STRENGTH ANISOTROPY ON SLOPE STABILITY AND BEARING CAPACITY OF CLAYS

L'ANISOTROPIE DE LA RESISTANCE SUR LA STABILITE DES PENTES ET LA FORCE PORTANTE DES ARGILES

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ABSTRACT Though it is well recognised that the strength of clays in nature varies with depth and direction of failure surface, there is paucity of experimental data to define even the nature of their interrelation and most of the analyses of stability problems ignore these aspects. This study consists of three phases. First is the experimental study to determine the strength on different failure planes using direct shear tests, the results of which have been used to confirm the rational hypothesis postulated for the functional relationship between strength and depth and direction of failure surface. Secondly, the influence of a useful range of values of the coefficients defining the anisotropy and strength increase with depth on bearing capacity of shallow strip foundation has been assessed numerically with the aid of a computer. Thirdly, it has been reasoned out that the stability analysis of a slope is best done in terms of a control chart which is defined to be a curve providing critical combination of stability number, N and coefficient of internal friction tan ϕ_{ho}. Numerical results show the control charts to be governed by the height and inclination of slope, the nature of strength variation and depth to hard surface and not by the actual magnitude of the strength.

INTRODUCTION

It is a matter of choice whether to adopt a rational but approximate method which can account for the complex and variable nature of the soil properties, or to adopt an exact analytical solution employing a simplified and idealized mathematical model of the soil. The idealized solution by the theory of plasticity provides for static equilibrium at all points in a failing mass whereas the approximate solution by assuming a single failure zone satisfies only the gross static equilibrium of the sliding mass. The assumption of a plausible failure surface will no doubt result in an upper bound solution which when minimized is known from principles of limit analysis to yield results close enough to those obtained by exact solutions.

When estimating the stability of foundations and slopes, it is often assumed that the soil is homogeneous and isotropic. But it is known that the shear strength increases with depth beyond the zone of desiccation and also that it is dependent on the direction of the failure surface. While it may be difficult to define the exact functional relationship between the shear strength and depth and direction of failure surface, this investigation will be concerned with: (i) obtaining a probable variation of the same from a carefully planned laboratory experimental study; and (ii) to use these findings to evaluate the influence of a practical range of such variations on slope stability and bearing capacity of clays on the basis of conventional method of analysis.

EXPERIMENTAL INVESTIGATION

As already stated the experimental work is to obtain the variation in shear strength (i) with direction of failure surface, keeping consolidation pressure constant, and (ii) with consolidation pressure, keeping the direction of failure surface constant. Also studied are: (i) the influence of trace additives of chemicals which will affect the initial structural state of the soil, and (ii) the effect of overconsolidation. Since the strength is to be determined on a predetermined failure plane, direct shear test is the obvious choice. Test samples have been obtained by consolidating soil slurry to a predetermined stress level in a big mould designed to reasonably simulate sedimentary deposits. Block sample so consolidated is transferred to a tilting frame which can be fixed in any position between vertical and horizontal, and a square mould of 6 cm. internal side is pushed vertically into the sample so that any predetermined inclination can be obtained between the failure surface and the direction of sedimentation.

The index properties of clay concentrated black cotton soil used in this study are:

L.L = 224 per cent, PL = 70 per cent, SL = 10 per cent.

Figure 1 reports the variation of undrained strength, C_u given by the radial distance, with the direction of failure surface, θ, for a sample normally consolidated to a typical pressure of 30 psi., each curve in it being for a particular type of pore fluid, viz., sodium oxalate treated, untreated and lime treated. While the experimental values are shown by points, the smooth curves are those given by the relationship:

$$C_\theta = C_h\,(\cos^2\theta + n\,\sin^2\theta) \quad \ldots (1)$$

where n is the ratio of the experimentally determined undrained strengths on vertical and horizontal failure surfaces(i.e., C_v/C_h). From [illegible] can be seen that Equation (1) (physical basis of which is given in ref.3) is a reasonably good representation of actual behaviour. With sodium oxalate the particles tend to get better oriented due to an increase in the repulsive force resulting in a greater anisotropic effect and a reduction in the overall strength level. When the soil is treated with lime (flocculent) there is an increase in the attractive force which resists deviation from the random structure resulting in a more or less isotropic condition and which also increases the overall strength level. Figure 2 summarizes the experimental values of strength anisotropy with different consolidation pressures. For the range of pressures tested, it is seen that lime treated soil is more or less isotropic whereas untreated and sodium oxalate treated soils exhibit anisotropy, the magnitude of which changes from a value greater than unity to one lower than unity. However, what is not brought out in the figure is the test result that lime treated soil has slightly greater strength on a failure plane inclined at about 45°. Also shown as dotted lines in the same figure is the variation in strength anisotropy values of over-consolidated clays (with and without sodium oxalate as additives) all of which have been originally consolidated to three times the effective normal stress at which they have been sheared. It is seen that overconsolidation enhances the degree of strength anisotropy both in magnitude and in its variation with effective stress.

