STRESS DEFORMATION AND STRENGTH CHARACTERISTICS, INCLUDING TIME EFFECTS

CARECTERISTIQUES CONTRAINTES DEFORMATIONS ET RESISTANCE, COMPTE TENU DE L'INCLUENCE DU TEMPS

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We will now, as they say, get the show on the road.... I wish to thank the Organizing Committee for the very nice gesture of having given me the honor to chair at the Opening Session of the business side of this Conference. I appreciate it very much because it is not only the First Session, but one that deals with the most important subject. Stress Deformation and Strength Characteristics Including Time Effects. We have had an excellent General Report prepared by professor Ronald F. Scott. Dr. Scott on my left, is professor of Civil Engineering at the California Institute of Technology in Passadena. He has been interested in the basic aspects of soil mechanics for many years; of course, we all know of his textbook on the subject. Particularly, he has been interested in soil mechanics not only on earth but on the ocean floor and on the moon, he has been working with one of our panelists, Prof. James K. Mitchell, as part of a team, with the Apollo XI program on the lunar-soil investigation. Prof. Scott had his bachelor's degree from Glasgow University in Scotland, his master's and dorctor's from M.I.T., despite that he can still think in practical terms. We are very fortunate to have him presents us today with his General Report. Please, professor Scott.

General Reporter R. F. SCOTT (U. S. A.)

Prof. Scott's State-of-the-Art Report appears on pp. 1 of the State-of-the-Art Volume.

Chairman J. G. ZEITLEN

Thank you very much Dr. Scott. We greatly appreciate your report; we will now have a short break.

RECESS

Chairman J. G. ZEITLEN

We will now hear the prepared discussion of the panel. Allow me a few minutes to introduce the gentlemen that you see before you; I shall introduce them in no particular order, I have on my right Dr. James K. Mitchell who is professor of civil engineering at the University of California at Berkeley. He is, as mentioned before, a member of the soil mechanics investigation team which is currently analyzing the Apollo XI results. Immediately on my left beyond Dr. Scott is Dr. Takeo Mogami who is professor of Civil Engineering at the University of Tokyo. Incidentally, he will be the new Vicepresident for the Asian Region of the International Society as soon as we conclude this meeting in México City. We have, again on my right, Dr. Jean Biarez, professor of the Faculté des Sciences, University of Grenoble, and at the Ecole Centrale de Paris. He is as well known, I think, in the field of applied mechanics as he is in soil mechanics. On my left again, Dr. Robert Gibson, professor of Engineering at King's College, University of London, and on the far right Dr. T. Cameron Kenney, professor of Civil Engineering at the University of Toronto in Canada. At the far left is Dr. Hon-Yim Ko who is assistant professor of Civil Engineering at the University of Colorado in Boulder, Colorado. He has been associated with Dr. Scott in the preparation of the excellent State-of-the-Art paper, which we have just heard in very brief form. And of course you all know Mr. Pedro de Alba who is the contribution of the Mexican Organizing Committee to help us run our session. May I now ask Dr. Mitchell if he would be kind enough to give us his discussion?

Panelist J. K. MITCHELL (U. S. A.)

SYNOPSIS

This contribution to the Panel Discussion for Session 1 deals with the following topics:

1. Applications of rate process theory. Phenomenological relationships are presented which describe the stress-strain-time behavior and time to failure under sustained loading.

2. **Three examples of the application of empirical constitutive relations to the prediction of stress-deformation behavior. These include stresses and deformations in a stabilized soil layer, settlement of footings on sand, and the heave and slope stability of a large excavation. The results show good agreement between predicted and observed behavior.**

3. **The validity of Darcy's law in saturated clays. The results of recent studies support the validity of Darcy's law in intact saturated clays. Some earlier work purported to show non-linearity between flow rate and hydraulic gradient. Data are accumulating, however, to show that under some conditions consolidation and swelling may be influenced by water flows induced by chemical gradients as well as hydraulic gradients.**

INTRODUCTION

Professors Scott and Ko have done an outstanding job in summarizing recent developments in the study of the stress-deformation and strength characteristics of soils, including time effects. The State-of-the-Art Reporter has suggested that panelists in Session 1 concentrate in particular on the application of soil mechanics to soil engineering, and has proposed several areas for discussion.

