CALCULATION OF FOOTINGS ON COMPRESSIBLE FOUNDATION BEDS

CALCULATION DES FONDATIONS SUR UNE COUCHE COMPRESSIBLE

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SYNOPSIS:

Problems of the calculation of circular or strip foundation considered. The rigid circular slab is in adhesion to the elastic stratum underlaid by the uncompressible bed. Computation formulae have been derived for the case of vertical external loads uniformly distributed over the circular slab. The deflection of the slab noticably depends on the thickness of compressible stratum of seil , i.e. on the thickness of given elastic stratum. As an example of strip foundation, pile foundation grill is considered which rests on separate immobile points (piles) and at the same time on the bed between the piles. The pile foundation grill is treated as a non-uniform continuous beam resting on piles. With this method it is possible to treat the continuous beam as consisting of a number simple uniform beams, resting on compressible bed. Numerical examples indicate that a consideration of the reaction of the soil under pile foundation grill gives a more economical solution to the design of structures.

I. Calculation of a round elastic plate (author: K.E. Egorov).

Let us consider the contact problem for a rigid circular foundation slab of radius R and thickness H, resting on a uniform elastic stratum of thickness H underlaid by the uncompressible part of the bed. The slab is in contact with the stratum and is symmetrical vertical load is applied to it, which is unifor mly distributed over the slab with an intensity q kg/sq.cm.

The components σ, w of stresses and settlements at contact plane of the slab with the elastic stratum may be determined from the formulae (K.E. Egorov, I96I).

$$\sigma = \int_0^\infty \frac{\alpha M(H\alpha)}{1 - g(H\alpha)} J_0(r\alpha)\, d\alpha \qquad (I)$$

$$w = \frac{2(1-\nu_0^2)}{E_0} \int_0^\infty M(H\alpha)\, J_0(r\alpha)\, d\alpha \qquad (2)$$

$$M(H\alpha) = \left[1 - g(H\alpha)\right] \int_0^R \varphi(t) \cos(H\alpha t)\, dt \qquad (3)$$

where

$J_0(r\alpha)$ - Bessel function the first type of zero order;

E_0 and ν_0 - average modulus of deformation and Poisson's ratio of the compressible stratum;

$\varphi(t)$ - unknown function to be determined from the condition that the settlements of the bed and of the foundation salb are equal ($w = W$).

In solving the problem, it is assumed that there is no friction between the slab and the stratum. If we suppose that the tangential forces are zero (W) at the boundary of the compressible (elastic) stratum with the uncompressible part of the bed, then we have (K.E. Egorov, I96I).

$$g(H\alpha) = 1 - \frac{sh^2(H\alpha)}{sh(H\alpha)\, ch(H\alpha) + H\alpha} \qquad (4)$$

However, if we assume that the compressible stratum is in complete adhesion to the uncompressible part of the bed ($\tau \neq 0$, $u = v = 0$) then the calculation more combersome due to the complicated expression of

$$g(H\alpha) = 1 - \frac{(3-4\nu_0)\, sh(H\alpha)\, ch(H\alpha) - H\alpha}{(3-4\nu_0)\, ch(H\alpha) + (H\alpha)^2 + (1-2\nu_0)^2}, \qquad (5)$$

where ν_0 - Poisson's ratio of the bed.

By solving the respective differential equation for the determination of the deflection (W) of a circular slab, freely laid on compressible (elastic) stratum, and using the condition $w = W$, we find that the unknown function can be determined from the following integral equation (V.D. Palmov, I960; O.D. Shilova, I963).

$$\omega(\xi) + \int_0^1 T(\xi,\eta)\, \omega(\eta)\, d\eta = 1 + \frac{\lambda S}{2\pi}\left[\frac{1-\nu_1}{1+\nu_1} - 4(\ln 2\xi - 1) + \frac{2}{3}\xi^2\right]\xi^2, \qquad (6)$$

where $\xi = \frac{x}{R}$, $\eta = \frac{t}{R}$

$$\varphi(x) = \frac{W_0}{\pi}\, \frac{E_0}{1-\nu_0^2}\, \omega\left(\frac{x}{R}\right) \qquad (7)$$

With a load uniformly distributed over

the whole area of the circuler slab and a summary load $P = R^2 q$, the vertical settlement (W_0) of the center of the slab is found from the formula given in the paper (V.D. Palmov, 1960).