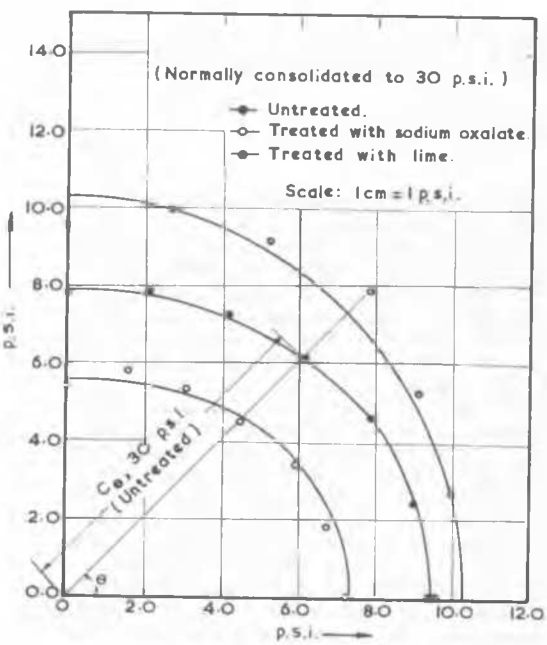

FIG. 1: UNDRAINED SHEAR STRENGTH ON DIFFERENT FAILURE SURFACES.

The same results have been used to relate the undrained shear strength either on horizontal or vertical failure surface with consolidation pressure in Figure 3. These results, while confirming the findings of earlier researchers in that undrained strength will increase almost linearly with consolidation pressure, focus attention that it is so when the direction of failure surface remains constant. These results also emphasize that except for lime treated soil, the variations in C_v and C_h with consolidation pressure follow distinctly different lines. It is thus logical to define the shear strength at any depth in relation to the direction of failure surface in the following manner:

$$C_{hz} = C_{ho}\left(1 + l_h \frac{z}{H}\right) \quad \ldots (2a)$$

$$C_{vz} = C_{vo}\left(1 + l_v \frac{z}{H}\right) = n_{co}\, C_{ho}\left(1 + l_v \frac{z}{H}\right) \quad \ldots (2b)$$

where l_v and l_h respectively define distinctly different coefficients of variation in C_v and C_h over a significant depth H. This form of defining the variation over a significant depth, such as the height of slope (H) in slope stability problems or half the width (b) in bearing capacity problems, helps to non-dimensionalise the coefficients. Combining Equations (1 and 2) an expression for undrained shear strength at any depth on any failure surface inclined at an angle θ to the horizontal is obtained.

$$C_{\theta z} = C_{ho}\left(1 + l_h \frac{z}{H}\right)\cos^2\theta + n_{co}\left(1 + l_v \frac{z}{H}\right)\sin^2\theta \quad \ldots (3)$$

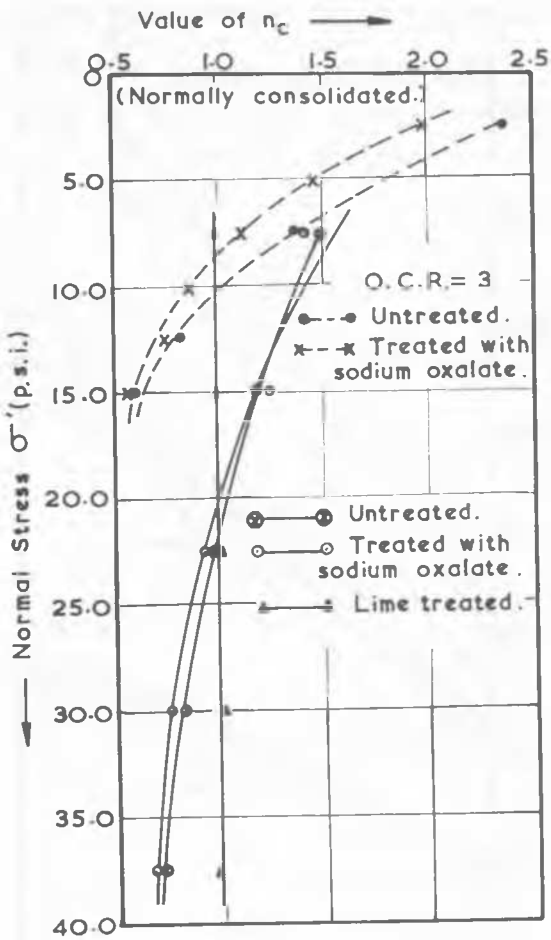

FIG. 2.

