STUDIES ON THE STRUCTURAL RIGIDITY OF REINFORCED CONCRETE BUILDING FRAMES ON CLAY

ETUDE DE LA RIGIDITE DES OSSATURES EN BETON ARME REPOSANT SUR ARGILE

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SYNOPSIS A method of settlement analysis is shown, in which the effects of the time - settlement - properties of the subsoil, the creep of concrete and the procedure of constructing the building frame can be considered. The method is based on the assumption, that the law of superposition is applicable to all processes and is limited to soils, in which the settlement occurs at a slower rate than the creep of concrete. The effects, which the influences of time exert upon the settlement stresses of the building frame and the loading and the deflection of the foundation, are shown by analysing a fictitious structure.

INTRODUCTION

The magnitude of differential settlements and the bending-stresses of statically indeterminate structures as well as the distribution of contact pressure are decisively influenced by the flexural rigidity of the structure and the compressibility of the subsoil. At the present the design of the foundation and the calculation of differential settlements are almost exclusively carried out, assuming, that forces imposed on the foundation by the superstructure are not influenced by differential settlements. On the other hand only a few papers , dealing with the foundation and the superstructure as one integral unit, have so far been published. All of these investigations deal only with the stresses and deflections, which arise in the structure, while assuming there are no settlements until the building is completed.

None of these assumptions actually includes the true conditions. During the time of construction the loads as well as the flexural rigidity increase continuously. Due to the consolidation, the settlements occur with a delay in time in soils with low permeability, as for instance in clay. As a result of the creep of concrete the stresses, caused in the structure by differential settlements, are lessened - lessening, which itself increases the differential settlements. It must be assumed, that every attempt to include the rigidity of the reinforced concrete frame, which does not consider these influences of time may lead to errors comparable to that of totally neglecting the rigidity of the superstructure.

In order to take these facts into account, Schultze (1962) proposed to determine the "true rigidity" of building frames from settlement observations. The "true rigidity" is defined as the rigidity of a fictitious foundation beam, which - under equal loads and on the same subsoil - shows the same maximum settlement difference, which has been measured on the actually erected construction. Proceeding in this way it is possible to estimate the stiffening effect of the superstructure. However no clue results for the design of the foundation and for estimating the secondary stresses of the building frame. Due to the influences depending on time, the knowledge of the deformations in the foundation is not sufficient for determining all unknowns.

The effects of the influences of time can only be estimated by calculating them theoretically, since the methods used up to now, make it neither possible to measure with sufficient precision the distribution of contact pressure nor the stresses which are present in the concrete.

GENERAL METHOD OF ANALYSIS OF REINFORCED CONCRETE BUILDING FRAMES ON CLAY CONSIDERING THE TIME DEPENDENT INFLUENCES

This method is based on the usual assumptions, that

1. the construction can be regarded as two dimensional
2. the flexural rigidity of the reinforced concrete members is constant within each member and does not depend on the magnitude of stress on it
3. the deformations due to shearing and direct forces as well as the frictional forces acting at the base of the foundation can be neglected
4. the final settlement of any point at the base of the foundation is equivalent to the consolidation of the vertical clay column under this point. This consolidation is computed - using a constant coefficient of volume change - from the increase in vertical stress determined by the theory of elasticity.

However, any other method of calculating settlements can be adopted without difficulty, provided that a linear relation is assumed between stresses and deformations.

The method is derived from the formulas stated by Sommer (1965). However, the meaning of some of the symbols used by Sommer have been changed in favour of a simpler way of writing. The foundation area is divided in n elements with the length a_i and the width B. In order to determine the deformations of the structure, the contact pressure q_i, which acts on the structure within every element i, is summed up in one single load Q_i acting at the center of the element. However, for the calculation of the settlements, a uniform contact pressure

$$q_i = \frac{Q_i}{a_i \cdot B} \tag{1}$$

is assumed within each element (Fig. 1a). Since it is postulated, that no settlement differences occur along the width B, the settlements are calculated at the points indicated in Fig. 1b.