$$P = 2\pi \int_0^R \varphi(t)\,dt \qquad (8)$$

The function $T(\xi, \eta)$ contained in (6) has the form

$$T(\xi,\eta) = \frac{2}{\pi} S\Big\{\frac{1-\nu_1}{1+\nu_1}\xi^2\eta^2 + \frac{1}{2}\Big[(\xi+\eta)^2 \ln|\xi+\eta| + (\xi-\eta)^2 \ln|\xi-\eta|\Big] - $$

$$- \xi^2 \ln|\xi| - \eta^2 \ln|\eta|\Big\} - \frac{1}{\pi}\Big[K(\xi+\eta) + K(\xi-\eta)\Big] \qquad (9)$$

The parameter S is the coefficient of flexibility for the elastic rigid circular slab and is equal to (M.I. Gorbunov-Posadov, 1941)

$$S = 3\,\frac{1-\nu_1^2}{1-\nu_0^2}\,\frac{E_0}{E_1}\left(\frac{R}{h}\right)^3 \qquad (10)$$

where
- E_0 and ν_0 - average modulus of deformation and Poisson's ratio of the compressible stratum;
- E_I and ν_I - modulus of elasticity and Poisson's ratio of the slab material.

The functions $K(\xi \pm \eta)$ characterise the deformation of compressible (elastic) stratum of thickness H and are found from the formula

$$K(\xi \pm \eta) = \frac{1}{m}\int_0^\infty g(\beta)\cos\left[\frac{(\xi\pm\eta)\beta}{m}\right]d\beta, \quad m = \frac{H}{R} \qquad (11)$$

When $K^0 = 0$, the problem reduces to that of surface settlement of an elastic semi-space, since $g(\beta) = 0$ when $H \to \infty$.

The integral equation (6) is solved by one of the approximate methods. However, when the load is uniformly distributed over the circular slab, it is possible to obtain an effective solution to the integral equation - (6) if the unknown function $\varphi(x)$ expressed in the form

$$\varphi(x) = \frac{P}{2\pi R^2 A}\sqrt{R^2 - k^2 x^2},$$

where

$$A = \int_0^1 \sqrt{1-k^2x^2}\,dx = \frac{1}{2}\left(\frac{\arcsin k}{k} + \sqrt{1-k^2}\right) \qquad (12)$$

and k is a parameter which varies from 0 to I.

The parameter k is found with the help of integral equation (6) depending on the given values of S and m = H/R. For this purpose equation (6) is integrated with respect to ξ from 0 to I, and then the value of K is determined numerically for the unknown function $\varphi(x)$. The values of K are given in table I for some S and m.

Table I.

S	k							
	m = 0	m = 0.25	m = 0.5	m = I	m = 2	m = 3	m = 5	m = ∞
0	I	0.95	0.85	0.60	0.30	0.I5	0.06	0.00
0.25	I	0.96	0.86	0.65	0.40	0.30	0.26	0.26
0.50	I	0.96	0.88	0.68	0.45	0.38	0.35	0.35
I	I	0.97	0.89	0.7I	0.54	0.49	0.48	0.47
2	I	0.98	0.92	0.77	0.65	0.6I	0.60	0.60
3	I	0.98	0.93	0.8I	0.7I	0.69	0.68	0.67
5	I	0.99	0.95	0.86	0.79	0.78	0.77	0.77
I0	I	0.99	0.97	0.92	0.88	0.87	0.87	0.87
∞	I	I.00	I.00	I.00	I.00	I.00	I.00	I.00

For these values of S and m not given in table I, parameter K is found with the help of the curve given in fig. I.

The reaction pressures are determined with the help of equalities (I) and (3) and are directly expressed in terms of $\varphi(x)$ by the formula

$$p(r) = \frac{\varphi(R)}{\sqrt{R^2 - r^2}} - \int_r^R \frac{\varphi'(t)\,dt}{\sqrt{t^2 - r^2}} \qquad (13)$$

Substituting the expression of $\varphi(x)$ from (12) in (13), we obtain

$$p(\rho) = \frac{P}{2\pi R^2 A}\left[\frac{\sqrt{1-k^2}}{\sqrt{1-\rho^2}} + \frac{k}{2}\arccos\frac{1-k^2(2-\rho^2)}{1-k^2\rho^2}\right] \qquad (14)$$

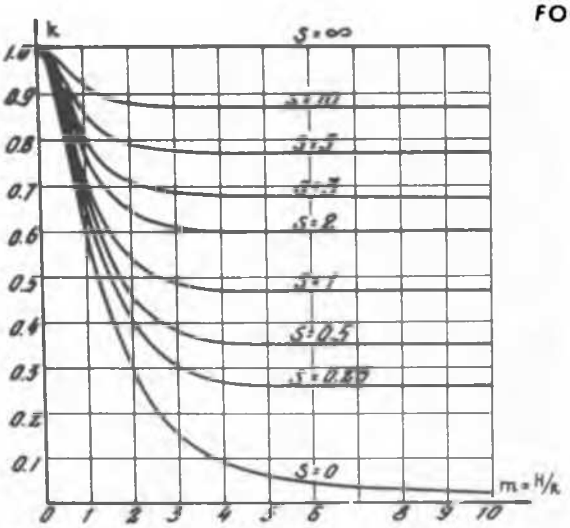

Fig. I. Values of k depending on the values of flexibility coefficient (S) and compressible thickness of bed ($m = H/R$) (load is uniformly distributed over the circular slab).