Professor Scott has pointed out that the individual papers will provide the intricate technical details, and the task of the panel is to assess the overall state-of-the-art of Session 1 with respect to practice. The writer concurs with this point of view, and the following discussion has been prepared in that light. This approach is not indended to imply, however, that "intricate technical details" or "minor" details may be bypassed in the solution of problems in soil engineering. On the contrary, experience has indicated clearly that proper attention to such details has often held the key to the successful solution of many difficult problems.

The following topics from those suggested by the State-of-the-Art Reporter have been selected for discussion herein:

1. Rate process theory and its applications.
2. Continuum constitutive relations and the application of empirical nonlinear expressions to the solution of stress-deformation problems.

In addition brief amplification of two additional topics covered by Scott and Ko in their report will be given. These are:

1. Property interrelationships in sensitive clays.
2. Consolidation and swelling: Darcy's law and water flows caused by other than hydraulic gradients.

1. RATE PROCESSES

Scott and Ko have summarized the development of rate process theory for application in soil mechanics to problems of creep and consolidation. The papers by Mitchell et al (1968, 1969), Murayama and Shibata (1964), Murayama (1969), Singh and Mitchell (1968), and Wu et al (1966) in their list of references provide details of the concepts advanced. Additional relevant papers include those by Murayama and Shibata (1961) and Christensen and Wu (1964).

1.1 Objectives of Studies Using Rate Process Theory

It appears that most soil behavior studies using rate process theory have had as their objective either or both of the following:

1. Development of an improved understanding of the factors controlling the strength and time-dependent deformation characteristics of soils.
2. Development of quasi-theoretical relationships for description and prediction of time-dependent deformation phenomena.

The writer's studies, which have been extensively quoted by Scott and Ko in their general report, were undertaken primarily with the first objective in mind.* As a by-product of this work certain empirical phenomenological relationships have emerged which appear useful for descriptive and predictive purposes, and these are presented in subsequent sections of this discussion.

Studies undertaken primarily to meet the second objective have generally used rate process theory for the development of a non-linear viscosity relationship for use with a dashpot which is in turn incorporated into a rheological model; e.g., Fig. 2.4.1(b) of Scott and Ko's report.

*On page 6 of their report the authors state that the writer's procedure for determination of the activation energy for creep of sand did not involve tests at different temperatures, and imply that because of this, the results obtained are not directly comparable to those obtained for clay. In fact, the tests did involve measurement of creep rates at different temperatures. Thus while ambient vibrations might have been responsible for a part of the observed creep deformations, as suggested by Scott and Ko, the strain rate change used to compute activation energy was temperature induced.

The authors state that the conclusion of Mitchell et al (1969), that the strength-generative and creep-controlling mechanisms may be similar in both sand and clay, may be too sweeping, because of the difference in the nature of the structures of the two materials and their response to applied loads. In the view of the writer, the facts that (1) activation energies, (2) number of interparticle bonds per unit area, (3) solid particle surface structure, and (4) the form of the stress-strain-time behavior are similar for sand and clay all argue in favor of such a conclusion, although the degree to which specific facets of the total deformation process manifest themselves in the two types of material may differ.

The tendency in soil mechanics has been to emphasize the difference between sands and clays. There may be merit in considering their similarities, particularly if the behavior of cohesive/frictional materials is to be correctly understood.

1.2 Rheological Models

A number of different rheological models containing linear springs, linear and non-linear dashpots, and sliders have been proposed for description of time-dependent deformation and volume change phenomena in soils. Difficulties associated with the application of these models to a description of real soil behavior include the following:

1. Once a particular arrangement of model elements has been chosen, the resulting model equations depend only on this arrangement and the characteristics of the different model elements.

2. The number of constants associated with a model of several elements may be large; the mathematics become cumbersome; and the solution may not be unique.

3. Because a particular model provides a reasonable description of some aspect of soil behavior, it does not follow that the physics represented by the model and that actually controlling the behavior of the soil are the same.