If only the final state is considered, the settlement for any element i results as

$$s_i = m_v \cdot \sum_{k=1}^{k=n} c_{ik} \cdot Q_k$$

or written as matrix equation

$$\mathbf{s} = m_v \cdot \mathbf{C} \cdot \mathbf{q} \tag{2}$$

in which m_v is the coefficient of volume change and c_{ik} is the final settlement of the element i - divided by m_v - resulting from the pressure acting under the element k

$$q_k = \frac{1}{a_k \cdot B}$$

In order to include the influences of time, however, it is necessary to consider the intermediary conditions in addition to the final ones. Therefore the additional assumption is made, that - keeping the external loading constant - the settlements at any time t can be calculated in the same way as the final ones, if the coefficient of volume change is diminished equivalently to the ratio between the average settlement up to this time $s_m(t)$ and the average final settlement $s_m(t=\infty)$. With

$$\mu(t) = \frac{s_m(t)}{s_m(t=\infty)} \tag{3}$$

one obtains instead of equation (2)

$$\mathbf{s}(t) = \mu(t) \cdot m_v \cdot \mathbf{C} \cdot \mathbf{q}(t) \,. \tag{4}$$

Objections can be raised to this assumption, since it implies that in every point of the subsoil at the time considered, the ratio between neutral and total vertical stress is the same. This ratio is assumed to be valid for the stresses, which arise immediately after loading as well as for the changes in stress arising in the following time due to contact pressure redistribution. Since it is not certain that better results can be obtained with the well-known theories of consolidation, it is assumed in the present case, to be of greater advantage to limit the numerical calculations by the choice of a very simple assumption. Furthermore the suggested assumption can only result in a different form of the settlement bowl in Fig. 1b. For this rea-

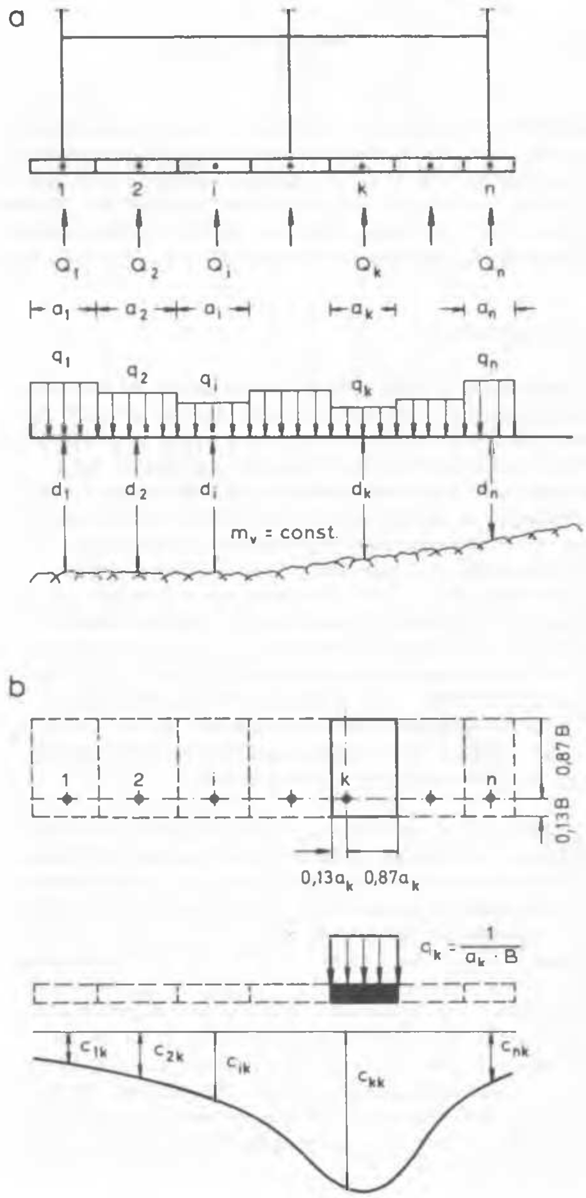


Fig. 1 Determination of the settlements:
a forces in the foundation level,
b $\frac{1}{m_v}$ -fold settlements resulting from $q_k = \frac{1}{a_k \cdot B}$.

son the application of the suggested assumption does not seem to lead to greater errors, than the application of the conventional method of calculating settlements usually employed for all types of soil.

