PORE PRESSURE DURING SHEAR FOR AN ANISOTROPIC SOIL

PRESSION INTERSTITIELLE PENDANT LE CISAILLEMENT D'UN SOL ANISOTROPE

A. ZUR, B.Sc., D.I.C., M.Sc., (Eng.), Instructor
G. WISEMAN, M.Sc., D.Sc., (Eng.), Associate Professor

Faculty of Civil Engineering, Technion-Israel Institute of Technology, Haifa, Israel

SYNOPSIS Consolidated undrained pure deviatoric stress controlled triaxial compression and extension tests have been carried out on fully saturated undisturbed clayey loess. Pore pressures have been measured during the entire shearing phase. For comparison some tests were carried out on artificially sedimented samples. Anisotropy of the soil was studied by testing both vertically and horizontally sampled specimens in the oedometer and the triaxial tests. A model has been postulated for predicting the pore pressure developed during the application of a deviator stress, considering the influence of anisotropic compressibility and dilatancy. This model has been found to be capable of predicting the relative pressure developed for various stress paths.

INTRODUCTION

This paper describes the pore water pressure behaviour of a fully saturated undisturbed clayey loess stressed to failure following different stress paths. This topic is part of a research program being carried out at the Israel Institute of Technology into the geological structure of undisturbed loess and its relation to the relevant engineering properties.

An attempt is made to understand the variables affecting the pore water pressure (u) parameters, and based on this understanding try and predict the pore pressure set up during an isotropically consolidated undrained (I.C.U.) test. A soil having geological structure is best suited for the present argument, as it introduces the largest possible number of variables. Pore pressure parameters, by their very definition, relate the changes in pore pressure to changes in the applied stresses. In order to define this relationship a model was used which takes the form:

$$\Delta u = \beta \Delta \sigma_{oct} + \alpha \Delta \tau_{oct} \qquad (1)$$

This equation was further modified allowing for anisotrophy in compressibility characteristics. The validity of the model was checked by operating on it with different stress paths (pure deviatoric axial compression and extension).

PORE PRESSURE AS AFFECTED BY SOIL STRUCTURE

Most soils in their natural, undisturbed, state posses a typical arrangement of the particles making them up. It is reasonable to assume that this arrangement imparts directional qualities to a soil sample (which is another way to define structure). Hence rotating the directions of the principal stresses and/or planes will result, inevitably, in different stress-strain characteristics, which in turn, may be used to interpret the soil structure. The same holds true for comparison of undisturbed samples having the same initial dry density and water content. As shown by Duncan and Seed (1966) both the inherent strength components (ϕ_e and C_e) as well as the pore pressure coefficient at failure A_f are affected by the soil structure. However it is the pore pressure which is most sensitive in this respect; this has been proven experimentally by Seed and Chan (1959) in earlier work. Since the structure alters during shear it is of interest to follow the development of the pore pressure during the entire shearing. A model for predicting pore pressure should possess the ability to account for these changes.

STRESS PORE PRESSURE RELATIONSHIP

The stress in an isotropic homogeneous elastic medium can be divided into two components: A hydrostatic stress accounting for volume change, and a deviatoric stress accounting for the change in shape. Octahedral normal and shear stresses give a measure of the hydrostatic and deviatoric components of the state of stress. If σ_1, σ_2, σ_3 are the principal stresses at a point, then:

$$\sigma_{oct} = \frac{\sigma_1 + \sigma_2 + \sigma_3}{3} \qquad (2)$$

$$\tau_{oct} = \frac{1}{3}\sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2} \qquad (3)$$

Thus a generalized equation for the pore pressure development during shear of a soil in an undrained test, in terms of octahedral and deviatoric stresses will be of the form (Henkel 1958):

$$\Delta u = \beta \left(\frac{\Delta\sigma_1 + \Delta\sigma_2 + \Delta\sigma_3}{3} \right) + \alpha \sqrt{(\Delta\sigma_1 - \Delta\sigma_2)^2 + (\Delta\sigma_2 - \Delta\sigma_3)^2 + (\Delta\sigma_3 - \Delta\sigma_1)^2} \quad (4)$$

where β and α are empirical coefficients. For a triaxial test $\sigma_2 = \sigma_3$ or $\sigma_2 = \sigma_1$; equation (4) reduces to:

$$u = \beta \left(\frac{\Delta\sigma_1 + \Delta\sigma_2 + \Delta\sigma_3}{3} \right) + \sqrt{2}\alpha \ (\Delta\sigma_1 - \Delta\sigma_3) \quad (5)$$