If we assume that k = 0, then we obtain the well known Boussinesq formula for the determination of reaction pressures in case of an absolutely rigid circular slab on an elastic semi-space

$$p(\rho)=\frac{P}{2\pi R^2}\,\frac{1}{\sqrt{1-\rho^2}} \qquad (15)$$

Supposing that k = I, we get the solution to a problem pertaining to an absolutely flexible circular slab under a uniformly distributed load (q)

$$A=\frac{\pi}{4}, \qquad \arccos(-1)=\pi,$$

$$p(\rho)=\frac{P}{\pi R^2}=q$$

The deflection of a circular slab can be determined in two ways: by making use of the formula for the determination of W or simply with the help of the values of vertical settlements w of the bed (under the condition $w = W$).

Thus, the deflection of a circular slab is found from the formula obtained from the equality (2)

$$W=\frac{(1-\nu_0^2)P}{\pi R E_0 A}\left[\int_0^{\pi/2}\sqrt{1-k^2\rho^2\sin^2\theta}\,d\theta+\psi(\rho,m)\right] \qquad (16)$$

where

$$\psi(\rho,m)=\frac{1}{\sqrt{2}}\sum_{i=1}^{4}B_i\int_0^1\frac{\sqrt{L+\rho^2-t^2+A_i^2m^2}}{L}\sqrt{1-k^2t^2}\,dt \qquad (17)$$

$$L^2=(\rho^2-t^2+A_i^2m^2)^2+4A_i^2m^2t^2$$

The coefficients A_i and B_i are obtained as a result of approximating the expression for $q(H\alpha)$ given in equality (4)

$$\frac{sh^2(H\alpha)}{sh(H\alpha)ch(H\alpha)+H\alpha}=\sum_{i=0}^{4}B_i e^{-(H\alpha)A_i}$$

where

$A_0 = 0$; $A_I = 0.8$; $A_2 = I.4$; $A_3 = 2.0$; $A_4 = 2.6$; $B_0 = I$; $B_I = 0.426$; $B_2 = -6.05I$; $B_3 = 7.395$; $B_4 = -2.770$.

When $m \to \infty$, we have $\psi(\rho,m)=0$
In this case the deflection of a rigid circular foundation slab resting on elastic semi-space is determined from formula (16).
When m = 0 we have

$$\psi(\rho,0)=\int_0^1\frac{\sqrt{1-k^2t^2}}{\sqrt{\rho^2-t^2}}\,dt=-\int_0^{\pi/2}\sqrt{1-k^2\rho^2\sin^2\theta}\,d\theta$$

consequently $W = 0$, i.e. the deflection of the slab on a uncompressible bed is equal to zero.

To determine the stresses arising in a uniformly loaded circular slab, formulae have been obtained to find the dimensionless quantities $\bar{M}_r$ and $\bar{M}_t$ by a method similar to the one used in paper (K.E. Egorov, I963).

Having the numerical values of $\bar{M}_r$ and $\bar{M}_t$ the values of radial M_r and tangential M_t moments are calculated by the formulas

$$M_r=qR^2\bar{M}_r, \qquad M_t=qR^2\bar{M}_t$$

The following tables 2 and 3 contain the values of $\bar{M}_r$ and $\bar{M}_t$ with respect to the changes in parameter k and the reduced radius ρ of the slab.

The stresses in the slab significantly depend on the thickness H of compressible stratum. When the rocky soil is situated very deep, it is necessary to know the value of H. Actual layer of by layer measurements on the soil settlements under circular foundations show that when $d \geqslant 20$ m, we can approximately take H = I/3 d and I/2 d (d - diameter of circular slab) as the thickness of the compressible stratum for sandy and clayey soils respectively.

2. Calculation on Pile Foundation Grill (author: I.A. Simvulidi)

In this paper the behaviour of pile foundation grill in joint action with soil bed is considered by making use of the plane problem in elasticity theory.

Table 2.