The loading of building frames is increased gradually by the adding of each single story. Since equation (4) is only applicable to a single loading stage, the effects caused by the load of each story j must be computed separately and then be superimposed. These quantities are characterized by the upper index mark j. Furthermore the rigidity of the superstructure changes by the addition of every further story, so that only the changes in the period between the construction of two successive stories can be computed.

The loads of the considered story j, at the time of their being added, produce the settlements

$$\mathbf{s}^j(t_j) = \mu^j(t_j) \cdot m_v \cdot \mathbf{C} \cdot \mathbf{q}^j(t_j) \tag{5}$$

according to equation (4). At the same time the relation is given between the settlements and the supporting reactions $Z^j{}_i(t_j)$ acting on the building - completed up to story j - by superposition of the restraining forces of every separate unit settlement condition pointed out in Fig. 2:

$$Z^j{}_i(t_j) = Z^j{}_{i0} + \sum_{k=1}^{k=n} Z_{ik}(t_j) \cdot s^j{}_k(t_j)$$

or written as matrix equation

$$\mathbf{z}^j(t_j) = \mathbf{z}^j{}_0 + \mathbf{Z}(t_j) \cdot \mathbf{s}^j(t_j) \tag{6}$$

The condition for equilibrium in the foundation level is:

$$\mathbf{z}^j(t_j) = \mathbf{q}^j(t_j) \tag{7}$$

From (7) with the use of (5) and (6) one obtains:

$$\left[\mathbf{Z}(t_j) \cdot \mathbf{C} - \frac{1}{\mu^j(t_j) \cdot m_v} \cdot \mathbf{E}\right] \cdot \mathbf{q}^j(t_j) = - \frac{1}{\mu^j(t_j) \cdot m_v} \cdot \mathbf{z}^j{}_0 \tag{8}$$

in which $\mathbf{E}$ indicates the unit matrix

$$E_{ik} = \begin{cases} 1 & (i = k) \\ 0 & (i \neq k) \end{cases}$$

The solution of the set of linear equations (8) supplies the unknown contact pressures $\mathbf{q}^j(t_j)$, with which the settlements can be calculated in (5).

In order to determine the upcoming changes of these quantities, it is sufficient to derive the relations valid in the period of time between the adding of any story l - 1 and the following story l. The changes of the settlements occurring in the time considered amount to

