EXPERIMENTAL EVALUATION OF YIELD SURFACES

DETERMINATION EXPERIMENTALE DES SURFACES DE GLISSEMENT

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SYNOPSIS A method is suggested for the experimental determination of yield surfaces (surfaces of sliding) in disperse media. The relative displacement between the grains is determined by X-raying bead threads or heavy metal chains placed in the medium. The method is free of the influence of friction between the side glass walls of the test box and the medium. It enables the solution of a number of three-dimensional problems of the stability of foundations and slopes and the theory of earth pressure.

INTRODUCTION

As is known from the statics of disperse media, a limit equilibrium sets in the points where the shear strength, characteristic for the medium, is exhausted, followed by a relative displacement between the grains. These points are usually infinitely close to each other and form a continuous surface of sliding.

The evaluation of the surfaces of sliding helps the solution of a number of problems in the field of the stability of foundations and slopes and earth pressure.

To our knowledge hitherto these surfaces have been determined only for two-dimensional cases by photographing through the limiting glass wall of the test box as reviewed by FRÖHLICH, 1934; TSYTOVICH, 1962; GORBUNOV-POSSADOV, 1962. This method cannot be applied for three-dimensional problems; for plane problems it is also insufficiently precise, due to friction between material and glass, which is considerable, as we shall see below. Nevertheless, the results obtained after this method have served as a basis for a number of theoretical conclusions (TERZAGHI, 1943; GORBUNOV-POSSADOV, 1962).

The size of the relative displacements between the grains can vary from infinitesimal to a few millimetres. It depends mainly on some properties of the loose medium (grain size, internal friction, etc.), on the value of the shear stresses due to external forces (loading, variation of support conditions within the medium, own dynamic impulses, temperature variations, etc.) and to a certain extent on the rate of their manifestation.

Therefore the ideal experimental setting for evaluating the surfaces of sliding would be as follows:

a) The size and quality of the material for the loose medium should be suitable for modelling, that is the grain size and the angle of internal friction should be relatively small, so that larger displacements could be obtained under smaller external action.

b) The determination of the displacement should occur within the medium in order to eliminate the influence of friction between glass wall and medium. Such a method would give the possibility of solving three-dimensional problems, too.

c) The smallest relative displacements should also be determined.

d) The continuity of the medium should be least possibly disturbed, in order to avoid a redistribution of the stresses in the points of interest.

An experimental method was drawn up which in our opinion almost fully satisfies the above requirements.

1. The central idea of the method suggested is as follows: within a loose medium, which relatively easily lets the X-rays through, thin plates, threads, chains or columns of a different substance of a lower X-ray penetration are placed during the filling of the medium in a given disposition at a certain distance between them. Then an external force is gradually applied; either the foundation is loaded or the supporting conditions of the loose medium are altered, the changes being registered in a series of X-ray pictures, taken at each displacement, corresponding to the respective loading stage. The comparison of the first (basic) X-ray picture with each of the following ones reveals the places of relative displacement between the grains at the corresponding stage.

The pictures can be taken in two ways:

a) with an X-ray apparatus,

b) with radioactivated bodies (it is possible to use tracer grains from the loose medium, too).

The first method is relatively safe for those working with it, while the second requires complex protective measures and is inconvenient in case of a greater number of tests, without having any important advantages.

2. After a series of tests with loose media of glass sand, quartz sand, glass spheres (size 1 to 4 mm), plastic spheres (size 0.4 to 0.8 mm), wheat, lentils, millet and others it was established that millet was most suitable for the purpose. Its advantages are as follows:
a) Low unit weight of the grains $\gamma_s = 1.37$ g/cm^3, almost twice as low as that of quartz glass; thus it shows a much better X-ray penetration.
b) Small angle of internal friction - 21.2^o and an angle of friction with glass - 11.3^o.
c) Relatively small non-deformable grains of a mean diameter of 1.1 mm, covered with a smooth and hard skin. The relation of the smallest to the largest size of a grain is not less than 0.8, so the form is near spherical.
d) The separate grains are almost equal in size, the variation being less than 10%, so that a uniform medium is easily obtained by passing the grains through 1 and 2 mm sieves. This circumstance explains the small difference in the unit weight: loose - 0.748 g/cm^3, compacted (after vibration, shaking, compression) - 0.782 g/cm^3, thus the maximum compaction is about 4.5%. This justifies the presumption that the formation of zones of different density, which would affect the stress distribution, is impossible during the filling of the material. The difference between the density of the immobile and the displaced zones of the material will be insignificant.

