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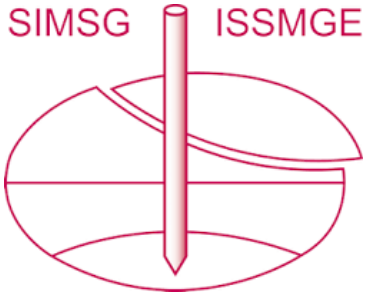

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STRAINS AND STRESSES IN EARTH AND ROCKFILL DAMS

CONTRAINTES DANS LES BARRAGES EN TERRE ET ENROCHEMENT

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SUMMARY:

In the USSR the research work on substation of adopted designs was developed in connection with construction of large earth and rockfill dams (Charvak, Nurek etc).

The first part of this paper, prepared by Nitchiporovitch, presents the results of stress strain control in dams undergoing statical loads and carried out in the "Vodgeo" Scientific Research Institute. Since the general solution of the three-dimensional problem is verycomplex, this problem is subdivided in a number of more simple two-dimensional and even one-dimensional problems.

The second part of the paper, prepared by Sinitsyn, deals with results of current research and selection of design criteria for aseismic structures.

The author presents a method for determining dynamic loads on principles of wave theory. The influence of horizontal and vertical displacements and besides those the cross-section rotation are taken into account. The conditions for the appearance of unstable movements and shears in the rock embankment is established.

Part I. Vertical Displacements/deformations/ of Rock Fill Shells.

In 1960 the "Vodgeo" Institute started developing methods to estimate the vertical deformations in rockfill shells of detritus material during the construction period and during the life-time of the structure. Extensive experimental studies on deformability of large-size rock material were carried out on life-size installations (Kandaurov, 1966).

Large specimens (d > 1.0 m) were tested for strains under pressures up to 60 kg per sq. cm, elastic-plastic and creep strains were studied, definite regularities were ascertained, methods of strain calculation were proposed and subsequently tested by field data.

It was ascertained, that the law of deformability of large-size rock materials can be well described by reological equations in the following form (2,3):

$$\varepsilon = \varepsilon_o - \left\{ a_{yn} + a_n \left[1 - \exp(-ct) \right] \right\} p \qquad (1)$$

or

$$\varepsilon = \varepsilon_o - \left\{ b_{yn} + b_n \left[1 - \exp(-ct) \right] \right\} \left[1 - \exp(-mp) \right] \qquad (2)$$

where ε and ε_o are the initial and design porosity factors; a_{yn}, a_n, b_{yn}, b_n, m and c are design factors depending upon deformability properties of material (elastic and elastic-viscous).

Under these conditions equation (I) being valid at the linear relation and equation (2) - at a non-linear relation $\varepsilon = f(P)$.

Differential equations for settlements of elementary layers were derived making use of the above-mentioned relations. In case of a one-dimentional problem, by integration these equations with respect to y and t, expressions for the settlement of large-size rockfill structures were derived under the linear condition of increasing load (increasing height) p = ut; where u is the rate of load increase.

On the basis of equation (I), the settlement of a point within the dam body, at the height y and at the instant of time $t \geq t_k$ will be:

$$S_{y,t} = \frac{uy}{1+\varepsilon_o}(a_{yn} + a_n)\left(t_\kappa - \frac{\gamma y}{u}\right) - \frac{u a_n}{(1+\varepsilon_o)c}\left\{ y\left[\exp - c(t - t_\kappa) - 1\right] - \frac{u}{c\gamma}\left[\exp\left(c\frac{\gamma y}{u}\right) - 1\right]\left[\exp(-ct) - \exp\left(-c\frac{\gamma y}{u}\right)\right]\right\} \qquad (3)$$

When the equation (2) is used, we obtain

$$S_{y,t} = \frac{mu}{1+\varepsilon_o}\left\{ -\frac{1}{mu}(b_{yn} + b_n)\frac{1}{m\gamma}(\exp(m\gamma y - 1)(\exp(-m\gamma H) - \exp(-m\gamma y) - \frac{b_n}{c - mu}\left[\frac{1}{m\gamma}(\exp(m\gamma y) - 1)(\exp - c(t - t_\kappa)\exp(-m\gamma H) - \exp(-m\gamma y) + \frac{u}{c\gamma}(1 - \exp(-c\frac{\gamma y}{u})(1 - \exp - c(t - \frac{\gamma y}{u})\right]\right\} \qquad (4)$$

Substituting the value $t = t_k$ in these equations we can obtain the settlement for any point y toward the end of construction period. For the case y = H these equations will give the settlement of the dam crest. Relations for the case $t > t_k$ were also obtained.

The experimental investigations of the large-size rock materials and data of field observations of strains and settlements in dams showed that these relations may be used for calculating the vertical settlements of

rockfill shells.

Similar formulae may be derived to estimate the settlements of rock fill shells for various periods of the dam life-time (Marsal R.I., Arellano L.R., 1965) e.g. during the water level rise. It should be kept in mind that this calculation for the moment prior to water level rise is to be carried out providing the upstream and downstream shells are in the same conditions but for any moment after the water level rise the upstream shell unit weight is to be reduced for buoyancy and the downstream shell is to be taken into consideration at vertical stresses $\sigma_z > \gamma h$.

The main coefficients in equations (I) and (2) should be obtained by processing compression curves plotted on the basis of testing the prototype or model materials, the modelling scale being n ≃ 6–8 . It should be borne in mind that deformations of rock fill shells of rounded particles (pebble) are 2 to I0 times less that those of sharp-edged particles. The creep strains constitute from I5 to 30 per cent of the elastic-plastic ones. Towards the end of the construction period the entire range of the elastic-plastic strains constitute up to 5 - I0 per cent of the creep strains.