The analogous equation for volume change in a drained test for a soil having anisotropic compressibility characteristics, should be:

$$\frac{\Delta V}{V} = m_1 \Delta\sigma'_1 + m_2 \Delta\sigma'_2 + m_3 \Delta\sigma'_3 \pm D(\Delta\sigma_1 - \Delta\sigma_3) \ldots\ldots \quad (6)$$

where:

$\frac{\Delta V}{V}$ = relative volume change

$\Delta\sigma'_1; \Delta\sigma'_2; \Delta\sigma'_3$ = effective principalstress increments,

m_1, m_2, m_3 = compressibilities in the direction of the 3 principalstresses.

D = dilation coefficient.

Expressing equation (6) in total stress terms and

setting $\frac{\Delta V}{V} = 0$,

the pore pressure developed in any undrained triaxial test can be expressed in terms of the compressibility coefficients m and the dilation coefficient D

$$\Delta u = \frac{m_1 \Delta\sigma_1 + m_2 \Delta\sigma_2 + m_3 \Delta\sigma_3}{m_1 + m_2 + m_3} \pm \frac{D}{m_1 + m_2 + m_3} (\Delta\sigma_1 - \Delta\sigma_3) \quad (7)$$

Equation (7) is a generalized pore pressure equation, and it implies that the pore pressure is generated by two distinct components: one due to the anisotropic compressibility of the soil and the other due to its dilatant properties. The generality of equation (7) may be shown by operating on it with different stress paths and compressibility properties.

Consider a conventional triaxial compression test, in which the axial stress is increased to failure while the lateral stress is kept constant, and assume an idealy isotropic and reversible material i.e. $m_1 = m_2 = m_3 = m$.

Equation (7) degenerates to:

$$\Delta u = \frac{\Delta\sigma_1}{3} \pm \frac{D}{3m} (\Delta\sigma_1 - \Delta\sigma_3) \quad (8)$$

which is the form suggested by Shibata and Karube (1966).

Considering still the same test and operating with different assumptions namely D = 0 and $m_2 = m_3$ Equation (8) degenerates to:

$$\Delta u = \frac{1}{1 + \left(\frac{2 m_3}{m_1}\right)} \Delta\sigma_1 \quad (9)$$

which is the form suggested by Bjerrum (1954).

In order to examine the predictive power of equation (7) it is instructive to consider each one of its components separately.

Assume D = 0, ie a non dilatant soil. For axial compression $\Delta\sigma_2 = \Delta\sigma_3$ and $m_2 = m_3$ hence:

$$\Delta u = \frac{m_1 \Delta\sigma_1 + 2 m_3 \Delta\sigma_3}{m_1 + 2 m_3} \quad (10)$$

Let $R = \frac{m_1}{m_3}$ and expressing Δu in terms of principal stress difference

$$\Delta u = \frac{R \Delta\sigma_1 + 2 \Delta\sigma_3}{R + 2} = \psi \ (\Delta\sigma_1 - \Delta\sigma_3) \quad (11)$$

where : ψ = apparent pore pressure coefficient. Hence:

$$\psi_{comp} = \frac{R \Delta\sigma_1 + 2 \Delta\sigma_3}{(R + 2)(\Delta\sigma_1 - \Delta\sigma_3)} \quad (12)$$

For the extension test $\Delta\sigma_1 = \Delta\sigma_2$ and $m_1 = m_2$ manipulating in a similar manner we obtain:

$$\psi_{ext} = \frac{2 R \Delta\sigma_1 + \Delta\sigma_3}{(2R + 1)(\Delta\sigma_1 - \Delta\sigma_3)} \quad (13)$$

For pure deviatoric compression $\Delta\sigma_3 = -\frac{1}{2}\Delta\sigma_1$ hence:

$$\bar{\psi}_{comp} = \frac{R - 1}{1.5 (R + 2)} \quad (14)$$

And for pure deviatoric extension $\Delta\sigma_3 = -2\Delta\sigma_1$ hence:

$$\bar{\psi}_{ext} = \frac{2 R - 2}{3 (2R + 1)} \quad (15)$$

It should be noted that only for the special case of R = 1.0 does a constant octahedral stress path yield ψ equal to zero. Hence any real soil, even if non dilatant, would be expected to develop pore pressure during application of pure deviatoric stresses.