These advantages indicate that millet is an exceptionally suitable material for modelling a loose medium.

3. Thanks to its high density ($\gamma_s = 10.49$ g/cm^3), strength and relatively accessible price, silver proved to be most suitable among the tested materials, for registering the relative displacements of the particles. Aluminium, zinc, brass and copper do not possess a sufficient density to obtain contrasting X-ray pictures.

Threads, plates (foil), chains and columns made of different materials were tested. It was definitely proved that chains and bead threads were most suitable (Fig.1). They have the following advantages:
a) They are best surrounded by the loose material and their separate chain links or beads can be assumed with sufficient approximation as grains of the medium, especially when they are chosen to have the same size. They do not disturb the continuity of the medium (unlike plates).
b) They secure a sufficiently contrasting and continuous picture.
c) Provided the chains are made of rectangular links and are covered in a loose state or the thread is drawn out after filling, the links or beads allow relative horizontal displacements without tensile forces, arising

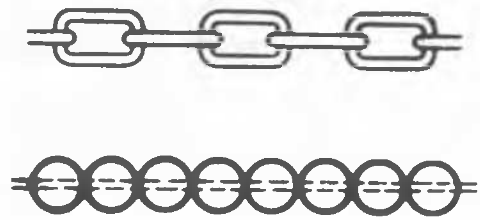

Fig.1 Chain and bead thread for evaluating the relative displacement of the grains

in the chain.
d) Placing, fixing, covering and taking out is easy and simple.

4. This combination of millet and silver chains secures sufficiently contrasting X-ray pictures under the following conditions:
a) The walls of the box through which the X-rays will pass should be of wood or glass.
b) The focal distance of the X-ray apparatus should be 1 m from the external wall of the box.
c) The thickness of the loose medium should be 65-70 cm.
d) The pictures should be made with 'hard' X-rays at a voltage of 130 KV and an amperage of 4 mA, and an exposure of about 35-40 sec.
e) Ordinary medical films can be used with reinforcing foils in the plate-holders.
f) Lead screens should be placed behind the plate-holders and besides the loose material in order to protect the films from the reflected and refracted rays.

5. The accuracy of the measured relative displacement depends on the contrast of the pictures and the skill of the operator. In case the contrast is obtained under the above-mentioned conditions, the accuracy depends mainly on the operator's experience, therefore the error is 0.5 - 0.8 mm in a direction perpendicular to the thread and 3-4 mm along its length. In case of very contrasting pictures (obtained at a loose medium thickness of up to 50 cm), after being enlarged, the accuracy can be raised to 0.2 and 2 mm, respectively. We consider this accuracy to be the limit for optical estimates; it can be still raised only if the measuring of the relative displacements is conducted by an electrical or other method.

6. As we know the existing X-ray apparatuses radiate focused rays, which spread in the form of a cone. This makes it necessary to conduct the X-raying and the reading of the pictures in the following way:
a) The box and the plate-holder are placed parallel to the axis of the X-ray tube.
b) A special device to the X-ray apparatus serves to define the position of points F_A and F_B in relation to the basis 0-0 and the retaining wall 0-V, that is, the place where the central ray F-F crosses the box walls and the film (Fig.2).

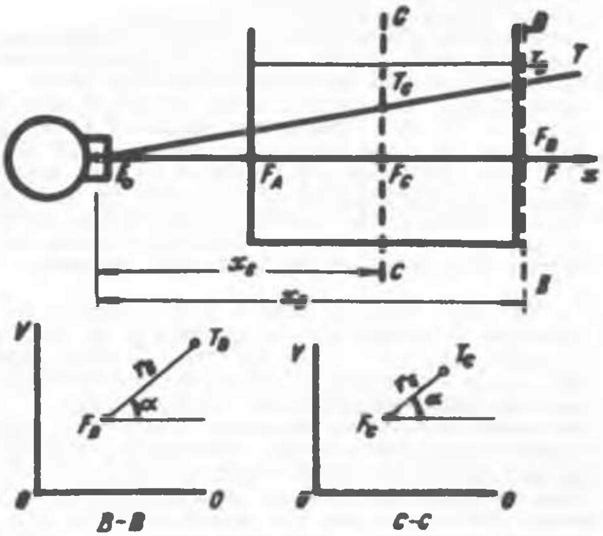

Fig.2 Pattern for reading the X-ray pictures

1 - box; 2 - X-ray apparatus; 3 - film; C-C - plane of the threads; B-B - plane of the film; a - longitudinal section; b - cross-section through film; c - cross-section through threads

c) After the first (basic) X-ray picture a drawing of the plane B-B (Fig.2-b) is made on tracing paper, plotting O-V, O-O, F_B and the initial disposition of the chains.

