# ONE-DIMENSIONAL CONSOLIDATION OF LAYERED SOILS

## CONSOLIDATION UNIDIMENSIONNELLE DES SOLS STRATIFIES

E.H. DAVIS, Professor
I.K. LEE, Senior Lecturer

University of Sydney, Australia

SYNOPSIS An analysis of the rate of settlement of layered soil deposits consisting of two layers, repeated layer pairs, and a general heterogeneous system of layers is discussed. To establish the degree of settlement-time factor relationship it is only necessary to specify two dimensionless parameters for each interface. Detailed numerical results for a two layer soil deposit and the repeated layer pair type of deposit, are presented. Two approximate methods are discussed and their accuracy examined in relation to the numerical and analytical results, and also in relation to two specific examples of four layer deposits.

INTRODUCTION

The mathematical treatment of the rate of consolidation of two contiguous soil layers under a one-dimensional strain state was first considered by Gray (1945) although he gave only a few numerical solutions. Subsequently, approximate methods for solving this problem have been suggested by several investigators, notably, Barber (1945), Richart (1957), Palmer and Brown (1957), Davis (1961), and Sridharan and Nagaraj (1962). These methods involve either a transformation in thickness of the layers in order to make use of the Terzaghi solution for a homogeneous layer, or the use in the Terzaghi equation of an equivalent coefficient of consolidation. Up to the present time there appears to have been no published detailed evaluation of the formal solutions for a soil deposit consisting of two layers, and the validity of the approximate methods for a two layer or any multi-layered soil has not been established.

It is shown in the present paper that the formal solutions for a two layer soil can be expressed in terms of two dimensionless parameters. For each extra layer there are two extra dimensionless parameters required to completely specify the relevant soil properties. Thus it has been found possible to give a full coverage of solutions for a two layer system but the presentation of settlement-time relationships for a deposit composed of a heterogeneous system of more than two layers is not practicable, and the latter require programming of the basic equations if a precise answer is required. However in many practical situations it is desirable to have available a method which allows rapid, but necessarily, approximate calculations for the settlement-time relationship.

BASIC PARAMETERS

From Gray's analysis, and the numerical method discussed below, it is evident that only two dimensionless parameters are required to define the interrelationship between the consolidation behaviour of two adjacent layers. The two parameters selected here are the ratio of the total final settlements of the layers, that is,

$$a_{i,(i+1)} = \frac{m_i}{m_{i+1}} \frac{h_i}{h_{i+1}}$$

and a parameter defined as

$$b_{i,(i+1)} = \frac{k_{i+1}}{k_i} \frac{h_i}{h_{i+1}}$$

where $m_i$, $m_{i+1}$ are the coefficients of volume decrease of the $i^{th}$ and $(i+1)^{th}$ layer, respectively, $h_i$, $h_{i+1}$ are the corresponding thicknesses, and $k_i$, $k_{i+1}$ are the corresponding coefficients of permeability. A further parameter useful in the presentation of results is the ratio "a" to "b" which is designated as $\alpha$.

For the two layer deposit and the Rowe model of a layered deposit consisting of identical layer pairs, the parameters "a" and "b" have only one value each. For the general case of a soil deposit composed of n different layers (n-1) "a" parameters, and (n-1) "b" parameters will define the rate of settlement.

When selecting the definition of the time factor for such a deposit it is logical to choose the total thickness of the deposit, H, and the equivalent coefficient of consolidation of the whole deposit, $\bar{c}$, in preference to the values of any particular layer. Thus, the time factor, $\bar{T}$, is defined as

$$\bar{T} = \frac{\bar{c}\,t}{H^2} \quad \text{where } \bar{c} = \frac{H}{(\sum^{n}_{1} m_i h_i)(\sum^{n}_{1} \frac{h_i}{k_i})}$$

For a two-layer deposit consisting only of layer 1 overlying layer 2, or a deposit having these layers alternating, as considered by Rowe (1964),

$$\bar{T} = T_1 \frac{ab}{(1+a)(1+b)}$$

where $T_1 = \frac{c_1 t}{p^2 h_1^2}$ and p = number of pairs.

Although for most purposes $\bar{T}$ is the most suitable definition of a time factor, $T_1$ is useful in bringing out some aspects of the theoretical results.

In general only three sets of drainage conditions at the upper and lower boundaries of the deposit need be considered. These are PTPB, PTIB and ITPB where P denotes permeable, I impermeable, T top, and B bottom.

Only vertical consolidation due to a vertical stress uniform with depth is dealt with in this paper.

## NUMERICAL ANALYSIS

In the following analysis the usual assumptions of one dimensional linear consolidation theory are made so that the Terzaghi equation is applicable within each layer. A complete solution is obtained by satisfying this equation and the appropriate pore pressure and continuity conditions of the external boundaries and the inter layer boundaries.

