DISPLACEMENTS AND INCLINATIONS OF RIGID FOOTINGS RESTING ON A LIMITED ELASTIC LAYER ON UNIFORM THICKNESS

DEPLACEMENTS ET INCLINAISONS DE FONDATIONS RIGIDES SUR UNE COUCHE D'EPAISSEUR FINIE ET UNIFORME

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SYNOPSIS In previous papers (Sovinc, 1954, 1961) solutions have been obtained for the distribution of contact pressures and the displacements of rigid footings; the soil was assumed to be a homogeneous elastic solid, defined by a modulus of deformation E and Poisson's ratio $\nu = 0.3$. Further formulas have been developed for stress distribution and displacements in an elastic layer of uniform thickness, resting on a rigid base; the loading of the elastic layer with flexible load of rectangular shape has been treated in detail.

In this paper a concise analytic procedure is presented enabling the calculation of settlements, inclinations and contact pressures of rigid square and rectangular shaped footings, resting on a limited elastic layer of uniform thickness, bounded by a rigid lower boundary.

INTRODUCTION

In a previous paper (Sovinc, 1954) an analitic procedure for determination of settlements, inclinations and contact pressures of rigid footings by supposing the soil as an isotropic, homogeneous semi-infinite elastic solid was evaluated. In this paper the procedure will be extended on the estimation of displacements and contact pressures in an elastic layer of uniform thickness, resting on a rigid base and loaded by a rigid load of rectangular shape.

Methods of estimating the vertical stress distributions and displacements in an elastic limited layer due to vertical uniformly distributed perfectly flexible load have received great attention during the past years (Burminster, 1956, Egorov, 1958, Tsytovich, 1963, Prikhodchenko, 1966). The solutions given in this paper were derived s some years ego (Sovinc, 1961) and are based on the following assumptions: the layer of uniform thickness is elastic and homogeneous medium obeying Hooke's low with Poisson's ratio $\nu = 0.5$; the contact between the layers is smooth and frictionsless; the ratio between the horizontal dimensions of the layer and of the loaded area is large enough to permit the application of resulting values to a compressible layer of infinite horizontal dimensions.

Using Lamee's differential equations of equilibrium and neglecting the weight of the layer itself, the displacements of an elastic layer with length a, width b and height h have been taken in the form of double trigonometrical series

$$u_1=\sum_m \sum_n U \cos\alpha x_2 \cos\alpha x_3$$
$$u_2=\sum_m \sum_n V \sin\alpha x_2 \cos\beta x_3 \quad \ldots (1)$$
$$u_3=\sum_m \sum_n W \cos\alpha x_2 \sin\beta x_3$$

The solutions, obtained by substituting expressions (1) in equilibrium equations

$$(1-2\nu)\nabla^2 u_i + \frac{\partial \gamma_0}{\partial x_i} = 0; \quad \ldots (2)$$

$i=1,2,3$

are biharmonic functions

$$U= \frac{1}{2G} \{[(3-4\nu)C_4-C_2\gamma]\sinh\gamma\psi_1+[(3-4\nu)C_3-$$
$$-C_1\gamma]\cosh\gamma x_1-\gamma x_1(C_1\cosh x_1+C_3\sinh\gamma x_1)\}$$
$$V= \frac{1}{2G}[(C_1\alpha+C_5\beta)\sinh\gamma x_1+(C_2\alpha+C_6\beta)\cosh\gamma x_1+$$
$$+\alpha x_1(C_3\cosh\gamma x_1+C_4\sinh\gamma x_1)] \quad \ldots (3)$$
$$W= \frac{1}{2G}[(C_1\beta+C_5\alpha)\sinh\gamma x_1+(C_3\beta+C_6\alpha)\cosh\gamma x_1+$$
$$+\beta x_1(C_3\cosh\gamma x_1+C_4\sinh\gamma x_1)]$$

where $\alpha= \frac{m\pi}{a}$, $\beta= \frac{n\pi}{b}$, $\gamma^2=\alpha^2+\beta^2$ and C_1-C_6 integration constants, resulting from the boundary conditions. In the next step the solutions of displacements u_i and stresses σ_{ii} resp. σ_{ik} were developed. The numerical evaluations of vertical stresses σ_{11} and displacements u_1 were carried out for a number of values of the basic parameters. As an example in Fig. 1 the vertical displacements of the surface of a layer of finite thickness h due to vertical flexible load of intensity p acting over a square area with half side $\frac{B}{2}$ for the ratio $\frac{h}{B}$ = 10 are presented. The settlement of any point i of the surface is given by the expression

$$u_{1,i} = \frac{pB}{2E} I_1$$

where E is modulus of linear deformation of the soil in the finite layer and I_1 influence factor given in Fig. 1.

