STABILITY OF SLOPES WITH CURVATURE IN PLANE VIEW

STABILITE DES PENTES AVEC TRACE COURBE EN PLAN

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SYNOPSIS Axially symmetrical vertical cuts are studied by means of a mathematical model of soil in order to acquire a quantitative knowledge of their behavior. Soil is assumed to be perfectly elasto-plastic, its strength is only determined by its cohesion as measured in an unconfined compression test. The failure criterion is that of von Mises. Two tentative laws of simple practical use are proposed for the relationship between the inverse of the stability factor and the ratio of the horizontal curvature radius to the height of the vertical slope, depending upon the concavity or convexity of the trace of the cut in said horizontal plane.

INTRODUCTION

The use of finite difference methods, applied to lumped parameter models, makes possible the quantitative interpretation of many questions whose behavior were only qualitatively understood. This is the case, for instance, of slopes with curvature in a horizontal plane. It seems experimentally proved that convex slopes fail at lower heights than concave slopes do; however, there is no definite relationship which permits a practical application of that fact.

Using the lumped parameter model developped by Ang & Harper [1], arranged for axially symmetrical conditions, several cases of circular elevations as well as excavations, with vertical slopes (i.e.,convex and concave vertical slopes) have been studied. The soil has been supposed to be homogeneous, isotropic and frictionless, obeying Hooke's law in the elastic region and yielding according to the von Mises criterion of failure, i.e.:

$$\sqrt{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_1-\sigma_3)^2} = \text{constant} \quad (1)$$

which may be rewritten

$$\sqrt{J_1^2 - 3J_2} = q_u \quad (2)$$

wherein J_1 is the first invariant of stresses

$$J_1 = \sigma_x+\sigma_y+\sigma_\theta \quad (3)$$

J_2 is the second invariant

$$J_2 = \sigma_x\sigma_y+\sigma_y\sigma_\theta+\sigma_\theta\sigma_x-\tau_{xy}^2 \quad (4)$$

and q_u is the unconfined compressive strength of the material. After the yield point is reached, stresses and strains are no longer uniquelly related but depend upon the loading history.

PROCEDURE

The Ang & Harper model (Fig. 1) consists of a grid of mass-points related by stress-points. Mass forces and displacements are defined at mass-points, whereas stresses and strains are only defined at stress-points. Strains are computed from the displacements of the four surrounding mass-points; using the correspondent set of equations according to the state of the stress-point (i.e., elastic or plastic), stresses are computed from those strains and, in turn, each mass-point equilibrated under the action of the internal forces developped at the four surrounding stress-points (which are simply equal to the correspondent components of the stress tensor multiplied by the appropiate fraction of the grid spacing) and its weight. In this way, at any stage of loading, the compatibility conditions are met both in terms of stresses and strains throughout the model and therefore, if the load is monotonically applied and the geometry of the model unchanged, the uniqueness of the solution is assured a priori. The plastic stress-strain relationships are derived according to the normality rule: the increments of plastic strain are normal to the failure surface and can be expressed as directly proportional to the partial derivatives of the failure function with respect to the corresponding stresses.For a perfectly plastic material, this rule implies that there is no plastic volume change during shear. The proportionality constant between partial derivatives and increments of plastic strain may be obtained from the energy produced during plastic flow, and therefore the instantaneous increments of stresses can be expressed as functions of the instantaneous increments of strains and the stresses acting in the preceding level of loading. This is, actually, a linearization of the differential equation governing the plastic behavior of the material.

In this way, the resulting set of equations is analogous to that derived by Prandtl & Reuss [2] for the perfectly plastic material under plane strain conditions.

As an excavation represents an extraction of mass, the geometry of the model has to be changed, and therefore the uniqueness of the solution cannot be guarantied. To avoid this problem, the increase of the excavation depth has been represented by an increase of the unit weight of the material, i.e., the excavation is assumed to have been previously opened in a weightless soil to its final depth and thereafter the unit weight of this soil is successively increased until failure occurs.

