ANALYSIS OF SOIL EMBEDED STRUCTURES

CALCULATION DES CONSTRUCTIONS ENFORCEES DANS LE SOL

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SYNOPSIS

The first part of the paper discusses structures operating near the surface of a base /anchor plates, sheet pile walls, piles/, taking into account that the anchor and sheet pile break the elastic half-space and that in driving the pile the soil is pushed of and solidified. To allow for the break, distributed double forces are used.

In the second part a method is proposed for analysing an arched structure deeply imbedded in a non-rooky ground operating in the elastic and the elastic-plastic stages. The soil is regarded as elastic layer /two-dimensional problem/, assuming zero displacements along the lower boundary of the layer. In the elastic stage the solution is given in the form of ready design expressions. In the elastic-plastic stage the associated features of structure design are demonstrated.

In analysing soil-embedded structures it is assumed that the soil operates as an elastic half-space. The first part of the paper (M.I.Gorbunov-Possadov, A.B.Ogranovich, L.N.Repnikov) discusses structures operating near the surface of a base (anchor plates, sheet pile walls, piles). Allowance is made for the facts that the anchors and the sheet pile break the elastic half-space and that when the pile is driven in, the soil is pushed of and solidified. In the second part S.S.Davydov proposes a method for analysing an arched structure deeply embedded in a nonrocky ground and operating in the elastic and elastic-plastic stages. When a structure operates inside the soil (sheet pile walls, piles, anchor plates, etc.) the utilisation of a model of an elastic half-space in strength and strain analysis yields results closely approximating actual conditions, since the appearance of plastic strains within the soil is impeded by the addition of its own weight, the soil is solidified by this weight, and the bearing areas of the structures are not so large (Gorbunov--Possadov, 1967).

In this case the advantage of the model of an elastic half-space over the Winkler hypothesis is that it takes into account not only the distributive ability of the soil, but also the decrease in the resistance to base strains near the surface.

The first attempts to use a model of an elastic half-space for analysis of structures near the surface of the ground were made in analysing piles for a horizontal load (Zhemochkin, 1948) and for a sheet pile wall (Krachmer, 1956), also for a rectangular anchor plate (Douglas and Davis, 1964). These authors, however, did not allow for the break in the continuity of the base (an opening) caused by the structure body. According to these solutions, therefore, in the upper portion of the struoture the soil was subjected to compression in front of the structure and to tension behind it. In the lower portion of the structure the situation was reversed (see diagram in fig. 1(a); tensile stresses are regarded as positive). In actual fact soil is not subjected to tension; for this reason the resistance of soil to displacement os overstated when using a diagram of a continuous elastic half-space.

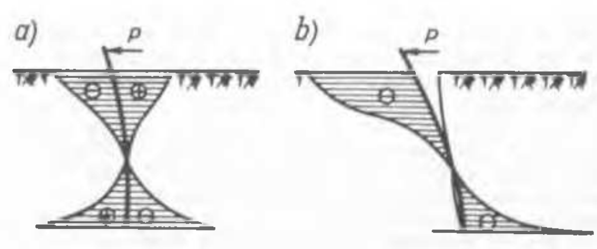

Fig.1. Diagram for analysing a structure operating within an elastic half-space: (a) without and (b) with allowance for a break in continuity.

A correct solution can be obtained by using a diagram according to which the structure is inserted in the slot in the elastic half-space (fig. 1(b)). In this case the soil will only be subjected to compression on that side of the structure to which it is displaced. On the other side there will be either no soil pressure on the wall (in the case of cohesive soils), or active pressure will manifest itself, or else pressure will arise from the heaping of the ground, i.e. from its elastic displacement toward the structure by gravity.

As proposed by M.I.Gorbunov-Possadov (Ogranovich and Gorbunov-Possadov, 1966; Gorbunov-Possadov, 1967), the effect of the opening in which the structure is inserted is obtained by using the double forces distributed continuously along the structure body, We will recall that a double force D, is a load obtained in the limit from two equal and opposite point forces, P, on yheir unlimited mutual approach and when the product D = Pa (where a is the distance between the points) is constant. The forces are directed along the line passing through the points of their application. If, for instance, the forces P are directed along the x-axis, the solution for the double force D is obtained from the solution for P by differentiation with respect to x (Timoshenko and Goodier,1951). It will be shown below that continuously distributed force couples break a half-plane or half-space and form an opening.