Comparison of equations (14) and (15) for a non dilatant soil shows that for equal R values an extension test will result in smaller pore pressure as compared to the compression test. By definition $R = m_1/m_3$ where m_1 is on virgin compression while m_3 is on the rebound, hence in most cases R > 1.0 . However the compression and extension tests result in rotation of the principalstresses through 90°, hence it is logical to assume that $R_{comp.} \neq R_{ext.}$, resulting in distortion of the afore mentioned picture with regard to the pore pressure build up during undrained compression and extension. Further inspection of equation (14) and (15) shows that the pore pressure due to the first term increases with R and it is further plausible to assume that during the course of shearing R varies.

Removing the restriction of a non dilating soil, we obtain for triaxial compression:

$$\Delta u = (\psi_{comp} \pm \frac{D}{m_1 + 2 m_3}) (\Delta \sigma_1 - \Delta \sigma_3) \qquad (17)$$

and for triaxial extension:

$$\Delta u = (\psi_{ext} \pm \frac{D}{2 m_1 + m_3}) (\Delta \sigma_1 - \Delta \sigma_3) \qquad (18)$$

In comparing the two it is important to remember that $2 m_1 + m_3$ for $\Delta\sigma_2 = \Delta\sigma_1$ (i.e. axial extension) $\neq$ $m_1 + 2 m_3$ for $\sigma_2 = \sigma_3$ (i.e. axial compression), based on the same reasoning given for the difference in R value.

TYPES OF TESTS USED AND TESTING PROCEDURES

All tests have been carried out on cylindrical samples brought to failure in pure deviatoric axial compression and extension. Undisturbed samples have been saturated by replacing the gas in the pores by CO_2 followed by saturation under back pressure. (5.0 kg/cm^2). It was found that this procedure resulted in B values of unity. All tests where stress controlled the axial load being applied in increments. The sample was allowed to come to equilibrium under every load increment which resulted in long test durations. An average test lasting between 5 and 6 days. The pure deviatoric condition was maintained by manipulating simultaneously the cell and the axial pressures. Four different test series have been carried out:

1) Isotropic consolidation followed by pure deviatoric axial compression and extension.

2) Anisotropic consolidation under two principal stress ratios, followed by axial compression. The anisotropic consolidation followed also a pure deviatoric stress path, so that the mean principle consolidation pressures were equal to those in the other series.

3) Isotropic consolidation of horizontally sampled specimens followed by axial compression.

   All above mentioned series involved undisturbed sample.

4) Isotropic consolidation of samples consolidated from a slurry to the same dry density as the undisturbed samples, followed by axial compression.

The samples in all the series where consolidated under three levels of all around pressure.

MATERIALS TESTED

The material investigated was an undisturbed clayey loess taken from a pit 1.5 m below ground surface. Specific properties of the soil are given in Table. 1.

TABLE 1. Properties of Clayey Loess

Grain Size Distribution			L. L.	P. L.	γ_d	Carbonate Content
Sand %	Silt %	Clay %	%	%	kg/m^3	%
9.8	59	29.5	31	16	1540	24.3

EXPERIMENTAL RESULTS AND ANALYSIS

The continuous variation of the pore pressure may be expressed in the dimensionless plot of $\Delta u/\sigma_{oct}$ vs. τ_{oct}/σ_{oct} and is shown in figures 1 through 5 for the various test series.

A measure of the anisotropy may be obtained from oedometer tests run on samples whose principal planes have been rotated through 90°. The results of such a test are shown in Fig. 6.