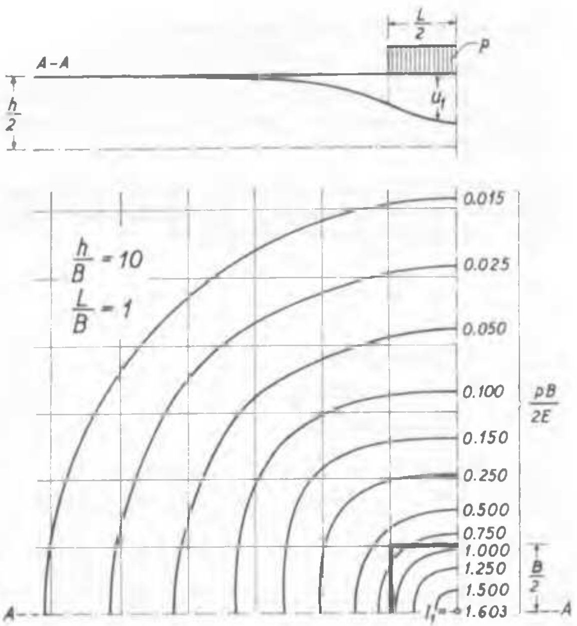

Fig. 1

METHOD OF PREDICTING OF SETTLEMENTS AND OF DISTRIBUTION OF CONTACT PRESSURES

The problem of predicting of settlements of rigid footings and of distribution of pressures at the contact between the foundation and the bearing soil, by using the assumption that the soil behaves as an elastic solid, is extensively treated in recent publications (Schultze, 1963, Kany, 1963, De Beer, Grasshoff and Kany, 1966, Grasshoff, 1966, Siemer, 1967, El-Kadi, 1968).

The present method for two-layer rigid-base system does not differ much from those used in author's solution for elastic half-space and is based on following consideration: A rigid footing connot follow the settlements as obtained for perfectly flexible load. It will settle for a medium value u and the contact pressures will concentrate more at edges. It is the intention of this paper just to estimate this average settlement as well as the distribution and magnitude of contact pressures for square shaped and all rectangular shaped footings with various side-and depth ratios.

The following procedure was decided upon: The footing is divided into small square load elements with half sides c. For such an element k the average settlement is given with equation (4). If the surface of the compressible layer is loaded on the element k, also the rest of square elements into which the footing is divided, will settle. Following this principle the settlements of all square elements due to loading of the entire footings resp. of every element, can be established gradually. For every position of the footing the influence coefficients a_{ik} representing the magnitude of the average settlement of the element i due to the loading X_k = 1 acting on the area k, should be summarized. For the reason of simmetry the computation of a quarter in rectangular shapes /m_1 = 4/ resp. an eighth in square shapes /m_2 = 8/ of the footing suffice.

The settlement of the yielding surface of the elastic layer must be equal to the displacement of the footing, e.h.

$$\sum_{k=1}^{n} a_{ik} \frac{q_k c}{E} = u_1 = u = \text{const.} \quad \ldots (5)$$

The contact pressure on the element k is

$$q_k = \frac{X_k}{4c^2} \quad \ldots (6)$$

/X_k = contact force on the element of square area $4c^2$/ and the expression (5) may be written in the form

$$\sum_{k=1}^{n} a_{ik} \frac{X_k}{4cE} = u = \text{const.} \quad \ldots (7)$$

The condition of equilibrium is expressed by the equation

$$m \sum_{k=1}^{n} X_k = P \quad \ldots (8)$$

P beeing the total force on the footing. We write the equations (7) and (8) in the form

$$\sum_{k=1}^{n} a_{ik} x_k = u' = \text{const.} \quad \ldots (9)$$

and

$$\sum_{k=1}^{n} x_k = 1 \quad \ldots (10)$$

For n elements k we obtain a system of linear non-homogeneous equations containing as many unknowns as the m-th part of the footing has elements. To these equations we add the equilibrium equation (8) which helps to determine the last unknown u representing the average settlement of the yielding surface of the compressible layer. The equations were solved for various parameters $\frac{L}{B}$ and $\frac{2h}{L}$ using an electronic computer.