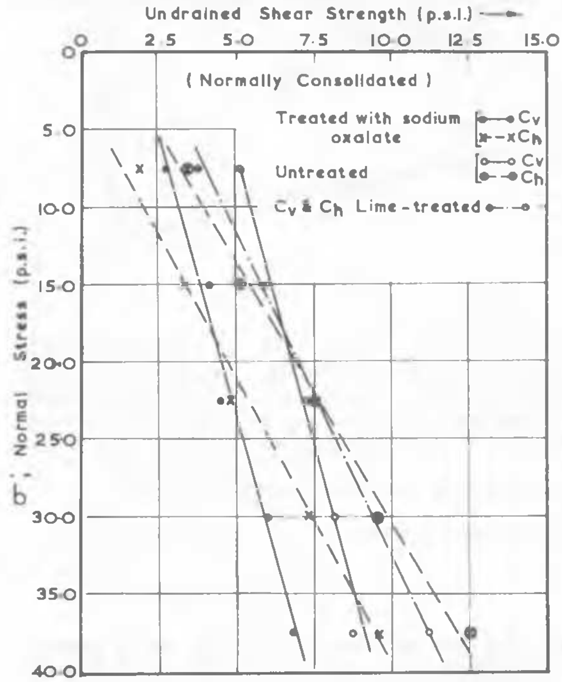

FIG. 3.

Consolidated undrained triaxial compression tests with pore pressure measurement have been carried out on 1½ in. diameter samples extracted at different inclinations with the use of tilting frame. The test results are reported in Table I which gives a maximum coefficient of anisotropy as 1.146 $\left(= \frac{\tan 33^\circ}{\tan 29.5^\circ}\right)$.

TABLE I

Inclination between sedimentation and sampling directions	0°	15°	30°	45°	60°	75°	90°
ϕ'	31°	33°	30.5°	30°	30.5°	29.5°	32°

This limited study is used only to get a qualitative assessment of the fact that the coefficient of anisotropy with respect to ϕ' will deviate rather slightly from unity. Hence the analysis incorporating the effect of anisotropy in ϕ' and that of increase in ϕ' with depth would remain an academic exercise until more extensive results warrant their use in practice.

BEARING CAPACITY FOR SHALLOW FOUNDATIONS

Bearing capacity of foundations on normally consolidated saturated clay is recognised to be critical at the end of construction owing to which the analysis is carried out in terms of undrained shear strength of the soil. While it may be difficult to determine the exact mechanism involved in a failure, a cylindrical failure surface is adopted for the analysis, since this study is primarily concerned with the effect of realistic variation in strength with depth and direction of failure surface on the ultimate bearing capacity. Figure 4 shows a typical failure surface which incidentally defines the

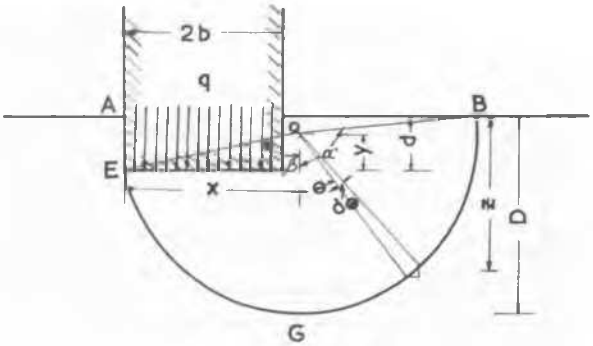

FIG. 4

various notations used. For limiting equilibrium of the mass above the potential surface of rupture AEGB, the total disturbing moment about O must be equal to the total resisting moment about the same point (Figure 4).

$$\text{Disturbing moment} = 2b\,q\,(x - b)$$

$$\text{Resisting moment} = \int_0^{\alpha+\beta} C_{\theta z} R^2\, d\theta + \int_0^d m\, C_{vz}(x-b)\,dz$$

where q is the uniform intensity of pressure at foundation level in excess of the overburden pressure.

$$C_{\theta z} = C_{ho}\left[\left(1 + l_h \tfrac{z}{b}\right)\cos^2\theta + n_{co}\left(1+l_v \tfrac{z}{b}\right)\sin^2\theta\right]$$

$$z = R\cos\theta - y + d,$$

$$\alpha = \cos^{-1}\left(\frac{y-d}{R}\right),\ \beta = \tan^{-1}\left(\frac{x}{y}\right),$$