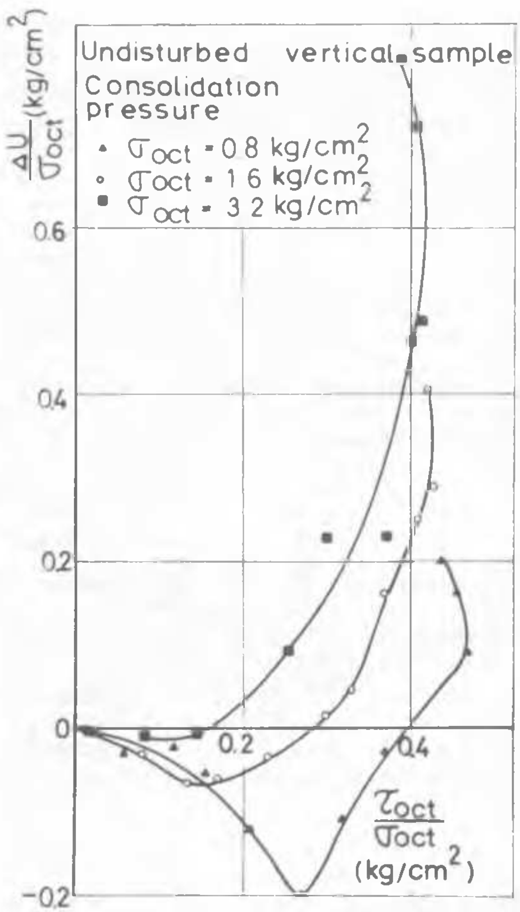

FIGURE 1. Pore Pressure vs. Shear Stress, Pure Deviatoric Axial Compression

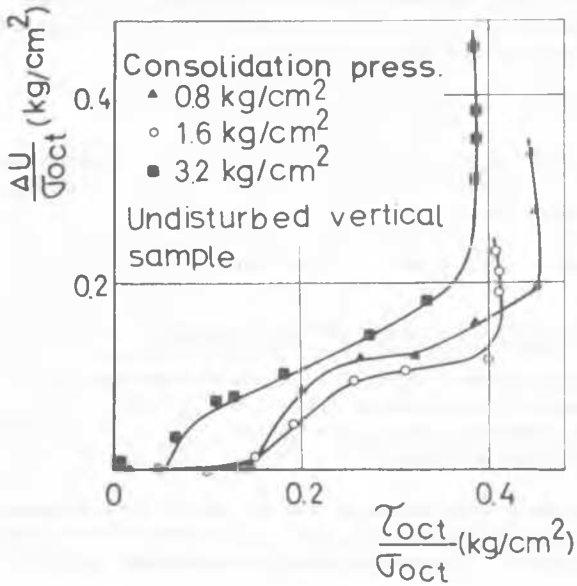

FIGURE 2. Pore Pressure vs. Shear Stress, pure Deviatoric Axial Extension

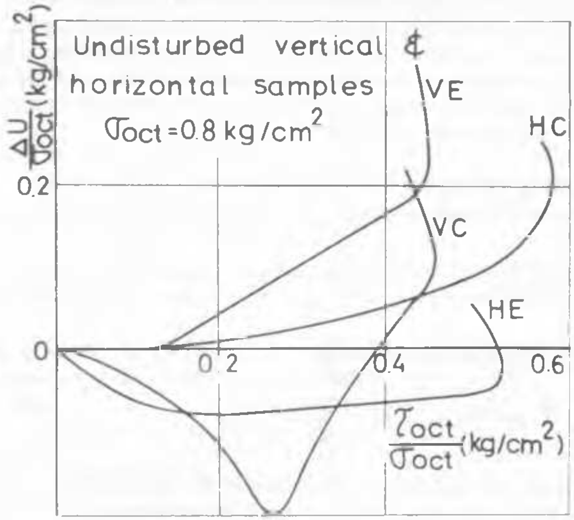

FIGURE 4. Pore pressure vs. Shear Stress, Pure Deviatoric Axial Compression-Extension, Vertical & Horizontal Samples.