ρ	k = 0	k = 0.2	k = 0.3	k = 0.4	k = 0.5	k = 0.6	k = 0.7	k = 0.8	k = 0.9	k=I
0	0.074	0.072	0.070	0.067	0.062	0.056	0.049	0.038	0.024	0
0.I	0.072	0.070	0.068	0.065	0.06I	0.055	0.047	0.037	0.023	0
0.2	0.069	0.068	0.066	0.063	0.059	0.053	0.046	0.036	0.022	0
0.3	0.065	0.063	0.06I	0.058	0.055	0.049	0.043	0.034	0.02I	0
0.4	0.059	0.056	0.055	0.053	0.049	0.045	0.039	0.03I	0.0I9	0
0.5	0.050	0.048	0.047	0.045	0.042	0.039	0.033	0.026	0.0I7	0
0.6	0.040	0.039	0.038	0.036	0.034	0.03I	0.028	0.022	0.0I4	0
0.7	0.029	0.028	0.028	0.026	0.025	0.023	0.020	0.0I6	0.0I0	0
0.8	0.0I8	0.0I7	0.0I7	0.0I6	0.0I5	0.0I4	0.0I2	0.0I0	0.006	0
0.9	0.007	0.006	0.006	0.006	0.006	0.005	0.005	0.004	0.003	0
I.0	0.	0	0	0	0	0	0	0	0	0

In order to obtain the general computation formulae, a low pile foundation on grill is regarded as a non-uniform beam with variable cross section resting on an elastic bed. Moreover, the beam rests on a number of fixed concentrated supports (piles) situated on the same level (fig. 2 a).Full scale and field observations on single piles show that in the majority of cases, the settlement of these piles within the limits of the assumed design load is insignificant, therefore without much error we can consider the pile foundation grill as resting on immovable piles.

Assuming the unknown bending moments at the intermediate supports I, 2, 3,......., n and n + I, the continuous beam under consideration can be treated as consisting of a number simple independent beams (I.A. Simvulidi I958 and I965), by installing hinges at the support points.

Let us consider two arbitrarily cut beams with the adjacent spans L_n and L_{n+I} (fig. 2 b).

If we cut away the left beam with a span L_n from the right beam with a span L_{n+I}, at the n-th support, we obtain two independent beams, on each of which in addition to the given loads, unknown support moments M_{n-1}; M_n and M_{n+1} and unknown support reactions Y_{n-1}, Y_n', Y_n'' and Y_{n+1} will act at the ends to replace the action of right and left discarded spans (fig. 2 c).

Such a scheme enables us to consider and to calculate each beam section as a finite uniform beam, resting on a compressible bed; moreover this beam rests on fixed supports at the ends.

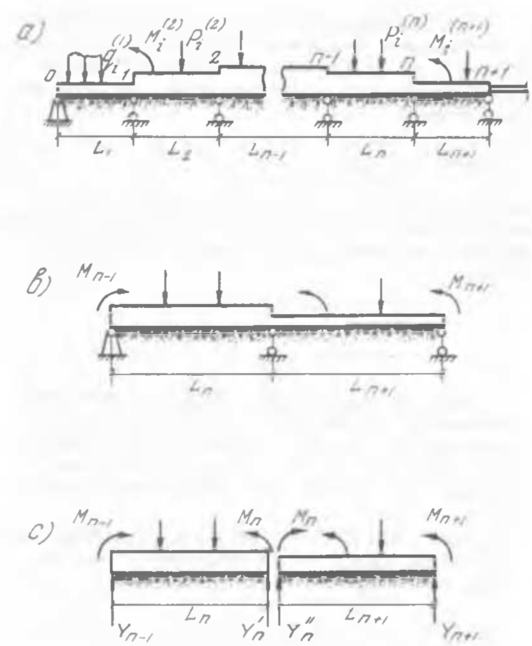

Fig. 2

For each beam let us derive the differential equation of bending

$$E_n J_n \frac{d^4 y_n}{dx_n^4} = \psi^{(n)} - p_x^{(n)} \qquad (I)$$

$$E_{n+1} J_{n+1} \frac{d^4 y_{n+1}}{dx_{n+1}^4} = \psi^{(n+1)} - p_x^{(n+1)} \qquad (2)$$

Table 3

ρ	k = 0	k = 0.2	k = 0.3	k = 0.4	k = 0.5	k = 0.6	k = 0.7	k = 0.8	k = 0.9	k = I
0	0.074	0.072	0.070	0.067	0.062	0.056	0.049	0.038	0.024	0.
0.I	0.074	0.072	0.069	0.066	0.062	0.055	0.048	0.037	0.023	0
0.2	0.072	0.070	0.068	0.065	0.06I	0.054	0.047	0.037	0.023	0
0.3	0.070	0.067	0.065	0.063	0.059	0.053	0.046	0.036	0.022	0
0.4	0.065	0.065	0.063	0.060	0.056	0.05I	0.044	0.034	0.02I	0
0.5	0.062	0.060	0.059	0.056	0.053	0.048	0.04I	0.033	0.020	0
0.6	0.057	0.056	0.054	0.052	0.049	0.044	0.038	0.030	0.0I9	0
0.7	0.052	0.05I	0.049	0.047	0.044	0.040	0.035	0.027	0.0I7	0
0.8	0.046	0.045	0.044	0.042	0.039	0.036	0.03I	0.024	0.0I5	0
0.9	0.040	0.039	0.033	0.036	0.034	0.03I	0.027	0.02I	0.0I3	0
I.0	0.034	0.033	0.032	0.03I	0.030	0.027	0.023	0.0I9	0.0I2	0