$R^2 = x^2+y^2$, b = half width of foundation,

d = depth of foundation

x and y are the coordinates of the centre of slip circle.

wall adhesion at depth z is taken as $m\,C_{vz}$ in which the multiplying factor m will generally be around 0.5 (Skempton 1959). Equating the two moments, after integration within appropriate limits, an expression for the ratio q/C_{ho} is obtained in terms of the various constants and the variables x and y.

$$\frac{q}{C_{ho}} = F\,(x, y, n_{co}, d, l_h, l_v, m, b) \qquad \dots (4)$$

To arrive at the minimum value of q at incipient failure, the expression is, therefore, minimised with respect to x and y.

$$\frac{\partial F}{\partial x} = 0\ ; \quad \frac{\partial F}{\partial y} = 0$$

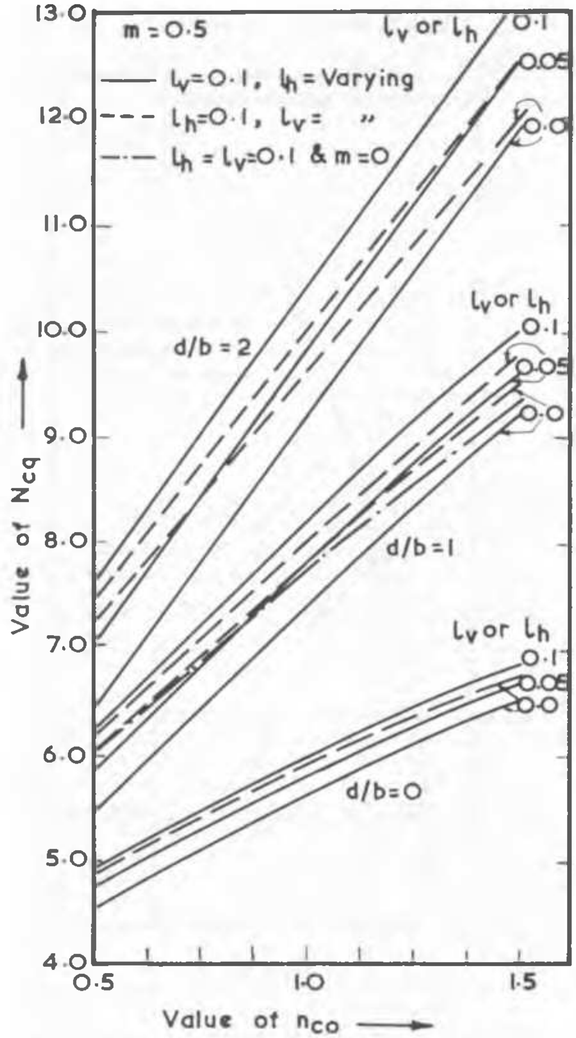

FIG. 5.

The values of x and y satisfying the above conditions are found by the use of a computer and by substituting these values in Equation (4) the ratio $\left(\frac{q}{C_{ho}}\right)_{min.} = N_{cq}$, (the bearing capacity factor for the combined effect of cohesion and surcharge) is obtained.

Figure 5 presents typical variation in N_{cq} values so computed, with strength anisotropy n_{co} deviating by 50 per cent on either side of unity (isotropic state) in terms of the following factors and their range; (i) depth factor, d/b — 0 to 2; (ii) coefficients defining the undrained strength increase respectively on the horizontal and vertical failure surfaces l_h and l_v — 0 to 0.1. Since N_{cq} by definition is related to C_{ho} an increase in n_{co} means an increase in C_{vo} which in turn increases the bearing capacity factor. These results have been used to relate the variation of N_{cq} with depth factor for the two limiting conditions of anisotropy herein studied in Figure 6. Also shown in this figure is the variation of N_{cq} with depth factor for isotropic soil to serve as a useful reference for comparison. It is evident that an increase in shear strength as well as its variation with depth on a vertical failure surface will result not only in enhanced values of N_{cq} but their increase with depth of foundation will also be more pronounced. The influence of anisotropy on N_{cq} increases with foundation depth, is a result that could not have been comprehended from those of the previous investigators. The dash and dot line in

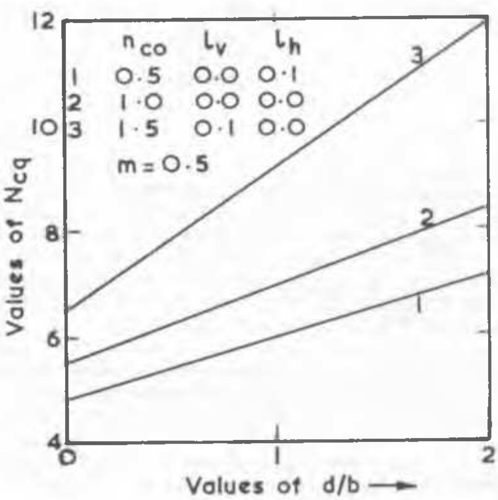

FIG. 6.

