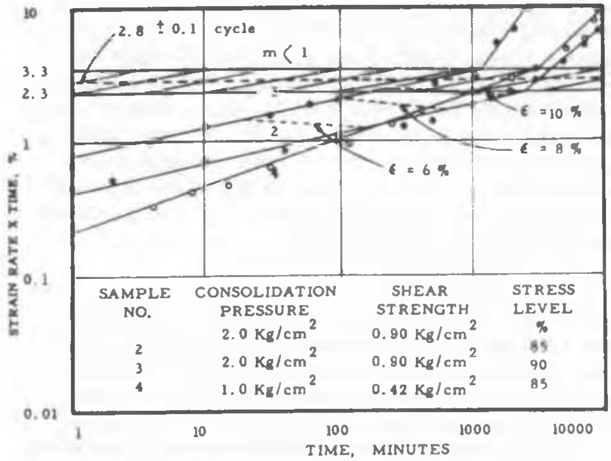

FIG. 4a STRAIN RATE X TIME VERSUS TIME: FOR CONSOLIDATED UNDRAINED CREEP TESTS OF UNDISTURBED REDWOOD CITY CLAY (m<1)

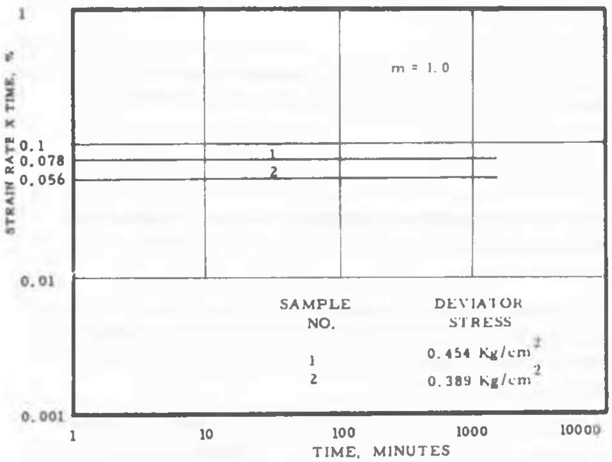

FIG. 4b STRAIN RATE X TIME VERSUS TIME: FOR UNDRAINED CREEP TESTS OF UNDISTURBED CLAY. *After S. Murayama and T. Shibata (1964) Cross Plot*

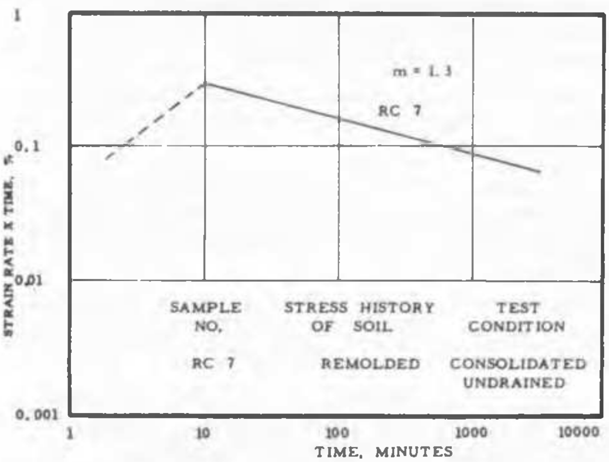

FIG. 4c STRAIN RATE X TIME VERSUS TIME: FOR UNDRAINED CREEP TESTS OF REMOLDED SAULT STE. MARIE CLAY. *After Christensen and Wu (1964)*

time t_1 = 10 minutes is 0.7. The slope of $\dot{\epsilon}t$ versus time plot is 0.24. If $(\dot{\epsilon}\,t)_f$ = 2.8, we could calculate failure of the specimen as

$$\left(\frac{t_f}{t_1}\right)^{0.24} = \frac{2.8}{\dot{\epsilon}\,t}$$

which gives $t_f = t_1\,(2.8/\dot{\epsilon}\,t)^{4.15}$ = 3100 min. The actual failure time was = 2500 min.

Figs. 5a through 5c show log $(\dot{\epsilon}\,t)$ versus log(t) plots at at various stress levels, for overconsolidated Seattle clay (Sherif (1959)); undisturbed Bangkok clay; and undisturbed Tonegawa loam (Saito and Uezawa (1961)) respectively. It may be observed that for a given soil the value of $\dot{\epsilon}\,t$ which causes rupture is fairly independent of stress level and $\dot{\epsilon}\,t$ at failure, $(\dot{\epsilon}\,t)_f$, would therefore appear to be a material property. At any given time a soil element or a soil specimen, under a sustained load will have an instantaneous value of $\dot{\epsilon}\,t$ which may increase (m< 1); remain constant (m = 1) or decrease (m > 1), as more time elapses. The moment this value reaches the $(\dot{\epsilon}\,t)_f$, which will be different for different materials, creep failure may be anticipated. Table II shows the range of $(\dot{\epsilon}\,t)_f$ values for some soils.

