THREE-DIMENSIONAL STABILITY OF FILL DAMS

STABILITE A TROIS DIMENSIONS DES BARRAGES EN TERRE

P. ANAGNOSTI, Dr. Sc.
Chief Engineer, Soil Mechanics Geotechnical Section,
Energoprojekt, Consulting & Engineering, Beograd, Yugoslavia.

SYNOPSIS The method of stability analysis of the threedimensional potential sliding mass having different sliding surface shapes is presented. The potential sliding mass is considered to consist of thin vertical slices and limit equilibrium is assumed on sliding sides of each slice. Different distributions of forces acting on sliding and lateral sides of slices can be examined and corresponding safety factors computed. Threedimensional equilibrium equations are used to determine unknown forces, their lever arms and factor of safety. Examination of physical meaning of possible mathematical solutions is the constitutive part of the analysis. The method is to be applied in cases when plane strain conditions are not warranted. Usefullness of the method is emphasized when seismic forces have to be considered. The method has been programmed for a digital computer and some examples of its application are presented.

INTRODUCTION

The presented paper is concerned to the stability analysis of earth and rock-fill dams with the basic assumption that a potential threedimensional sliding mass is formed in the dam body. Such an analysis is justified in all cases where plane-strain conditions are not warranted, i.e. for dams placed in more or less narrow valleys or canyons. However, the most of high dams are usually located in such places.

The presented method of stability analysis is based on recently published works in connection with the analyses of the equilibrium of the potential sliding mass in plain-strain conditions /see Ref. (2) and (3)/. The current analyses of threedimensional stability are limited to that cases where shape of sliding surface is plane or a special geometrical surface of revolution /see Ref. (1), (4), (5)/. The possibilities as regards the selection of shapes for trial sliding surface are considerably broadened by the presented method. The all shapes of sliding mass approximated by thin vertical slices having two plane sliding sides can be now analysed. The particular advantage of the method of thin vertical slices, which is fully conserved for the case of threedimensional sliding mass, is the possibility to investigate different distributions of internal forces acting on interslice surfaces, and resistive forces acting on the sliding surfaces. The possibility to select different shapes of sliding surfaces allows the investigation of the most unfavourable shapes of potential sliding mass. It may be also stressed that the presented method is based on all threedimensional equilibrium conditions, that was not the case for other mentioned methods /see Ref. (1) and (5)/.

DEVELOPMENT OF THE METHOD

The stability of the potential sliding mass is analysed by setting equilibrium equations for the series of thin vertical slices assuming limit equilibrium conditions on sliding sides of each slice. It has been proved that the method of thin slices /Ref. (2)/, provides the satisfactory base in current design practice in cases where plane-strain conditions are valid. The background of the threedimensional analysis will be briefly explained herebelow starting from the plane-strain case.

The potential sliding mass in plane-strain conditions is assumed to consist of "k" thin vertical slices, and the following unknowns are introduced into the analysis:

- interslice forces : P_n
  ($n=1,2,\dots,k-1.$) i.e. ($k-1$) forces
- lever arms : r_n
  ($n=1,2,\dots,k-1.$) i.e. ($k-1$) arms
- inclination angles : β_n
  ($n=1,2,\dots,k-1.$) i.e. ($k-1$) angles
- base normal forces : N_n
  ($n=1,2,\dots,k-1,k.$) i.e. ($k-1$) forces
- base shear forces : S_n
  ($n=1,2,\dots,k-1,k.$) i.e. (k) forces
- factor of safety : F_s
  i.e. one
- constant of assumed distribution of inclination angles : A
  i.e. one

Total number of unknowns: $5k-1$

The known forces are represented by the components: X_n, Y_n, M_n which depend on the weights of slices, surface loads, inertial forces caused by earthquake actions, etc. Pore pressure forces could be easily put in the equations as known forces.