4. Viscoelastic models represent only one-dimensional behavior.

5. Non-linear material response is often neglected; superposition is then not valid.

6. No model proposed thus far is sufficiently general to account for the range of behavior exhibited by different soils

Barden and Poskitt (1969) have presented arguments in support of the use of rheological models:

1. Simple three parameter models can be used to represent the development of pore pressure-time as well as compression-time relations.

2. Rheological models can be extended to stress states more general than that corresponding to one-dimensional behavior; e.g., axisymmetric and plane strain.

Little application of rheological models to the solution of soil engineering problems appears to have been made. This is not surprising, however, because only since the last International Conference has significant progress been made in the solution of non-linear stress-deformation problems in soil mechanics. Most prior work was concentrated on the application of equilibrium methods and examination of conditions at failure. With the exception of simple one-dimensional consolidation, stress-deformation-time problems are considerably more complex than most stress-deformation analyses.

Whether or not with the aid of new computational methods; e.g., finite element and finite difference, adequate solutions may be obtained for problems involving time-dependent deformations, remains to be seen. In the view of the writer success is likely, provided adequate representation of soil properties is used in the analysis. Whether this representation should be by means of a rheological model or by means of phenomenological relationships derived using curve-fitting methods will perhaps in the long run be largely a matter of convenience. Since both approaches are empirical, the choice should depend on which is the simplest.

1.3 General Stress-Strain Time Function

From the results of a large number of sustained stress creep tests Singh and Mitchell (1968) suggested a simple three parameter equation relating strain rate, stress, and time; i.e.,

$$\dot{\varepsilon} = Ae^{\alpha D}\left(\frac{t_1}{t}\right)^m \tag{1.3.1}$$

where $\dot{\varepsilon}$ = strain rate
A = strain rate at time $t = t_1$ and $D = 0$ (extrapolated value Fig. 1a)

α = slope of the mid-range linear portion of a plot of logarithm of strain rate versus stress for a fixed time after load application. (In Fig. 1a strain rate is plotted vs. stress level. In this case the slope is αD_{max}, where D_{max} is the strength.)

t_1 = unit time.
m = slope of a plot of logarithm of strain rate versus logarithm of time (Fig. 1b).

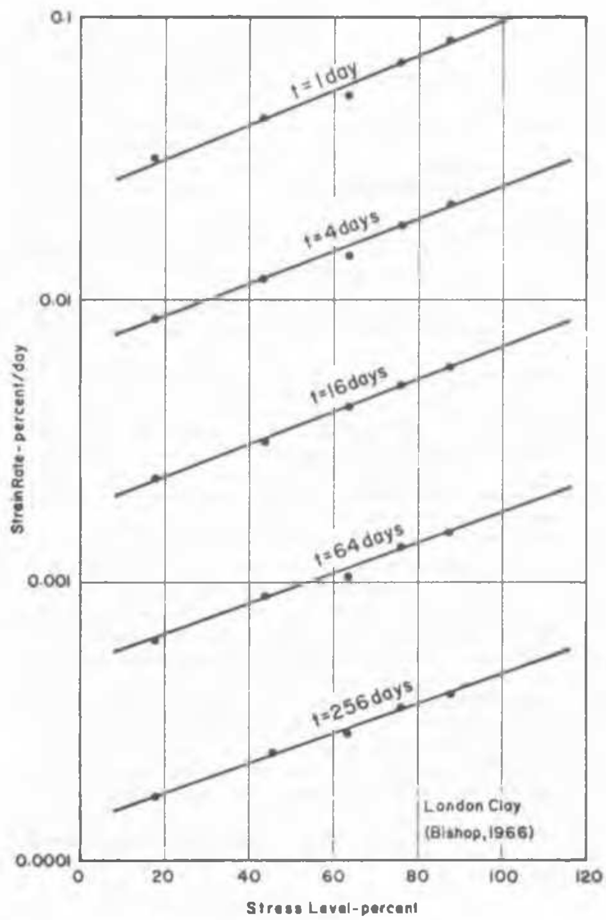

Fig. 1a - Variation of Strain Rate with Deviator Stress Level for Drained Creep of London Clay

This equation has been found to account well for the creep behavior of a variety of soils and conditions; e.g., undisturbed and remolded saturated clay, dry clay, dry sand, overconsolidated clay, and drained and undrained loading conditions. No simple rheological model has yet been reported which adequately represents such a variety of conditions over the range of stress intensities above that needed to cause measurable creep strains (25-30% of the failure stress in normal strength tests) and below that which failure is likely within a short time (80-90% of the normal strength).