The equation (I) with linear stress strain relation, may be used for evaluating the vertical deformation of dams 60 to 70 m high of gravelly soils, at stresses up to I0 - I5 kg per sq. cm.

The equation (2) provides the same data for the dams of greater height, erected of angular materials.

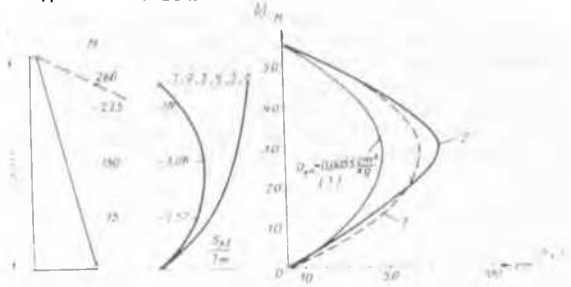

Fig. I.

The table I shows approximate values of design coefficients in equations (I) and (2) for various soils. Fig. Ia shows the results of calculation of settlements of the Nurek Dam during construction, while Fig. 2 shows the comparison of measured and computed settlement values for the Infernillo Dam at a_{yn} = 0.00068 and a_n = 0.00003.

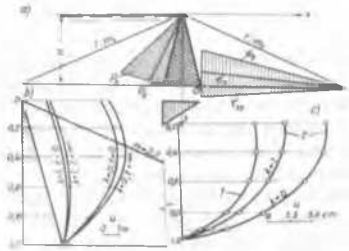

Fig. 2.

The settlement of the upstream shell after the water level rise in the reservoir can be computed not only by the previous method, i. e., by substituting in the equations the parameters obtained for the conditions of wetting, but also by making use of the relationship between the relative strain and the load.

$$e = \alpha a p^{k} \qquad (5)$$

where e - is the unit deformation of the material;
a and k - parameters of wetting conditions;
p - load;
α - empirical factor, wetting degree.

The settlement of a rock fill body at any height can be obtained by deriving a differential equation for the settlement of an elementary layer (dy), and integrating it with respect to y with due account of corresponding weights of suspended and non-suspended material.

The settlement of a layer for different wetting processes can be computed with the help of experimentally obtained coefficients a and k, which vary within the following limits: a - from 0.00I to 0.007, and k - from 0.3 to 0.07.

In the case of erection of a dam with partial wetting and subsequent filling of the reservoir, the settlement caused by wetting may be expressed by the following equation:

$$S_{(y)} = \frac{a\gamma^{k}}{k+1} H^{k+1} \left(1-\left(1-\bar{y}_i\right)\right)^{k-1} \qquad (6)$$

where $\bar{y}_i = \frac{y_i}{H}$ -reduced ordinates of a particular point;
$S_{(y)}$ - settlement caused by wetting.

2. Horizontal Displacements of the Core and the Downstream Shell under the Action of Water Pressure.

The laboratory tests showed that the active pressure of rock fill shells upon the core reaches its maximum at such strains in the shell which are considerably less than those at which the passive pressure can reach its miximum. The above said made it possible to assume the following scheme for the statical analysis of dams with a central core (Fig.2)

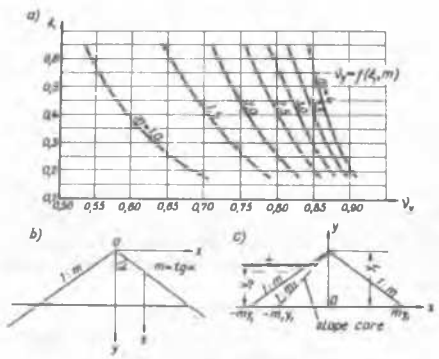

Fig. 3.

Values of coefficients in equations (I) and (2) Table I.

Material	Density indices			Deformability indices					
	n %	ε	γ g/cu cm	a_{yn} sq. cm/kg	a_n sq. cm/kg	b_{yn}	b_n	m sq. cm/kg	E kg/sq. cm
With angular particles (broken stone, rock muck, crushed stone):									
a) homogeneous	42-50	0.7-I.0	I.3-I.6	6.10^{-4} $I.10^{-3}$	6.10^{-5} $I.10^{-4}$	0.4-0.7	-	0.I5-0.40	I50-300
b) non-homogeneous	30-45	0.4-0.8	I.4-I.9	3.10^{-4} $I.10^{-3}$	$I.10^{-5}$ 9.10^{-5}	0.2-0.4	8.10^{-4} $I2.10^{-3}$	0.0I-0.035	I20-400
With rounded particles (gravel, boulders with pebbles):									
a) homogeneous	30-40	0.4-0.7	I.7-I.9	2.10^{-5} 6.10^{-5}	$I.10^{-6}$ 5.10^{-6}	0.02-0.06	$I.10^{-4}$ 4.10^{-4}	0.0I-0.02	500-6000
b) non-homogeneous with fine aggregates	20-30	0.25-0.4	I.9-2.2	$I.10^{-5}$ 6.10^{-5}	5.10^{-7} 5.10^{-6}	0.0I-0.06	5.10^{-5} 4.10^{-4}	0.0I-0.02	500-6000

When the water level in the reservoir rises rapidly, the core is subject to the active pressure P_a (Fig.2) of the upstream shell in addition to the water hydrostatic pressure P_w. Therewith it is evident that a part of the total pressure will be taken up by the core.