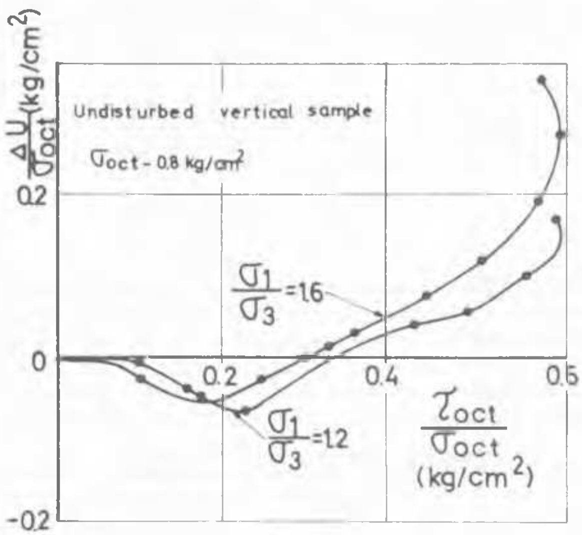

FIGURE 3. Pore Pressure vs. Shear Stress, Pure Deviatoric Axial Compression, Anisotropic Consolidation.

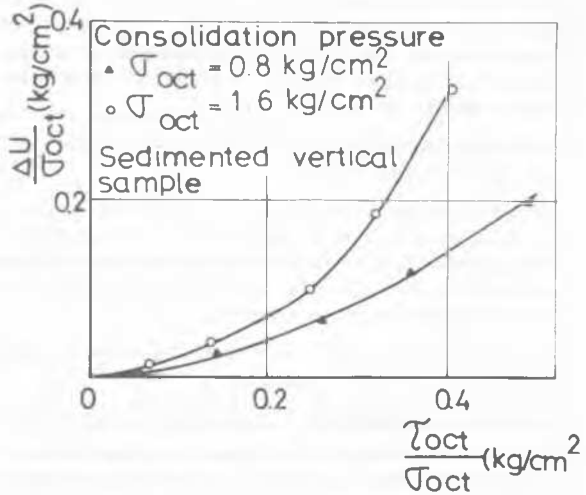

FIGURE 5. Pore Pressure vs. Shear Stress, Pure Deviatoric Axial Compression, Sedimented Samples.

a) General Observations

Comparing the different pore pressure curves it is observed that the different boundary conditions and stress paths result in three different response patterns:

1. Negative pore pressure in the initial stages of the test followed by positive pore pressure;
2. Zero pore pressure in the initial stages followed by positive pore pressure;
3. Positive pore pressure from the very start of the shear stress application.

From the consolidation tests (Fig. 6) it can be observed that R in a horizontally sampled specimen is roughly 5 times as large as in a vertically sampled one.

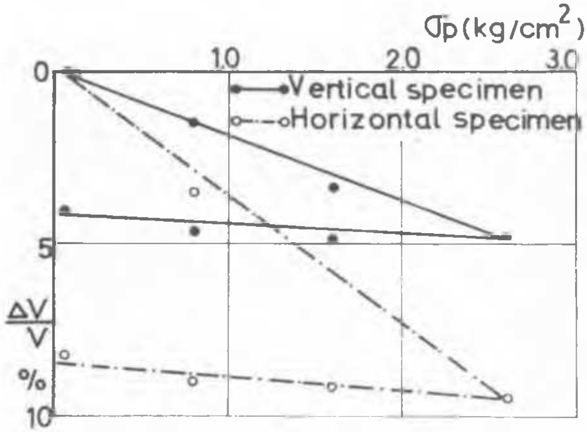

FIGURE 6. Oedometer Consolidation, Vertically and Horizontally Sampled Specimens.

b) Analysis of Test Results

It has been shown that usually $R > 1.0$ and hence the first term in the pore pressure equation may contribute only positive pore pressures. Since the test results on all the undisturbed samples (Fig. 1 to 4) exhibit either negative or zero pore pressure in the early stages of the shear phase, it may be concluded that the pore pressure due to the dilation term is negative for the soil investigated (at least initially), in its undisturbed state. By similar logic it may be assumed that this is not the case for the artificially sedimented soil (Fig. 5). The increase of the pore pressure in the latter stages of the test can be definitely attributed to the increase of the virgin compressibility m_1 due to collapse of the structure of the loess. This mechanism was suggested by Juarez-Badillo (1963). (It may also be attributed to the D term turning positive due to the same reason, this effect, however, has not been separated out in the present experimental program).

