THE ANALYSIS OF DOWNDRAG IN END-BEARING PILES

FROTTEMENT NEGATIF SUR DES PIEUX A POINTE PORTANTE

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SYNOPSIS An analysis is made of the magnitude and distribution of downdrag force induced in an end-bearing pile when placed in a consolidating clay layer underlain by a rigid base. For purely elastic conditions within the soil, influence factors are presented for the downdrag force at the toe of the pile for a wide range of values of length-to-diameter ratio L/d and pile stiffness K. Both L/d and K are found to have a major influence on the magnitude of this downdrag force.

The elastic analysis is extended to the case where the surface settlement of the soil layer is sufficient to cause yield between the pile and the soil. It is shown that the distribution of maximum adhesion τ_a between pile and soil influences the relationship between downdrag and settlement, the downdrag force for a given settlement being greater when τ_a is uniform with depth than when a triangular distribution of τ_a with depth exists.

INTRODUCTION

It has long been recognised that when end-bearing piles are placed in a consolidating soil mass, a downward force is induced in the pile. This downdrag effect is commonly termed "negative friction". Measurements on long steel end-bearing piles in soft clay by Johannessen and Bjerrum (1965) have revealed that negative friction may induce a downdrag force in a pile sufficient to cause the design load to be exceeded. In consequence, additional settlement of the pile occurs, due partly to elastic compression of the pile and partly to penetration of the pile tip into the bearing stratum.

Approximate methods of calculating the downdrag force due to negative friction have been suggested by Terzaghi and Peck (1948) and Zeevaert (1959). A summary of these methods is given by Locher (1965) who also quotes an empirical expression for the downdrag force given by Elmasry (1963). Locher has found that for a particular case involving a circular pile, the estimates of downdrag force from the above three methods agree closely, but no measurements of the actual downdrag force are reported. A theoretical analysis based on the elastic theory has been presented by Salas and Belzunce (1965), but no account has been taken of pile compressibility or of the influence of the underlying rigid base on the displacement of the soil adjacent to the pile.

In this paper, an analysis is made of the effects of negative friction on a single compressible pile of circular cross-section. The tip of the pile is assumed to rest on a perfectly rigid base, and the surrounding soil is assumed to be a homogeneous isotropic elastic material. By employing elastic theory, solutions are obtained for the relationship between the surface settlement of the soil and the downdrag force induced in the pile. The influence on this relationship of the relative stiffness of the pile, the length to diameter ratio and Poisson's ratio of the soil is investigated.

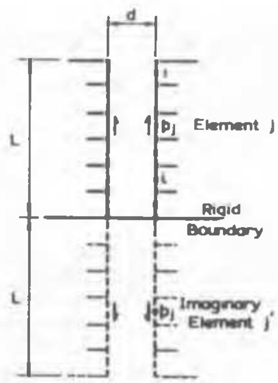
Several approximate solutions for the downdrag force-settlement relationship are also obtained for the case where local yield occurs between the pile and the soil.

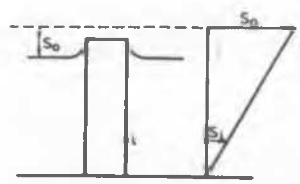
METHOD OF ANALYSIS

A circular pile of length L and outer diameter d is considered, the area of the pile section being A_p. The pile is divided into n equal cylindrical elements, any element j being acted upon by a uniform vertical shear stress p_j on the periphery. The surrounding soil layer is assumed to have constant elastic parameters E_s and ν_s. It is also assumed that the consolidation settlement of the soil remote from the pile varies linearly with depth from S_o at the surface to zero at the base of the layer.

The problem is solved by equating the displacements of the soil adjacent to the pile to those of the pile itself at various points along the pile. It has been found that only compatability of vertical displacements need be considered to obtain solutions of satisfactory accuracy for practical problems. The vertical displacements at the mid-point of the outer surface of each element are considered in the subsequent analysis.