A numerical technique was used as it could be readily adapted to the general case of a large number of layers. The relevant finite difference approximation of the Terzaghi equation was the explicit form,

$$u_{r+1,s} = u_{r,s} + \beta_i (u_{r,s+1} - 2u_{r,s})$$

where r is an integer defining time, and s is an integer defining position within a layer.

If the above equation is applied to the $i^{th}$ layer, then the expression for the $(i+1)^{th}$ layer is identical provided $\beta_i$ is replaced by $\beta_{i+1}$, where

$$\beta_{i+1} = \beta_i . a_{i,i+1} . b_{i,i+1}$$

The latter expression involves the logical assumption that there are an equal number of nodes in each layer. $\beta_i$ and the corresponding β values for other layers must be restricted to 1/2 to avoid instability in the numerical solution. $\beta_i$ is defined as

$$\beta_i = \frac{q^2}{n} \frac{c_i t}{h_i^2}$$

$$\text{where } q = \frac{h_i}{\delta h_i} = \frac{h_{i+1}}{\delta h_{i+1}} = --- \quad \text{and} \quad n = \frac{t}{\delta t}$$

In the quoted numerical results q is equal to 10. This value was derived from a consideration of the effect of number of nodes on the numerical values of pore pressure.

The necessary and complete inter-layer boundary conditions are

$$(u)_i = (u)_{i+1}$$

$$k_i (\frac{\partial u}{\partial z})_i = k_{i+1} (\frac{\partial u}{\partial z})_{i+1}$$

As in any numerical technique several possible approximations can be obtained. If the Terzaghi equation is combined with the continuity condition then the finite difference expression along the i, i+1, layer boundary is

$$u_{r+1,s} = u_{r,s} + \beta_i \left(\frac{2a}{1+a}\right) \times \left(u_{r,s-1} + u_{r,s} + b(u_{r,s+1} - u_{r,s})\right)$$

where the integer "s" here defines the location of the boundary and the subscript (i,i+1) has been omitted from the "a" and "b" for brevity. However, use of this expression can lead to an instability in the numerical solution for values of a in excess of 0.1. The expression finally adopted satisfies the continuity condition, that is

$$u_{r,s} = \frac{4}{3(1+b)} \times \left((bu_{r,s+1} + u_{r,s-1}) - (bu_{r,s+2} + u_{r,s-2})\right)$$

The degree of settlement is given by the expression

$$U_s = 1 - \frac{\frac{1}{2qu_0}\left(s(1) + \frac{1}{a_{1,2}} s(2) + \frac{1}{a_{1,2}.a_{2,3}} s(3) + ---\right)}{\left(1 + \frac{1}{a_{1,2}} + \frac{1}{a_{1,2}.a_{2,3}} + ----\right)}$$

where $u_o$ is the initial uniform pore pressure, and s(1), s(2), --s(i) are defined as

$$s(i) = u_{s'} + 2u_{s'+1} + --- u_{s''}$$

where s' is an integer defining the upper node of the $i^{th}$ layer (=(i-1)(q+1)), and s" is an integer defining the lowest node of the $i^{th}$ layer (=(i)(q+1)).

For a two layer soil the expression simplifies to

$$U_s = 1 - \frac{1}{2u_o\, q(1+a)} \left( a\, s(1) + s(2)\right)$$

An alternative numerical procedure has been given by Abbott (1960) and an electrical analogue technique has been described by Christie (1966).

## SPECIAL CASES

There are two special cases in which the settlement time relationship can be derived from the solution for a homogeneous layer.

Special Case A

It was first suggested by Glick (1945) in the discussion to Gray's paper that if the depth, z within the $i^{th}$ layer is transformed by the relationship

$$z_T = z\sqrt{\frac{\bar{c}}{c_i}}$$

then the diffusion equation

$$\frac{\partial u}{\partial t} = \bar{c}\,\frac{\partial^2 u}{\partial {z_T}^2}$$

becomes common to all layers, that is, the transformed single layer of total thickness

$$H_T = \sum_{i=1}^{i=n} h_i \sqrt{\frac{\bar{c}}{c_i}}$$

is equivalent to the layered system with respect to the basic diffusion equation. For complete equivalence the inter-layer continuity condition must also be satisfied. This leads to the condition that

$$\frac{k_i}{k_{i+1}} = \sqrt{\frac{c_i}{c_{i+1}}}$$

or $a_{i,i+1} = b_{i,i+1}$ or $\alpha_{i,i+1} = 1$ for all i.

It can then be shown that the total transformed thickness is equal to the actual total thickness.

$$H_T = H$$

Thus the degree of settlement for this special case, when all the α's are unity, is given directly by the ordinary Terzaghi solution, provided $\bar{c}$ is used as the coefficient of consolidation.