TABLE II

Soil	Stress History	Test Type	$(t)_f$
Overconsolidated Seattle Clay m≈0.5 *[Ref. 8]*	undisturbed	consolidated undrained	0.56±0.2 log cycles
Bangkok Mud m 0.70	undisturbed	consolidated undrained	1.35±0.125 log cycles
Tonegawa Loam m≈0.8 *[Ref. 6]*	undisturbed	compressive	1.55±0.3 log cycles
Redwood City Clay m = 0.75	undisturbed	consolidated undrained	2.8±0.1 log cycles

For a given soil if $(\dot{\epsilon}\,t)_f$ is a constant, equation (3) shows that the time to failure t_f will be given by

$$\log_e(t_f) = \frac{1}{1-m}\left[\log_e(\dot{\epsilon}\,t)_f - \log_e(At_1^m) - \alpha D\right] \quad (7)$$

or

$$\log_e(t_f) = \frac{1}{1-m}\,(c_2 - \alpha D) \quad (8)$$

For m < 1 the logarithmic time to failure is a linear function of the stress intensity, and the higher the stress level, the shorter will be the time to failure. This is in agreement with the work of Murayama and Shibata (1964).

CONCLUSIONS

Depending upon the creep potential of a soil, creep may cease, continue or culminate in creep rupture under a given value of sustained stress. If the stress-strain-time behavior is expressed in the

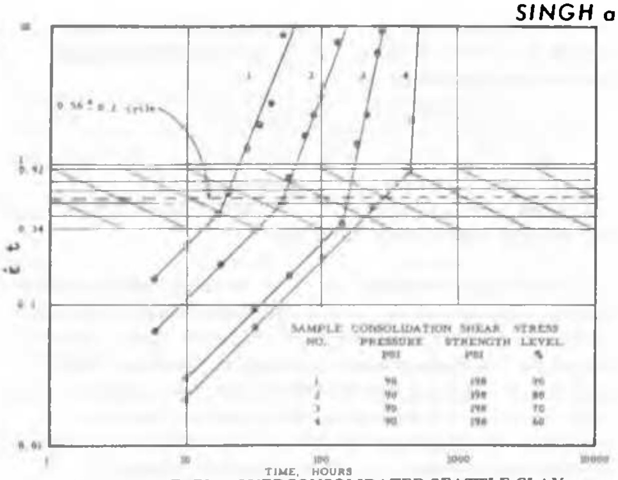

FIG. 5a CREEP TESTS – OVERCONSOLIDATED SEATTLE CLAY.
After M. A. Sherif (1965), Creep Series No. 1, Figs. 29 and 30, Cross Plot

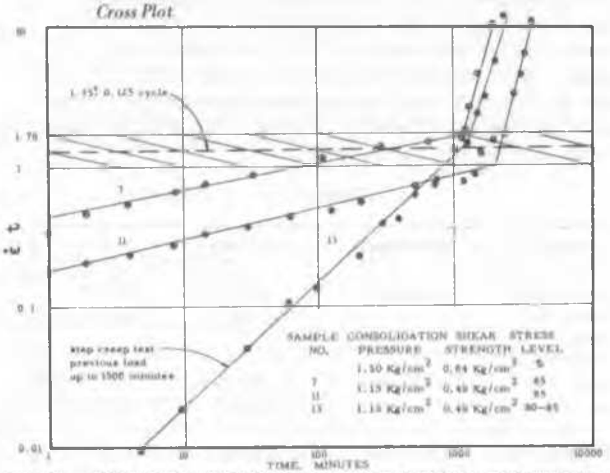

FIG. 5b CONSOLIDATED UNDRAINED CREEP TESTS–UNDISTURBED BANGKOK CLAY

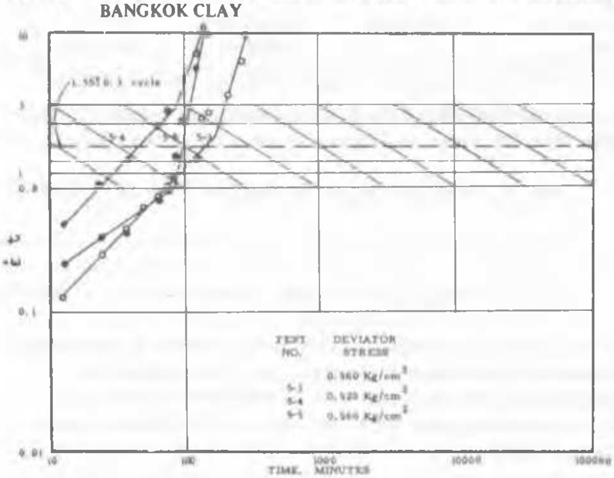

FIG. 5c CREEP TESTS – UNDISTURBED TONEGAWA LOAM.
After M. Saito and H. Uezawa (1961), Fig. 2, Cross Plot

form of equation (1) or (2), the parameter 'm' quantifies the creep potential. The lower the value of 'm' the larger are creep strains and the greater is the probability of creep rupture. Creep rupture appears to be associated only with values of $m \leqq 1$. The instantaneous creep coefficient $\epsilon^{\cdot} t$ is an increasing function of time for $m < 1$; a constant for $m = 1$ and a decreasing function of time for $m > 1$. There exists a value of creep coefficient, i.e., $(\epsilon^{\cdot} t)_f$ which determines the creep rupture of a soil. Such a value, though variable from soil to soil, is a constant for a given soil. The lower the value of $(\epsilon^{\cdot} t)_f$ the lower the strain rates or the shorter the creeping time needed prior to rupture. Knowledge of the value of $(\epsilon^{\cdot} t)_f$ and the $\epsilon^{\cdot} t$ versus t behavior of a soil affords a method for computing the time to failure.

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