$$\Delta\mathbf{s}^j(t_l) = \mathbf{s}^j(t_l) - \mathbf{s}^j(t_{l-1}) \tag{9}$$

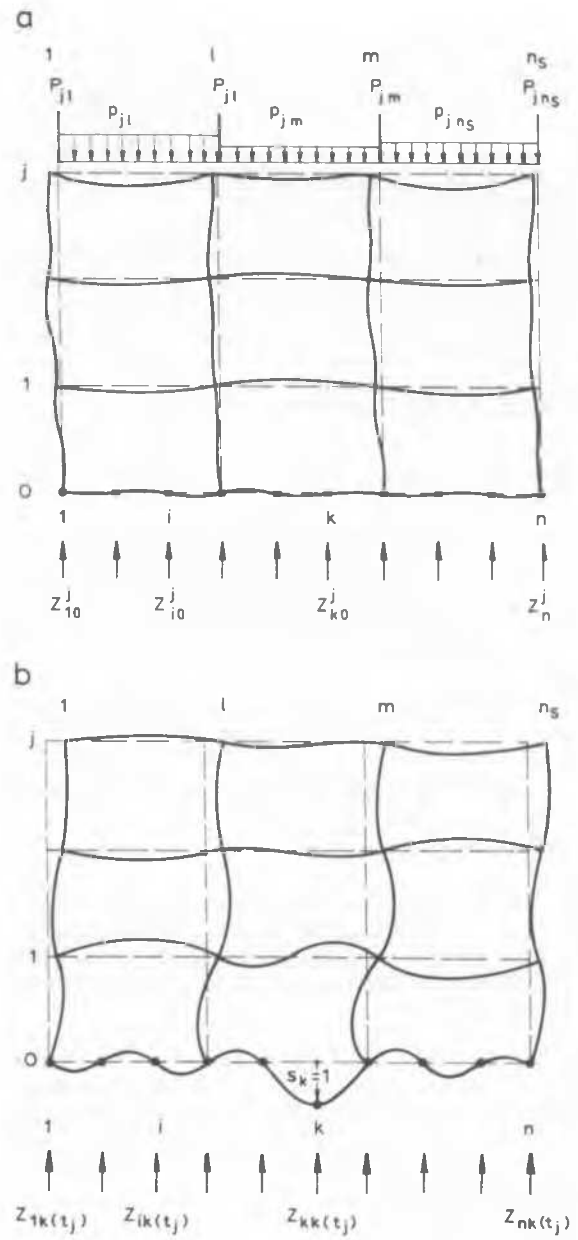


Fig. 2 Restraining forces of unit settlement conditions of the construction completed up to story j
a zero position (no settlements)
b unit settlement condition $s_k = 1$ ($s_i = 0$ for $i \neq k$)

Equation (4) substituted in (9) leads to

$$\Delta \mathbf{s}^j(t_l) = \mu^j(t_l) \cdot m_v \cdot \mathbf{C} \cdot [\mathbf{q}^j(t_l) - \frac{\mu^j(t_{l-1})}{\mu^j(t_l)} \cdot \mathbf{q}^j(t_{l-1})] \tag{10}$$

In the same period of time the change of the supporting reactions of the building, completed up to story $l - 1$ $(l > j)$

$$\Delta \mathbf{z}^j(t_l) = \Delta \mathbf{z}^j_0(t_l) + \mathbf{Z}(t_l) \cdot \Delta \mathbf{s}^j(t_l) \tag{11}$$

has to be determined considering the different creep properties in the separate stories. $\Delta \mathbf{z}^j_0(t_l)$ stands for the change due to the creep of concrete in the zero position (no change in the settlements) and $\mathbf{Z}(t_l)$ for the change due to the unit settlement conditions.

The condition for equilibrium is:

$$\Delta \mathbf{z}^j(t_l) = \mathbf{q}^j(t_l) - \mathbf{q}^j(t_{l-1}) \tag{12}$$

By introducing (10) and (12) in (11) one obtains:

$$[\mathbf{Z}(t_l) \cdot \mathbf{C} - \frac{1}{\mu^j(t_l) \cdot m_v} \cdot \mathbf{E}] \cdot \mathbf{q}^j(t_l) = \frac{\mu^j(t_{l-1})}{\mu^j(t_l)} [\mathbf{Z}(t_l) \cdot \mathbf{C} - \frac{1}{\mu^j(t_{l-1}) \cdot m_v} \cdot \mathbf{E}] \cdot \mathbf{q}^j(t_{l-1}) - \frac{1}{\mu^j(t_l) \cdot m_v} \cdot \Delta \mathbf{z}^j_0(t_l) \tag{13}$$

If the contact pressure distribution is known at the time t_{l-1}, with this relation it is possible to calculate the contact pressures $\mathbf{q}^j(t_l)$ at the time t_l. The change of the settlements in this period of time then results in (10).

