OPTIMIZATION OF FOOTINGS IN COMPRESSIBLE SOILS

DIMENSIONNEMENT OPTIMUM DES SEMELLES EN TERRAINS COMPRESSIBLES

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SYNOPSIS. An approach for the optimization of foundations, in the case of footings underlain by soils subject to consolidation settlements, is described in this paper. The total expected building cost as a function of foundation cost, consists of the sum of initial cost plus the maintenance cost which can be attributed to settlements in excess of those prescribed by the designer. The curve relating this total expected building cost with the probability of exceeding a maximum total allowable settlement of a footing (probability which depends on the size of the footing) has a minimum cost point, if the ratio of maintenance cost to initial cost exceeds a certain small value. The approach suggested in this paper permits the engineer to take into account the variability of natural soils in the design of footings with regard to settlements due to consolidation, and use this variability to his advantage.

INTRODUCTION

It is usual in practice to design the size of footings on the basis of two major criteria, so that the imposed unit stresses will not 1) exceed the bearing capacity of the underlying soil, and 2) result in detrimental total or differential settlements.

Both criteria are evaluated on the basis of equations which involve the use of laboratory or field tests of different soil properties, and the computation of the stresses in the underlying soils.

The results of laboratory or field tests of soil properties constitute statistical populations, not only because of errors inherent in testing methods and equipment, as well as human errors, but mainly because all soil properties in nature have statistical distributions (Lumb, 1966).

The computed results of the stresses imposed by the footing on the underlying soils are of a deterministic nature. However, the difference between the computed values and the real stresses, again results in a statistical population, due to the inadequacy of the existing theories of stress distribution in soils.

At present, the engineer is subconsciously aware of these facts, but he cannot take them into account when designing the size of a footing. The so-called judgment, consists precisely in applying those values of laboratory or field test results, which conform best to the past experience of the engineer. The range of values applied goes anywhere from average to extreme values, for each one of the factors which enter into consolidation settlement computations.

The purpose of this paper is to suggest and present schematically an approach by means of which the engineer can design the size of footings, so that the expected total building cost as a function of foundation cost, will be a minimum.

This approach permits the engineer to place his settlement computations and the resultant design, within the the proper framework of the variability and statistical nature of soil properties.

DESCRIPTION OF THE SUGGESTED APPROACH

If the probability distributions of the factors which enter in the consolidation settlement equation are known, the marginal probability distribution of the computed settlement can sometimes be derived mathematically (for example see, Wadsworth and Bryan, 1960, Chapter 6).

In most cases this derivation is too complicated, especially when two or more of the factors are not independent, as occurs for example in the case of an "infinitely" thick clay layer below the footing, in which case the depth of the foundation's bulb of influence depends on the size of the foundation. Where the mathematical derivation is unattainable, numerical methods or simulation techniques together with probability distribution fitting methods with a high level of statistical significance, have shown promise.

The conditional probability distribution of the sizes of a footing (on the basis of the probability of exceeding a given maximun allowable total or differential settlement), can be obtained from the marginal distribution of the computed settlement. The determination of this conditional distribution for a maximum allowable differential settlement, requires obtaining the distribution of the difference between the computed settlements for two or more footings.

If the probability distributions of the factors which enter the time-settlement computations are known, it is also possible to determine the probability of exceeding a given maximum allowable total or differential settlement within an interval of time.

The distribution of the probability of a computed settlement, and of the computed settlement within a given amount of time, is being investigated at present for various soil conditions, for different types of probability distributions of each one of the factors included in the respective equation, and for different conditions of dependence and correlation between two or more of these factors. The results of this investigation will be made available when completed.

The total expected cost of a building consists of the sum of the initial cost plus the maintenance cost during the economic life of the building. The total expected building cost as a function of foundation cost, consists of the sum of the initial cost, plus only those items of maintenance cost which are imputable to excessive settlement of the foundation. The maintenance cost items which can be attributed to excessive settlement of the foundation, are:

a. Underpinning of the footings where excessive settlement has occurred.
b. Repair or replacement of those parts of the structure which have been damaged due to excessive settlement of footings.
c. Repair or replacement of pipes, conduits, or other interior and exterior installations, damaged due to excessive settlements.
d. Repair or replacement of exteriors and walls, damaged due to excessive settlements.
e. Repair or replacement of connecting elements, systems, or structures, or of equipment with levelling tolerances, mainly in industrial or commercial buildings.
f. Others.