The displacement of the soil arises from both the shear stresses along the pile and from the consolidation of the soil itself. Because a rigid boundary exists at a depth L below the surface, the condition of zero vertical displacement at this boundary must be satisfied. An approximate method of satisfying this condition has been outlined by D'Appolonia and Romualdi (1963), in which a mirror-image element j' of element j, loaded by an equal and opposite shear stress p_j, is also considered when calculating the soil displacement at point i (see Fig.1a)*.





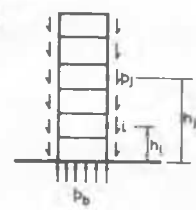


Fig.1. Stresses & Displacements in soil-pile system

*The Steinbrenner approximation (Steinbrenner, 1934) could equally well be used to satisfy the zero boundary displacement condition.

Taking downward displacements as positive, the displacement at i due to the shear stresses along the pile may be expressed as

$$\rho_i = -\frac{d}{E_s}\sum_{j=1}^{n} p_j\,(I_{ij} - I'_{ij}) \quad \ldots\ \ldots(1)$$

where I_{ij}, I'_{ij} are the influence factors for the displacement at i due to shear stresses on the real element j and the imaginary element j', respectively.

Both I_{ij} and I'_{ij} are obtained by integration of the Mindlin equation for the vertical displacement of a point within a semi-infinite mass, as described by Poulos and Davis (1968) in relation to the analysis of a floating pile.

Referring to Fig.1b, the displacement at i due to consolidation of the soil is S_i, and therefore, the net downward displacement $_s\rho_i$ of the soil at i is

$$_s\rho_i = S_i - \frac{d}{E_s}\sum_{j=1}^{n} p_j(I_{ij} - I'_{ij}) \quad \ldots (2)$$

In calculating the displacement of the pile itself, only pure axial compression is considered. Referring to Fig.1c, the displacement $_p\rho_i$ of the pile at i can be expressed as

$$_p\rho_i = \frac{1}{E_p R_A}\sum_{j=1}^{n} p_j \,.\, D_{ij} \quad \ldots\ \ldots (3)$$

where $R_A = \frac{A_p}{\pi d^2/4}$

= the area ratio (for the special case of a solid pile, $R_A = 1$).

A_p = area of pile section

$D_{ij} = \frac{4\delta h_i}{d}$ for $i \leq j$ or

$\frac{4\delta h_j}{d}$ for $i < j$

E_p = Young's modulus of pile material

$\delta = L/n$

h_i and h_j are defined in Fig.1c.

If no local yield occurs between the pile and the soil, the displacements of the pile and soil at any point are equal, and thus from equations (2) and (3),

$$\sum_{j=1}^{n} p_j \,.\, d \,.\, \left(\frac{D_{ij}}{Kd} + I_{ij} - I'_{ij}\right) = E_s \,.\, S_i \quad \ldots\ \ldots\ \ldots\ \ldots (4)$$

where $K = \frac{E_p}{E_s} \,.\, R_A \quad \ldots\ \ldots\ \ldots\ \ldots (5)$

= the pile stiffness factor.

The equations for displacement compatibility at all n elements of the pile may therefore be written in matrix form as follows:

$$d[\frac{D}{Kd} + I - I'] . [p] = E_s . [S] \quad .. .. (6)$$

Equation (6) may be solved to obtain the unknown shear stresses p_j on the pile while conditions remain purely elastic. The load per unit area p_b transferred to the base of the pile may then be obtained by considering the equilibrium of the pile viz:

$$p_b = \frac{4P}{\pi d^2} - \frac{4\delta}{d} . \sum_{j=1}^{n} p_j \quad .. .. (7)$$