The available equations are:
Equilibrium condition
in x-direction $\Sigma X_n = 0$ i.e. (k) equations
Equilibrium condition
in y-direction $\Sigma X_n = 0$ i.e. (k) equations
Moment equilibrium condition
$\Sigma M_n = 0$ i.e. (k) equations
Limit equilibrium condition on the base of each slice:
$F_s S_n = N_n tg\Phi'_n + C'_n$
(n=1,2,...,k) i.e. (k) equations
Assumed distribution of angles:
$A\sin\Psi_n = \sin\beta_n$
(n=1,2,...,k-1.) i.e. (k-1) equations

Total number of equations: 5k-1

It can be seen that the number of equations is equal to the number of unknowns.

The full set of equations for plane strain conditions is:

$\Sigma X = 0$:

$$P_{n-1}\cos\beta_{n-1} - P_n\cos\beta_n + N_n\sin\alpha_n - S_n\cos\alpha_n + X_n = 0 \quad \ldots(1)$$

$\Sigma Y = 0$:

$$P_{n-1}\sin\beta_{n-1} - P_n\sin\beta_n - N_n\cos\alpha_n - S_n\sin\alpha_n + Y_n = 0 \quad \ldots(2)$$

$\Sigma M = 0$:

$$P_{n-1}(r_{n-1} + \frac{1}{2}\, l_n tg\alpha_n + \frac{1}{2}\, l_{n-1} tg\alpha_{n-1})\cos\beta_{n-1} -$$

$$-P_n r_n \cos\beta_n - P_{n-1}\, \frac{1}{2}\, l_n \sin\beta_{n-1} -$$

$$-P_n\, \frac{1}{2}\, l_n \sin\beta_n + M_n = 0 \quad \ldots(3)$$

$$F_s S_n = N_n tg\Phi'_n + C'_n \quad \ldots(4)$$

$$A\sin\Psi_n = \sin\beta_n \quad \ldots(5)$$

where: $P_{n-1} = 0$ and $r_{n-1} = 0$ for $n = 1$
$P_n = 0$ and $r_n = 0$ for $n = k$
because all these forces have to be included in X_1, Y_1, M_1 and X_k, Y_k, M_k respectively.

In equations (1) to (5) are:
l_n is the thickness of the n^{th} slice
α_n is angle between base of the slice and horizontal line
C'_n is cohesion - force i.e. $C'_n = c'_n \frac{l_n}{\cos\alpha_n}$ ...(6)
Φ'_n is angle of internal friction in terms of effective stress
c'_n is cohesion intercept in terms of effective stress.

All forces and their positions are shown on the Fig. 1

The described method of stability analysis for plane - strain conditions is in essence similar to that published by Morgenstern and Price /see Ref. (2)/. The departure is made as regards the position of base force N_n which is assumed to act in the center point of the base intercept. This departure actually postulates the smooth change of N_n forces along the sliding line.

The development of the proposed method in threedimensional conditions has been done in the following way:

The sliding mass is assumed to consist of "k" thin vertical slices, and the following unknowns are introduced in the analysis:

- interslice force components: P_{xn}, P_{yn}, P_{zn}
  (n=1,2,...,k-1) i.e. (3k-3) equations
- lever arm of interslice force: R_n
  (n=1,2,...,k-1) i.e. (k-1) arms
- angle between lever arm R_n and Z axis measured in the center plane of the slice: ρ_n
  (n=1,2,...,k-1) i.e. (k-1) angles
- Normal forces on sliding sides: N_{bn}, N_{sn}
  (n=1,2,...,k) i.e. (2k) forces.
- Shear forces on sliding sides: S_{bn}, S_{sn}
  (n=1,2,...,k) i.e. (2k) forces
- lever arms of forces acting on sliding sides: r_{bn}, r_{sn}
  (n=1,2,...,k) i.e. (2k) arms
- angles between forces S_{bn}, S_{sn} and center lines of the slice sides: Θ_{bn}, Θ_{sn}
  (n=1,2,...,k) i.e. (2k) angles
- factor of safety: F_s
  one
- constants of distribution of interslice force components P_{yn} and P_{zn}: A_y, A_z
  two
- constants of distribution of directions of shear forces on the sliding sides: b, s
  two
- constant of distribution of directions of lateral interslice force arm: A_ρ
  one

Total number of unknowns is: 13k + 1

The known forces are represented by components: X_n, Y_n, Z_n, M_{xn}, M_{yn}, M_{zn}; these forces depend on the weights of slices, surface loads, inertia forces, etc. also pore pressure forces can be easily put in the equations as external loads.