A solution for a vertical and a horizontal forces applied inside a half-plane is given by E.Melan (Melan, 1932). As regards the horizontal force, however, this solution is in error. In the formula for horizontal stresses the term

$$\frac{y^2 + 6dx + 6d^2}{z_2^4}$$

should be raplaced by

$$\frac{y^2 - 4dx - 2d^2}{z_2^4}$$

Besides, E.Melan did not supply a formula for displacements. A correction of Melan's formulas for stresses and the deduction of displacement formulas have been made by us (Gorbunov-Possadov, Shekhter, Kofman, 1954; Gorbunov-Possadov, 1964).

A solution for a point force in an elastic half-space was obtained by R.Mindlin, 1936; Mindlin and Chen, 1950). The same author supplied the expressions for kernels for force couples in an elastic half-space through the Galerkin vector.

Analysis of a sheet pile wall has been performed by us in two versions: for a rigid wall (Ogranovich and Gorbunov-Possadov, 1966) and for a flexible wall (Ogranovich, 1967). We restricted ourselves to a scheme where a point horizontal force P and a moment M are applied at the upper portion of the wall coinciding with the free surface of the ground. The heaping of the ground is taken into account as a uniformly distributed load on the surface of the half-plane to one side of the wall.

Now we will state briefly a more general case of a flexible wall (fig.2). We will assume that the ground is dense clay, the weight of the mound is insufficient for closing up the opening, and complete closing up of the opening is not achieved (we have also investigated other cases where the opening closes up but a break in the stresses occurs just the same).

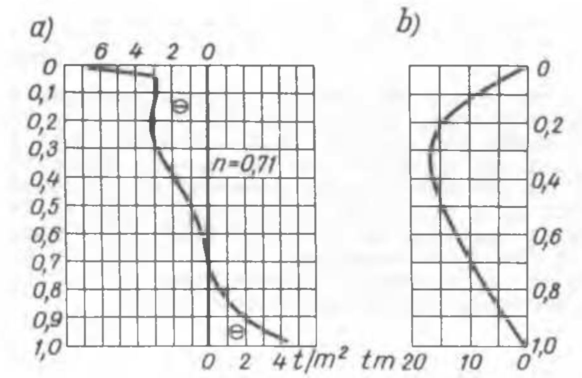

Fig.2. Analysis of a flexible sheet pile wall.

The wall thickness is assumed to be negligibly small, although, as will be shown below, it is quite possible to make allowance for the wall thickness. The effect of the zone of limiting state of the ground at the top of the wall is also ignored; the error introduced by this assumption is partly smoothed out by the above-mentioned increase in the deformation of the medium at its surface as in the solutions of Melan and Mindlin.

The boundary conditions on the surface of the ground are satisfied automatically by using Melan's solution. The boundary conditions to the right (y = +0) and left (y = -0) of the wall (fig.2) will be different. Assume that the point at which the reaction pressures pass over from one side of the wall to the other and where the width of the opening is zero is located at a depth nh, where h is the depth of the embedding of the wall. Then the boundary conditions for a smooth wall will be:

$$\begin{aligned}
&\text{at } x \le nh, \quad y = +0, \quad \sigma_y = 0, \quad \tau_{xy} = 0 \\
&\phantom{\text{at } x \le nh,} \quad y = -0, \quad Y = v, \quad \tau_{xy} = 0 \qquad (1) \\
&\text{at } x \ge nh \quad y = +0, \quad Y = v, \quad \tau_{xy} = 0 \\
&\phantom{\text{at } x \ge nh} \quad y = -0, \quad \sigma_y = 0, \quad \tau_{xy} = 0
\end{aligned}$$

Here Y and v are the deflection and displacement of the soil, respectively. Since in a two-dimensional problem displacements are determined to within the additive constant, it is conventionally assumed that the lower end of the wall suffers no displacement.

We will now use reduced coordinates $\xi = x/h$ $\eta = y/h$. To fulfil the boundary conditions (I) we apply to a continuous half-plane, along the x-axis. over a segment $0 \leq \xi \leq 1$, fictitious loads in the form of usual horizontal forces distributed according to the law

$$q(\xi) = \sum_{i=0}^{m} q_i \xi^i \tag{2}$$

and of horizontal double forces

$$d(\xi) = \sum_{i=0}^{m} d_i \xi^i \tag{3}$$

The solution can be made more precise by introducing a fictitious load from the vertical self-balanced forces.

The power of the polynomials (2) and (3) depends on the desired accuracy. In our solution, m was taken as 4.

The unknown coefficient q_i and d_i should be so determined that the boundary conditions will be best satisfied (I).