and the equation of deformation for the soil surface under each beam from Plamann equation (I.A. Simvulidi, I958). The origin is taken at the the left end of beams. X - axis is directed towards the right and the Y - axis downwards.

In equations (I) and (2)

$$\left.\begin{aligned}\psi^{(n)} &= \sum \Gamma_{\ell^{(n)}_{H_i}}^{\ell^{(n)}_{K_i}} f_i^{(n)}(z) + \sum \Gamma''_{\ell^{(n)}_{2i}} M_i^{(n)} + \\ &+ \Gamma'_{\ell^{(n)}_{3i}} P_i^{(n)} - \Gamma''_0 M_{n-1} + \Gamma''_{L_n} M_n - \\ &- \Gamma'_0 Y_{n-1} - \Gamma'_{L_n} Y'_n\end{aligned}\right\} \tag{3}$$

$$\left.\begin{aligned}\psi^{(n+1)} &= \sum \Gamma_{\ell^{(n+1)}_{H_i}}^{\ell^{(n+1)}_{K_i}} f_i^{(n+1)}(z) + \sum \Gamma''_{\ell^{(n+1)}_{2i}} \cdot M_i^{(n+1)} + \\ &+ \sum \Gamma'_{\ell^{(n+1)}_{3i}} P_i^{(n+1)} - \Gamma''_0 M_n + \Gamma''_{L_{n+1}} \cdot M_{n+1} - \\ &- \Gamma'_0 \cdot Y''_n - \Gamma'_{L_{n+1}} \cdot Y_{n+1},\end{aligned}\right\} \tag{4}$$

$p_x^{(n)}$ and $p_x^{(n+1)}$ are soil reaction pressure on the n and (n + I) - th beams; $\Gamma_{\ell^{(i)}_{H_i}}^{\ell^{(i)}_{K_i}}, \dots, \Gamma'_{\ell_{3i}}, \dots, \Gamma'_{\ell_{2i}}$ - double sided and instantaneous first and second order Gersevanov's breakers.

In deriving the computation formula, the soil bed under each beam is supposed to be different and is considered as a continuous elastic medium characterised by modulus of deformation and Poisson's ratio.

Equation of the plane problem of the elasticity theory (plane deformation) has been made use of to find out the deformation of soil under each beam.

Each beam is considered as a thin elastic rod subjected to linear deformation along itslength. In derlving the general computation formulas, it is supposed that each beam adheres to its bed in such a manner that the elastic line of each deflected beam and the settled surface under it are approximately coincident.

For the sake of obtaining a simple and wieldy solution for practical use with a sufficient garantee accuracy, let us assume the following four contact conditions.

a) that the deflections of both curves at the left end of beam are equal;

b) that the deflections of both curves in the center of beam are equal;

c) that the deflections of both curves at the right end of beam are equal;

d) that the third differential of both functions are equal at the center of beam.

The reaction of soil under each beam is given by the equation (I.A. Simvulidi, I958).

$$p_x^{(i)} = a_0^{(i)} + \frac{2a_1^{(i)}}{L_i}\left(x_i - \frac{L_i}{2}\right) + \frac{4a_2^{(i)}}{L_i^2}\left(x_i - \frac{L_i}{2}\right)^2 + \frac{8a_3^{(i)}}{L_i^3}\left(x_i - \frac{L_i}{2}\right)^3 \tag{5}$$

where $a_0^{(i)}, a_1^{(i)}, a_2^{(i)}$ and $a_3^{(i)}$ and unknown parameters, whose values depend on the rigidity of the beam, its length, modulus of deformation of the elastic bed, nature of load and its point of application.

After integrating for four times, twelve unknown parameters are contained in each of equations (I) and (2) (the unknown quantity L_T contained in both equation
In order to find these parameters the following conditions are used in addition to the four above mentioned contact conditions:

a) two conditions of statics;

b) two boundary conditions at the beam ends;

c) sincc the beam is supposed to be continuous (fig. 2 c) the tangent to the elastic line at the center of the n-th support should be identical for the left and right spans;

d) since the ends of each dismembered beam rest on fixed immovable supports, it is supposed that the reaction pressure of soil at concentrated supports (piles) are equal to zero , therefore we obtain two equations for each beam.