The above analysis may be modified to take account of local yield between the pile and the soil in a similar manner to that described by D'Appolonia and Romualdi (1963) in relation to end-bearing piles without negarive friction. When the shear stress on an element reaches the maximum adhesion τ_a between the pile and the soil for that element, local yield occurs and compatibility of pile and soil displacements at that element no longer holds. The surface settlement required to cause local yield of the first element may be calculated. Any further settlement results in redistribution of stresses between the remaining n - 1 elastic elements. This stage is analysed by considering the compatibility of pile and soil displacements at these elements, and setting the stress on the yielded element to the value of maximum adhesion τ_a at its centre. The resulting n - 1 equations are solved to give the distribution of shear stress on the pile and the surface settlement at which local yield of the next element occurs. The above procedure is repeated until all elements of the pile have yielded. At this stage, the ultimate downdrag force P_u in the pile is reached. Any further settlement of the soil does not result in any increase in downdrag force on the pile. The value of P_u at any depth z is given by

$$P_u = \pi d \int_0^z \tau_a . dz \quad .. .. (8)$$

It will be seen from equation (8) that the maximum value of P_u occurs at the toe of the pile ($z = L$).

Although, as discussed previously by Poulos and Davis (1968), this procedure is only approximate, it does nevertheless enable a more complete estimate to be made of the relationship between downdrag force and settlement than does a purely elastic analysis.

In obtaining the subsequent solutions described in this paper, the pile has been divided into ten elements. This number has been found to give results of satisfactory accuracy by Poulos and Davis (1968).

ELASTIC SOLUTIONS FOR DOWNDRAG FORCE IN THE PILE

For purely elastic conditions in the soil, it has been found that the maximum downdrag force occurs at the toe of the pile. In Fig. 2 the influence factor I_N for this maximum downdrag force P_N is plotted against K for various values of L/d and for $\nu_s = 0$ and 0.5. The actual downdrag force P_N is related to the influence factor I_N as follows:

$$P_N = I_N . E_s . S_o . L \quad .. .. .. (9)$$

where I_N is the downdrag influence factor

E_s is the Young's modulus of the soil

S_o is the surface settlement of the soil

L is the length of the pile.

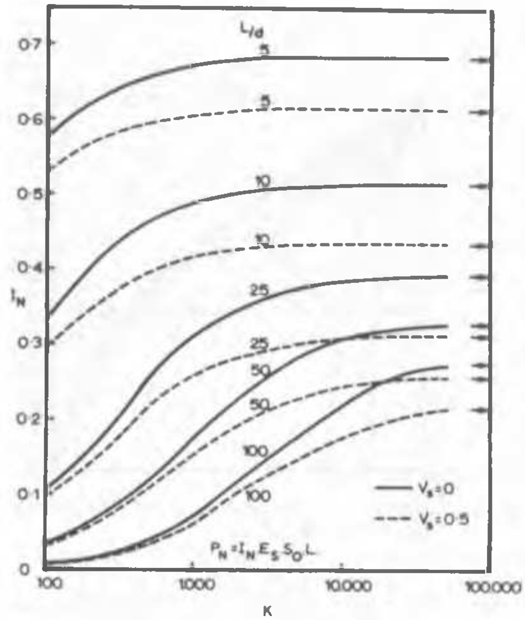


Fig.2. Influence factors for downdrag at toe of pile

From Fig.2, it will be seen that L/d has a major influence on P_N. For a pile of given length L, P_N increases as L/d decreases. This might be expected since a decrease in L/d will result in an increase in the surface contact area between the pile and soil. The other factor which has a major influence on P_N is the pile stiffness factor K. P_N increases from zero for K = 1 (a pile of stiffness equal to the soil) to a maximum for $K = \infty$ (an incompressible). The rate of increase of P_N with increasing K is largely dependent on the value of L/d. P_N is almost unaffected by K when L/d is small, but for slender piles (e.g. L/d > 25), P_N depends largely on K. It is interesting to note that for L/d = 10, a pile having K = 2,000 behaves as an incompressible pile whereas for L/d = 100, the value of K at which the pile is effectively incompressible is much greater (about 40,000).

Poisson's ratio ν_s of the soil also affects P_N, which increases as ν_s decreases. However, in relation to the effects of L/d and K, the effect of ν_s is generally of minor importance, and thus, in the subsequent solutions presented in this paper, attention is confined to the case

$\nu_s = 0$. The application of these solutions to real soil (for which Poisson's ratio of the soil skeleton is generally greater than zero) will tend to give a slight over estimate of P_N.