According to Melan's solution the horizontal stresses from loading by forces (2) have a discontinuity along the x-axis; therefore, if the direction from right to left is considered positive for these forces, the stresses are determined by the equation

$$\sigma_y(\xi)_{\mp 0} = \mp \sum_{i=0}^{4} q_i \xi_i \tag{4}$$

(Compressive stresses are taken as positive, tensile as negative).

The horizontal stresses from the distributed double forces are determined by integrating the stress formula for double forces D distributed according to the law (3) along the vertical axis of the sheet pile wall. In addition, account is taken of the "elastic" pressure of the soil due to its own weight and that of the mound, by multiplying the corresponding vertical stresses by the coefficient of lateral pressure $\xi_o = \nu/(1-\nu)$, where ν is Poisson's ratio of the soil.

The distributed double forces do not cause any tangential stresses along the surface of the sheet pile. In determining horizontal displacement use is made of the fact that double forces produce a break in displacements which is determined by the equation

$$\nu(\xi)_{\pm 0} = F \frac{h(1-\nu^2)}{2E} \sum_{i=0}^{4} d_i \xi^i \tag{5}$$

The tangential stresses to either side of the sheet pile are thus determined by integrating, over the length of the sheet pile, of the formula for tangential stresses due to horizontal forces distributed according to the law (2). Horizontal displacements are determined by integrating the formula for these displacements due to the double forces (3), adding the displacements due to the distributed horizontal forces.

The displacement (deflection) of the wall is described by the conventional differential equation:

$$\frac{D^*}{h^4} \frac{\partial^4 Y}{\partial \xi^4} + \sigma_y(\xi) = 0 \tag{6}$$

where D^* is the cylindrical rigidity of the wall.

The value $\sigma_y(\xi)$ is determined in equation (6) at the upper and lower portions of the sheet pile, allowing for all the above-listed features.

By integrating equation (6) four times in succession we obtain the values of displacements and substitute it in the boundary conditions (I).

Thus, all boundary conditions are expressed through the unknown parameters q_i and d_i. These parameters are so determined that the boundary conditions are expressed to the best advantage in the sense of the least squares. To this end we use the method suggested by one of the present authors for any boundary conditions (Gorbunov-Possadov, Shekhter, Kofman, 1954).

This method imposes, not the condition of the minimum for each integral (taken separately along the boundary) over the squares of deviation of the given magnitude, included in the boundary conditions, from its true value, but the condition of the minimum of the sum of all these integrals. Besides, depending on one's wish, each of the separate conditions can be assigned a higher or lower relative value by introducing the corresponding weights. The value of the weight for each of the conditions may also be variable over the length of the segment along which the boundary condition is imposed. This is particularly important, because in our problem a number of integrands convert to infinity at $\xi = 1$; the weight function was taken as $f(\xi) = \sqrt{1-\xi^2}$. Subsequently the problem is solved with the aid of the theory of the conditional Lagrange extreme, by adding those conditions which should be fulfilled accurately: the equilibrium condition, the condition that the horizontal stresses at point n should be zero, and the condition that the width of the opening at the lower end of the sheet pile should also be zero.

Taking partial derivatives from this sum over the unknown coefficients q_i and d_i and equating these derivatives to zero we obtain a system of simultaneous equations for determining the unknowns. We have to preassign the value of n and, by solving a number of systems for different values of n, to choose the one at which the width of the opening at the point $\xi = n$ is zero, negative values of the width being absent. When taking into account the thickness of the sheet pile the width of the opening at the point $\xi = n$ should be equal to this thickness. The work is performed on electronic computers.

The analysis should be made for the sum of both loads, P and M, simultaneously, since the law of independent action of the forces is not fulfilled any longer when the opening is taken into account.

If only a system of distributed vertical and horizontal forces is applied to a segment, we achieve the effect of taking into account, the appearance of a hair crack in the base without a structure inserted in it.

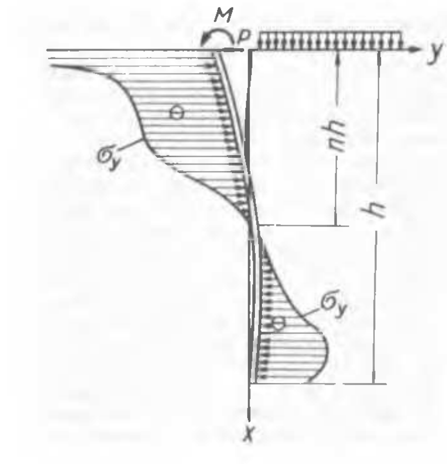

Fig.3. Results of analysis of a rigid wall: (a) curve of horizontal pressures on the wall; (b) curve of bending moments

Fig.3 shows the results of an analysis of a rigid wall subjected to the action of a force P = 5 t/m at the surface of the ground No mound is present. The depth of embedding h = 2m, the unit weight of soil γ = 1.6 t/m^3, ξ_o = 0.54. It was established by trial and error that n = 0,71. The moments are referred to a section 0.4 m wide.