By integrating the differential equations (I) and (2), and with the above mentioned conditions and by simultaneously solving the obtained linear equations, we obtain a theorem on the three moments for the foundation beam resting on an elastic bed, moreover the beam rests on concentrated supports (piles):

$$D_n M_{n-1} + (D'_n + D'_{n+1}) M_n + D_{n+1} M_{n+1} = \\ = (\lambda_n U^{(n+1)} - U^{(n)}) + (\omega^{(n)} - \omega^{(n+1)}) \qquad (6)$$

Application of the basic equation (6) presents no difficulty. To this, it is sufficient to derive the equations for each pair of adjacent spans (as it is done in the application of well known theorem on three moments for a simple continuous beam) and then these equations should be simultaneously solved with respect to the support moments M_0 , M_I M_2 , ... M_{n-I} , M_n and M_{n+I} .

The quantities D_n; D_{n+I}; D'_n; D'_{n+1}; D_{n+1}; λ_n, $U^{(n)}$, $U^{(n+I)}$, $\omega^{(n)}$, and $\omega^{(n+I)}$ contained in equation (6) are determined from the following formulae:

$$D_n = \left[(H_1^{(n)} - H_5^{(n)}) m_3^{(n)} - (H_1^{(n)} + H_3^{(n)}) m_4^{(n)} + \\ + \left(\frac{H_2^{(n)}}{16} + 2 H_3^{(n)} + H_4^{(n)} - \frac{1}{6} H_5^{(n)} \right) \right] \frac{1}{L_n} \qquad (7)$$

$$D'_n = \left[(H_1^{(n)} + H_3^{(n)}) m_3^{(n)} - (H_1^{(n)} - H_3^{(n)}) m_4^{(n)} + \\ + \frac{H_2^{(n)}}{16} - 2 H_3^{(n)} - H_4^{(n)} - \frac{1}{3} H_5^{(n)} \right) \Big] \frac{1}{L_n} \qquad (8)$$

The formulae for the determination of D'_{n+1} and D_{n+1} are obtained from formulac (8) and (7) respectively by substituting (n + I) for n and then multiplying each formula by λ_n

For the determination of $U^{(n)}$, $U^{(n+I)}$, $\omega^{(n)}$ and $\omega^{(n+I)}$, the following formulae have been obtained:

$$U^{(n)} = \left[H_1^{(n)} A^{(n)} + H_2^{(n)} \varkappa^{(n)} + H_3^{(n)} (2C^{(n)} - A^{(n)}) + \\ + H_4^{(n)} N^{(n)} + H_5^{(n)} \varphi^{(n)}_{x_n = L_n} \right] b_n L_n ,$$

$$U^{(n+1)} = \left[- H_1^{(n+1)} \cdot A^{(n+1)} - H_2^{(n+1)} \varkappa^{(n+1)} + H_3 (2C^{(n+1)} - A^{(n+1)}) + \\ + H_4^{(n+1)} \cdot N^{(n+1)} - H_5^{(n+1)} \left(\frac{K}{6 L_{n+1}} + \Phi^{(n+1)} \right) \right] b_{n+1} L_{n+1} \qquad (9)$$

$$\left. \begin{aligned} \omega^{(n)} &= T_1^{(n)} \Delta_1^{(n)} - T_2^{(n)} \Delta_2^{(n)} , \\ \omega^{(n+1)} &= T_2^{(n+1)} \Delta_1^{(n+1)} - T_1^{(n+1)} \Delta_2^{(n+1)} \end{aligned} \right\} \qquad (I0)$$

$$\lambda_n = \frac{\alpha_n E_n J_n \rho_n}{\alpha_{n+1} E_{n+1} J_{n+1} \rho_{n+1}} \cdot \frac{L_{n+1}^2}{L_n^2} \qquad (II)$$

$$\left. \begin{aligned} \gamma_n &= 2048 + \alpha_n \\ g_n &= 13440 + 38{,}5 \alpha_n \\ \rho_n &= \gamma_n \cdot g_n \end{aligned} \right\} \qquad (I2)$$

$$\left. \begin{aligned} H_1^{(n)} &= (16319520 + 5880 \alpha_n) \gamma_n \alpha_n , \\ H_2^{(n)} &= 4515840 \gamma_n \cdot \alpha_n^2 , \\ H_3^{(n)} &= \left[(80640 + 112 \alpha_n)(3840 - 3 \alpha_n) + \right. \\ &\quad \left. + (53760 + 48 \alpha_n)(3840 + 10 \alpha_n) \right] g_n , \\ H_4^{(n)} &= (215040 - 768 \alpha_n) \alpha_n \cdot g_n , \\ H_5^{(n)} &= 8! \rho_n \alpha_n \end{aligned} \right\} \qquad (I3)$$