Assuming that the stress in the downstream shell varies according to linear law and abiding the regularities of the elasticity theory as well as the boundary conditions (in particular the stresses are equal to zero on the slope surface, and are known at the point 2) we obtain the values of all stress components in the shell. Further, the interrelation of stresses being known, distribution of horizontal displacements in the downstream shell can be determined:

$$\bar{u} = \frac{a_x}{2}\bar{x}^2 + b_x\bar{x}\bar{y} + \frac{b_{xy} - a_y}{2}\bar{y}^2 + b\bar{y} + d \qquad (7)$$

$$\bar{v} = a_y\bar{x}\bar{y} + \frac{b_y}{2}\bar{y}^2 + \frac{a_{xy} - b_x}{2}\bar{x}^2 + a\bar{x} + e$$

where $\bar{u} = \frac{uE}{H^2}$ - reduced horizontal displacement;

$\bar{v} = \frac{vE}{H^2}$ - reduced vertical displacement;
u - true horizontal displacement;
v - true vertical displacement;
E - modulus of deformation of material in kg/sq. sm;

$\bar{x} = \frac{x}{H}$, $\bar{y} = \frac{y}{H}$ - reduced coordinates;
H - height of the dam;
b and d - coefficients characterizing the conditions of restraint;
a_x ... b_{xy} - coefficients characterizing strain distribution in the downstream shell.

All coefficients were computed proceeding from the condition of maintaining tho stability and the physical relations given by the elasticity theory:

$$\begin{aligned}
a_x &= (1+\mu)\left[\mu\alpha\frac{\gamma}{m_1} - (1-\mu)(\alpha+1)\frac{\gamma m_1}{10}\right]\\
a_y &= (1+\mu)\left[\mu(\alpha+1)\frac{\gamma m_1}{10} - (1-\mu)\alpha\frac{\gamma}{10m_1}\right]\\
b_x &= (1+\mu)\left[(1-\mu)(\alpha+1)\frac{\gamma m_1}{10} - \mu\alpha\frac{\gamma}{10}\right]\\
b_y &= (1+\mu)\left[(1-\mu)\alpha\frac{\gamma}{10} - \mu(\alpha+1)\frac{\gamma}{10}m_1^2\right] \qquad (8)\\
a_{xy} &= -2(1+\mu)(\alpha+1)\frac{\gamma}{10}\\
b_{xy} &= 2(1+\mu)(\alpha+1)\frac{\gamma m_1}{10}
\end{aligned}$$

The coefficients b and d can be determined from the following relations:

$$\begin{aligned}
b &= (a_{xy} - b_x)\bar{x}_o + a_y\\
d &= \frac{a_x}{2}\bar{x}_o - a_{xy}\bar{x}_o - \frac{a_y}{2} - \frac{b_{xy}}{2} \qquad (9)
\end{aligned}$$

where $\bar{x}_o$ - a conventional point of restraint of the downstream shell and is selected at the intersection point of the design verticals with the foundation surface; γ_o - unit weight of the downstream shall material; μ - Poisson's ratio for the downstream shell material; m_1 - ratio of the downstream slope; α - coefficient characterizing the stress distribution in the downstream shell.

For the purpose of determining the displacement in the downstream shell caused by water level rise in the reservoir, the design values of coefficients a_x ..... b_{xy} should be calculated as the differences of their magnitudes after the reservoir water level rise

and prior to this rise:

$$a_x = a_x'' - a_x' \qquad (10)$$

where a_x'' and a_x' are determined from the equation (8), assuming the following values of α :

after head water rise:

$$\alpha = \frac{\lambda F - P_x \frac{H_1}{2}}{\frac{\gamma_n}{10}(m_1 - m_2)^2 \frac{H^2}{2}} \qquad (11)$$

prior to head water rise:

$$\alpha = \frac{-1 + \frac{m_2}{m_1 - m_2}\left[\frac{4m_2\frac{\gamma_\pi}{\gamma_n} - 4(m_1 - m_2) - \frac{\gamma_\pi}{\gamma_n}}{\zeta_\pi + 2m_2}\right]}{1 + \frac{m_2}{m_1} + \frac{4m_2(m_1 - m_2)}{\zeta_\pi\, 2m_2}} \qquad (12)$$

where m_2 - inclination of the core downstream slope;

ζ_π - coefficient of the lateral pressure of the core material;

γ_π и γ_n - units weights of the core and shell material respectively, g per cu sm;

λ - coefficient characterizing the degree of utilization of the core bearing capacity ranging from 0 to I;

P_x - horizontal component of the total pressure of the upstream shell and water upon the core at the point of intersection of the upstream slope of the core and of the foundation surface. The value can be determined from the formula:

$$P_x = \left(\gamma_n - \frac{\gamma_n}{\gamma_{y\partial}}\right)\frac{bH}{10} + \frac{\gamma_b H}{10\sqrt{1 + m_2^2}} \qquad (13)$$

where $\gamma_{y\partial}$ - specific gravity of the shell material;

γ_b - specific gravity of water;

b - reduced pressure determined from the corresponding formulae or tables;

F - core bearing capacity determined from the following formula:

$$F = \frac{2\gamma_\pi - 1}{10}\, m_2 H^2 \operatorname{tg}\varphi_\pi \qquad (14)$$

where φ_π - angle of internal friction of the core material.

Horizontal displacements in a rock-and-earth fill dam were calculated proceeding from the following initial data: $m_1 = 2.2$; $m_2 = 0.25$; $H = 300$ m; $\gamma_\pi = 2.01$ g/sq. sm; $\zeta_\pi = 0.5$ $\varphi_\pi = 26°$; $\gamma_n = 2.15$ g/sq. cm; $\gamma_o = 2.70$ g/sq.sm $\mu = 0.2$; $\zeta_n = 0.25$; $a_{yn} = 0.0001$; $a_n = 0.00005$; $\varepsilon_o = 0.26$.

Having determined the coefficients of the system from the equations II, I2, 9 and 8 and taking the conditions of restraint $X_o = 0.59$ the core specific displacement can be derived from the equation (7). Fig. 26 shows the horizontal displacements of 300 m high dam, at $\lambda = 0$ and $\lambda = 1$.