d) Comparison is made with any of the following X-ray pictures, taken after a certain stage of the displacement of the wall by placing the picture over the tracing paper drawing. The points where the initial relative displacement has occurred in each chain is defined (the boundary between the immobile and the displaced part of the loose medium). These points lie in plane B-B (for instance T_B).

e) In order to define the actual position(T_C) of these points in plane C-C, passing through the threads (Fig.2-c), we have to bear in mind that X-rays spread like a cone with its top in F_0; points F_0, F_C and F_B lie on the central ray F-F, points F_0, T_C and T_B - on the straight line F_0-T, the lines $F_BT_B = r_B$ and $F_CT_C = r_C$ lie in the plane FF-F_0T, therefore the angle α is identical for both planes B-B and C-C and $r_C = r_B \frac{x_F}{x_B}$.

f) The points thus obtained in Fig.2-c for each chain are joined and give the wanted curve (or line) of sliding in plane C-C.

g) Similarly the curves of sliding for the other planes, in which chains have been placed, are defined. In practice it is possible to evaluate the pictures using chains placed in 5 or 6 planes.

h) In case of three-dimensional problems it is possible to take X-ray pictures along all three axis; the fixing of the points of relative displacement can be made with thread rows placed in advance in the respective planes. The evaluation of the pictures for displacements which are perpendicular to the rays of the apparatus is conducted similarly.

7. There are special reticular lead X-ray refractors, beyond which the rays spread in almost parallel direction. Such a device would considerably simplify the evaluation of the pictures; absolutely parallel rays would also give a higher accuracy.

A number of tests were conducted after this method in order to get an idea of its applicability and accuracy. Pictures were taken for defining the surfaces of sliding in plane problems, for instance in the case of:

- stability of the subgrade under a model strip foundation (Fig.3)

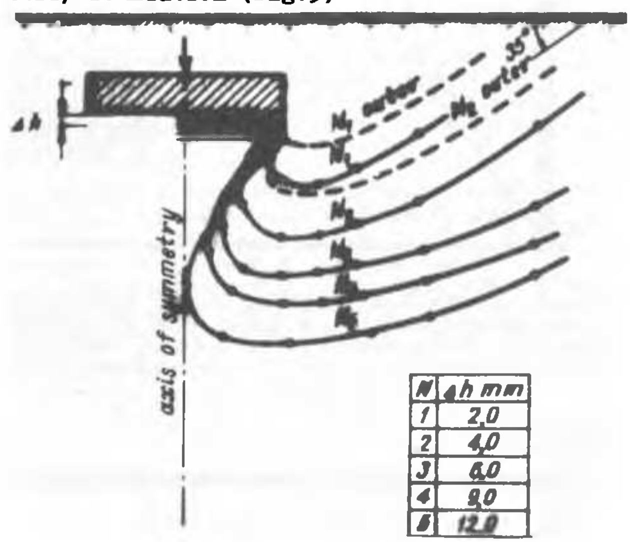

Fig.3 Surface of sliding under a model strip foundation

- active earth pressure in the case of forward parallel displacement of the wall (Fig.4)

- passive earth pressure in the case of backward parallel displacement of the wall (Fig.5)

The tests were carried out with a vertical glass wall, a horizontal surface of the loose medium and a box width of 50 cm. The height of the wall during the tests was 55 cm. Each study was conducted with two or more X-ray pictures series. The deviations obtained in the surfaces of sliding in the identical test series for the separate cases do not exceed 5-6 mm, and this only for some of the points along which the curves are described. The general character, the position, the curvature and the inclination of the surfaces of sliding are entirely identical, which proves their truthfulness and the absence of a casual error.