Special Case B

Employing the thickness transformation of special case A to a two layer soil deposit subject to two-way drainage (PTPB), the inter-layer continuity condition is satisfied irrespective of the value of α if the transformed thicknesses of the two layers are equal. This occurs when a.b = 1.

It then follows that the degree of settlement is given by the ordinary Terzaghi theory for two-way drainage of a homogeneous single layer of total thickness $H_T$ and having a coefficient of consolidation $\bar{c}$. For example the time for 50% settlement is given by

$$\bar{T}_{50} = \frac{\bar{c}t_{50}}{H^2} = 0.0493\,\frac{{H_T}^2}{H^2} = \frac{0.197a}{(1+a)^2}$$

The two special cases can be used as a partial check on the accuracy of the numerical analysis.

MULTIPLE LAYER PAIRS

The relationship between degree of settlement and time factor for a soil deposit composed of a finite number of identical layer pairs lies between that for a single pair and that for an infinite number of pairs. The latter solution is given by the ordinary Terzaghi analysis for a single homogeneous layer by equating the coefficient of consolidation in this analysis to $\bar{c}$ of the multiple layer pairs.

The transition from the curve for a single layer pair to that for an infinite number of pairs can be seen in Figure 1 in which some of the results of the numerical analysis for the boundary conditions PTPB with a value of the parameter "b" of 10 and various values of the parameter "a" are plotted. Further results for the drainage conditions PTPB are shown in Figure 2, and for the conditions PTIB in Figure 3. Figures 1 and 3 bring out the point that relatively few pairs of layers are required to ensure that under these one-dimensional consolidation conditions, a varved or laminated deposit effectively behaves as a single homogeneous layer. In Figures 1, 2 and 3 the time factor $T_1$ has been employed. This is convenient in separating the families of curves for different values of "a". It also demonstrates in Figure 2 that the curve tends to the ordinary solution for one-way drainage of a single layer ($T_1$ = 0.197 for $U_s$ = 0.5) when "b" is small compared with "a", since in these circumstances, layer 2 is in effect only an incompressible impermeable plate.

In Figures 4 and 5 some of the numerical results for a single pair of layers are plotted with the time factor defined as $\bar{T}$. These figures show how, with this definition of time factor, the curve for an infinite number of layer pairs is always identical with that for a single homogeneous layer whatever the values of the parameters "a" "b" or "α", and furthermore, that this particular curve is the curve for a single (or any number) pair of layers when α = 1 as explained under the heading special case A. This last point is also apparent in Figures 1, 2 and 3 (when α = 1, a = b), the slight discrepancy in Figure 2 being an indication of the order of accuracy of the numerical analysis.

The curves in Figure 4 (PTPB) also apply to values of "a" and "α" which are both the inverse of those specified in the figure, whereas inversion of these quantities for Figure 5 (PTIB) renders the curve applicable to the drainage conditions ITPB.

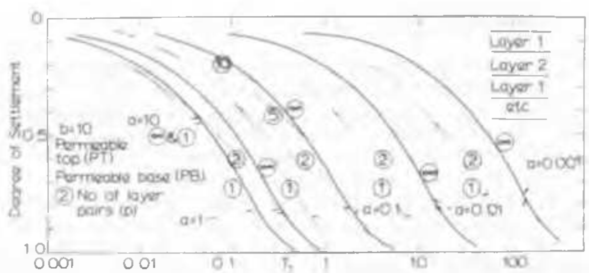
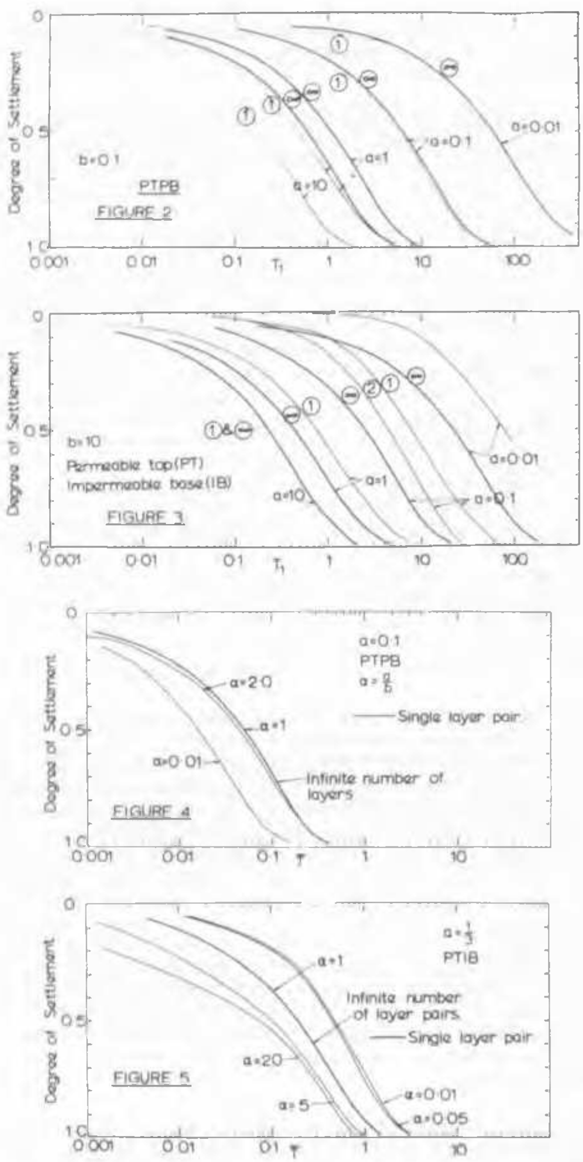