The solution fo anchor plates was performed by L.N.Repnikov. If a plate is embedded parallel to the horizontal surface of the ground, has the shape of an elongated rectangle and is being pulled by a vertical symmetrical load, analysis is made in conditions of a two-dimensional problem of the theory of elasticity. For smooth rigid plate the conditions are imposed that vertical displacements at the upper boundary be constant, and no vertical normal stresses be present at the lower boundary; besides, there should be no tangential stresses to either side of the slot. The summary area of the diagram of normal stresses at the upper boundary of the slot should be equal to the pulling force. To fulfil these boundary conditions, fictitious loads from vertical and horizontal forces, as well as from vertical double forces, are distributed along the opening according to polynomial laws For small depths of plate embedding use is made of Melan's formulas, for large depths similar formulas for a continuous half-plane (Timoshenko and Goodyear, 1951) are used. For the case of a plate embedded at a depth equal to half-width of the plate the results of the solution through fourth-power polynomials are given in fig.4.

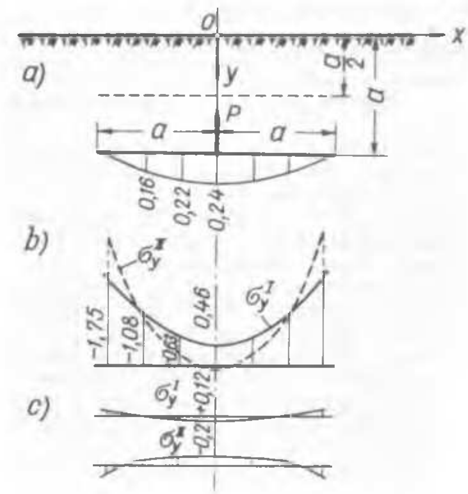

Fig.4. Analysis of an anchor plate is conditions of plane strain (Poisson's ratio of the soil ν = 1/3): (a) layout diagram; I - at a depth equal to half-width; II - at a depth equal to quarter-width. For case I the width of the slot is shown in fractions P/E; (b) diagrams of reactive pressures for cases I and II in fractions of their mean value, p_m; (c) diagrams of stresses in fractions of p_m for cases I and II at the lower side of the slot, serving as criteria of the accuracy of the solution.

Two cases of embedding are discussed: I - at a depth equal to quarter-width of the plate and II - at a depth equal to half-width of the plate (fig.4a). It can be seen from fig.4b that the closer the plate to the surface, the more the reactive pressures concentrate towards the edges of the plate. Figure 4c shows the design values of the vertical stresses along the lower boundary of the slot. Their true value is zero, therefore the diagrams serve as the evaluation of the general accuracy of the solution.

In solving a three-dimensional problem on pulling a round horizontal anchor use is made of Mindlin's formula for small depths of embedding and of Kelvin's formula for large depths (Timoshenko and Goodier, 1951).

Analysis of round-section driven piles for a vertical load is made with the aid of Mindlin's solutions for a continuous half-space, but as regards the half-space occupied by the pile we apply a fictitious load which provides the necessary boundary conditions on the surface of the pile. For driven piles, these conditions are:

(I) Under the assumption of complete adhesion between the piles and soil

a) equality of displacements of any point of the surface and end-face of the pile to the displacements of the contacting point of the base ($W_p = W_B$), and

b) displacement of soil particles which before the driving of the pile, had been located at the axis at a distance equal to the pile radius ($U = r_o$).

(2) If in solving the problem according to the first version of the boundary conditions the inequality $\tau_{rz} > \sigma_r tg\varphi + C$ (where φ - friction angle and c adhesion between the pile and soil, or between the soil jacket and the soil) obtains at individual areas of the pile surface, in refined calculations one should take into account the slipping of the pile relative to the soil at these portions, and the condition $W_p = W_s$ should be replaced here by the conditions $\tau_{zr} = \sigma_r tg\varphi + C$. At the other portions the condition $W_p = W_B$ remains valid. The precise position of the boundaries of the areas is found by successive approximations. With this formulation of the problem the relationship between the load on the pile and its subsidence ceases to be linear (Gorbunov-Possadov and Sivtsova, 1966).