The maintenance costs due to excessive foundation settlement, plus the loss of earnings during maintenance and repairs, can be predicted with a certain degree of accuracy for a given building.

In the case of a foundation consisting of footings, the total expected building cost as a function of the foundation cost, is given for each footing by the following expression:

$$C_{Tj} = A_j c_{Bj} + C_{Fj} + (C_{Mj}) \cdot p_j \quad \ldots\ldots\ldots\ldots (1)$$

where:

C_{Tj} = Total expected cost of the building as a function of the foundation cost, for the area of influence of column "j".
A_j = Area of influence of column "j".
c_{Bj} = Initial cost per area of the building, excluding cost of foundation.
C_{Fj} = Initial cost of the footing of column "j" (a function of the size of the footing).
C_{Mj} = Maintenance cost imputable to excessive settlement of the footing of column "j".
p_j = Conditional probability (from 0 to 1) of exceeding a given maximum total settlement for footing "j".

Equation 1 can be simplified by the use of the ratio R_j:

$$R_j = \frac{C_{Mj}}{A_j c_{Bj} + C_{Fj}}$$

Substitution in Equation 1, gives the following:

$$C_{Tj} = A_j c_{Bj} + C_{Fj} + Rj\,(A_j c_{Bj} + C_{Fj}) \cdot p_j$$

Which becomes:

$$C_{Tj} = (A_j c_{Bj} + C_{Fj})(1 + R_j \cdot p_j)$$

And finally:

$$C_{Tj} = C_{Oj} \cdot (1 + R_j \cdot p_j) \quad \ldots\ldots\ldots\ldots (2)$$

where:

C_{Oj} = Initial building cost for the area of influence of column "j", including footing cost.
R_j = Ratio of maintenance cost directly attributable to excessive settlement to initial building cost, for the area of influence of column "j". This ratio has a unique value only at the point where the allowable settlement is exceeded, if it is assumed that no maintenance cost will be incurred until this point is reached.

The total expected cost "C_T" of the whole building, as a function of foundation cost, can be obtained from equation 2, thus:

$$C_T = \sum_{j=1}^{n} C_{Tj} \quad \ldots\ldots\ldots\ldots (3)$$

If C_{Tj} is a minimum for each footing, then the total expected building cost will also be a minimum.

ANALYSIS OF A SQUARE FOOTING

As an example of some of the ideas discussed in this paper, the results for the square footing of column C-3 of an apartment building in Bogotá are described in the figures and text which follow.

Except for a thin layer (average thickness of 20 centimeters) of normally consolidated clay which is found 40 centimeters below the base of the foundation, footing C-3 rests on granular material which is assumed as not being subject to consolidation settlements. The footing's base is at a depth of 1.0 meters below ground surface. The relation between consolidation settlements and the size of footing C-3 is shown on figure 1, for its column load of 50 tons.

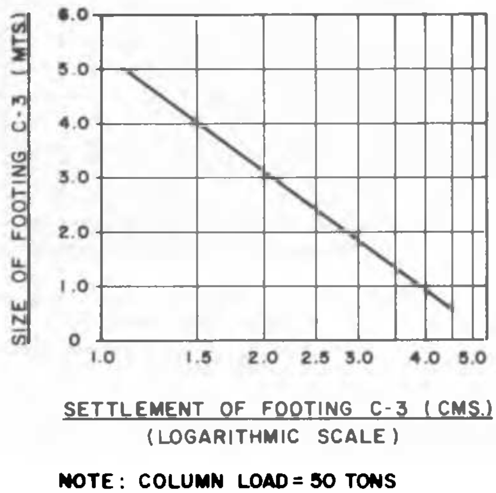

Fig. 1 Size of footing vs. settlement. Footing C-3.

The probability distribution of the computed settlement of footing C-3 fits a normal distribution with a 95% level of statistical significance, and with a standard deviation "Ss" of 0.5 centimeters. The conditional probability of exceeding a maximum total allowable settlement of 2.5 centimeters of the size of footing C-3, fits a log-normal distribution with the same level of significance, and is shown on figure 2.