Figure 1. Rate of Settlement. Multiple Layer Pairs. b=10 (PTPB)




Figures 2 to 5. Rate of Settlement for Multiple Layer Pairs.

## TWO LAYER SYSTEM

The presentation of a sufficient number of time-settlement curves to cover the full range of practical values of "a" and "α", even for a two layer system, would take too much space. However, Figures 1 to 5 show that, at least for degrees of settlement in excess of 0.2 for PTPB and 0.4 for PTIB, the curves for a single layer pair are all geometrically similar in shape and similar to that for one homogeneous layer. Thus it should be adequate for most practical purposes to have the time for 50% settlement and to rely on the shape similarity to obtain the rest of the time-settlement curve. The numerical results have been used to derive contours of $\overline{T}_{50}$ (for $U_s = 0.5$) on a plot of "a" and "α" as shown in Figure 6 for PTPB and in Figure 7 for PTIB.

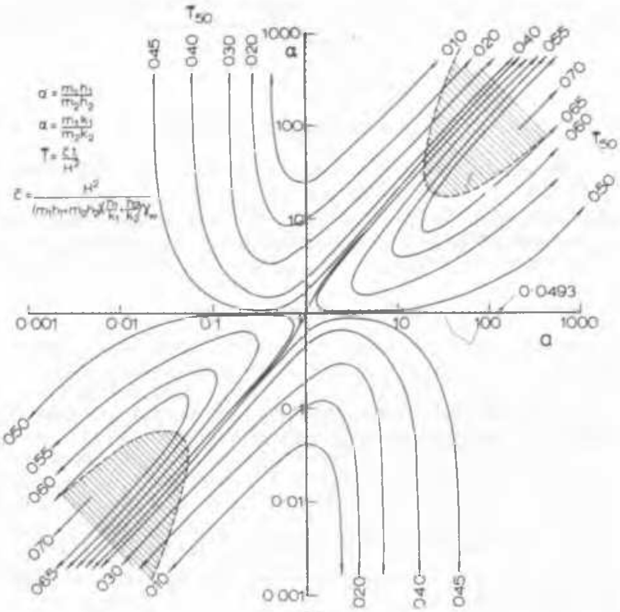



Figure 6. Contours of $\overline{T}_{50}$ for Two Layer Soil. (PTPB)

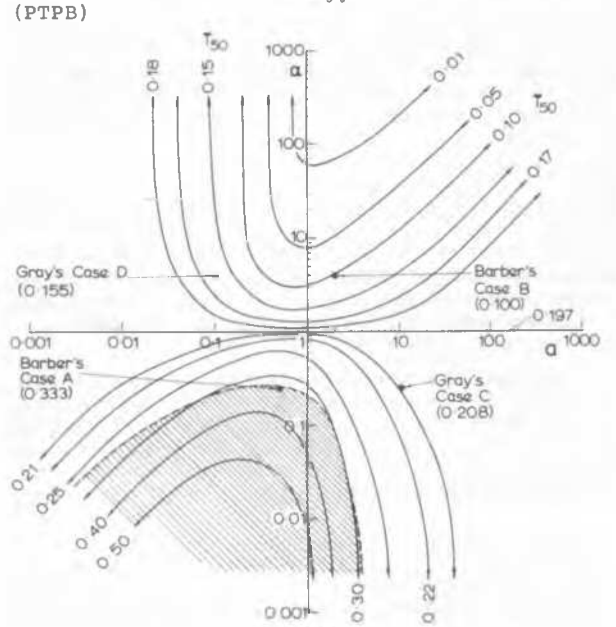



Figure 7. Contours of $\overline{T}_{50}$ for two layer soil. (PTIB)

For the condition PTPB, inversion of both "a" and "α" must lead to the same value of $\overline{T}_{50}$. The symmetry following from this requirement is evident in Figure 6. There is no corresponding symmetry in Figure 7 (PTIB) but inversion of both "a" and "α" on this figure gives the values of $\overline{T}_{50}$ for the condition ITPB.