$$\left.\begin{aligned}\Delta_1^{(n)} &= (H_1^{(n)} - H_3^{(n)})\, m_1^{(n)} - (H_1^{(n)} + H_3^{(n)})\, m_2^{(n)},\\ \Delta_2^{(n)} &= (H_1^{(n)} - H_3^{(n)})\, m_2^{(n)} - (H_1^{(n)} + H_3^{(n)})\, m_1^{(n)}\end{aligned}\right\} \qquad (I4)$$

$$\left.\begin{aligned}m_1^{(n)} &= \frac{t_1^{(n)} + t_3^{(n)}}{4 t_1^{(n)} \cdot t_3^{(n)}}\\ m_2^{(n)} &= \frac{t_1^{(n)} - t_3^{(n)}}{4 t_1^{(n)} \cdot t_3^{(n)}}\\ m_3^{(n)} &= \frac{1}{2 t_1^{(n)} \cdot t_3^{(n)}}\left(2 t_1^{(n)} t_3^{(n)} + t_1^{(n)} t_4^{(n)} + t_2^{(n)} \cdot t_3^{(n)} \cdot \frac{1}{16}\right)\\ m_4^{(n)} &= \frac{1}{2 t_1^{(n)} t_3^{(n)}}\left(2 t_1^{(n)} t_3^{(n)} + t_1^{(n)} t_4^{(n)} - t_2^{(n)} \cdot t_3^{(n)} \cdot \frac{1}{16}\right)\end{aligned}\right\} \qquad (I5)$$

$$\left.\begin{aligned}t_1^{(n)} &= (32070 + 213{,}5\,\alpha_n)\,\gamma_n,\\ t_2^{(n)} &= 13440\,\alpha_n \cdot \gamma_n,\\ t_3^{(n)} &= (7680 + 7\alpha_n)\, g_n,\\ t_4^{(n)} &= 16\, g_n \cdot \alpha_n\end{aligned}\right\} \qquad (I6)$$

$$\left.\begin{aligned}T_1^{(n)} &= \left[t_1^{(n)} A^{(n)} - t_2^{(n)} \mathcal{H}^{(n)} - t_3^{(n)} (2C^{(n)} - A^{(n)}) - t_4^{(n)} N^{(n)}\right] b_n L_n\\ T_2^{(n)} &= \left[t_1^{(n)} A^{(n)} - t_2^{(n)} \mathcal{H}^{(n)} + t_3^{(n)} (2C^{(n)} - A^{(n)}) + t_4^{(n)} N^{(n)}\right] b_n L_n\end{aligned}\right\} \qquad (I7)$$

For the determination of $H_1^{(n+1)}, \dots, H_5^{(n+1)}$; ρ_{n+1}; $\Delta_1^{(n+1)}$; $\Delta_2^{(n+1)}$, $m_1^{(n+1)}, \dots, m_4^{(n+1)}$, $t_1^{(n+1)}, \dots, t_4^{(n+1)}$; $T_1^{(n+1)}$, $T_2^{(n+1)}$ should be substituted for (n) everywhere in $n+1$ formulae I3–I7. For the determination of flexibility coefficient α_n, we make use of the formula (I.A. Simvulidi, I965)

$$\alpha_n = \frac{\pi\, b_n\, E_{0_n}\, L_n}{E_n\, J_n}$$

For the determination $\varphi^{(n)}_{x_n = L_n}$ the following formula has been derived:

$$\varphi^{(n)}_{x_n=L_n} = \left\{ \sum \frac{M_i^{(n)}}{b_n L_n^2}\left(1 - \frac{\ell_{2i}^{(n)}}{L_n}\right) + \frac{1}{2}\sum \frac{P_i^{(n)}}{b_n L_n}\left(1 - \frac{\ell_{3i}^{(n)}}{L_n}\right)^2 + \frac{1}{L^3}\left[\sum \int_{\ell_{H_i}^{(n)}}^{L_n} f_i^{(n)}(z)\, \frac{(L_n - z)^2}{2!}\, dz - \sum \int_{\ell_{K_i}^{(n)}}^{L_n} f_i^{(n)}(z)\, \frac{(L_n - z)}{2!}\, dz\right] + \frac{K^{(n)}}{3L_n} - \Phi^{(n)} \right\} \qquad (I8)$$

Knowing $H_1^{(i)}, \dots, H_5^{(i)}$; ρ_i; $\Delta_1^{(i)}, \Delta_2^{(i)}$; $m_1^{(i)}, \dots, m_4^{(i)}$; and $t_1^{(i)}, \dots, t_4^{(i)}, T_1^{(i)}, T_2^{(i)}$; $\varphi^{(n)}_{x_n = L_n}$, we can find D_n; D_n'; D_{n+1}' and D_{n+1}, and the support moments can be determined from equation (6).