Comparison of Fig. 1 and 2 shows that the extension tests result in higher pore pressures than the compression tests; this, of course, is a departure from the predictions of our model for equal R values. This discrepancy, however, may be explained when considering the actual R values as pointed out previously. Such difference more than balances the difference predicted by equations (14) and (15) for equal R values. Further more since the experimental data is in terms of the total pore pressure, examination of equation (16) and (17) will show that the negative pore pressure contribution of the dilatancy term in the case of the $\sigma_1 = \sigma_2$ test, is much smaller then that of the $\sigma_2 = \sigma_3$ test, since the term $m_1 + 2m_3 < 2m_1 + m_3$. Tests were done also on horizontally sampled specimens. Shown in Fig. 4 are the test results for pure deviatoric axial compression and extension for both vertically and horizontally sampled specimens (labeled VC; VE; HC and HE respectively). All tests are for the same consolidation pressure of $\sigma_{oct.} = 0.8$ kg/cm^2. Since for the horizontally sampled specimens no radial symmetry exists for the compressibility the equations developed above cannot be directly used for predicting the relative magnitude of the pore pressure developed. However, using those equations it is possible to predict that the HC tests should be similar to the VE tests and the HE tests similar to the VC tests, the latter group showing lower pore pressures. These general trends may be indeed observed in Fig. 4.

The effect of the consolidation stress level may be best shown by comparing the relative pore pressure i.e. the ratio of the developed pore pressure to the effective consolidation pressure (maximum value being unity). It may be observed that for all cases this quantity increases with the consolidation stress level. No unique explanation may be offered in terms of the suggested model, since it is plausible to assume that both m_1 increases and the negative D term decreases upon increase of the effective consolidation stress. This is most clearly brought out in Fig. 1.
The effect of anisotropic consolidation is shown in Fig. 3 and it may be observed the either R is increased or the negative D term is decreased or both of them, due to the application of deviator stress during consolidation, resulting in less negative pore pressure in the initial stages of the test. The two different principal stress ratios ($\sigma_1/\sigma_3 = 1.2$ and 1.6) did not result in any appreciable differences in pore pressure.

CONCLUSIONS

1. Application of a deviator stress to a fully saturated undrained soil sample results in generation of pore pressure which is related to the deviator stress by dilatancy term and a compressibility term which in turn is dependant on the anisotropy of the soil.
2. No constant proportionality exists between changes in pore pressure and changes in deviator stress since both the proportionality coefficients vary in the course of shearing.

ACKNOWLEDGEMENT

The testing refered to in this paper was executed in the Soil Mechanics Laboratory Faculty of Civil Engineering, Israel Institute of Technology.

REFERENCES

Bishop, A.W. and Henkel, D.J., (1962) - "The measurement of soil properties in the triaxial cell" - E.Arnold Ltd., London.

Bjerrum, L., (1954) - "Theoretical and experimental investigation on the shear strength of soils" - N.G.I. Pub. No.5, Oslo.

Duncan, J.M. and Seed, H.B., (1966) - "Strength variations along failure surfaces in clay" - Proceedings S.M.F.E., Div. A.S.C.E. vol. 92, No. SM6, pp. 64-85.

Henkel, D.J., (1958) - "Correlation between deformation pore water pressure and strength characteristics of saturated clays" - Thesis, Ph.D. in engineering, Imperial College of Science & Technology, London.

Juarez-Badillo, E., (1963) - "Pore pressure functions in saturated soils" - A.S.T.M. Special Technical Publication No. 361, pp. 226-240.

Klausner, Y., (1967) - "Certain properties of the pure deviatoric test" - Proc. 3rd Asian Regional Conf. on S.M.F.E., Haifa, pp. 282- 288.

Mitchell, J.K., (1956) - "The fabric of natural clays and its relation to engineering properties" - H.R.B. vol.35 pp. 693-713.

Seed, H.B. and Chan, C.K. (1959) - "Structure and strength characteristics of compacted clays" - Proceedings S.M.F.E. Div. A.S.C.E., vol. 85, No. SM-5, pp. 87-128.

Shibata, T. and Karube, D., (1965) - "Influence of the variation of the intermediate principalstress on the mechanical properties of normally consolidated clays"- Proc. 6th International S.M.F.D. Conference vol. I pp. 354-359.

Skempton, A.W., (1948) - "A study of the immediate test on cohesive soils" - Proceedings 2nd International Conf. on S.M.F.E., Vol.II pp. 192-196, Zurich.