For footing C-3, the estimated ratio "R" of maintenance to initial cost is 0.1. Figure 3 shows the curves for initial cost and for total expected cost as a function of the conditional probability of exceeding a maximum total allowable settlement of 2.5 centimeters. The initial cost curve decreases continuously as the probability of exceeding the allowable settlement increases. On the other hand, even for a small value of the ratio "R", the total expected cost curve has a minimum cost point. The probability corresponding to this minimum cost point is the optimum design probability "Popt.", and its value for footing C-3 is 5%.

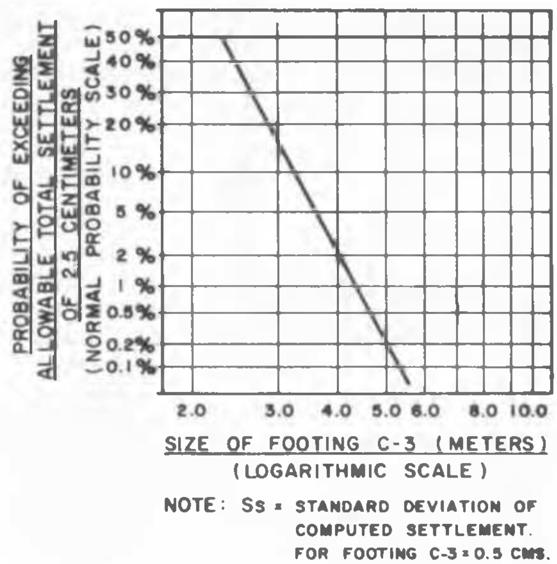

Fig. 2 Conditional distribution of the probability of exceeding an allowable total settlement of 2.5 centimeters, of the size of footing C - 3.

The optimum size for the square footing C-3, is obtained from figure 2 on the curve representing the conditional distribution of the probability of exceeding an allowable total settlement of 2.5 centimeters, of the size of footing C-3. This optimum size is 3.60 x 3.60 meters.

It is of interest to explore a bit further the influence of changes in the variance (or standard deviation) of the distribution of the computed settlements and in the ratio "R" of maintenance to initial building cost, on the optimum design probability "Popt." and on the total expected building cost. Figure 4 shows the results for footing C-3.

For a given ratio "R", the distribution with a higher variance, implies a higher optimum design probability as well as a higher optimum cost, and hence a larger footing size.

As is to be expected, for a given variance, an increase in the ratio "R" diminishes the optimum design probability and increases the optimum cost. In other words, as the ratio of maintenance to initial building cost increases, there is a decrease in the risk one can afford of exceeding a given maximum total allowable settlement.

If the variance of the computed settlement is small, the optimum total expected cost is only slightly sensitive to changes in the ratio "R", which implies that a very accurate estimate of maintenance cost is not required for soils of low variability.