Certain limiting cases and the two special cases A and B already explained can help to confirm the contours obtained from the numerical analysis. For a = b, that is $\alpha = 1$, $\bar{T}_{50}$ must be equal to 0.0493 for PTPB and 0.197 for PTIB so that the contours of these values are the abscissa of each plot. For PTPB the second special case of ab = 1 allows the determination of $\bar{T}_{50}$ along a straight line at a slope of 2 to 1 on a log-log plot ($a^2 = \alpha$).

The limiting cases are as follows

| | | |
|---|---|---|
| PTPB (i) | $m_2 \to 0$, $a \to \infty$, $\alpha \to \infty$ | but b finite |
| PTPB (ii) | $k_2 \to \infty$, $\alpha \to 0$ | but a finite |
| PTIB (i) | $m_2 \to 0$, $a \to \infty$, $\alpha \to \infty$ | but b finite |
| PTIB (ii) | $m_1 \to 0$, $a \to 0$, $\alpha \to 0$ | but b finite |
| PTIB(iii) | $k_1 \to \infty$, $\alpha \to \infty$ | but a finite |
| PTIB (iv) | $k_2 \to \infty$, $\alpha \to 0$ | but a finite |

The analytical expressions for these limiting cases are given in an Appendix. The contours in Figures 6 and 7 merge into the limiting lines given by these expressions. The contours also agree with spot values derived from analytical results given by Gray (1945) and by Barber (1945). Thus there is adequate confirmation of the reliability of the main numerical calculations.

## IMPERFECT POROUS PLATES IN OEDOMETER TESTS

As recognized by Gray (1945), the limiting case PTIB(ii) is relevant to an oedometer test in which the soil specimen is loaded between two equally imperfect porous plates. Curve A of Figure 8 gives the results of calculations for this limiting case in a form suitable for assessment of the effect of inadequate permeability of the porous plates. For example, if the thickness of each plate is equal to half that of the specimen, the coefficient of permeability of the plates has to be less than twenty times that of the soil for the time for 50% consolidation to be increased by more than 20%.

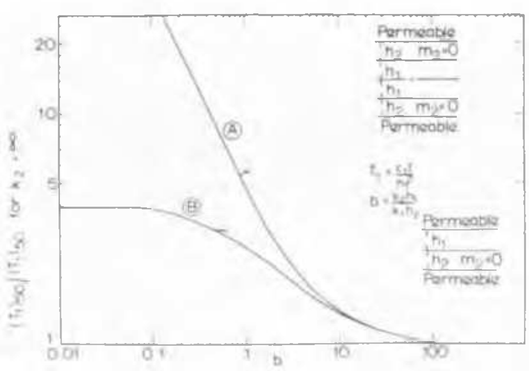



Figure 8. Oedometer test - effect of imperfect porous plates.

Curve B in Figure 8 has been calculated from limiting case PTPB(i) and, in the present context, is relevant to an oedometer test with only one imperfect plate.

## APPROXIMATE METHODS FOR MULTIPLE LAYER SOIL DEPOSITS

There are several possible approximate methods for predicting the rate of settlement of a layered soil deposit. Two of these methods will now be considered in detail.

The simplest method is to treat the whole deposit as a single homogeneous layer having a coefficient of consolidation equal to $\bar{c}$, the rate of settlement being given by the ordinary Terzaghi theory. This is the method proposed by Terzaghi (1940).

The second approximate method is to employ the thickness transformation specified under Special Case A even when the values of $\alpha$ are not unity. The distribution of pore pressure within the total transformed depth can then be taken as that given by the ordinary Terzaghi theory. However, in order to take into account the different values of the coefficient of volume decrease, m of the several layers, calculation of the degree of settlement requires evaluation from the Terzaghi theory of the degree of consolidation within only a portion of the total depth. On this basis, Figure 9 enables the degree of consolidation, $U_i$ for the i th layer to be calculated. The degree of settlement of the whole soil deposit is then given by

$$U_s = \frac{\Sigma m_i h_i U_i}{\Sigma m_i h_i}$$

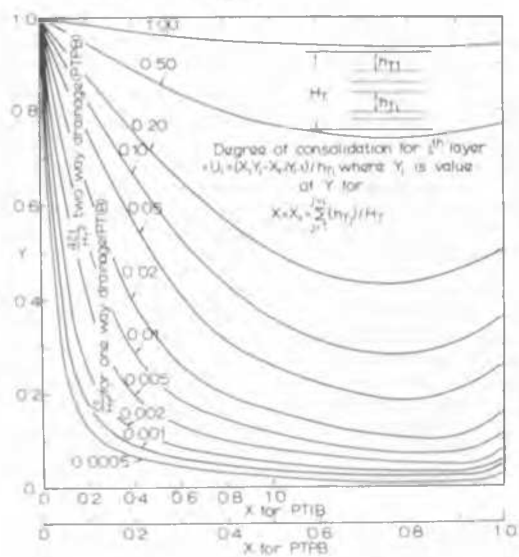



Figure 9. Chart for layer transformation method

As already explained, this second approximate method is only exact when it reduces to Special Case A, i.e. for all layers $\alpha$ is unity or $\frac{mk}{\gamma_w} = m^2c$ = constant.