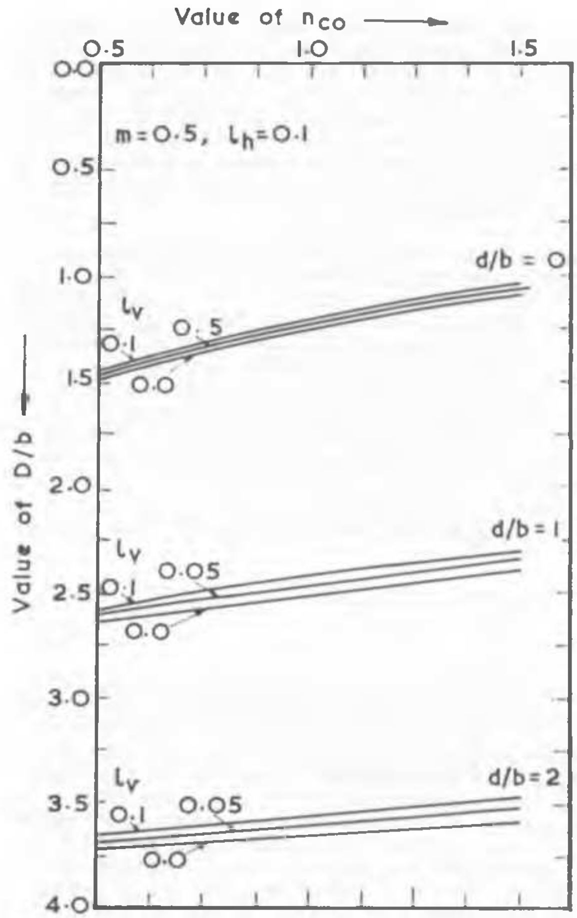

FIG. 7

Figure 5 reports the influence of ignoring wall adhesion (i.e., m = 0) instead of assuming it to be half the value of cohesion on vertical surface. As can be expected, ignoring m reduces N_{cq}, the reduction being proportional to the level of anisotropy (about 4 per cent at n_{co} = 0.5 and about 8 per cent at n_{co} = 1.5). Figure 7 reports the variation in the non-dimensional factor defining the deepest level touched by the failure surface with anisotropy for footings at surface, and for those founded at depths equal to half and full width of the foundation. A decrease in n_{co} and l_v means a decrease in C_v which in other words means the vertical plane to be relatively weaker with the tendency to push the failure surface deeper. Another result discernible from the results in Figure 7 is that the variation in the depth between the foundation level and the deepest level of the failure surface is negligible and if any, increases slightly with depth.

SLOPE STABILITY

The process of evaluating soil behaviour under various conditions of external loading (no load state either at the end of construction or after drawdown, water load and wave action during full reservoir condition) and internal stresses (effective and neutral stresses, seepage forces) together with the fact that changes take place in the constituents of the soil complex (changes in water content and/or degree of saturation, ion exchanges, deposition and/or internal erosion of fine particles) and the process of incorporating the shear strength to best represent these conditions with particular reference to the most critical combination of these at any time, are as important as that of making correct stability analysis. Because of this, a rational method of assessing the factor of safety of a slope at any time would be to relate the strength that is likely to be mobilized compatible with the then existing field conditions with a control chart which is nothing other than a curve providing the critical combinations of the stability number N (= $C_{ho}/\gamma H$) and the coefficient of internal friction tan ϕ_{ho}. It may be noted that the control chart is influenced both by the assumptions on which the method of analysis is based and by those considered regarding the strength mobilised along the failure surface.