Fig. 26 shows the measured horizontal displacements in the I-VIII measuring cross-section of the Infernillo Dam and the displacements calculated in the above described way.

The above method can be recommended to evaluate approximately the deformations expected in the downstream shell after the reservoir filling.

3. Distribution of Stresses in the Cross-Section.

For a dam body composed of detritus, it is more expedient to consider the problem of stress from the viewpoint of granular medium mechanics. The system of differential equations derived by Kandaurov - Müller (Nazarov A.G., 1962) for the physical state of granular medium can be written as

$$\begin{aligned} \frac{\partial \sigma_y}{\partial y} &= \zeta y \frac{\partial^2 \sigma_y}{\partial x^2} - \gamma \\ \tau_{xy} &= -\zeta y \frac{\partial \sigma_y}{\partial x} \\ \sigma_x &= \zeta \sigma_y + \zeta^2 y^2 \frac{\partial^2 \sigma_y}{\partial x^2} \end{aligned} \qquad (15)$$

where σ_x, σ_y, τ_{xy} are the stress components, γ - unit weight, ζ - coefficient of the the distributing ability of the medium equal to the coefficient of the lateral pressure at rest.

Solving the first equation of the system (15) for a symmetrical embankment of a triangular cross-section (Fig. 3b) under the action of the embankment material dead weight and assuming that

$$\sigma_y(x,y)\Big|_{x=\pm my} = 0 \quad \text{and} \quad \frac{\partial \sigma_y}{\partial x}\Big|_{x=\pm\infty}$$

the general relation for vertical stresses σ_y can be written in the following form:

$$\sigma_y = \frac{\gamma}{2}\int_y^{\frac{H}{m}} \left[\Phi\left(\frac{x + mz}{\sqrt{\zeta(y^2 - z^2)}}\right) - \Phi\left(\frac{x - mz}{\sqrt{\zeta(y^2 - z^2)}}\right)\right] \qquad (16)$$

where $\Phi(t)$ is the tabulated function of the integral errors. The stresses σ_x and σ_{xy} can be computed from the equations (15) and (16).

Fig. 3a shows the values of the factor of stresses reduction under the centre-line of the embankment for different values of ζ , m and γ = I, obtained from the formula:

$$\nu_y^o = \frac{\sigma_y^o}{\gamma\, y_o} = \int_0^1 \Phi\left(\frac{mz}{\sqrt{\zeta(1 - z^2)}}\right) dz \qquad (17)$$

The experiments permitted to derive simple semi-empirical relations for calculation of the vertical stresses σ_y caused by the dead weight and the hydrostatic pressure of water; at various location of antiseepage devices. These relations take the forms:

from the dead weight: $\frac{1}{2\nu_y - 1}$

$$\sigma_y^{cb} = \nu_y^o \gamma y_1 \left[-\left(\frac{x}{my_1}\right)^{\frac{1}{2\nu_y - 1}} + 1\right] \qquad (18)$$

from the hydrostatic pressure:

$$\sigma_y^{\delta} = y_o \Bigg|_{-my_1 \le x \le m_1 y_1} + \left[\frac{y_o x^2}{y_1^2 (m_1+m_2)^2} - \frac{2 y_o m}{y_1 (m_1+m)} + \right.$$

$$\left. + \frac{y_o m^2}{(m_1+m)} \right] \Bigg|_{-m_1 y_1 \le x \le m y_1} \qquad (19)$$

where y -height of the embankment, y_o - depth of water, m - ratio of embankment slope, m_1 - ratio of the impervious core slope (Fig. 3b).

With the advent of modern electronic computers, the problem in stresses in the cross section can be solved by grid method or by finite elements method (Finn V.D.L, 1967). At present, a programm is being compiled to solve the problem of stresses in the cross-sections of dams with central cores.

4. Stress Distribution in the Longitudinal Profile of the Dam.

Displacements of the dam body from the banks towards the canyon axis are observed when the dams are being built in relatively narrow canyons with sufficiently steep banks ($m \le$ I-I.5). This causes cracking in the impervious cores of dams, mostly in their portions adjoining to the banks. Therefore, it is necessary to reveal the causes of cracking and to study the stress distribution in the longitudinal sections of the dam (of the core).

The problem under consideration can be approximated (if some simplifying assumptions are made) to the two-dimensional problem in the theory of elasticity which considers a heavy symmetrical wedge resting on an infinitely rigid inclined foundation, the displacements along the contact surface being absent (I).

Of all the methods of solving the two-dimensional problem of the elasticity theory, the nets method was chosen, which permits the use of modern computing machines, taking into account the effect of elasticity constants E and μ , and if necessary, to solve the problem at variable E and μ (the problem was solved by M. Dolezhalova).

For the sake of simplifying the expression of boundary conditions in finite differences, along the contact with the rigid foundation, the problem was solved in the terms of displacement, with the use of the rectangular net. The net was shifted for half of the pitch, this net gives equal accuracy of the finite difference approximation of all terms in the Lame's equations. The net pitch was equal to $\frac{H_{max}}{14}$, number of unknown quantities - I56. The system of equations was solved with the help of electron digital computer, using Seydel method.

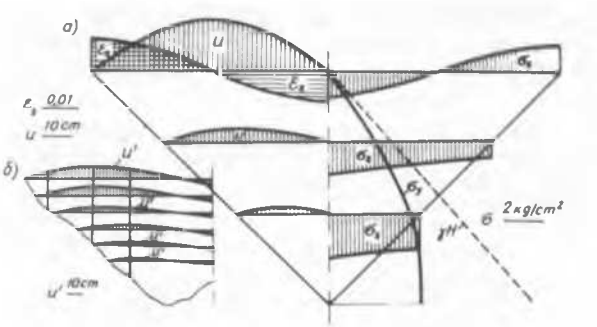

Fig. 4.