A study of the available data on a variety of soil types suggested that the value of $m^2c$ is of the order of $0.1 \times 10^{-4}$ ft$^6$/tons$^2$ day. The range of $m^2c$ in this study was 0.01 to $1 \times 10^{-4}$ ft$^6$/tons$^2$ day so that, on this evidence, $\alpha$ can range from 0.01 to 100 if the extreme combinations of soil types are taken.

To establish the extent to which the lack of a correct interlayer continuity condition affects the time factor calculated by the second or transformation method, reference is made to Figures 6 and 7. In these figures regions are indicated by hatching where this approximate method leads to a $t_{50}$ less than two thirds of the correct value. Elsewhere within the figures the approximate value is within two thirds to three halves of the correct time.

The first approximate method leads to an anser, to within the same accuracy range, for combinations of "α" and "a" bounded by the contour $\bar{T}_{50}$ = 0.0329 for PTPB, and between the 0.131 and 0.296 contours for PTIB. Thus it is evident that the second or transformed layer method has a greater range of applicability.

This evidence suggests that the transformed layer method could find application in a practical case of heterogeneous multiple layers provided the "α" and "a" values of each adjacent layer combination lie within the range where the errors in the two layer soil deposit are deemed to be acceptable. A complete coverage of results for multi-layered deposits is, of course, a practical impossibility. If theoretically correct settlement rates were essential for any particular problem then special solution of that problem by a computer programme would be necessary. However, if a rapid manual calculation is required and some degree of approximation is acceptable, then the transformed layer method can be considered.

To examine further the effectiveness of the two approximate methods in particular cases of multiple layers, two four layer soil deposits were analysed by the finite difference programme. The curves of degree of settlement versus time factor are shown in Figures 10 and 11 for the boundary drainage conditions PTPB, PTIB, and ITPB.

Case 1 (Fig.10) is for a fairly extreme range of α's but except for the drainage condition PTIB, the second approximate method is in satisfactory agreement with the correct solution. The results of the comparison, including the first approximate method are summarized in Table 1 on the basis of the time for 50% settlement. Here it can be seen that for Case 1, the first approximate method can be greatly in error.

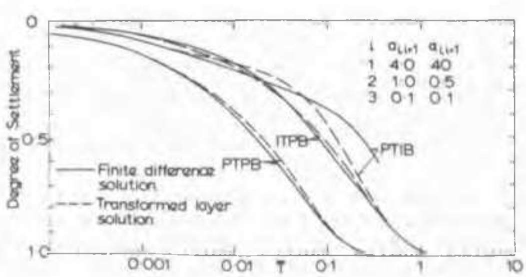



Figure 10. Rate of Settlement for four layer soil. Case 1.

Table 1 $\bar{T}_{50}$ values for two cases of four layer deposits

| Case | Drainage | Approx. Method 1 | Approx. Method 2 | Correct Solution |
|---|---|---|---|---|
| 1 (Fig.10) | PTPB | .049 | .019 | .016 |
| | PTIB | .197 | .123 | .230 |
| | ITPB | .197 | .088 | .078 |
| 2 (Fig.11) | PTPB | .049 | .033 | .057 |
| | PTIB | .197 | .155 | .185 |
| | ITPB | .197 | .170 | .220 |

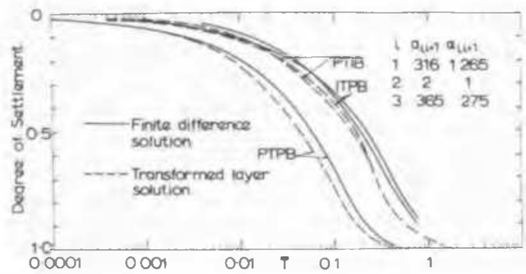



Figure 11. Rate of Settlement for four layer soil. Case 2.

Case 2(Fig.11) constitutes a much less severe combination of layer parameters and from Fig. 11 and Table 1 it is evident that both approximations might be considered acceptable.