When solving the problem in any of these formulations we apply, along the entire length of the pile, a frictitious vertical load distributed according to the polynomial law with unknown coefficients, similarly to the manner the horizontal load was applied in analysing the sheet pile wall. In additions, a load of unknown intensity q distributed uniformly over the surface area of the end-face is applied at the point of the pile.

This solution of the problem makes it possible to establish separately which part of the external load is transmitted to the soil by the surface of the pile and which by its point.

A second boundary conditions on the surface is the constant value of the horizontal displacement of the soil. For driven piles one should make allowance for the fact that pile driving is accompanied by the displacement of the soil away from the axis of the cylindrical volume filled by the pile after the completion of the driving, towards its surface. This effect can be obtained by using a cylindrical vector field of radial double forces. This field cannot be uniform, because otherwise a break in the continuity of the medium would occur at the boundary of the point being displaced. To avoid a break, the double forces at the cylinder surface should vanish. As a first approximation we used the distribution of double forces of constant intensity along the vertical, which, however, obeyed the triangle law (with a zero ordinate at the edge) along the horizontal (Gorbunov-Possadov, 1968). Tentative calculations showed that transition from a three-dimensional axisymmetrical problem to the conditions of plane strain is possible by using the stresses from double forces in the continuous plane of S.P.Timoshenko as starting formulas (Timoshenko and Goodyear, 1951). It was found that in this case the displacement of the soil particles which had previously been located near the cylinder edge was equal to one third of its radius. An accurate law of the distribution of double forces can be obtained by proceeding from the requirement of the minimum displacement of the extreme points, which is associated with the condition of the minimum of work of the whole soil massif during displacements. Thus, the precise value of the displacement of the extreme points should be less than one-third of the radius, this being in agreement with the experimental data on the thickness of the soil jacket.

It is not necessary to solve the problem in this rigorous formulation for taking into account the solidification of the soil associated with the driving of the piles, and the corresponding increase in friction along the surface of the piles in the group. It is quite permissible to use the Saint-Venant principle for replacing the cylindrical vector field by a system of pairs of mutually perpendicular continuously distributed along the pile axis, provided that the intensity of these forces is equal to the sum of volumetric double forces at each cross section of the pile.

Now we proceed to analysing an arched structure daaply embedded in a non-rocky or half-rock ground operating in an elastic and elastic-plastic stage

A monolithic or assembled monolith-like structure erected in a soft soil is characterised by the following features: reinforcement of the roof which always has curvilinear outlines; rigid walls, whose strains may be neglected; the presence of a trough which may be flat (its weight together with preparation counter balances the pressure from the side of the pit foot), or may be assumed to have the shape of an inverted arch.

Such a structure usually consists of the following basic elements: the upper elastic arch (II), the lateral rigid walls (I), and the lower flat (IV) or arched trough (III) (fig.5).

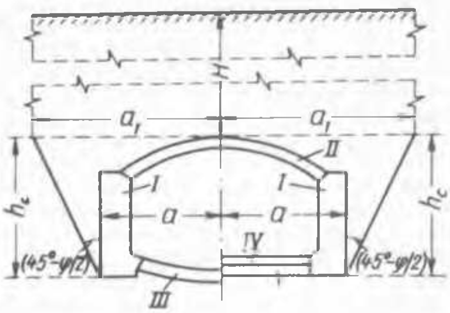

Fig.5. Elements of a structure in a non-rocky ground.

By deep embedding we mean such embedding where the rock pressure on the structure does not depend on it any longer. According to K.Tertsagi this will take place at (fig.5):

$$H \geqslant 5a_1$$

$$a_1 = a + h_c\, tg(45° - \varphi/2) \qquad (7)$$

where H - is the thickness of the ground above the pit, a_1 - the size of the jacketing.

An underground structure involves in its work a certain part of the surrounding ground, resulting in the formation of a compressed layer of ground whose thickness is determined by the methods of soil mechanics.

In 1934-35 the author for the first time suggested and published a method for analysis of an underground structure as a statically indeterminate system operating in an elastic medium, obeying the Winkler hypothesis. This method has gained wide acceptance, however it was based on indeterminancy depending on the bedding value of the ground.