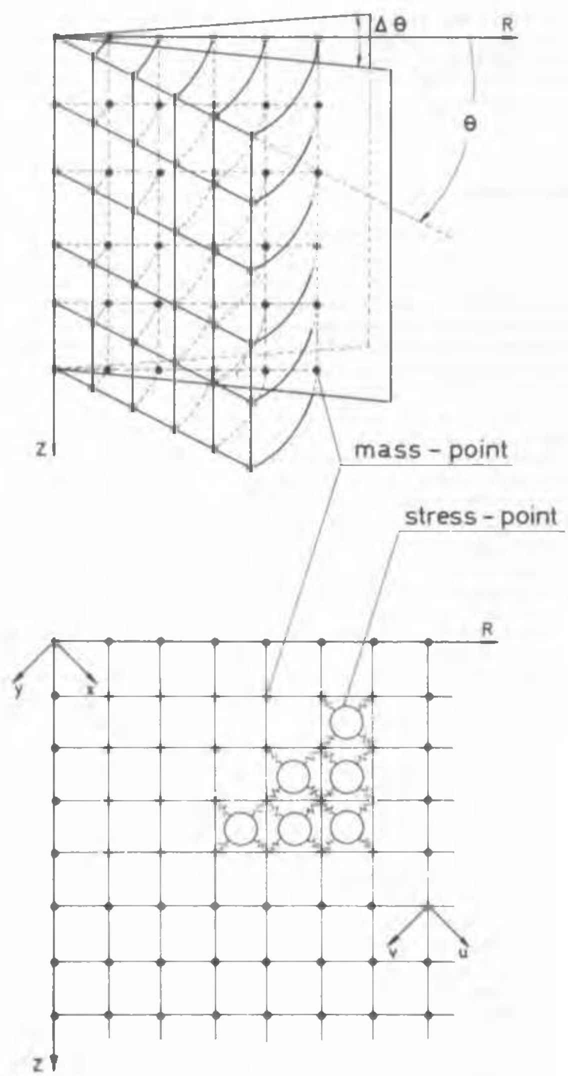

Fig. 1 The lumped parameter model for axially symmetrical conditions.

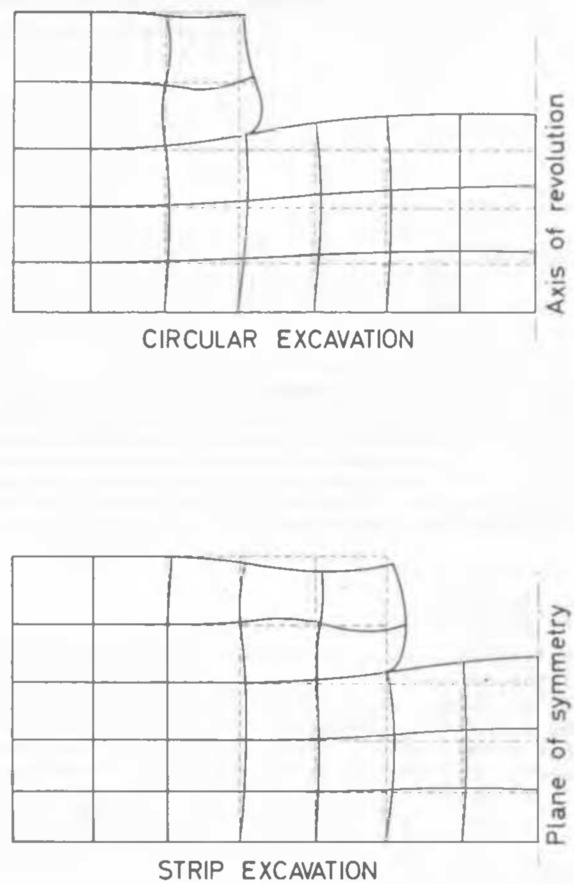

Fig. 2 Deformations of a particular model at failure (concave slope) as compared with those of a strip excavation.

It should be pointed out that, as all computations have been carried out in terms of dimensionless quantities, which is possible when no fixed external forces are applied, as in the present study, the plastic zone is independent from the unit weight, the shear modulus and the cohesion of the material, being only a function of the Poisson's ratio.

As a result of the use of the von Mises criterion of failure, first yielding will allways develop at the bottom of the model. As a matter of fact, with increasing depth, (so that the effect of the presence of the cut is negligeable and $\sigma_2 = \sigma_3$) the von Mises criterion coincides with that of Tresca, i.e., failure is determined by the difference of the principal stresses, difference which increases with depth unless the earth pressure coefficient at rest, k_o, equals one, and this is not the case, since by hypothesis

$$k_o = \frac{\nu}{1-\nu} \qquad (5)$$

and Poisson's ratio ν has been taken equal to 0.25 in all studied examples. To avoid this difficulty, the shear strength has been assumed to increase linearly with depth from the surface of the excavation.

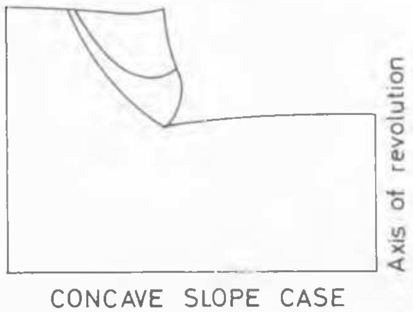

Fig. 3 Typical plastic zones at failure.