Knowing the support moments we can find the support reactions:

$$Y_{n-1} = T_1^{(n)} \cdot m_1^{(n)} - T_2^{(n)} m_2^{(n)} - m_3^{(n)} \frac{M_{n-1}}{L_n} + m_4^{(n)} \frac{M_n}{L_n}, \qquad (I9)$$

$$Y_n' = T_2^{(n)} m_1^{(n)} - T_1^{(n)} m_2^{(n)} + m_4^{(n)} \frac{M_{n-1}}{L_n} - m_3^{(n)} \frac{M_n}{L_n} \qquad (20)$$

Formulae for the determination of Y_n'' and Y_{n+1} are derived from formulas (I9) and (20) respectively by substituting (n + I) for (n).

The load terms $A^{(i)}$, $C^{(i)}$, $K^{(i)}$, $N^{(i)}$, and $\Phi^{(i)}$ contained in the formulae are determined from the formulae given in the paper

With the help of the obtained theorem on three moments (I.A. Simvulidi, I965) and formulae (I9) and (20), we can easily calculate any low pile foundation grill of both constant and variable cross section and loaded in any arbitrary manner. After having found the support reactions and moments, these forces and moments are included in the given (known) forces and moments; and each separate beam is considered and calcualted as an independent beam on an elastic foundation bed.

Example.

A low pile foundation grill (fig. 3) is given. It is required to determine the support reactions Y_0, Y_I and Y_2 in the general form

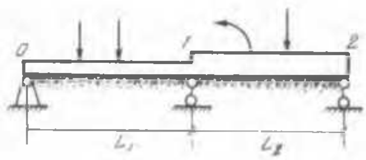

Fig. 3

According to the given conditions of the problem $M_{n-1} = 0$, $M_n = M_1$, $M_{n+1} = 0$

Therefore from equation (6), we have:

$$(D_1' + D_2')\, M_1 = (\lambda_1 U^{(2)} - U^{(1)}) + (\omega^{(1)} - \omega^{(2)})$$

The formulae (I9) and (20) for this particular case assume the form:

$$Y_0 = T_1^{(1)} m_1^{(1)} - T_2^{(1)} m_2^{(1)} + m_4^{(1)} \frac{M_1}{L_1},$$

$$Y_1' = T_2^{(1)} m_1^{(1)} - T_1^{(1)} m_2^{(1)} - m_3^{(1)} \frac{M_1}{L_1},$$

$$Y_1'' = T_1^{(2)} m_1^{(2)} - T_2^{(2)} m_2^{(2)} - m_3^{(2)} \frac{M_1}{L_1}$$

$$Y_2 = T_2^{(2)} m_1^{(2)} - T_1^{(2)} m_2^{(2)} + m_4^{(2)} \frac{M_1}{L_1}$$

where $Y_1 = Y_1' + Y_1''$

REFERENCES:

K.E. Egorov, A.A. Nichiporovich, 1961, Studies on the deformation of foundation beds. Report read at V Inter. Congress on Soil Mechanics and Foundation Engineering, GOSSTROYIZDAT, Moskva (In Russian).

K.E. Egorov, R.V. Serebryanii, 1963. Determination of Stresses in Rigid Circular-Foundation. Collection of articles No 53, Scientific Research Institute of Foundation and Underground Structures, Moskva (In Russian).

M.I. Gorbunov-Posadov, 1941. Slabs on elastic foundation beds (Theory and computation tables) STROYIZDAT, Moskva (In Russian).

V.D. Palmov, 1960. Contact problem for a plate on elastic stratum. PMM AN SSR. Vol. XXIV, issue 3, Moskva (In Russian).

O.D. Shilova, 1963. Remarks on the paper by V.D. Palmov. Vol. XXVIII, issue 6, Moskva (In Russian).

I.A. Simvulidi, 1958. Calculation of structures on elastic foundation bed. ROSVYZIZDAT, Moskva (In Russian).

I.A. Simvulidi, 1965. Calculation of non-uniform beams on elastic foundation bed. Vol. III, IV Inter. Congress, Canada.

I.A. Simvulidi, 1965. Calculation on continuous beams resting on elastic beds. Izves. Vys. Ucheb. Zavsdeniy "Stroitelstva i arkhitektura" No 4, Moskva (In Russian).

B.N. Zhemochkin, A.P. Sinitsyn, 1962. Practical methdos for the calculation of foundation beams and slabs on elastic foundation bed. GOSSTROYIZDAT, Moskva (In Russian).