Fig. 4a shows computation results for a two-dimensional deformation with the following initial data: H = 52.0 m; γ = 2 tons/cu. m; E = I.50 kg/sq. cm; μ = 0.32; m = I. The vertical stresses σ_y obtained by calculations were substantially less than those caused by arch effect; the displacements u reach their maximum value approximately at $\frac{L}{4}$. The maximum relative horizontal strains (tension) occur near the slopee and reach the value of ε_x = 0.0I. Fissures are possible in thie place. Fig. 4b ehowe the measured strains in the Infiernillo Dam site. As may be seen, the data are very close as to their qualitative features.

Part II. The Seismic Load.

Statement of the problem.

The theory of the aseismic structures is highly developed by the Soviet scientists (K.S. Savriev, I964; A.G. Nazarov, I959; S.V. Medvedev, I962; Sh.G. Napetvaridze, I959; A.P. Sinitsyn, I967) and others. The general method of the dams designing was worked out previously (A.P. Sinitsyn, I967). The application of this method to deesigning of large dams was possible by means of a special model in which the dam is considered as a system with many degrees of freedom.

The distributed dams mass is substituted by several concentrated masses, as it is shown in Fig.5. The solution of the problem may be found by using the displacement method and the differential equations of motion would be written for each concentrated mass. On the rockfill dams made of local materials the vertical component of seismic load exert a great influence as the profile of such dams is more stretched out. In paper (A.P. Sinitsyn, I964) we have established, that by small amplitudes of seismic waves the deformations of the dam profile and the corresponding displacements can not exit the elastic limit, and the profile may be considered as anelastic triangular wedge. By heavy earthquakes the bonds between some sections of the dam profile would be broken and the motion should be described by

the nonlinear equations. The slope slide and the fall off of some dams parts occur. The necessity of determination the unstability areas of motion arises.

Spectral analysis.

For calculation of the dams spectrum, as a system of many degree of freedom it is necessary to determine preliminary its dynamic characteristics.

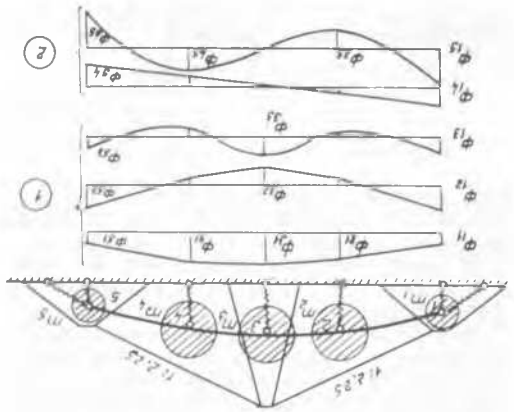

Fig. 5.

The circular frequencies $\omega_1, \omega_2 \cdots \omega_n$ must be calculated by solving the frequencies equation, after that the normal modes of vibration $\phi(x,y,z)$, $\phi(x,y,z) \cdots \phi_n(x,y,z)$ corresponding to these frequencies are determined. It must be noted, that for the dam with a complicated profile these preliminary calculations are sometimes very laborious, but the general methods to solving such problems are highly elaborated now.

Displacements of the dam are calculated in the vektorial form:

$$\bar{u}(x,y,z,t) = \sum q_n(t)\,\Phi_n(x,y,z) \qquad (20)$$

This solution corresponds to decomposition of the motion in the natural modes of vibration. To get the higher limit of the displacements spectrum the following unequality is used:

$$\left|\bar{u}_{max}\right| \le \sum \left|q_{n\,max}\right| \left|\bar{\Phi}_n(x,y,z)\right| \qquad (21)$$

In those cases when the time of seismic forces increase is long enough the unequality (21) convert into the equality:

$$max\ \bar{u}(x,y,z,t) = \sum q_{n\,max}\,\bar{\Phi}_n(x,y,z) \qquad (22)$$

The spectrum of the displacements which correspond to the given generalized force is determined by solving the equation of the generalized coordinate.

$$\left|q_{n\,max}\right| = D_n(\omega_n) \qquad (23)$$

Substituting this quantity in the formulae (21) and (22) we obtain:

$$\left|\bar{u}_{max}\right| \le \sum \left|D_n(\omega_n)\,\bar{\Phi}_n(x,y,z)\right| \qquad (24)$$

If the seismic acceleration is a vektor which have a fixed direction and a magnitude S (t), then the generalized force would determined by formula (25) where $\bar{\kappa}$ is the unite vektor in direction of the acceleration.

$$\frac{Q_n(t)}{m_n} = \ddot{S}(t)\,\frac{\int \mu_o \bar{\kappa}\,\bar{\Phi}_n\,dv}{\int \Phi_n^2\,dm} \qquad (25)$$

The ratio of two integral in formula (25) is an abstract number, which may be called the coefficient of distribution y_n .

$$y_n = \int \mu_o \bar{\kappa}\,\bar{\Phi}_n\,dv \Big/ \int \bar{\Phi}_n^2\,dm \qquad (26)$$

The formula (25) may be written so:

$$\frac{Q_n(t)}{m_n} = S(t)\,y_n \qquad (27)$$

But from formula (27) is clear:

$$D_n(\omega) = D_S(\omega_n)\,y_n \qquad (28)$$

If by D_S is described the displacements spectrum due by $S(t)$. This spectrum should be common to all generalized coordinates:

$$\left|q_n\right|_{max}^{\cdot} = y_n\,D_S(\omega_n) \qquad (29)$$

$$\left|\bar{u}\right|_{max} \le \sum \left|y_n\,D_S(\omega_n)\,\Phi_n\right| \qquad (30)$$

$$\left|\ddot{\bar{u}}\right|_{max} \le \sum \left|y_n A_S(\omega_n)\,\Phi_n\right| \le \sum \left|y_n \omega_n^2 D_S(\omega_n)\,\Phi_n\right| \qquad (31)$$

The formulae (10) - (12) showed that for the dam as a system of many degree of freedom the full spectrum is obtained by substitution of the separate spectres calculated from each general mode of vibration.