Indeed, let us assume that two monolithic structures, one with a large span and the other with a small one are erected in the same ground (fig.6). The structure

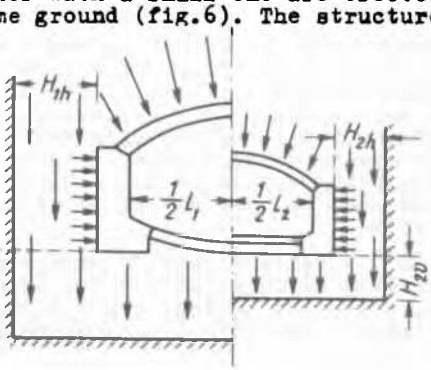

Fig. 6. Compressed layer of an arched structure with a large and a small spans.

with a large span forms a compressed layer of considerable thickness, $H_{1,h}$ and $H_{1,v}$, the structure with a small span - a thinner layer, $H_{2,h}$ and $H_{2,v}$. Assigning a physical meaning to the Winkler hypothesis and comparing it with Hooke's law, we get

$$\sigma = Ky\,, \quad \sigma = E\varepsilon = E\frac{y}{H} = Ky \qquad (8)$$

As a result we find the bedding value:

$$K = E/H \qquad (9)$$

For our cases we obtain:

$$K_1 = E_0/H_1\,; \quad K_2 = E_0/H_2 > K_1 \qquad (10)$$

Taking into account that the soil is the same in both cases and has a modulus of deformation of E_0^h and E_0^v, we see that the bedding value depends on the structure and is not objective characteristic of the soil.

For this reason methods of analysing which are based on the use of the bedding value cannot evaluate the stresses and deformed state of the system.

In 1939 a method was published which used the theory of elasticity in analysis of underground structures, this eliminating the above-mentioned indeterminacy. Simultaneously a solution for an elastic layer was given which was free from the assumption previously introduced by Melan.

According to the author's solution (Davydov, 1950) the displacements of the elastic layer, in conditions of a two-dimensional problem, multiplied by $\frac{\pi E_0}{1-\nu_0^2}$ are equal to (fig.7):

$$y = T + \Pi \qquad (11)$$

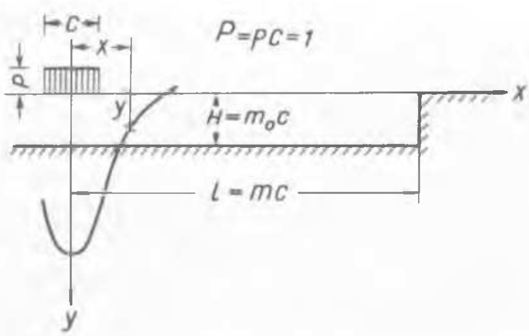

Fig.7. Settlements of the surface of an elastic layer

where

$$T = \frac{2m}{\pi}\sum_1^\infty \frac{1}{n^2}\left\{\sin\left[\frac{n\pi}{2m}\left(2\frac{x}{c}+1\right)\right] - \sin\left[\frac{n\pi}{2m}\left(2\frac{x}{c}-1\right)\right]\right\} \qquad (12)$$

$$\Pi=\frac{4m}{\pi}\cdot\sum_{1}^{\infty}\frac{1}{n^2}\cdot\sin\left[\frac{n\cdot\pi}{2m}\left(2\frac{x}{c}-1\right)\right]\cdot K \tag{13}$$

$$K=\frac{(3-4\nu)\cdot\eta-\alpha H(\eta^2-1)}{4\eta^2(1-\nu_o)^2-(1-2\nu_o)^2-(\alpha H)(\eta^2-1)} \tag{14}$$

$$\eta=cth(\alpha H);\ \alpha=\frac{n\cdot\pi}{l} \tag{15}$$

Tables of settlements of an elastic layer calculated by formula (11) at l = 10 c and H = m_0c = c, 2c, 3c, 4c, and 5c have been compiled. Previously (Melan, 1919) the problem of the elastic layer had been solved under the assumption that at the basic of the layer the shear stresses are zero, i.e.

$$y=H,\quad X_y=Y_x=0 \tag{16}$$

This assumption enabled Melan to replace the elastic layer by a plate counterbalanced by two forces and thus made it impossible to take into account the specific features of the elastic layer. In the author's solution there is no such assumption, and the displacements along the lower and lateral boundaries of the layer are taken to be zero. Comparing both solutions for the case H = c and l = 1oc we will find the settlements of the surface of the elastic layer which are shown in fig.7. The difference in settlements is considerable and exceeds 25% for their maximum values. Settlements after Melan are naturally always greater than the actual settlements of the surface of the elastic layer.

The height of the elastic layer H will be found from the condition

$$\sigma_{const} = 1,2\sigma \tag{17}$$

i.e. that the stressesat the foot of the elastic layer at y = H caused by the structure should not exceed the stresses at this point which acted before the structure had been erected, by more than 20%.

In the elastic stage of operation the basic system of a monolithic underground structure according to the author's solution is such as shown in fig.8.