After having determined the settlements, the computation of the bending moments is not fundamentally difficult, if the creep of concrete is handled by the rate of creep method. In this point the indicated method corresponds completely to the step by step procedures usually used for creep analysis.

The equations (9) to (13) are also valid from the time of completion of the structure until all processes depending on time die out. During this time it is necessary to keep the time intervals small enough to guarantee a sufficient numerical accuracy. During the period of construction, the time intervals between the adding of two successive stories can be considered small enough.

For the application of this method it is necessary to consider one aspect in detail. The stresses produced in a structure by differential settlements are largely diminished in course of time by the creep of concrete. If during this time no further loads are added, the hereby conditioned redistribution of contact pressure causes relief in some areas of the subsoil, while in others the stresses are increased. In all points of the subsoil, in which the compression, due to the stresses working up to this moment, is greater than the final compression under the new stresses, this stress reduction causes an increase in volume. According to equation (4) this increase in volume is calculated with the same coefficient of volume change as the compression. This is contradictory to the behaviour of real soils. Hence it may be assumed, that without modification this method is only applicable to soils, in which the settlements occur at a slower rate than the creep deformations of the concrete. This proves to be correct for clay.

APPLICATION

In order to show the effects of the influences of time, a fictitious structure - of which the dimensions, loads and properties are given in Fig. 3 - is analysed in three different ways:

α The superstructure and the foundation are treated separately, as it is usually done in practice. The analysis of the frame is conducted assuming that the supports are perfectly rigid and that the loads are added after the construction is completed. The analysis of the foundation is based on the assumption that the supporting reactions of the superstructure are not influenced by differential settlements.

β The superstructure and the foundation are considered as a unit. It is assumed - according to the previous theoretical studies - that the loads are added after the construction is completed and that the construction is only subject to elastic deformation.

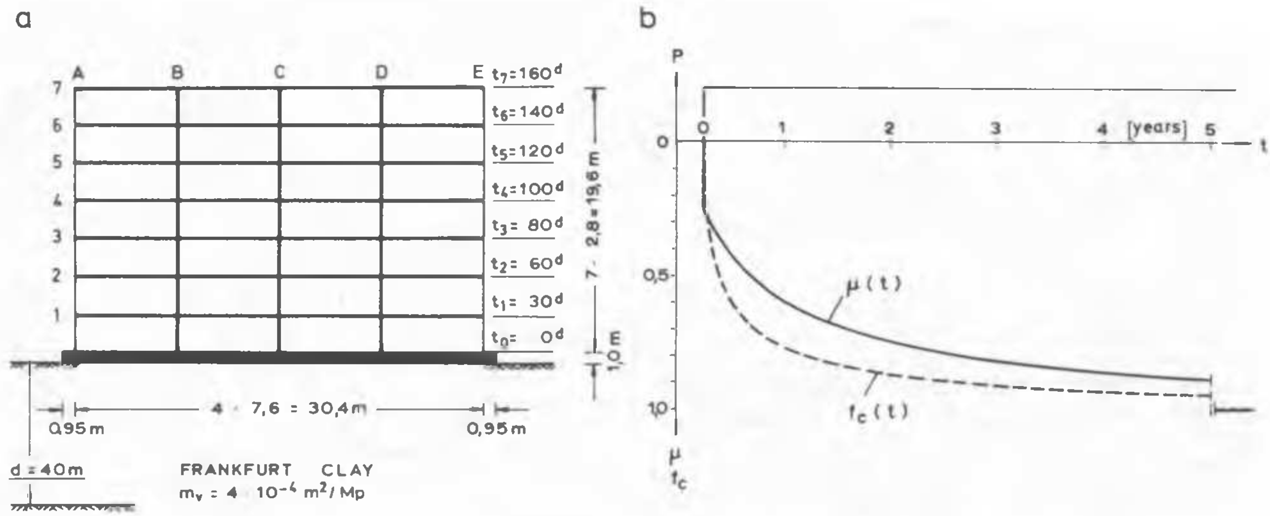
γ The construction is analysed considering the influences of time by means of the proposed method.