It is recognized that the evidence from the above cases of four layers, and that previously presented for two layers, is inadequate for definite conclusions to be drawn on the accuracy of the approximate methods for all possible multi-layer deposits. It would frequently be necessary to exercise personal judgement. However, there is the fundamental objection to the first approximate method that no account is taken of the layer sequence. The transformed layer method on the other hand does take this aspect into account, although not necessarily giving the sequence the right emphasis. It is worth noting that, for the four-layer cases considered above, the approximate transformed layer method has an acceptable accuracy when the pairs of values of "a" and "α" all plot as points outside the hatched areas in Figures 6 and 7, but, when at least one of the points lies within the hatched area, the accuracy may or may not be acceptable. It may well be that this is generally true and not only for the examples given. Certainly if all pairs of values of "a" and "α" for a particular multilayer deposit plot within a hatched area, the approximate transformed layer method is very unlikely to be satisfactory.

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REFERENCES


ABBOTT, M.B. (1960) "One Dimensional Consolidation of Multi-layered Soils". Geotechnique, Vol.10,p.151.

BARBER, E.S. (1945) Discussion of paper by Gray. Trans.ASCE, Vol.110,p.1345.

DAVIS, E.H. (1961) Foundation Engineering Course,University of Sydney (unpublished).

CHRISTIE, I.F. (1966) "The Solution of Consolidation Problems by General Purpose Analogue Computer". Geotechnique, Vol.16,p.131.

GRAY, H. (1945) "Simultaneous Consolidation of Contiguous Layers of Unlike Compressible Soils" Trans.ASCE, Vol.110,p.1327.

PALMER, L.A. and BROWN P.P. (1957) "Settlement Analysis for Areas of Continuing Subsidence". Proc. I.C.S.M.F.E., Vol.1,p.395.

RICHART, F.E. (1957) "Review of the Theories for Sand Drains". Proc. ASCE, Vol.83, Paper 1301.

ROWE, P.W. (1964) "The Calculation of the Consolidation Rates of Laminated, Varved or Layered Clays,with Particular Reference to Sand Drains". Geotechnique, Vol.14,p.321.

SRIDHARAN, A. and NAGARAJ. T.S. (1962) "One Dimensional Consolidation of Layered Soils". Inst.Engr. (India), Vol.42, No.11,p.616.

TERZAGHI, K. (1940) "Sampling Testing and Averaging". Proc.Conf. Soil Mechanics and its Applications, Purdue,p.151.


APPENDIX

LIMITING CASES FOR TWO-LAYER DEPOSITS

PERMEABLE TOP, IMPERMEABLE BOTTOM (PTIB)(i)

$m_2 = 0$

Hence $a = \frac{m_1 h_1}{m_2 h_2} = \infty$ and $\alpha = \frac{m_1 k_1}{m_2 k_2} = \infty$

but $b = \frac{h_1 k_2}{h_2 k_1}$ is finite

i.e. layer 2 has a finite permeability and thickness but is incompressible. Therefore, because the bottom boundary of layer 2 is impermeable, it effectively supplies an impermeable boundary to the bottom of layer 1. The consolidation of layer 1 must then be governed by the ordinary Terzaghi theory for one-way drainage.

Thus $U_s = U_T$

where $U_T$ is given by the Terzaghi theory for a time factor,

$$T_1 = \frac{c_1 t}{h_1{}^2}$$

and $\bar{T} = \frac{b}{1+b} T_1$ where $\bar{T} = \frac{\bar{c}t}{H^2}$

For example, for $U_s = 0.5$, $\bar{T}_{50} = \frac{0.197b}{1+b}$

PERMEABLE TOP, IMPERMEABLE BOTTOM (PTIB)(ii)

$m_1 = 0$

Hence $a = \alpha = 0$ but b is finite

i.e. layer 1 has a finite permeability and thickness but is incompressible. It therefore impedes the consolidation of layer 2 but does not contribute to the total settlement.

In layer 2 $\frac{\partial u}{\partial \bar{T}} = (1+b)\frac{\partial^2 u}{\partial z^2}$

where distance from bottom of layer 2 is $zh_2$.

With boundary conditions

$\left(\frac{\partial u}{\partial z}\right)_{z=0} = 0$, $\left(\frac{\partial u}{\partial z}\right)_{z=1} = -\frac{1}{b}(u)_{z=1}$ and $(u)_{\bar{T}=0} = 1$,

solution is

$$u = 2\sum_{n=1}^{\infty} \frac{\sin\lambda_n \cos(\lambda_n z)}{(\lambda_n + \sin\lambda_n \cos\lambda_n)} \exp\left(-\lambda_n{}^2 (1+b)\bar{T}\right)$$

and

$$U_s = 1 - 2\sum_{n=1}^{\infty} \frac{\sin^2\lambda_n}{\lambda_n(\lambda_n + \sin\lambda_n \cos\lambda_n)} \exp\left(-\lambda_n{}^2 (1+b)\bar{T}\right)$$

where the $\lambda_n$'s are the roots of $\cot\lambda_n = b\lambda_n$

PERMEABLE TOP, IMPERMEABLE BOTTOM (PTIB)(iii)

$k_1 = \infty$

Hence $\alpha = \infty$ and $b = 0$ but a is finite

i.e. layer 1 is compressible but fully permeable so that it does not impede the drainage of layer 2 but makes an immediate contribution to the total settlement. Thus consolidation of layer 2 is governed by the ordinary Terzaghi theory for one-way drainage.