The correct value of the settlement of the footing u is obtained by the equation

$$u = u' \frac{P}{m4cL} = \beta \frac{pL}{E} \qquad \ldots (11)$$

and the contact pressure on the element k by the expression

$$q_k = \frac{n x_k P}{m} \qquad \ldots (12)$$

The diagram in the Fig. 2 shows the values of coefficient β computed for various side ratios $\frac{L}{B}$ of the footings and for various ratios between the thickness of the compressible layer and the longer half side of the footing $\frac{2h}{L}$. By means of the diagram the average settlement of rigid foundations are determined according to equation (11) where p is the uniform loading of the footing, L its longer side and E the modulus of linear deformation of the soil.

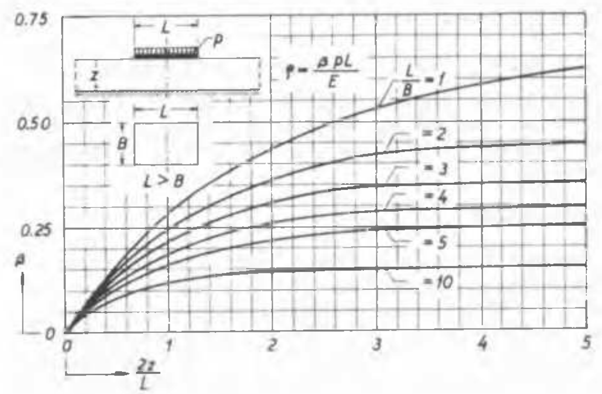

Fig. 2

METHOD OF PREDICTION OF INCLINATIONS AND OF DISTRIBUTION OF CONTACT PRESSURES

The surface of an elastic layer loaded by the moment M on square or rectangle flexible area is deformed approximately to the form shown in Fig. 3. In this case of loading likewise, such different deformations cannot be followed by a rigid footing. The inclination of the footing will be linear and the contact pressures will concentrate on the edges.

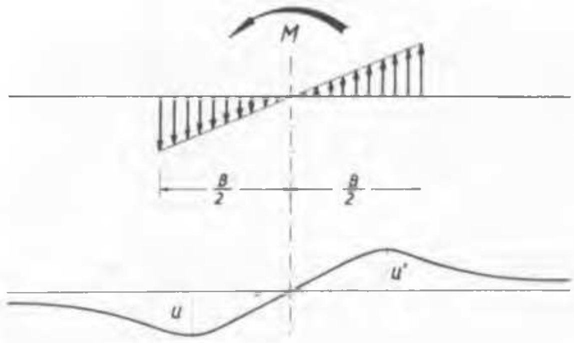

Fig. 3

The procedure is similar to that applied in the case of uniformly distributed load. The footing is divided into small square elements. Settlements and pressures on the elements lying at equal distances on both sides from the central line, about which the moment M is acting, are equal but have opposite signs. The displacements increase linearly. For reason of simmetry the n-th part of the footing will be dealt with.

The condition that the displacement of the yielding subsoil is equal to the displacement of the footing, has an analitical form

$$\sum_{k=1}^{p} a_{ik} \frac{X_k}{4cL} = \varepsilon_i 2c\phi \qquad \ldots (13)$$

where

a_{ik} is the average displacement of the element i due to the load X_k acting on the element k,

$\varepsilon_i 2c$ is the distance of the center of the element i from central axis of the footing and

ϕ angle of rotation.