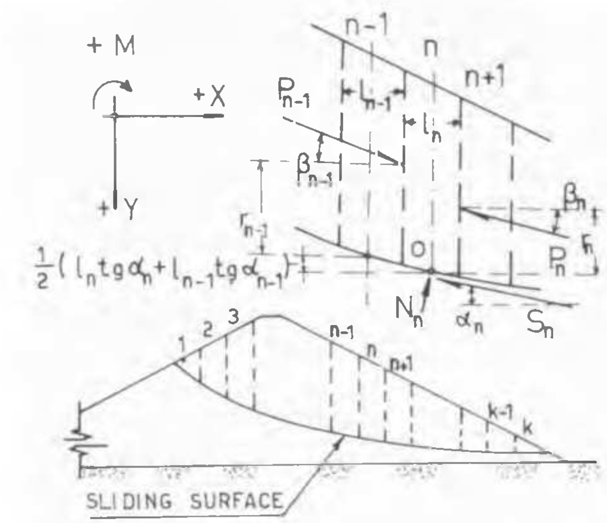

Fig. 1

The available equations are:
Equilibrium equations:
$\Sigma X = 0 \quad \Sigma Y = 0 \quad \Sigma Z = 0 \quad \Sigma M_x = 0 \quad \Sigma M_y = 0 \quad \Sigma M_z = 0$
(n=1,2,...,k) i.e. (6k) equations
Limit equilibrium conditions on sliding sides:
$F_s S_{bn} = N_{bn} tg\Phi'_{bn} + C'_{bn}, \quad F_s S_{sn} = N_{sn} tg\Phi'_{sn} + C'_{sn}$
(n=1,2,...,k) i.e. (2k) equations
Distribution equations:
$\cos\Theta_{bn} = A_b \cos\Psi'_{bn}$
(n=1,2,...,k) i.e. (2k) equations
$\cos\Theta_{sn} = A_s \cos\Psi_{sn}$
(n=1,2,...,k)
$\cos\rho_n = A_\rho \cos\Psi_{\rho n}$
(n=1,2,...,k-1) i.e. (k-1) equations
$P_{yn} = A_y P_n tg\Psi_{yn}$
(n=1,2,...,k-1) i.e. (k-1) equations
$P_{zn} = A_z P_n tg\Psi_{zn}$
(n=1,2,...,k-1) i.e. (k-1) equations

Total number of available equations is : 13k-3

To provide the equal number of unknowns to the number of available equations one shall select four suitable unknowns as the known constants. It has been found that the most suitable procedure for solving this problem is the selection of: $r_{s1}, r_{b1}, r_{sk}, r_{bk}$, or R_1 and R_{k-1} instead of r_{s1} or r_{b1} and r_{sk} or r_{bk} respectively, as the known constants. In fact this selection assure the position of exit points of base or interslice forces on the first and last slice. It has been also proved that this procedure has negligible effect to the value of the safety factor. The full set of equations for threedimensional conditions of sliding is:

$\Sigma X = 0$:
$$\frac{tg\Phi_n}{F_s}(N_{bn}\cos\alpha_{bn} + N_{sn}\cos\alpha_{sn}) + N_{bn}\cos\alpha_{1n} + N_{sn}\cos\alpha_{2n} - P_n + P_{n-1} + X_n = 0 \quad \ldots(7)$$

$\Sigma Y = 0$:
$$\frac{tg\Phi_n}{F_s}(N_{bn}\cos\beta_{bn} + N_{sn}\cos\beta_{sn}) + N_{bn}\cos\beta_{1n} + N_{sn}\cos\beta_{2n} - P_n A_y tg\Psi_{yn} + P_{n-1} A_y tg\Psi_{y\,n-1} + Y_n = 0 \quad \ldots(8)$$