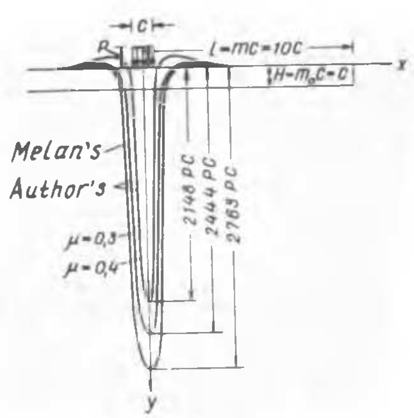

Fig.8. Settlement of the surface of an elastic layer multiplied by $\frac{\pi E_o}{1-\nu_o^2}$

The vertical and horizontal rock pressures, as well as the pressure from the side of the pit foot, is determined by methods of rock mechanics.

The horizontal components of the volumetric forces of the soil which act on the wall are calculated by the formula

$$\mathcal{E}_y=\frac{\nu_o}{1-\nu_o}\,\gamma H_y \tag{18}$$

where H_y is the distance of the cross section from the top of the soil pressure arch. The resultant of these forces - E_w counterbalances part of the structure thrust.

The elastic effect of soft soil on the wall is replaced by point forces, the number of which along each plane of the wall should be taken at least as 4 as indicated by investigations. In practice, a fairly accurate solution can be obtained with 5 forces X_i and Y_i. This figure is assumed in further calculations.

The basic system, when symmetrical relative to the vertical axis (the basic case), will have 17 unknowns - 4 stresses M_1 and H_1; 10 forces of soil counterthrust X_i and Y_i; the force of friction along the foot X and 2 - displacement at the point of embedding - φ_o and y_o . In the absence of symmetry the system has 32 unknowns. A wall I embedded in several different soils is made sufficiently rigid and its own strains may be neglected; All the unknowns of a symmetrical system can be expressed as a function of the rotation angle φ_o (Davydov, 1950). We have (fig.9):

$$Q_3 = Q_3^* = 0 \tag{19}$$

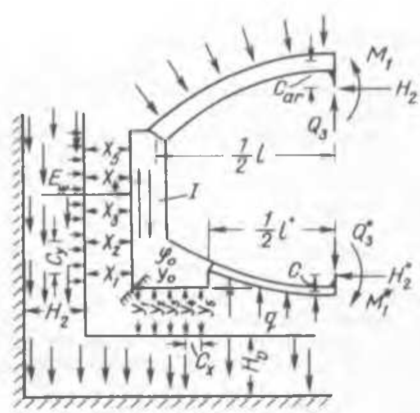

Fig.9. Basic system of monolithic underground structure

The stresses in the elastic centre of the upper arch will be:

$$M_1=M_1^\circ+A_1\varphi_o;\ H_2=H_2^\circ+A_2\varphi_o \tag{20}$$

For the lower arch we obtain:

$$M_1^{\bullet}=M_1^{o\bullet}+A_1^{\bullet}\varphi_o\,;\quad H_2^{\bullet}=H_2^{o\bullet}+A_2^{\bullet}\varphi_o \qquad (21)$$

Here M_1^o and H_1^o - moment and thrust for an arch with embedded abutments;

A_i - coefficients depending on the geometrical dimensions of the structure.

The stresse from the elastic counter-thrust of the ground along the lateral surface of the wall are found from the solution of simultaneous equations of the type

$$a_{\kappa 5}X_5+a_{\kappa 4}X_4+\ldots+a_{\kappa 1}X_1+(\kappa-0{,}5)c_y\varphi_o=0 \qquad (22)$$

Unit displacements are determined from the expression:

$$a_{\kappa y}=y_{\kappa i} \qquad (23)$$

where $y_{\kappa i}$ - tabulated settlements of the surface of the elastic layer.

By solving there simultaneous equations we obtain

$$X_i=\chi_i h_y\varphi_o \qquad (24)$$

where χ_i = numerical coefficient.

The elastic counter-thrust along the foot of the wall is found from the solution of the system

$$a_{\kappa 5}Y_5+a_{\kappa 4}Y_4+\ldots+a_{\kappa 1}Y_1-Y_o-(\kappa-0{,}5)c_x\varphi_o=0$$
$$-Y_5-Y_4-Y_3-Y_2-Y_1+Q_w=0 \qquad (25)$$

The last given equation is a supplementary one. Unit displacements are also determined from (23).

Solving this simultaneous equation we obtain:

$$Y_i=\lambda_i h_x\varphi_o+\delta_i Q_w \qquad (26)$$

Here λ_i and δ_i = numerical coefficient,

$$Q_w=\sum_1^5 Y_i=Q_w^o-\mu_1\sum_1^5 X_i$$

where Q_w^1 - vertical component of all known forces,

μ - coefficient of friction between the soil and the wall.