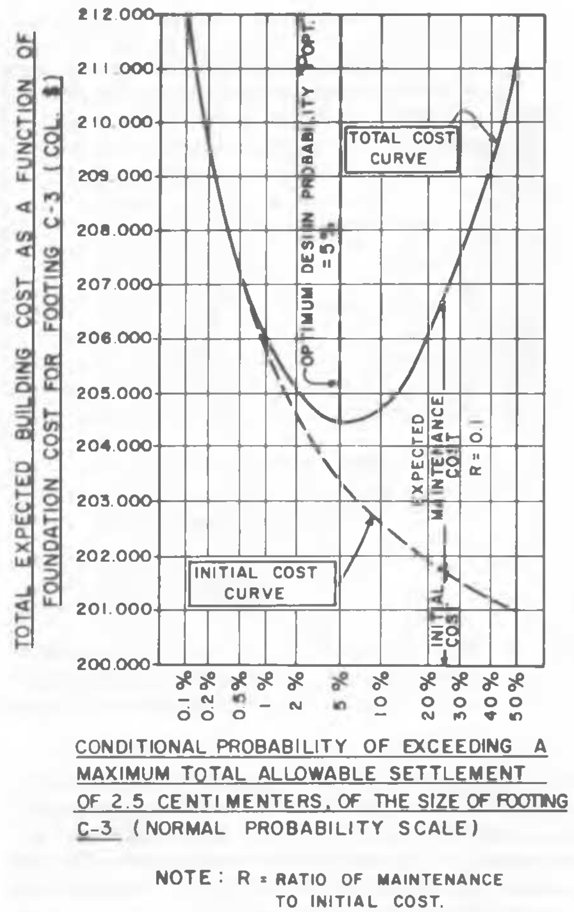

Fig. 3 Relationship between the total expected building cost as a function of foundation cost, and the conditional probability of exceeding a maximum total allowable settlement of 2.5 centimeters, of the size of footing C-3.

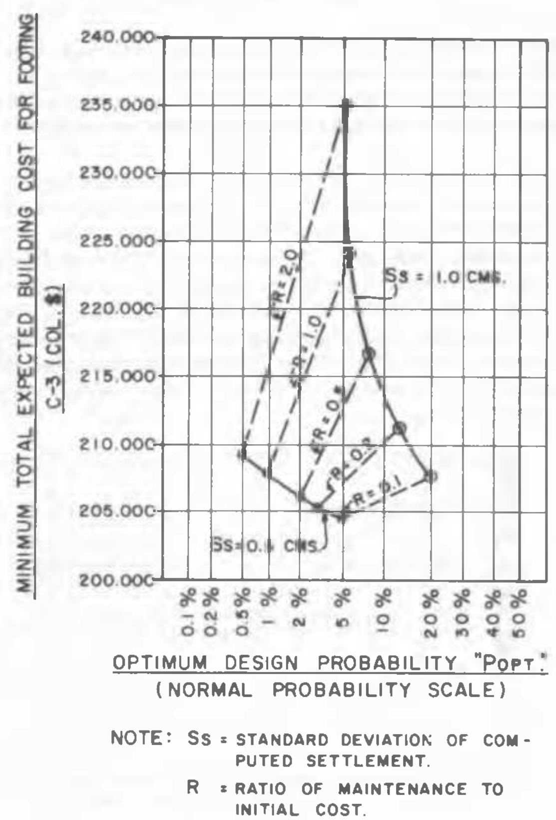

Fig. 4 Relationship between the minimum total expected building cost for footing C-3, and the optimum design probability, for different values of Ss and R.

The results shown on figure 4, should hold for all types of probability distributions of the computed settlement.

SUMMARY OF THE SUGGESTED PROCEDURE

To optimize the cost of a foundation with regard to consolidation settlement, the approach suggested in the previous discussion can be summarized in the following steps:

1. The marginal probability distribution of the computed settlement, and the conditional distribution of the probability of exceeding a maximum allowable total or differential settlement of the size of a given footing, are obtained either mathematically or by approximate methods.
2. The total expected building cost for each footing is then determined as a function of foundation cost, for the conditional probability of exceeding the given maximum allowable total or differential settlement. The total expected building cost curve for each footing, will have a minimum cost point, if the ratio of maintenance cost (only that imputable to excessive settlement) to initial building cost exceeds a certain small value. The probability corresponding to this minimum total expected cost point, is the optimum design probability.
3. The optimum (or minimum cost) size of the footing is obtained from the size corresponding to the optimum design probability, on the curve giving the conditional distribution of the probability of exceeding the given maximum allowable total or differential settlement, of the size of a given footing.
4. The total expected cost of the whole building as a function of foundation cost, is minimized by repeating the previous steps for each footing.

The accuracy required in the estimate of maintenance cost attributable to excessive settlement, depends on the variance of the marginal distribution of the probability of the computed settlement. For soils of low variability, a great accuracy of this estimate is not required.

CONCLUSIONS

On the basis of the foregoing discussion, the following conclusions are pertinent:

-. It is possible to minimize the total expected cost of a building as a function of foundation cost, on the basis of the probability of exceeding a maximum total or differential settlement, along the lines of the approach suggested in this paper.

-. The approach suggested in this paper gives the engineer the possibility of basing his judgement on a more tangible and practical feature of the design of footings for consolidation settlement criteria, as is the optimization from an economic point of view of his design. It also permits him to place this design within the proper perspective of the variability and statistical nature of soil properties.

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