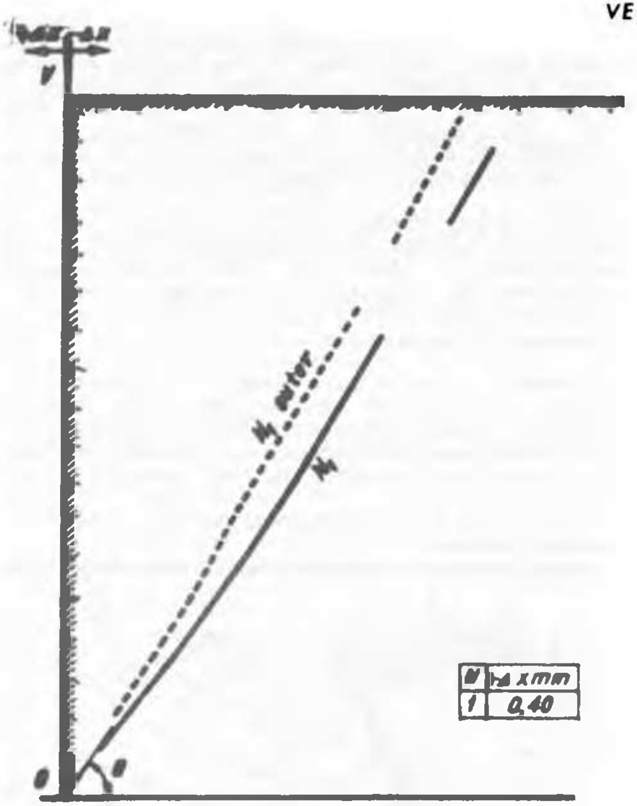

Fig.4 Surface of sliding in the case of active earth pressure, arising from the forward parallel displacement of the wall, next to the glass (dotted line) and in the medium (full line)

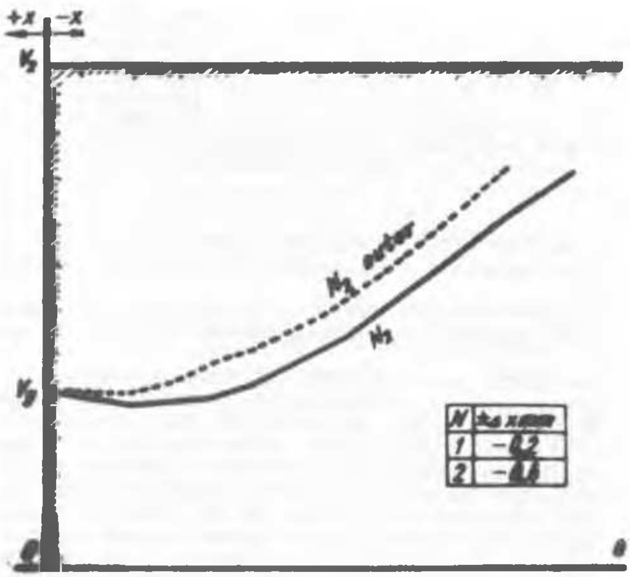

Fig.5 Surface of sliding in the case of passive earth pressure, arising from the backward parallel displacement of the wall, next to the glass (dotted line) and in the medium (full line)

8. Besides with a slightly compacted loose medium (such as millet) tests were alsoconducted with glass sand of varying grain size and with pure quartz sand. In order to obtain contrasting pictures under the above conditions, the width of the test box was reduced to 26 cm. Since the angle of internal friction in these media is bigger - 29° to 34°, the influence of friction on the side walls was considerable. In order to reduce this influence the width of the medium should be increased by using an X-ray apparatus of a higher capacity, as well as lead threads.

9. The best results with a compressible medium are obtained by using crushed millet of a grain size of 0.1 to 1.1 mm. In this case the angle of internal friction is about 27.5° and the angle of friction with glass - 13°. The unit weight is: loose - 0.62 g/cm^3, compacted (compaction with a pressure of 0.055 kg/cm^2 on a 3 cm thick layer) - 0.72 g/cm^3, that is, it possesses a compressibility of about 17%. The results obtained with this medium practically coincide with the results from the non-compressible medium in Fig.4 (case after loosening), while in Figs. 3 and 5 the results coincide only after 3-3.5 times greater displacements, that is, after the necessary compaction is attained.

10. A loose medium with a certain degree of cohesion was also experimented. A medium of millet grains covered with mineral oil of a higher viscosity was tested. Mineral oil is not absorbed by the grains and forms a durable thin film around them, which does not run down. Thus a medium of a constant angle of internal friction 16° and a cohesion of 0.007-0.01 kg/cm^2 can be obtained, these values being very suitable for models.

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