For the whole deposit, $U_s = \frac{a + U_T}{1 + a}$

where $U_T$ is given by the Terzaghi theory for a time factor defined as $\frac{c_2 t}{h_2{}^2} = (1+a)\bar{T}$.

PERMEABLE TOP, IMPERMEABLE BOTTOM (PTIB)(iv)

$k_2 = \infty$

Hence $\alpha = 0$ and $b = \infty$ but a is finite

i.e. layer 2 is compressible but fully permeable so that at all depths it has a pore pressure equal to that in layer 1 at the interface, and the rate of flow of water out of layer 2 into layer 1 is proportional to the rate of compression of layer 2.

In layer 1 $\frac{\partial u}{\partial \bar{T}} = \frac{(1+a)}{a}\frac{\partial^2 u}{\partial z^2}$

where depth below top $= zh_1$ .

With boundary conditions

$(u)_{z=0} = 0$, $a\left(\frac{\partial u}{\partial z}\right)_{z=1} = \left(\frac{\partial^2 u}{\partial z^2}\right)_{z=1}$ and $(u)_{\bar{T}=0} = 1$,

solution is

$$u = 2\sum_{n=1}^{\infty} \frac{(1-\cos\lambda_n)\sin(\lambda_n z)}{(\lambda_n - \sin\lambda_n \cos\lambda_n)} \exp\left(\frac{-\lambda_n{}^2 (1+a)\bar{T}}{a}\right)$$

and

$$U_s = 1 - \frac{2}{1+a}\sum_{n=1}^{\infty} \frac{(1-\cos\lambda_n)(a - a\cos\lambda_n + \lambda_n \sin\lambda_n)}{\lambda_n(\lambda_n - \sin\lambda_n \cos\lambda_n)} \times \exp\left(\frac{-\lambda_n{}^2 (1+a)\bar{T}}{a}\right)$$

where the $\lambda_n$'s are the roots of $\cot\lambda_n = \frac{\lambda_n}{a}$.

PERMEABLE TOP, PERMEABLE BOTTOM (PTPB)(i)

$m_2 = 0$

Hence $a = \alpha = \infty$ but b is finite i.e. layer 2 has a finite permeability and thickness but is incompressible. It therefore impedes the consolidation of layer 1 but does not contribute to the total settlement.

In layer 1

$$\frac{\partial u}{\partial \bar{T}} = \frac{1+b}{b}\frac{\partial^2 u}{\partial z^2}$$

where depth from top is $zh_1$.

With boundary conditions

$$(u)_{z=0} = 0, \left(\frac{\partial u}{\partial z}\right)_{z=1} = -b(u)_{z=1}, \text{ and } (u)_{\bar{T}=0} = 1,$$

Solution is

$$u = 2\sum_{n=1}^{\infty}\frac{(1-\cos\lambda_n)\sin(\lambda_n z)}{(\lambda_n-\sin\lambda_n\cos\lambda_n)}\exp\left(\frac{-\lambda_n^2(1+b)\bar{T}}{b}\right)$$

and

$$U_s = 1-2\sum_{n=1}^{\infty}\frac{(1-\cos\lambda_n)^2}{\lambda_n(\lambda_n-\sin\lambda_n\cos\lambda_n)}\exp\left(\frac{-\lambda_n^2(1+b)\bar{T}}{b}\right)$$

where the $\lambda_n$'s are roots of $\tan\lambda_n = \frac{-\lambda_n}{b}$

PERMEABLE TOP, PERMEABLE BOTTOM (PTPB)(ii)

$$k_2 = \infty$$

Hence $\alpha = 0$ and $b = \infty$ but a is finite i.e. layer 2 is compressible but fully permeable so that it does not impede the drainage of layer 1 but makes an immediate contribution to the total settlement of the whole deposit. Thus consolidation of layer 1 is governed by the ordinary Terzaghi theory for two-way drainage.

Degree of settlement of whole deposit,

$$U_s = \frac{1+aU_T}{1+a}$$

where $U_T$ is degree given by Terzaghi theory, the time factor being $4c_1t/h_1^2$ if this factor is defined in terms of the half thickness for two-way drainage in the conventional way.

Then $\bar{T} = \frac{\bar{c}t}{H^2} = \left(\frac{a}{1+a}\right)\frac{c_1t}{h_1^2}$ .