The parameter D in equation (1.3.1) has been taken as the deviator stress under triaxial loading; other

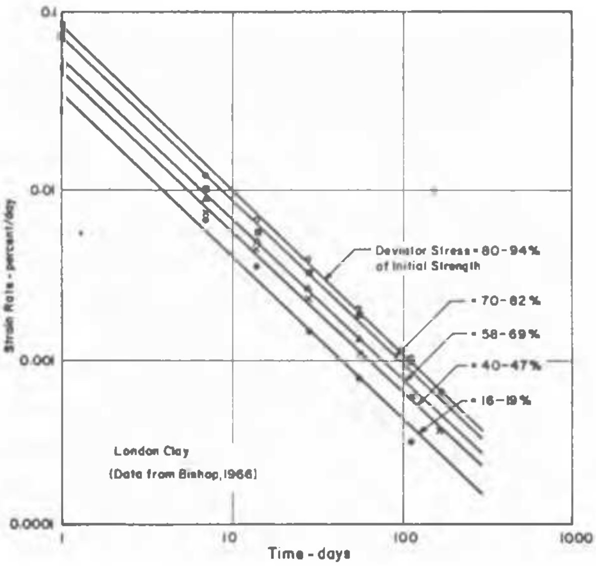

Fig. 1b - Strain Rate Versus Time Relationship During Drained Creep of London Clay.

measures of shear stress may be just as suitable. If stress is expressed as a stress level; i.e., the ratio of creep stress to strength at the onset of deformation, D/D_{max}, then α in equation (1.3.1) is replaced by (αD_{max}), a dimensionless parameter which has been found to have essentially constant value for a given soil over a range of water contents. It appears possible, then, to predict the strain rate-time behavior for any stress and at any water content for a given soil from the results of creep tests at any other water content (a minimum of two tests are needed in order to determine the parameters in equation (1.3.1)), provided the strength versus water-content relationship is known. Since normal strength tests are considerably simpler and less time-consuming to perform than creep tests, the uniqueness of (αD_{max}) may be of considerable usefulness in predicting creep behavior over a wide range of conditions from the results of a limited number of creep tests.

Integration of equation (1.3.1) yields expressions for strain as a function of time under sustained stress. These are:

$$\varepsilon = \varepsilon_1 - \frac{Ae^{\alpha D}}{1-m} + \frac{A\, t_1^{\,m}\, e^{\alpha D}\, t^{1-m}}{1-m} \qquad (m \neq 1) \qquad (1.3.2)$$

and $$\varepsilon = \varepsilon_1 + Ae^{\alpha D} \ln t \qquad (m=1,\ t>1) \qquad (1.3.3)$$

These equations provide a basis for estimation of the influences of stress and time on both rate of strain and total strain. They can be used for prediction of future time-dependent shear deformations from past observations. Since the influence on creep of other than triaxial loading conditions has not yet been determined, and field problems usually involve both shear and volumetric time-dependent deformations, the present state-of-the-art does not allow for direct application of laboratory determined values of A, α, and m to field problems, however.

1.4 Time to Failure Under Sustained Loading

In a paper to this conference Singh and Mitchell (1969) have proposed a method for determination of the time required to achieve a specified strain or creep rupture under a sustained load. It has been observed that soils with a value of m, equation (1.3.1), less than 1.0 fail eventually under sustained stresses less than their short-term strength. For such soils a plot of $\dot{\varepsilon}$ (log scale) versus time (log scale) has the characteristic form shown by Fig. 2(a). At some time after loading the strain-rate ceases to decrease and begins to increase, thus signalling the onset of failure. A plot of $\dot{\varepsilon}t$ (log scale) versus time (log scale) exhibits the form shown by Fig. 2(b). The break in the slope of the line represents the point of strain rate reversal shown in Fig. 2(a).