Influence of vertical displacement and rotation.

In generalized case the dam acting by seismic forces received the horizontal and vertical displacements and besides those get a rotation. The model of motion is shown in Fig.2 and be analysis is made for the schematic profile of Nurek dam.

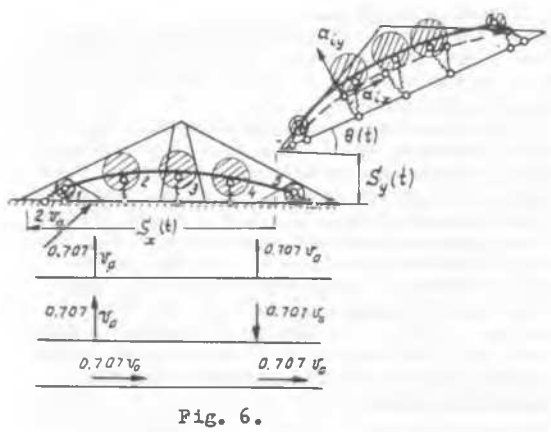

Fig. 6.

The concentrated masses m_i of the dam are affected by accelerations a_{ix} and a_{iy} .

Forcalculation the relative displacements the dam may be considered as system loaded by inertial forces. The vertical components of the acceleration are calculated by addition of the given vertical acceleration of foundation $\ddot{S}(t)$ and the vertical component of acceleration $a_{i\theta}$ which arises as result of foundation rotation θ (t). The influence of rotation on the horizontal component of the acceleration is now usually neglected the Coriolis acceleration is neglected too. The vertical component of the acceleration $a_{i\theta}$ is different to each mass m_i and is proportional to the ratio of distance h_i from left end of the dam to the mass m_i and the full breadth of foundation. Thus the influence of foundation rotation would be taken into account by corresponding modification the values of the vertical accelerations $\ddot{S}_x(t)$ are obtained by differenciation of the foundations horisontal displacements $S_x(t)$. Forcalculation the vertical acceleration the formula is:

$$a_{iy}(t)=\ddot{S}_y(t)+h_i\ddot{\theta}(t) \qquad (32)$$

The vertical displacements and accelerations spectrum should be obtained by using the frequencies and the general modes of the transverse bending profiles vibrations. The spectrum of the horizontal vibration have its own frequencies and modes, which are corresponding to the longtudinal vibrations of a bar. The generalized coordinats q_n (t) of the system without damping should be obtained from the independent equation:

$$\ddot{q}_{n\kappa}+\omega_{n\kappa}^2 q_{n\kappa}=-y_{n\kappa}a_{i\kappa}(t),\ (\kappa=x,y,\theta) \qquad (33)$$

The values y_n are determined by formula:

$$y_{n\kappa}=\frac{\sum m_i \Phi_{n\kappa}}{\sum m_i \Phi_{n\kappa}^2},\quad (\kappa=x,y,\theta) \qquad (34)$$

The formula of $q_{n\kappa}$ is:

$$q_{n\kappa}=-\frac{y_{n\kappa}}{\omega_{n\kappa}}\int_0^t a_{i\kappa}(t)\sin\omega_n(t-u)du,(\kappa=x,y,\theta) \qquad (35)$$

Now we write:

$$R_{n\kappa}(t)=\frac{1}{\omega_{n\kappa}}\int_0^t a_{i\kappa}(t)\sin\omega_n(t-u)du \qquad (36)$$

and obtain:

$$q_{n\kappa}=-y_{n\kappa}R_{n\kappa}(t) \qquad (37)$$

The displacements would be calculated by formula:

$$u_{i\kappa}(x,t)=\sum q_{n\kappa}\Phi_{n\kappa}(x)=-\sum y_n\Phi_{n\kappa}(x)R_{n\kappa}(t),(\kappa=x,y,\theta) \qquad (38)$$

The spectral curve correspond to $\left|R_n(t)\right|_{max}$

$$D_{n\kappa}=\left|R_n(t)\right|_{max} \qquad (39)$$

$$\left|u_{i\kappa}(x,t)\right|_{max}\le\sum\left|\Phi_{n\kappa}y_{n\kappa}D_{n\kappa}\right| \qquad (40)$$

The absolute maximum of the accelerations is calculated by formula:

$$\left|\ddot{W}_{i\kappa}\right|_{max}\le\sum\left|\Phi_{n\kappa}y_{n\kappa}A_{n\kappa}\right|,\ (\kappa=x,y,\theta) \qquad (41)$$

The formula (41) allows to determine the components of the absolute acceleration on the two perpendicular axes. To calculate the absolute maximum of acceleration limit it is necessary to made a geometrical addition of these two components.

$$\left|\ddot{W}_i\right|_{max}\le\sqrt{\left[\left|\ddot{W}_{ix}\right|_{max}\right]^2+\left[\left|\ddot{W}_{iy}\right|_{max}\right]^2} \qquad (42)$$