The variation of the downdrag for P with depth, from zero at the surface to P_N at the toe, is shown in Fig.3 for the case L/d = 25.

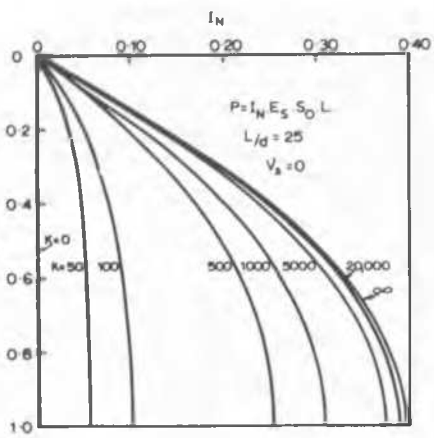


Fig.3. Elastic Distribution of Downdrag force along pile

The influence of K on P is clearly shown. For very compressible piles (K = 50), P is virtually constant over the lower half of the pile, reflecting the very small shear stresses developed over this portion of the pile. The curves in Fig.3 may be used to estimate the butt movement of the pile, but it must be borne in mind that tip movements are not taken into account in this analysis.

SOLUTIONS TAKING INTO ACCOUNT LOCAL YIELD BETWEEN PILE AND SOIL

In a practical problem, the maximum adhesion τ_a between the pile and soil at any point may be conveniently estimated from the Coulomb expression (Thurman and D'Appolonia, 1965). However, in the present paper, attention is confined to the following idealized distributions of τ_a which may be relevant to field problems:

(i) constant τ_a with depth

(ii) linear variation of τ_a with depth, from $\tau_a = 0$ at the surface to a maximum value at the base.

The consideration of case (ii) is possibly inconsistent in that a varying value of τ_a with depth implies a similar variation in soil modulus E_s, whereas E_s is assumed to remain constant in the analysis. However, in the absence of a more satisfactory elastic displacement theory in which account can be taken of varying E_s, it appears justifiable to ignore this inconsistency.

Solutions are shown in Fig.4 for L/d = 25 and K = 10,000 and for the two cases considered.

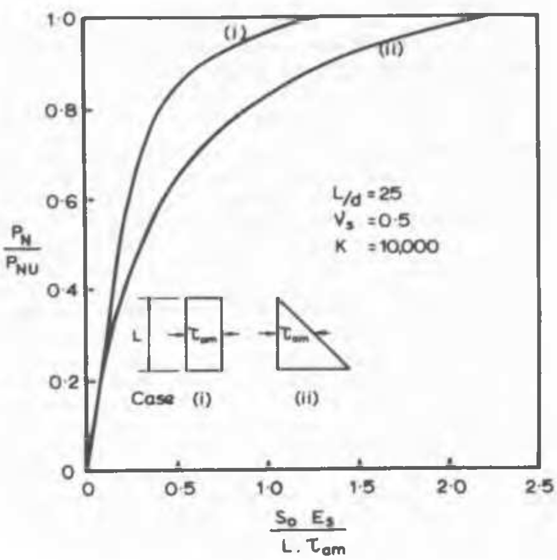


Fig.4. Influence of distribution of τ_a on downdrag force-settlement relation ship

P_N is expressed in dimensionless form as $P_N P_{Nu}$ where P_{Nu} is the ultimate downdrag force developed at the toe of the pile (given by equation (8) with z = L). The surface settlement of the soil is also expressed in dimensionless form as S_o/L . E_s/τ_{am} where τ_{am} is the mean soil adhesion along the pile. The value of τ_{am} has been taken to be equal for both cases so that P_{Nu} is the same. The relationship between P_N and S_o is influenced considerably by the τ_a distribution. For the triangular distribution (case (ii)) local yield occurs near the top of the pile at very small settlements, whereas for the uniform distribution (case (i)), local yield commences at much larger settlements. Consequently the downdrag force for a given surface settlement is generally greater for case (i) than for case (ii).