$\Sigma Z = 0$:
$$\frac{tg\Phi_n}{F_s}(N_{bn}\cos\gamma_{bn} + N_{sn}\cos\gamma_{sn}) + N_{bn}\cos\gamma_{1n} + N_{sn}\cos\gamma_{2n} - P_n A_z tg\Psi_{zn} + P_{n-1} A_z tg\Psi_{2n-1} + Z_n = 0 \quad \ldots(9)$$

$\Sigma M_x = 0$:
$$r_{bn} N_{bn}\left[\frac{tg\Phi_n}{F_s}(l_{1n}\cos\beta_{bn} + m_{1n}\cos\gamma_{bn}) + l_{1n}\cos\beta_{1n} + m_{1n}\cos\gamma_{1n}\right] - r_{sn} N_{sn}\left[\frac{tg\Phi_n}{F_s}(l_{2n}\cos\beta_{sn} + m_{2n}\cos\gamma_{sn}) + l_{2n}\cos\beta_{2n} + m_{2n}\cos\gamma_{2n}\right] + A_y\left[P_n tg\Psi_{yn} R_n A_\rho \cos\Psi_{\rho n} - P_{n-1} tg\Psi_{y\,n-1}(R_{n-1} A_\rho \cos\Psi_{\rho\,n-1} + \Delta z_{n\,n-1})\right] - A_z\left[P_n tg\Psi_{zn} R_n \sqrt{1 - A_\rho^2 \cos^2\Psi_{\rho n}} + P_{n-1} tg\Psi_{z\,n-1}\left(R_{n-1}\sqrt{1 - A_\rho^2 \cos^2\Psi_{\rho\,n-1}} + \Delta y_{n\,n-1}\right)\right] + M_{xn} = 0 \quad \ldots(10)$$

$\Sigma M_y = 0$:
$$-r_{bn} N_{bn}\left(\frac{tg\Phi}{F_s} l_{1n}\cos\alpha_{bn} + l_{1n}\cos\alpha_{1n}\right) + r_{sn} N_{sn}\left(\frac{tg\Phi}{F_s} l_{2n}\cos\alpha_{sn} + l_{2n}\cos\alpha_{2n}\right) - P_n\left(R_n A_\rho \cos\Psi_{\rho n} + \frac{\lambda_n}{2} A_z tg\Psi_{zn}\right) + P_{n-1}\left(R_{n-1} A_\rho \cos\Psi_{\rho\,n-1} + \Delta z_{n\,n-1} + \frac{\lambda_n}{2} A_z tg\Psi_{z\,n-1}\right) + M_{yn} = 0 \quad \ldots(11)$$

$\Sigma M_z = 0$:
$$-r_{bn} N_{bn}\left(\frac{tg\Phi}{F_s} m_{1n}\cos\alpha_{bn} + m_{1n}\cos\alpha_{1n}\right) +$$

$$+r_{sn}N_{sn}\left(\frac{tg\Phi}{F_s}m_{2n}\cos\alpha_{sn}+m_{2n}\cos\alpha_{2n}\right)+$$
$$+P_n\left(R_n\sqrt{1-A_\rho^2\cos^2\Psi_{\rho n}}-\frac{\lambda_n}{2}A_y tg\Psi_{yn}\right)-$$
$$-P_{n-1}\left(R_{n-1}\sqrt{1-A_\rho^2\cos^2\Psi_{\rho\ n-1}}+\Delta y_{n\ n-1}-\right.$$
$$\left.-\frac{\lambda_n}{2}A_y tg\Psi_{y\ n-1}\right)+M_{zn}=0 \qquad \ldots(12)$$

where:

$\cos\alpha_{bn}$, $\cos\beta_{bn}$, $\cos\gamma_{bn}$, are the components of the unit vector of force S_{bn}

$\cos\alpha_{sn}$, $\cos\beta_{sn}$, $\cos\gamma_{sn}$, are components of the unit vector of force S_{sn}

$\cos\alpha_{1n}$, $\cos\beta_{1n}$, $\cos\gamma_{1n}$, are the components of the unit vector normal to the "base" sliding side "b"

$\cos\alpha_{2n}$, $\cos\beta_{2n}$, $\cos\gamma_{2n}$, are the components of the unit vector normal to the "side" sliding side "s".

l_1, and m_1 .... are the components of the unit vector of the center line "b-b" of the side "b" in direction of Y and Z axis, respectively.

l_2, and m_2 .... are the components of the unit vector of the center line "s-s" of the side "s" in direction of Y and Z axis, respectively.