Let us consider a case when the foundation of the dam (Fig; 6), received displacements due by impulse of velocity v_o suddenly applied t = 0. If the velocity vektor have the inclination angle to the horizont 45°, the components of the velocity on the coordinateaxes are:

$$\dot{S}_x(t)=v_x=v_y=0,707\,v_o \qquad (43)$$

The rotation of the foundation is due by suddenly applied angular velocity:

$$\theta_o=\frac{v_o}{r} \qquad (44)$$

The spectral curves connected with transversal motion are calculated by formula (36) taking into account that:

$$a_{i\kappa}(t)=\dot{S}_\kappa(t)=0,707\,v_o\delta(t)$$

We obtain:

$$R_{n\kappa}(t)=-\frac{0,707v_o}{\omega_{n\kappa}}\sin\omega_{n\kappa}(t) \qquad (45)$$

The spectral curve of the vertical displacement connected with the rotation motion is obtained from formula:

$$a_\theta(t)=\ddot{\theta}(t)=\frac{v_o}{r}h_i\delta(t) \qquad (46)$$

$$R_{n\theta}(t)=-\frac{v_o}{\omega_{n\kappa}}\frac{R_i}{r}\sin\omega_{n\theta}t \qquad (47)$$

In general case it is necessary to use three groups of the frequencies and the modes of vibrations. The relative vertical displacements of any point of the dams profile may be calculated by formula (38):

$$u_{ix}=-0,707\frac{v_o}{\omega_{1x}}\sum_{n=1}^{n=3}\Phi_{in}y_{nx}\frac{\omega_{1x}}{\omega_{nx}}\sin\omega_{nx}t\mp \mp\frac{v_o}{\omega_{4\theta}}\sum_{n=4}^{n=5}\Phi_{in}y_{n\theta}\frac{2h_i}{l}\frac{\omega_{4\theta}}{\omega_{n\theta}}\sin\omega_{n\theta}t \qquad (48)$$

The first item in this formula is the vertical relative displacement of the mass m_i due by vertical component of the velocity v_o suddenly applied to the bottom of the dam. The second item is the vertical displacement of each mass which depend upon the rotation of foundation. At the left part of the dam these two items are negative. At the right part of the dam the second item have a positive sign In the formula (48) h_i - mean the distance from dam axis to the given mass. The calculation of the absolute accelerations is made by

formula:

$$\ddot{W}_{ik} = v_0 \omega_1^2 \sum^{n=N} \Phi_{in} y_n \frac{\omega_n}{\omega_1} \sin \omega_n t \qquad (49)$$

The spectral values of the components of relative displacements corresponding to the limit value were obtained by formulae (24) and (25) by addition the absolute values of the coefficients in formulae (48) and (49):

$$\left|U_{\substack{1x \\ 5x}}\right|_{max} \leq \frac{v_0}{\omega_1^2}[0{,}587+0{,}025+0{,}00005+0{,}798+0{,}026]=1{,}436\frac{v_0}{\omega_1^2}$$

$$\left|U_{3x}\right|_{max} \leq 0{,}789\frac{v_0}{\omega_1^2};\quad \left|U_{1y}\right|_{max}=0{,}65\frac{v_0}{\omega_1^2}$$

The total values of the relative displacements are:

$$\left|U_1\right|_{max} \leq \sqrt{1{,}436^2+0{,}65^2}\,\frac{v_0}{\omega_1^2}=1{,}57\frac{v_0}{\omega_1^2} \text{ and } \left|U_3\right|_{max}=1{,}00\frac{v_0}{\omega_1^2}$$

Now we compare the obtained values of total relative displacements with those which were calculated when the vektor of the external velocity was directed vertical and was applied in the centre of mass of the whole system.

$$K = \frac{|U|_{max} \text{ (with rotation)}}{|U|^0_{max} \text{ (without rotation)}} = \frac{1{,}570}{1{,}117}=1{,}41$$

It is clear that the rotation of foundation magnify the displacements at about of 40%. But the mass situated in the middle of the dams profile has almost equal displacements in both cases. The rotation motion do not influence on this mass because the assymmetric modes of vibration have in the middle of the span the ordinate equal nil. The maximum values of acceleration componsnts would be obtained by summation of the absolute value of coefficients in formulae of acceleration.

$$\left|\ddot{W}_{\substack{1x \\ 5x}}\right|_{max} \leq v_0\omega_1^2[0{,}587+0{,}525+0{,}019+2{,}414+2{,}990]=6{,}535\,v_0\omega_1^2$$

$$\left|\ddot{W}_{3x}\right|_{max}=1{,}124\,v_0\omega_1^2;\quad \left|W_{1y}\right|_{max}=0{,}780\,v_0\omega_1^2$$

The increase of the acceleration in the extreme points is caused by rotation motion of the profile. The overloading which occur as result of the rotation motion must be taken into account. The higher modes of vibration produce an increase of acceleration too. The higher modes of vibrations add a few percents to the value of displacements calculated for the first mode only. On the accelerations the influence of higher modes is more perceptible. The accelerations are proportional to the square of the corresponding frequencies have great value.

Dynamical stability of dams profile.

The aseismicity of the rookfill dams depends upon a phenomenon which was observed by us in I942. When the propagation of a shock impulse through the sand layer on small models was investigated. The earthfill body is like an elastic body when the value of shock impulse is small and the rigidity ratio of the system have a certain value. If the ratio of rigidity would be changed to a critical value that the remarkable shear strain arise and the deformations increase. The instability of the system occur, when the ratio of the rigidity would be changed in the same direction afterwards the system worked as an elastic system again; but its rigidity coefficient should be another (Fig. 7). The nonlinear law correlating stresses and deformations have been obtained from specially organized experiments. To draw conclusions of the dam stability we substitute a three linear sections, for a non-linear law of the change of the system rigidity (Fig. 8).