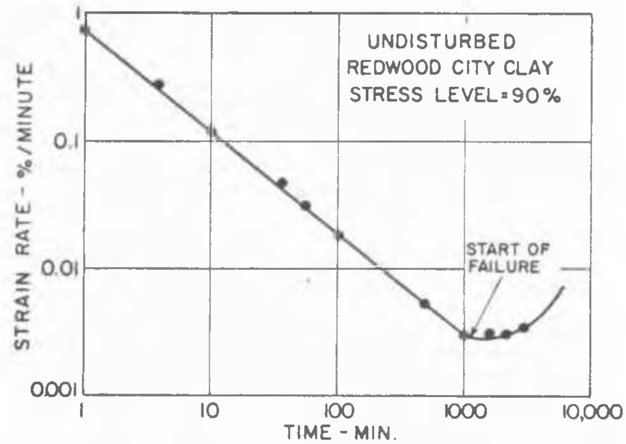

Fig. 2a - Characteristic Form of Strain-Rate vs. Time Relationship for Sample that Fails Under Sustained Stress.

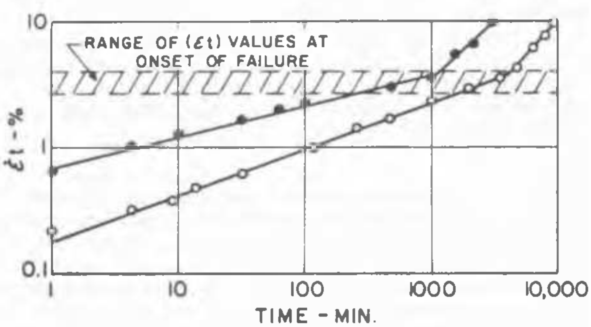

Fig. 2b - Characteristic Form of ($\dot{\varepsilon}$t) vs. Time Relationship for Clays that Fail Under Sustained Stress.

It has been found that the value of $(\dot{\epsilon}t)_f$ corresponding to the onset of failure varies within rather narrow limits for a given soil subjected to tests at different stress levels, (see Figs. 4 and 5 in the Singh and Mitchell paper). Thus if the value of $(\dot{\epsilon}t)_f$ is determined for a test at high stress level, times to failure can be estimated for lower stress levels provided either A, α, and m have been determined or a test of sufficient duration to establish the first part of the relationship between $(\dot{\epsilon}t)$ and (t) for the lower stress levels has been conducted.

With the aid of equation (1.3.1) and the fact that $(\dot{\epsilon}t)_f$ is a constant for a given soil, it may be shown that the relationship between time to failure and stress is given by

$$\ln(t_f) = \frac{1}{1-m}\,(\text{const.} - \alpha D) \tag{1.3.4}$$

This relationship is of the same form as that developed by Murayama and Shibata (1961) using a rheological model with a dashpot coefficient derived using rate process theory.

2. CONTINUUM CONSTITUTIVE RELATIONS

In their report, Scott and Ko discuss the problems associated with the proper mathematical description of the relationship between stress and deformation in soils for use in the solution of boundary value problems. Limitations of classical linear elastic theory are noted, the inadequacy of failure theories when applied to sub-failure conditions are pointed out, and principles which a constitutive relation has to satisfy to insure that the solution of a boundary value problem applied to the material will be unique are listed.

It appears that no completely suitable constitutive relationship for soil which satisfies all of the listed criteria has yet been advanced. It may be some time before such a relationship is developed, and even then mathematical complexities may limit its usefulness. Nonetheless problems are continually encountered for which solutions are needed, and considerable effort has been expended in the search for empirical nonlinear stress-deformation relationships that will give a reasonable representation of actual behavior. While fundamental criticisms, as listed by Scott and Ko, can be levelled at the use of such expressions, considerable success has been achieved in the prediction of stresses and deformations in complex boundary value problems. Three examples are presented in the following paragraphs to illustrate the usefulness of such an approach. In each case a non-linear, stress-dependent modulus has been incorporated into the analysis, as well as limiting strength criteria for different soil materials. In each case the solutions were obtained using the finite element method.