$\Delta y_{n,n-1}$ and $\Delta z_{n,n-1}$ ..... are the changes of the coordinates of the local coordinate centers for two adjacent slices.

λ_n .... is the thickness of the slice.

This set of equations in its general form has been programmed on digital computer (Elliott 803) applying the suitable variational method to the constants: F_s, A_b, A_s, A_ρ, A_z, A_y, which are present in all equations.

On the Fig. 2 all assumed forces and their lever arms are shown. It may be noted that only torsional moments on the sliding and interslice sides were not introduced in the analysis.

Considerable saving in computer work can be achieved by taking as the constants the directions of shear forces S_{sn} and S_{bn}. These directions can be assumed as equal to the direction of the intersection line of the slice sides "b" and "s" i.e. to the direction of the sliding of each slice. The result of this approximation is the reduction of unknowns to 11 k - 1 (because the values of Θ_{bn} and Θ_{sn} are fixed now), and the number of available equations is 11k-3. Two constants that shall be introduced in the equations are R_1 and R_{k-1} i.e. the entrance and the exit lever arms of the interslice forces. The solutions of the equations is readily obtained

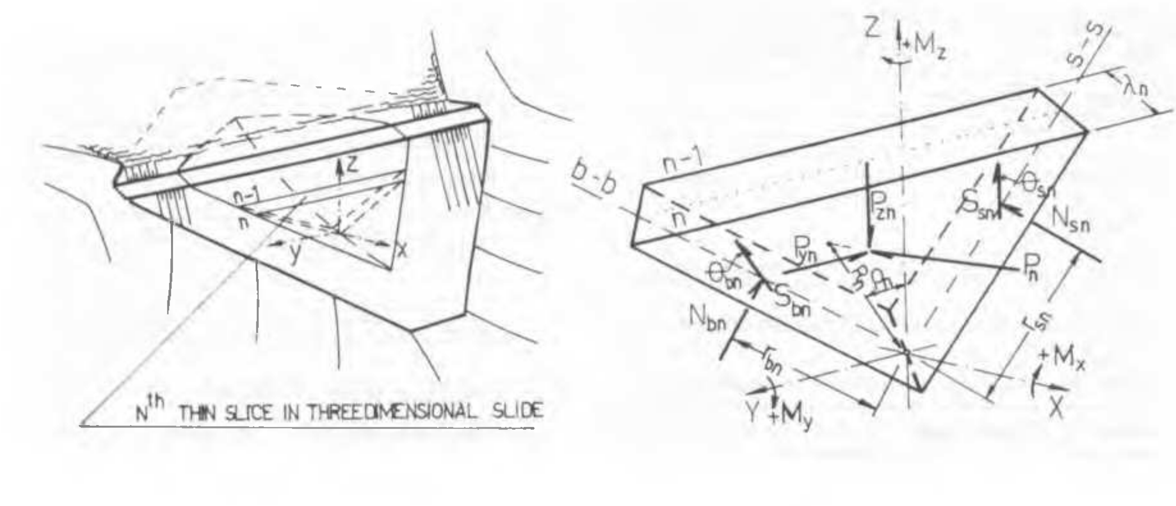

Fig. 2

by the suitable trial end error method applied to the constants A_y, A_z, and F_s.