The force of friction along the foot of the wall is equal to

$$X=\sum_1^5 X_i-H_w^o \qquad (27)$$

where H_w^o - horizontal component of all known force.

Thus, the solution of the system is reduced to the determination of one unknown - the angle of rotation of the wall - φ_o which we find from the condition

$$\Sigma M=0$$

Solving tnis equation we obtain

$$\varphi_o=\frac{2M_w^o+hQ_w^o}{\alpha_1 h_y^2+\alpha_2 h_x^2+\mu_1\alpha h_x h_y+2A} \qquad (28)$$

where

$$A_o=A_1+\kappa A_2+A_1^{\bullet}+\kappa_2 A_2^{\bullet} \qquad (29)$$

M_w^o = moment of all known forces relative to embedding,

α_i and κ_i = numerical coefficients.

The values of the coefficients and L_1 have been calculated by the author and are tabulated (Davydov, 1950). A diagram of the counter-thrust of a soil operating in the elastic stage is shown in fig. 10.

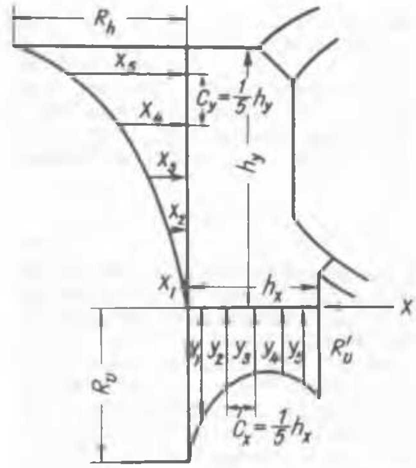

Fig.10. Diagram of elastic counter-thrust of soil

<u>In the elastic-plastic stage</u> of soil operation which develops in time, diagrams of soil counter-thrust acting on the wall will change their shape.

This will take place at considerable pressures of the wall on the soil, R_h and R_v, causing structural changes in the soil, as a result of which plastic strains develop. We will denote by R_h and R_v the limiting values of these pressures which correspond to the linearly deformed state of the soil. The basic system of the structure (fig.9) retains its nature.

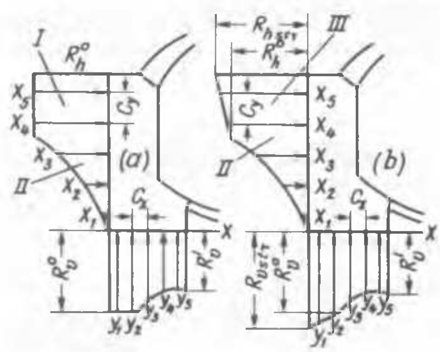

Fig.11. Diagrams of elastic-plastic counter-thrust: (a) without and (b) with allowance for soil solidification

Depending on the difference in pressures

$$\Delta R_h=R_h-R_h^o\,,\quad \Delta R_v=R_v-R_v^o \qquad (30)$$

Diagram of elastic-plastic counter-thrust of the soil will take the shape displayed in fig.11(a). In this case the forces X_i and Y_i which are in the plastic zone, will be known and the corresponding equations of the form (22, 25) will convert into identities.

Solving the remaining equations we will find forces X_i and Y_i of the elastic zone of soil operation.

From the equilibrium equation M_i we find the angle of rotation of the wall and calculate all stresses of our basic strcture system. If we take into account soil solidification in the course of development of plastic deformation we will obtain the soil counter-thrust diagrams given in fig. 11(b).

Having Prandtl's diagram for a given soil and allowing for its solidification, we find $R_{h\,str}$ and $R_{v\,str}$ corresponding to wall strains at its angle of rotation of φ_0. In this case the values of the stresses, X_i and Y_i, which are in the plastic zone, will change, but the general principle of the problem solution remains.

Other values of rock pressure will also correspond to the elastic-plastic stage of structure operation. The vertical and horizontal pressures of the soil on the structure should in this case be multiplied by the coefficient (Davydov, 1954)

$$n = \frac{h_{max}}{h_1} \qquad (31)$$

Here

$$h_{max} = \frac{S_{max}}{p}\ ; \quad h_1 = \frac{a_1}{f_h} \qquad (32)$$

where S_{max} = limiting settlement of underground structure

p = loosening coefficient of soft soil

f_h = soil hardness coefficient after Protodyakonov (Davydov, 1950)

a_1 = dimension shown in fig.5.

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