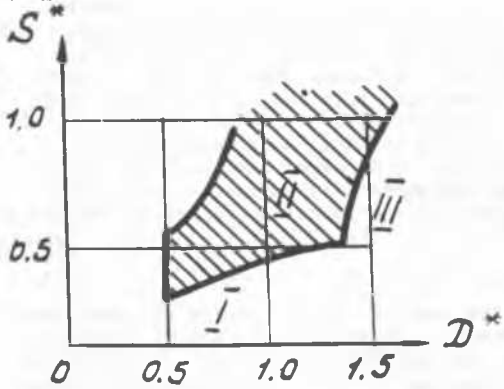

Fig. 7.

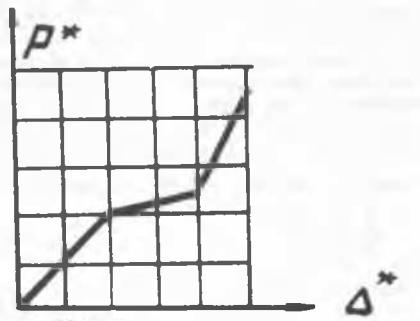

Fig. 8.

Now the problem can be solved with the aid of the so-called "skeleton curves" which correspond to the appearance of the resonance and a considerable rise in displacements between the parts of the dam. As is known, when dealing with a linear system of one degree of freedom the "skeleton curves" are in fact straight lines which are parallel to an axis of ordinates. These are shown for each frequency with dotted lines in Fig. 9.

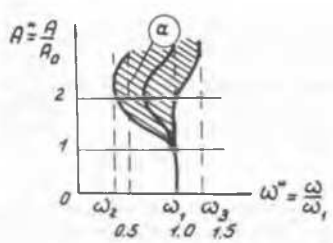

Fig. 9.

On the diagrams, the ratios of frequencies are plotted as absoissas and the amplitudes as ordinates. When during movement the rigidity of the system drops the "skeleton ourve" bends first to the left and then to the right following a rise in the rigidity of the system. The unstable movement and shears in the rock embankment appears: this area is shaded in Fig.9.

The diagrams shown in Fig.8 and 9 permit to made estimation of the stability of the dam, as a first approximation, determining the relationship which characterises physical properties of the rockembankment and constructing the stresses - strain diagram of the system considered. This diagram is then to be approximated with the three linear sections as shown in Fig.8. The simplified diagram is used for determination of three values of frequency corresponding to three different sections of the diagram. Thus, the "skeleton curve" of the whole system can be plotted and the areas of unstable movement corresponding to this curve may be obtained. This investigation corresponds to studying the properties of the dam profile. To answer the question whether the dam is stable or not under a seismic action, a study of the separate harmonics which make up a spectrum of a seismic wave and concern to the vertical component of the surface wave should be made. SOme of the harmonics, whose frequencies are within the limits of the unstable area, shown in Fig. 9, may cause shears in the dam profile if an additional condition is fulfilled namely, if the amplitudes of these harmonics are above the horisontal straight line which correspond to the ratio $A/A_o = A^* = 1$. If the spectrum of the seismic wave has the amplitudes which are outside the unstable range plotted for the given profile of the dam this profile is considered to be reliable.

Conclusions.

We described the calculation method as applied to the system with one degree of freedom which is the most simple one, and as experience has shown, it is accurate enough for practical estimation of the dynamic properties of a dam.

As for the certain proximity of the above described static loads control (part I) it is believed that along with other methods of investigations (optical and centrifugal modelling etc) and well planned field observations, this method can provide reliable data on behaviour of dams constructed of local materials.

REFERENCES:

FINN V.D.L. 1967, "Behaviour of Earth-Fill Dams During Earthquakes", IX International Congress on Large Dams, Istanbul, R. 2I, Q 35.

KANDAUROV I.I.1966, "Granular Media Mechanics and its Applications in the Construction", Stroi izdat, Moscow (in Russian).

MARSAL R.G., ARELLANO L.R. 1965, "El Infiernillo dam observation during the construction period and first filling of the reservoir".

MEDVEDEV S.V. 1962, "Engineering Seismology", Gosstroyizdat, Moscow (in Russian).

NAPETVARIDZE Sh. G. 1959"Aseismicity of hydrotechnical structures", Gosstroyizdat, Moscow (in Russian).

NAZAROV A.G. 1962, "Method for engineering analysis of seismic forces", Armenian SSR, Academy of Sciences, Erivan (in Russian).

NITCHIPOROVITCH A.A. 1961, "Deformations and Stability of Rockfill Dams", VIII International Congress on Large Dams, R 19, Q 31.

NITCHIPOROVITCH A.A. 1965 "On Deformations of Dams of Large-Size Detritus", Trudy Instituta Vodgeo, Gidrotekhnika, Moscow, N II (in Russian)

NITICHIPOROVITCH A.A., ZHUREK Ya. , 1967, "Horizontal Displacements and Settlements of Shells of Rock-and-Earthfill Dams with Central Cores During the Reservoir Filling", Gidrotekhnicheskoe Stroitelstvo, Moscow, N 10 (in Russian).

SAVRIEV K.S. 1964, "Dynamic of Structures Gostransjeldorizdat, Moscow (in Russian).

SINITSYN A.P. 1967, "Practical Methods for designing of Structures on Seismic Loads", Gostroyizdat, Moscow (in Russian).

SINITSYN A.P. 1964, "Stability of Rockfill Dam due by Seismic Load", Magazine "Engineering seismology", Tadjik SSR Academy of Sciences, Dushanbe-Leninakan; N 1-2 (in Russian).

TEITELBAUM A.I. 1968, "Stressed State of Large-Size Detritus Dams", Trudy Instituta Vodgeo, Gidrotekhnika. N 19, (in Russian).