The distribution of downdrag force P with depth for cases (i) and (ii), is shown in Figs.5a and 5b for various ratios of the downdrag force at the toe to the ultimate value, P_N/P_{Nu}. The limiting distribution for $P_N P_{Nu} = 1$ is linear for case (i) and parabolic with depth for case (ii). Measurements made by Johannessen and Bjerrum (1965) have revealed a distribution of P with depth similar in shape to the limiting curve for case (ii).

The surface settlement S_{om} required to cause local yield along the whole length of the pile (and thus to mobilize P_{Nu}) is shown in Fig.6. As indicated in Fig.4, a larger value of S_{om} is required in case (ii) than in case (i). Also, as a pile becomes more compressible S_{om} increases.

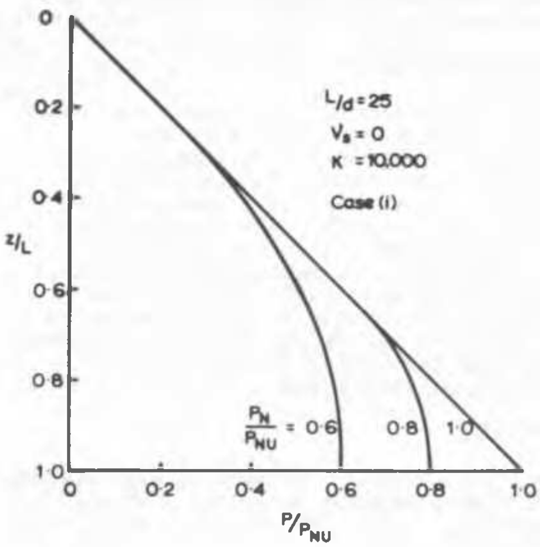


Fig.5a. Distribution of Downdrag force along pile - uniform τ_a distribution

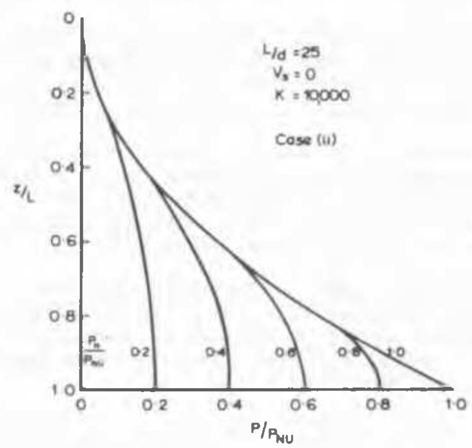


Fig.5b. Distribution of downdrag force along pile - triangular τ_a distribution

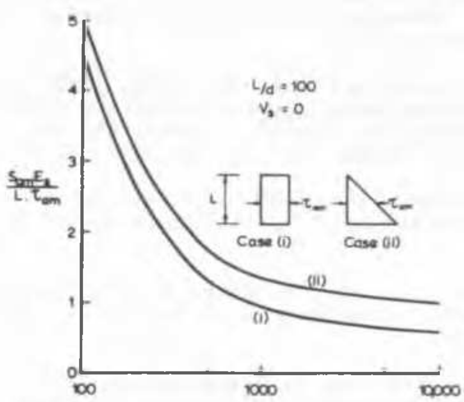


Fig.6. Surface settlement required to mobilise maximum downdrag force

For example, for K = 100, S_{om} is about five times that for K = 10,000.

The influence of L/d on the P_N - S_o relationship is shown in Fig.7 for case (ii) and for K = 10,000.