n equations of type (13) with /n+1/ unknowns and the equilibrium equation will be used:

$$n \sum_{k=1}^{n} X_k \varepsilon_i 2c = M \qquad \ldots (14)$$

We write equation (13) in the form

$$\sum_{k=1}^{n} a_{ik} x_k = \varepsilon_i \phi' \qquad \ldots (15)$$

and

$$\sum_{k=1}^{n} x_k \varepsilon_i = 1 \qquad \ldots (16)$$

By solving the system of equations (15) and (16) the following expression for inclination of rigid foundations in the directions of the acting moments M_x and M_y are obtained:

$$\varphi_x = \frac{\phi' M_x}{m 16 c^3 E} = \gamma \frac{M_x}{(L/2)^3 E} \quad \ldots (17)$$

$$\varphi_y = \frac{\phi' M_y}{m 16 c^3 L} = \gamma \frac{M_y}{(B/2)^3 E} \quad \ldots (18)$$

For this case likewise, Fig. 4 shows the coefficient γ as function of the footings side ratio $\frac{L}{B}$ and of the depth ratio $\frac{2h}{L}$.
Contact pressures on the surfaces k are computed according to expression

$$q_k = \frac{n x_k M}{m \beta c^3} \quad \ldots (19)$$

Contact pressures for centrically and eccentrically loaded footings have been calculated for various base parametres. They are in comprehensive tables and are in course of publication.

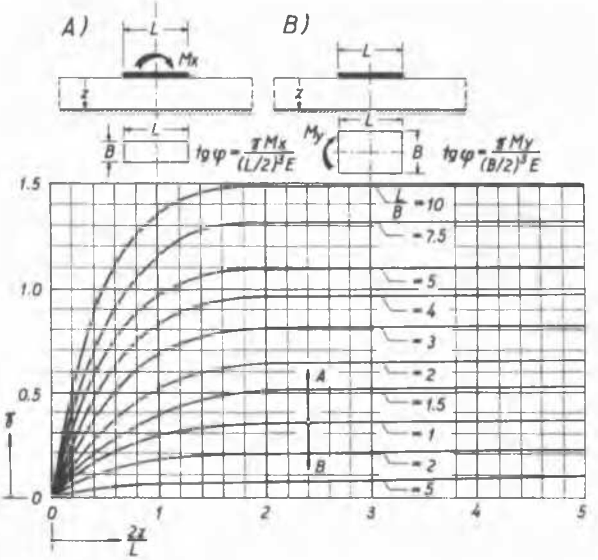

Fig. 4

NOTES ON ESTIMATING RESULTS

In cases where the basic surface is divided into more or less square elements, parallel computations show that the magnitude of displacement does not essentially depend upon the density of basic surface division. Contact pressures, however, depend a great deal upon the density of division. Nevertheless, it must be empasized, that stress condition remain in the proximity of computed conditions only until the soil is considered to be still an elastic medium. Accurate theories on division of contact pressures lead to the conclusion that the contact pressure on the edges of a rigid footing is infinite for every final footing load. For this reason plastic deformations occur on slab edges as soon as the loading begins to act. By the incerase of loading the plastic zone widens, and the differences between the theoretically computed and real contact pressures increase. The stresses at the footing edges are more and more rounded off and consequently the contact pressures assumed in our calculations represent at least a slight but desired approximation to the real stress condition. In practice plastic deformations of material underneath the foundation edges at a proper foundation depth will occur rarely. Therfore the expressions derived for settlement computation in most cases yield satisfactory data for estimating safety of foundations. In the calculation of inclinations of deep foundations, however, the earth layers resting about the foundation base, which oppose such deformations, must be taken into account.

The present method might be applied to the analysis of deformable footings. The condition of equating the subsoil settlements and elastic footing deformations has the following form:

$$\sum_{k=1}^{n} a_{ik} X_k - u + w_{ip} = 0 \quad \ldots (20)$$

in which w_{ip} is the footing deflection in place i due to the effect of external load p which is quoted by literature dealing with the theory of elasticity for various types of loading p.

CONCLUSIONS

A method for calculating the displacements and contact pressures in an elastic layer of uniform thickness, resting on a rigid base and loaded by a rigid load of rectangular shape has been developed. The coefficients β and γ, presented in Fig. 2 and 4, permit a very quick determination of the average settlement and the angle of rotation by means of equations (11), (17) and (18).

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