The present study is devoted primarily to demonstrate the procedure of obtaining control charts for typical clay slopes whose shear strength varies with depth and direction of failure surface. The method follows the analysis of Janbu (1954) (based on dimensionless parameters for base failure). The shear strength $S_{z\theta}$ on an element at depth z along the failure surface inclined at θ to the horizontal can be expressed as follows:

$$S_{z\theta} = S_{zh} \cos^2\theta + S_{zv} \sin^2\theta$$

$$= (C_{zh}+\sigma_{zv} \tan \phi_{zh})\cos^2\theta + (C_{zv}+\sigma_{zh} \tan \phi_{zv}) \sin^2\theta \quad \ldots (5)$$

To facilitate elegant presentation and not to cloud the method with too many a constant, the problem has been limited to the following particular case when either of the anisotropic coefficients n_c and n_ϕ do not vary with depth which incidentally makes the variation of a strength parameter with depth on horizontal and vertical failure surfaces the same. The analysis is with respect to a slope in critical state unlike the methods generally used for slopes in which the strength mobilised is made a fraction of actual strength (i.e., $S_m = S/F$) in order to postulate a limiting state. Hence the slope in effect is in an active state which condition gives rise to the following result:

$$\sigma_{zh} = K_a \sigma_{zv} = K_a \gamma z \quad \ldots (6)$$

in which K_a is the coefficient of active earth pressure in a backfill with the same strength characteristics. The method of obtaining K_a has been suggested in ref. 2.

The expression for $S_{z\theta}$ will now become:

$$S_{z\theta}=(1+l_c \tfrac{z}{H}) C_{ho}(\cos^2\theta+n_c\sin^2\theta) + (1+l_\phi \tfrac{z}{H}) \tan \phi_{ho}(\cos^2\theta+K_a n_\phi \sin^2\theta) \quad \ldots (7)$$

where l_c and l_ϕ are dimensionless coefficients defining the strength increase with respect to C and tan ø respectively. For critical equilibrium the overturning and resisting moments about O (centre of rotation, Figure 8) are equated which on simplification (details given in ref. 3) will yield:

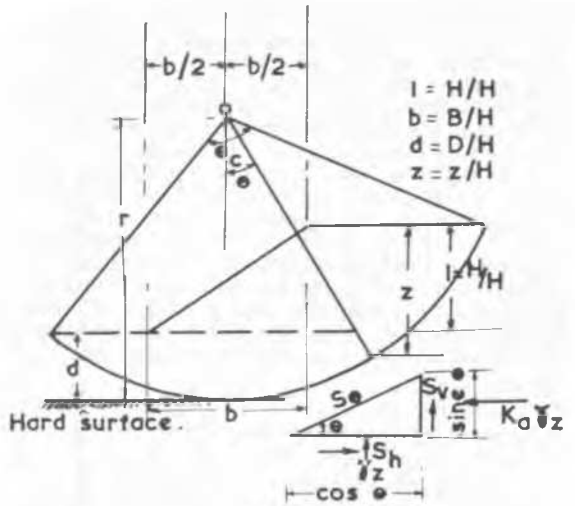

FIG. 8.

$$N = f_{rc}(n_c, r, d, l_c) + \tan \phi_{ho}\, f_{r\phi}(n_\phi, r, d, l_\phi, K_a) = \frac{1}{2r^2} f_c(r, b, d) \quad \ldots (8)$$

Equation (8) gives the functional interrelation between the stability number, N and the coefficient of internal friction tan ϕ_{ho}. For a particular slope with the values of b, d, n_c, l_c, n_ϕ and l_ϕ defined, it is possible to obtain critical combinations of N and tan ϕ_{ho} using an iterative process. However, for the ø = 0 case N is explicitly defined. The iterative process in brief consists of assuming a value of N (less than that for ø = 0) for a particular value of tan ϕ_{ho}, the value of K_a is computed with m = 1 and substituted in Equation (8) to yield a new value of N. This process is repeated in a computer programme till the difference between two successively computed values of N is less than any desired small value ϵ. The curve relating so computed values of N and tan ϕ_{ho} is the control chart for the above defined slope.

Figure 9 reports the control charts for a typical slope with b=2.5 (1 vertical : 2.5 horizontal) for an isotropic deposit when it is homogeneous with respect to only coefficient of internal friction and the hard surface is at half the height of the slope. This figure in addition to demonstrating the control charts to be linear when depth to hard surface is fixed, indicate the influence of increase in undrained strength with depth on control chart. Judged from the fact that closer is the control chart to the origin, the higher will be the factor of safety of the slope for a particular value

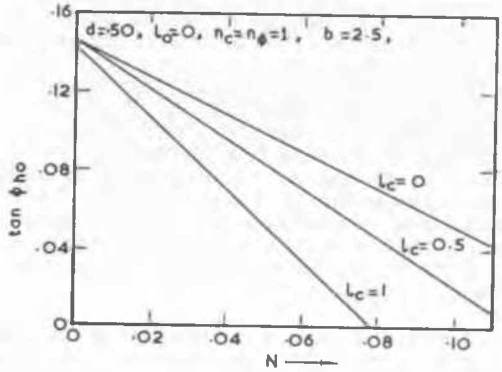

FIG. 9.