By applying the proposed method of analysis it is assumed that the final creep strain of concrete amounts to two times the elastic strain, while the shape of the creep curve is assumed according to the recommendations of the Comité Européen du Béton (1964).

The settlement ratio $\mu(t)$ is chosen as follows:

$$\mu(t) = \frac{0{,}218 + t\ [\text{years}]}{0{,}882 + t\ [\text{years}]} \tag{14}$$

This corresponds to the results obtained from settlement observations of structures on Frankfurt clay. In Fig. 4 the average settlements calculated by means of equation (14) are compared with those measured at a 22-storied building.



FLEXURAL RIGIDITY EI [Mp · m²]				LOADING [Mp/m]		WIDTH B = 12,5m
RAFT	$2{,}2 \cdot 10^6$	COLUMNS B, C, D	$2{,}8 \cdot 10^4$	RAFT	25,0	
BAYS 1....7	$9{,}8 \cdot 10^4$	COLUMNS A, E	$1{,}0 \cdot 10^4$	BAYS 1....7	15,0	

Fig. 3 Fictitious structure:

a dimensions, loading and time of construction,

b settlement ratio $\mu(t) = \dfrac{s_m(t)}{s_m(t=\infty)}$ and creep curve of concrete $f_c = \dfrac{\text{creep strain}}{\text{final creep strain}}$ for a single stage of loading.

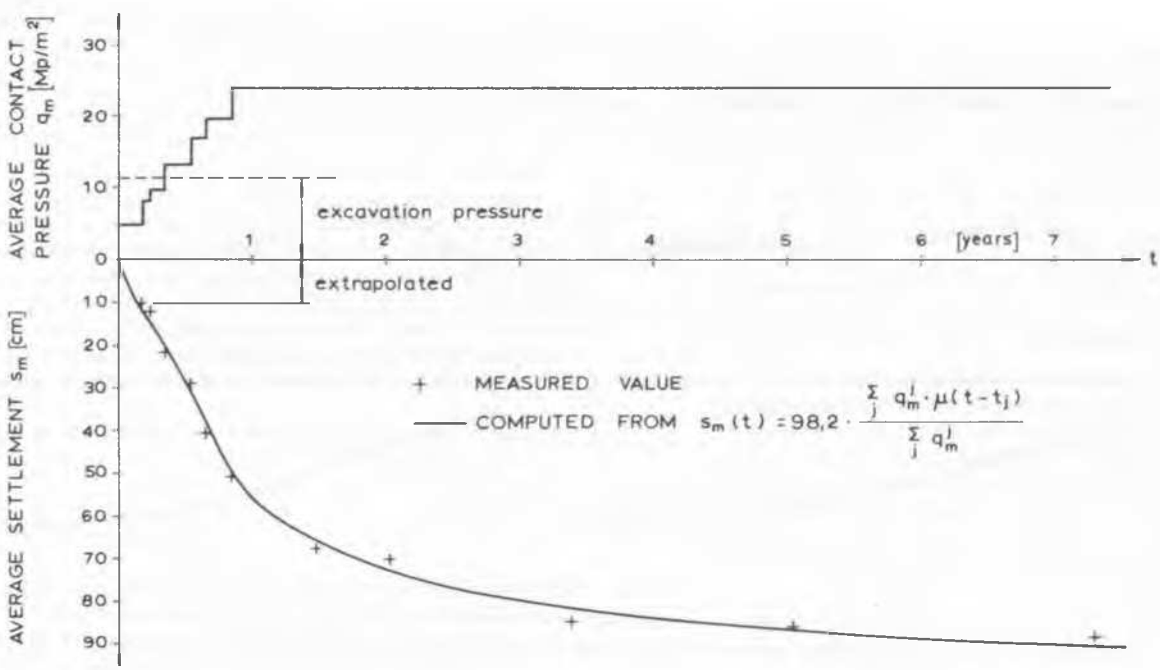


Fig. 4 Comparison between measured and computed average settlement.