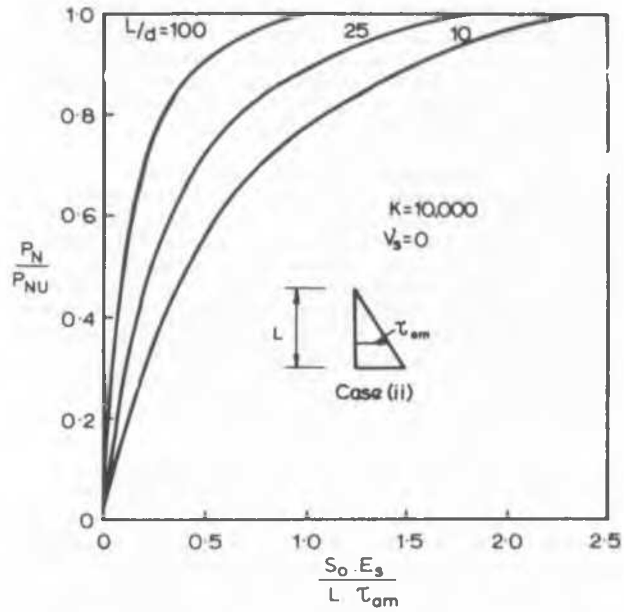


Fig.7. Influence of L/d on downdrag force-settlement relationship

L/d has little influence on the general shape of the P_N - S_o curve, but the settlement required to mobilize a given proportion of the ultimate downdrag force decreases as L/d increases. The effect of L/d for case (i) is similar to that shown in Fig.7 for case (ii), so that the curves in Fig.7 may be used as a basis for correcting Figs.4 and 6 for values of L/d other than 25.

APPLICATION TO PRACTICAL PROBLEMS

Relationships between P_N and S_o such as those shown in Figs.2, 4 and 7 enable the total final downdrag on the toe of the pile to be determined. For this determination values of S_o, E_s and ν_s of the soil at the end of consolidation are appropriate together with the final magnitude and distribution of τ_a.

The value of τ_a may conveniently be obtained from the Coulomb expression

$$\tau_a = c_a + K_o \cdot \sigma_v \cdot \tan\phi_a \quad .. \quad .. \quad (10)$$

where c_a = soil-pile adhesion

K_o = coefficient of earth pressure at rest

σ_v = overburden pressure

ϕ_a = angle of friction between pile and soil.

A summary of typical values of K_o, c_a and ϕ_a is given by Broms (1966). For relatively deep layers of soft normally consolidated clay, equation (10) will yield a distribution of τ_a approaching case (ii). For stiffer clays, relatively thin layers of soft clay or layers which are heavily surcharged, the distribution of τ_a at the end of consolidation is likely to approach the uniform distribution of case (i).

It should be borne in mind that the influence factors presented in this paper will give an upper limit to the downdrag force in practical problems, since the base underlying the soil layer will generally have a finite compressibility. A preliminary analysis indicates, however, that the compressibility of the bearing stratus will have little influence on pile behavious unless the pile is relatively short (e.g. L/d ≤ 15) or the ratio of base to soil modulus is less than 100.

Although pile groups are not considered specifically in this paper, the solutions for a single pile may be used to estimate the average downdrag force on a pile in the group by considering the group as an equivalent single pile of circumference equal to that around the outer piles of the group. The soil adhesion τ_a can be taken as equal to the shear strength of the soil. In such an approximate analysis, no account can be taken of the non-uniform distribution of downdrag force likely to exist within the group. A more refined analysis of downdrag in pile groups, similar to that employed by Poulos (1968) for floating pile groups, is at present being carried out.

CONCLUSIONS

Theoretical relationships have been obtained between the downdrag force P_N developed at the toe of an end-bearing pile in a consolidating soil layer and the surface settlement S_o of the layer. The effects of various factors on these relationships have been examined and the conclusions reached may be summarised as follows:

i) For a pile of given length, P_N decreases as L/d increases. The soil settlement S_{om} required to mobilize the ultimate downdrag force P_{Nu} also decreases as L/d increases.

ii) As the pile stiffness factor K decreases, P_N decreases while S_{om} tends to increase.

iii) The distribution of soil adhesion τ_a with depth has a considerable influence on the relationship between P_N and S_o. For a given settlement and the same average value of τ_a along the pile, P_N is greater when τ_a is uniform than when the distribution of τ_a is triangular, and consequently S_{om} is less for a uniform τ_a.

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