equally on either side of that for isotropic case and in keeping with the physical behaviour, an increase in the value of anisotropic coefficient will make the slope safer. Also shown dotted is the control chart for $n_c = 2$ and $n_\emptyset = 1$ which is clearly seen to approach the control chart for $n_c = n_\emptyset = 2$ at large values of N and $n_c = n_\emptyset = 1$ at large values of tan $\emptyset_{ho}$. Thus it confirms the fact that effect of anisotropy with respect to a strength parameter gets enhanced as the absolute value of that parameter increases. Figure 12 reports the variation in tan $\emptyset_{ho}$ for a fixed value of N at 0.001 with anisotropy, with respect to either of the strength parameters keeping the other strength parameter isotropic, different lines indicating different depths to hard surface. The full lines are for the soil isotropic with respect to $\emptyset$ and the dotted lines for a soil isotropic with respect to c. This also confirms that anisotropy with respect to only $\emptyset$ affects the value of tan $\emptyset_{ho}$ for

of actual strength of the soil, it is evident that ignoring the effect of strength increase with depth (i.e., making $l_c = 0$) will make the factor of safety computation err on the conservative side, the error increasing with the value of l_c. Also evident is the fact that as N approaches zero, the location of the control chart becomes almost invariant (i.e., converging to a point). Figure 10 reports the influence of depth to hard surface for the typical slope with b = 2.5 when the soil is homogeneous and isotropic with respect to $\emptyset$ but anisotropic and non-homogeneous with respect to cohesion. Figure 11 reports the influence of anisotropy deviations on either side of isotropic state (i.e., either $n_c = n_\emptyset = 0.5$ or $n_c = n_\emptyset = 2$) on control charts for the same slope when the soil is homogeneous with respect to only $\emptyset$ (i.e., $l_\emptyset = 0$) and l_c = 0.25. The lines get shifted more or less

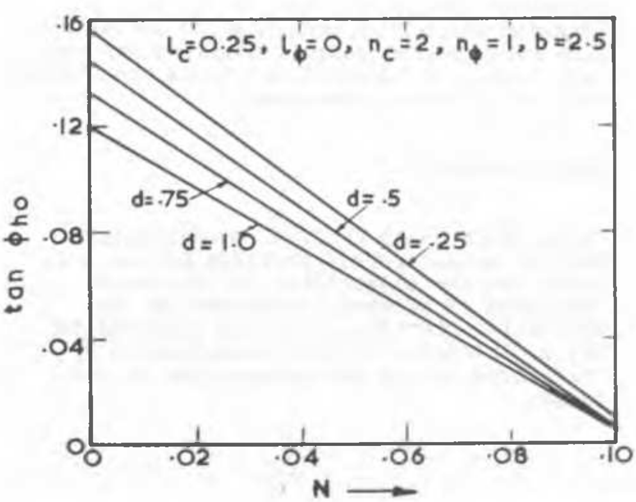

FIG. 10.

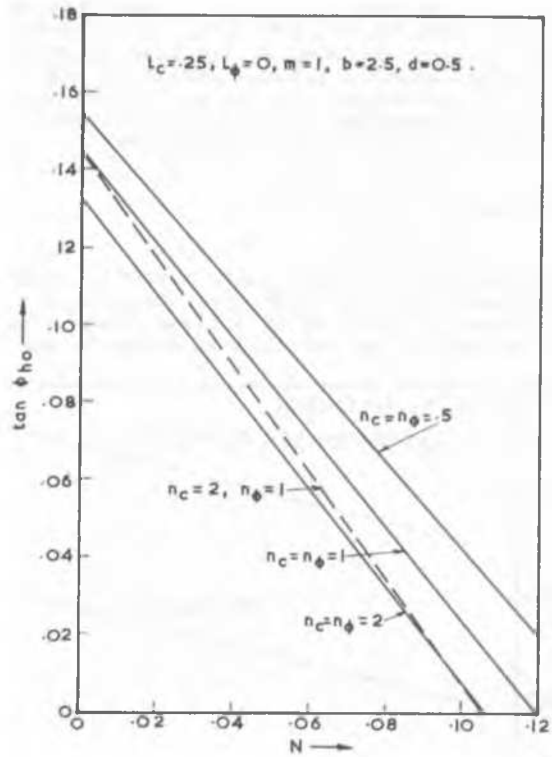

FIG. 11.