Different heights of excavation (or elevation) and different radii of horizontal curvature of the vertical slope have been considered. The bottom boundary has been considered at such a depth, and the left boundary at such a distance from the symmetric axis, that the presence of the excavation (or elevation) would not appreciably affect their behavior. Therefore, nor horizontal neither vertical movements are allowed at the bottom boundary; left boundary moves only vertically like a layer of soil resting on a hard stratum. The upper boundaries are assumed to be free, as well as the wall of the vertical cut. The right boundary has been the axis of symmetry.

(The calculations involved in the resolution of these problems were carried out on the IBM 1130 computer of the Entrecanales y Távora firm.)

RESULTS

The obtained results are represented in Figures (2), (3) and (4).

Fig. (2) shows the deformations of a particular model (circular excavation) at failure as compared with the results obtained for a plane strain situation [3], it can be seen that the horizontal and vertical displacements of the top of the cut are lower in this case of concave slope than were in the case of the strip excavation. (Although not shown, in the case of a convex vertical cut those displacements were greater.)

Fig. (3) shows the typical plastic zones at failure for the two cases: concave and convex vertical slopes.

Finally, Fig. (4) shows the values of the inverse of the stability factor obtained from the computer versus the ratio of the radius of horizontal curvature to the height of the slope as compared with the simple hyperbolic relationships tentatively proposed.

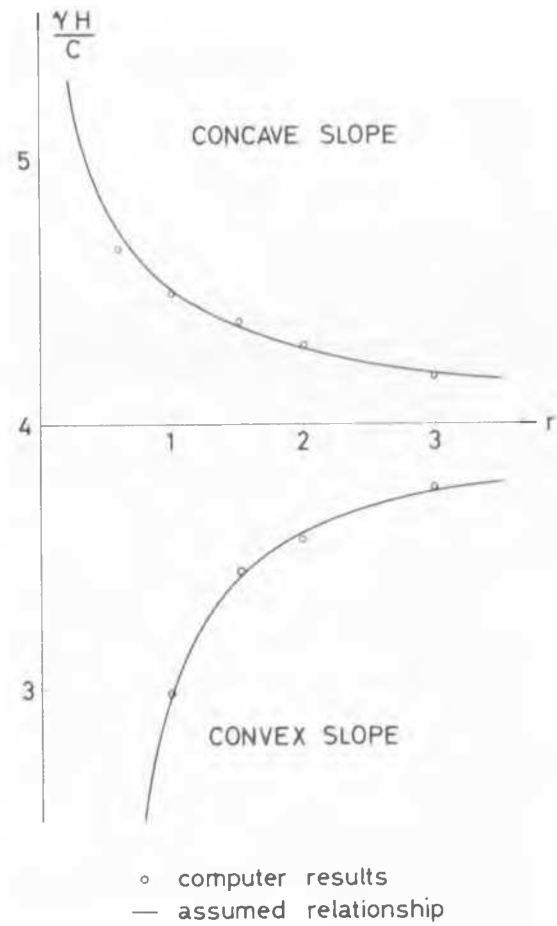

Fig. 4 Inverse of the stability factor versus the ratio of horizontal curvature to the height of the vertical slope as compared with the tentatively proposed hyperbolic laws.

CONCLUSIONS

Plotting the obtained results in a system of ortogonal coordinates whose axis are the inverse of the stability factor:

$$f = \frac{\gamma H}{c} \qquad (6)$$

and the ratio between the curvature radius and the height of the slope:

$$r = \frac{R}{H} \qquad (7)$$

wherein

γ = unit weight
H = height of the vertical slope
c = cohesion, equals one half of the unconfined compressive strength
R = radius of horizontal curvature

it can be seen that these results coincide very approximatively with the simple hyperbolic laws:

$$f = 4 + \frac{2}{3r+1} \qquad (8)$$

for concave vertical slopes and

$$f = 4 - \frac{2}{3r-1} \qquad (9)$$

for convex vertical slopes.

The first law (equation (8)) has been tested for values of $r \geq 0.6$, whereas the second law (equation (9)) has been tested for $r \geq 1$. In the case of concave cuts, it would have been required a finer net to obtain reasonably accurate values of f for $r < 0.6$, but such a grid would exceed the capacity of the used computer. In the case of convex cuts, for $r<1$ the plastic flow reaches the axis of symmetry before it can fully develop to the surface, creating a tensional zone around that axis which will cause a failure by the combined action of shear and a tension crack along said axis of symmetry. At present, this type of failure has not been analyzed, since tension cracks would involve problems of discontinuities and geometric changes.

In the case of concave slopes, the plastic wedge, beginning at the toe of the cut, seems to develop toward the surface of the model making an angle less than $\pi/4$ with the vertical, whereas for convex slopes this angle seems to be greater than $\pi/4$. For the strip excavation, this same angle was approximatively $\pi/4$.

REFERENCES

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[3] "Plastic Flow behind a Vertical Excavation", by C. Lorente de Nó, Thesis for the Master degree at the Massachusetts Institute of Technology, June 1966.