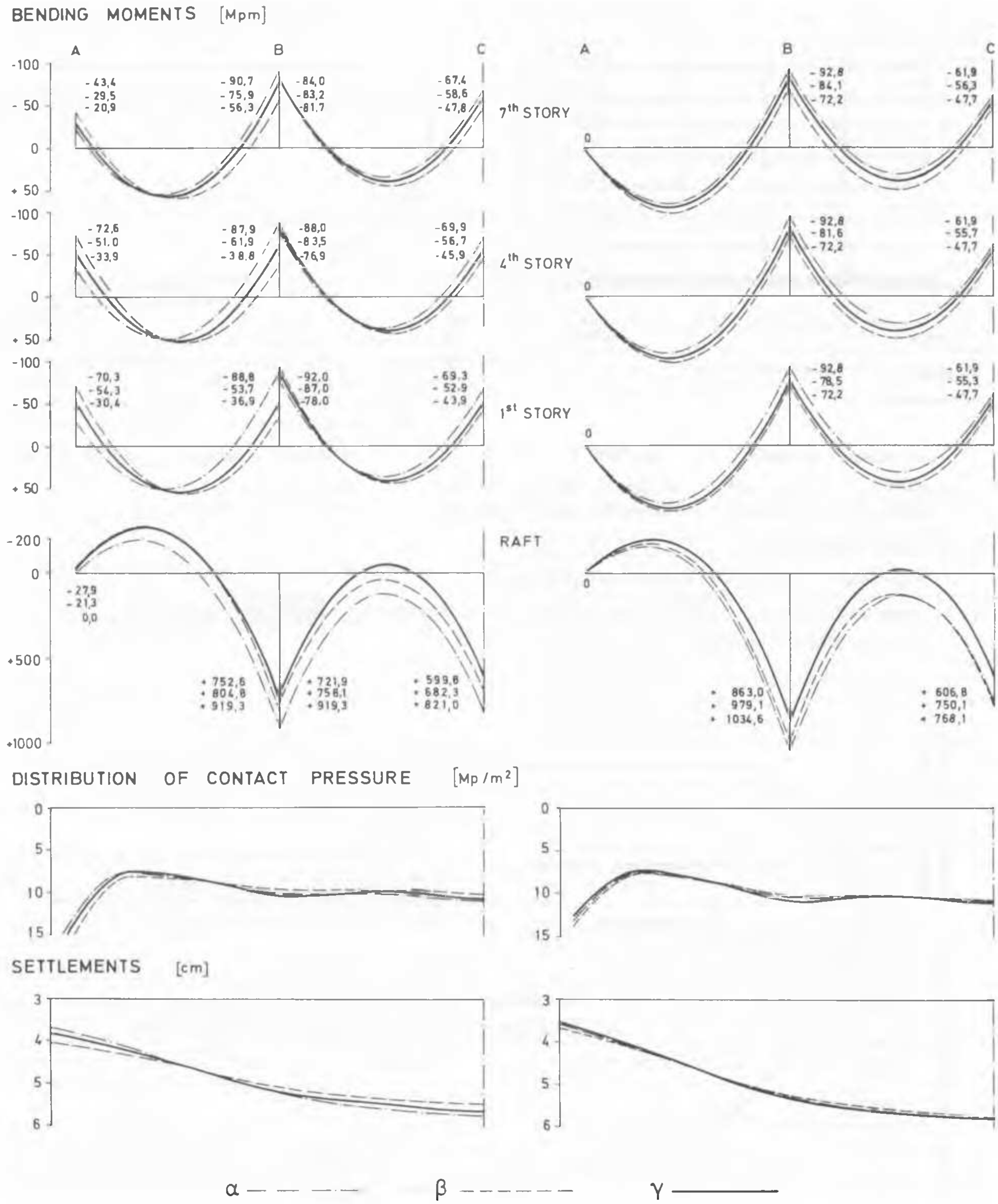


Fig. 5 Bending moments, distribution of contact pressure and settlements as obtained by different methods of analysis.