APPLICATION EXAMPLE

The 106 m high rock-fill dam Tikves in Macedonia has been analysed by the described method. The valley where dam is located has 40 m wide terrace on the bottom and the width at the dam crest level is appr. 340 m. The plane-strain conditions for the stability analysi are not warranted. The dam is situated in earthquake active region of moderate degree, appr. VIII according to MMS scale. The most unfavourable sliding surface for plane - strain conditions in the deepest dam section was determined, and relevant factor of safety computed. Dam section and the determined sliding surface are shown on the Fig. 3. Factor of safety for plane - strain condition was found equal to 1.58, and line of interslice forces remined well within the sliding mass. The computations of three-dimensional stability performed by the described procedure having as constants the values of angles Θ_{bn} and Θ_{sn}, proved that the factor of safety is larger for appr. **50%** compared to the plain-strain case. In case that a strong earthquake motion of the ground is assumed the difference of factors of safety is enlarged.

CONCLUSIONS

A method has been developed for the determination of the factor of safety of a potential sliding mass of different shapes, with varying shear strength parameters. It is based solely upon the equations of limiting equilibrium on sliding sides and satiasfaction of all equations of equilibrium in space. The shape of sliding surfaces must be chosen and the assumptions must be made regarding the distribution of internal forces. The factor of safety appear to be very insensitive to the last assumptions and this is the most valuable feature of the proposed method. The physically admissible solutions have to be selected between other, mathematically possible ones, that might be obtained from the outlined set of the equations. The method is usefull not only for the designs of earth-rockfill dams, but also for the analysee of the stability of natural slopes in earth or rock foundations of other dams. The solution ensures that all equilibrium and boundary conditions are satisfied. Comparison with the current analyses assuming plane-strain conditions reveals that the actual safety factor may be higher for **50%**. Further computations are sertainly required to investigate these differences in more detail. The method can be readily applied either for

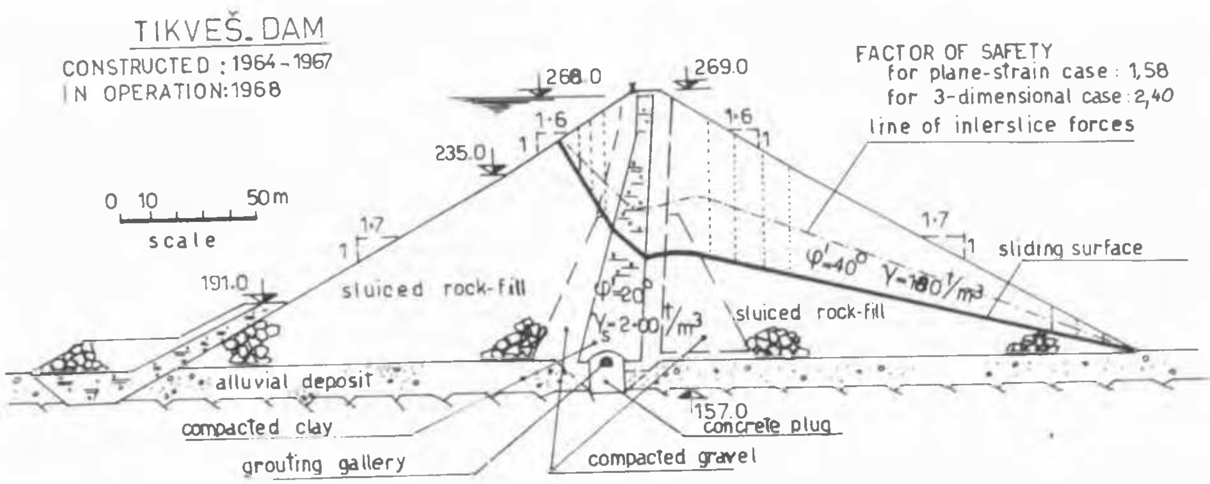

Fig. 3

analysis in terms of effective stress or in terms in total stress. The meaning of factor of safety in the method is the ratio between available shear strength on sliding surface and required shear strength to obtain the equilibrium with external loads. It may be pointed out that earth pressure and bearing capacity can be also considered by the proposed method.

AKNOWLEDGEMENT

The author gratefully aknowledge the assistance of Mr.M.Maksimovic, Civ.Eng. and members of the staff of the Digital Computer Section of Energoprojekt in programming the solutions and carrying out the computations.

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