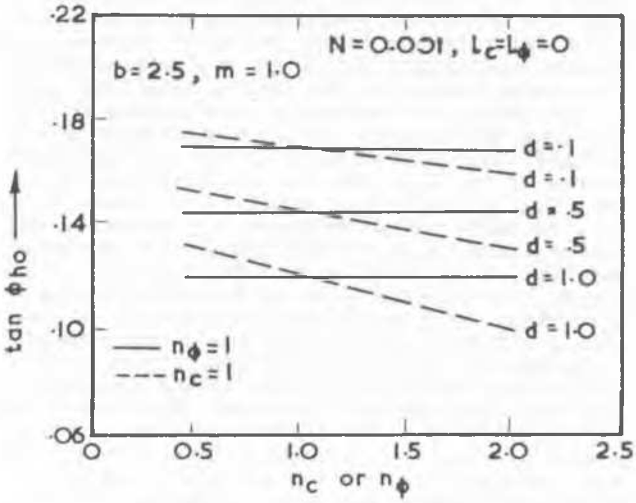

FIG. 12.

constant depth to hard surface. Figure 13 reports the variation in critical radius with depth to hard surface for the two limiting combinations of anisotropy studied (i.e., $n_c = n_\phi = 0.5$ and $n_c = n_\phi = 2$). When viewed from the geometrical fact that for a fixed depth to hard surface, an increase in critical radius means a lateral spread of the failure surface the results do confirm the age-old observation that the failure will follow the path of least resistance.

CONCLUSIONS

The experimental study lends support to the hypothesis that the undrained strength of an element of soil along a plane other than the horizontal or vertical is equal to the vectorial sum of those acting on the projected areas of the element on the vertical and horizontal planes. Further when the soil is in a flocculated state with a strongly binding calcium ion, the soil is more or less isotropic with respect to strength when consolidation pressures are in the neighbourhood of 2 tons/ft^2. The increase of strength on horizontal and vertical failure surfaces though linear with consolidation pressure follow distinctly different lines. This is probably due to the complex influence of interparticle forces as the distance between particles changes due to consolidation. In one-dimensional consolidation the distance between particles in the vertical direction changes with little or much less change in the lateral direction.

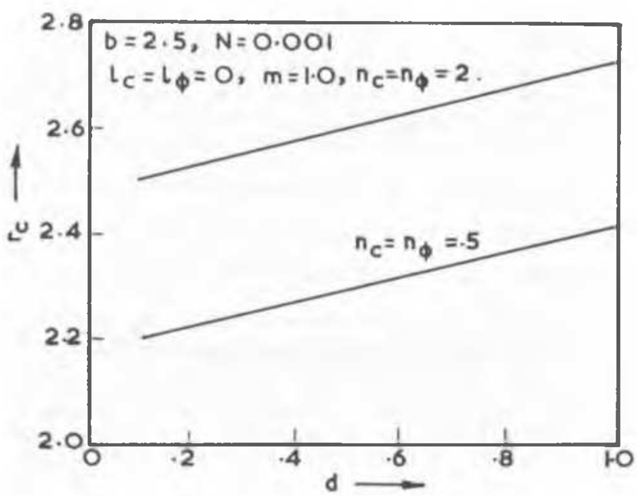

FIG. 13.

Numerical results have been presented in the form of graphs relating the combined bearing capacity factor of a shallow strip foundation in terms of coefficients defining strength anisotropy and strength increase with depth on vertical and horizontal failure surfaces. The value of ultimate bearing capacity for an anisotropic medium changes as the coefficient of anisotropy but at a lesser rate when comparisons are made with an isotropic medium having the same strength on the horizontal plane. The bearing capacity increase consequent on linear increase in shear strength with depth is more or less of the same order as the strength increase. The influence of anisotropy as well as strength increase becomes more pronounced on bearing capacity factor as foundation depth increases.

A method of analysis of stability of slopes which can be used to evaluate the slope in relation to the actual strength that can be mobilised at any time in its life is developed. This is done in terms of a control chart which is a curve providing the critical combinations of N and tan ϕ_{ho} and this curve is found to be a straight line when depth to hard surface is fixed. Numerical results presented demonstrate the influence of strength anisotropy and strength increase with depth on the control charts which have been logically reasoned out to be consistent with the physical behaviour.

ACKNOWLEDGMENT

The authors thank Prof. K. Seetharamiah for the encouragement provided during this study and the authorities of the Indian Institute of Science, Bangalore for the facilities provided. They are indebted to Dr. A. Sridharan for his constructive criticisms during the preparation of the paper.

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