Building frame shown in Fig. 3: columns with fixed ends.

Building frame shown in Fig. 3: columns with pin-jointed ends.

TABLE I Column loads and maximum differential settlement

Column-Beam Connection	Method of Analysis	Column Loads A [Mp]	Column Loads B [Mp]	Column Loads C [Mp]	Maximum Differential Settlement [cm]
fixed ends	α	346,8	857,7	783,0	1,88
	β	424,4	811,4	720,5	1,33
	γ	386,1	836,2	747,4	1,66
pin-jointed ends	α	313,5	912,0	741,0	2,00
	β	332,5	887,0	752,9	1,92
	γ	323,9	896,8	750,5	2,04

The results obtained by the given methods are shown in Fig. 5 and in Table I, in which a fixed-ended as well as a pin-jointed connection between columns and beams is considered.

By comparing the results obtained with the first two methods without regard to the influences of time, it can be realized that the rigidity of the superstructure exerts a discernable effect on the rigidity of the entire building if the columns are connected by fixed-ends to the beams, while in case of a pin-jointed connection the rigidity of the structure is hardly increased. The importance of the stiffening effect, produced by the flexural rigidity of the columns, was stated already by Meyerhof (1953).

The results obtained by considering the influences of time show, that the settlement stresses in the superstructure have been overestimated by the previous theoretical studies. This deviation is greater in the upper stories than in the lower ones. This can be attributed to the fact, that the upper stories were added at a point at which a considerable part of the differential settlements had already occurred.

The bending moments in the foundation show the greatest deviation. Considering the influences of time for the fixed-ended as well as the pin-jointed connections, much smaller positive bending moments are obtained, than by the other two methods. This is due exclusively to the influence of creep of concrete. Additional calculations have shown, that the moments obtained while neglecting creep of concrete lie between the values obtained by methods α and β.

The settlements and contact pressure distributions obtained by the three methods differ only slightly.

CONCLUSIONS

Despite the effects of the influences of time the rigidity of the superstructure - in building frames on clay - exerts a discernable effect on the bending moments in the frame and in the foundation. The extent of this effect is mainly determined by the flexural rigidity of the columns.

By neglecting the creep of concrete positive bending moments in the foundation are overestimated. This is especially true when determining the bending moments in the foundation, assuming that the flexural rigidity of the superstructure can be neglected. In this case a smaller modulus of elasticity can be adopted to approximately allow for the effect of creep of concrete. Although more usable bending moments can be obtained hereby, the differential settlements are then overestimated.

It must be expected, that in building frames on sand the effect of the creep of concrete is much more significant, since in this case the differential settlements occur at an earlier time, at which the structure possesses a greater ability to reduce secondary stresses. The same applies to the effect of the method of erecting the structure, since a greater share of the differential settlement then occurs before the structure is completed.

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REFERENCES

Comité Européen du Béton, 1964, Recommandations pratiques unifiées pour le calcul et l'exécution des ouvrages en béton armé, Paris

Meyerhof, G.G., 1953, Some recent foundation research and its application to design, The Structural Engineer 31, No. 6, pp. 151 - 167

Schultze, E., 1962, Probleme bei der Auswertung von Setzungsmessungen, Vorträge der Baugrundtagung 1962 in Essen, Deutsche Gesellschaft für Erd- und Grundbau e. V., Essen, pp. 343 - 386

Sommer, H., 1965, A method for the calculation of settlements, contact pressures and bending moments in a foundation including the influence of the flexural rigidity of the superstructure, Proc. 6th Int. Conf. Soil Mech., Montreal, Vol. 2, pp. 197 - 201