2.1 Plate Load Tests on Cement-Treated Silty Clay Overlying Clay Subgrade

Wang (1968) performed circular plate repeated load tests on the surface of cement-treated silty clay (CL) slabs of 8-inch thickness overlying a clay (CH) subgrade. Undisturbed samples from the slab and subgrade were tested in triaxial compression and flexure in the laboratory for determination of the stress-deformation and strength characteristics.

From the results of these tests it was established that the elastic modulus in compression could be expressed as a bilinear function of deviator stress, D, according to the following relationships:

$$M_R = K_1 + (K_2 - D)K_3 \qquad (D<K_2) \tag{2.1.1}$$

and

$$M_R = K_1 + (D - K_2)K_4 \qquad (D>K_2) \tag{2.1.2}$$

The modulus in tension was found to be a constant, independent of stress intensity. Table I lists the values of coefficients for equations (2.1.1) and (2.1.2), modulus in tension, and strength for the subgrade soil and the cement-treated layer after a curing period of 2 days.

These values of soil properties were then used, in conjunction with the axisymmetric finite element program developed by Duncan, Monismith, and Wilson (1968), for prediction of stresses and deflections under repeated plate load stresses. Fig. 3 shows a comparison between predicted and measured compressive strains in the stabilized layer for a range of applied stresses and three plate sizes. It may be seen that agreement is good for two of the plate sizes. Predicted and measured values of compressive stress at the boundary between the treated and untreated soil are compared in Fig. 4. Agreement in this case is excellent.

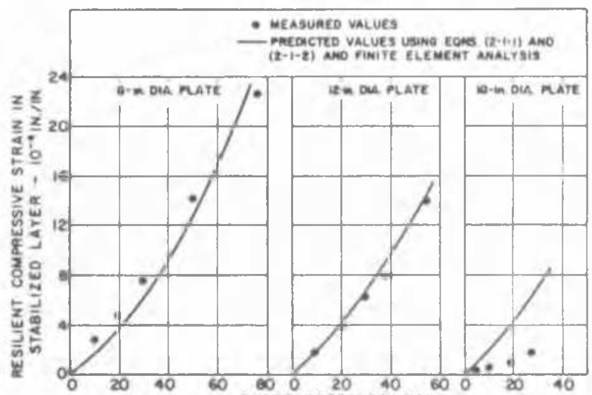

Fig. 3 - Predicted and Measured Compressive Strains in Cement-Stabilized Silty Clay Overlying Clay Subgrade (Wang, 1968).

Thus an empirical non-linear elastic constitutive relationship has been utilized to provide a reasonable basis for prediction of stresses and strains for the case of axisymmetric loading on a layered system.

2.2 Load-Settlement Behavior of Footing on Compact Sandy and Clayey Soils

2.2.1 Hyperbolic Stress-Stratin Relationships. Duncan and Chang (1969) observed that the simple, hyperbolic stress-strain relationship proposed by Kondner (1963) and Kondner and Zelasko (1963)

$$(\sigma_1 - \sigma_3) = \frac{\epsilon}{a+b\epsilon} \tag{2.2.1}$$

where $(\sigma_1 - \sigma_3)$ is the normal principal stress difference, ϵ is the axial strain, and a and b are related to the initial tangent modulus and ultimate strength at large strain, respectively, represented

TABLE I

PROPERTIES OF SUBGRADE AND CEMENT-TREATED SILTY CLAY

	Compression Modulus Constants, psi				Tension Modulus psi	Compressive Strength psi	Tensile Strength psi	Poisson's Ratio
	K_1	K_2	K_3	K_4				
Clay Subgrade	3,500	5.39	0.16	-	0	40	0	0.48
Silty Clay Treated with 3% Cement	4.5	14,500	1,000	-67.0	30,000	26	17	0.20

the stress strain behavior of a wide range of soils very well up to failure. They observed that the actual stress at failure $(\sigma_1 - \sigma_3)_f$ is usually somewhat less than the value predicted according to equation (2.2.1), $(\sigma_1 - \sigma_3)_{fh}$, and defined a hyperbolic failure ratio by

$$R_f = \frac{(\sigma_1 - \sigma_3)_f}{(\sigma_1 - \sigma_3)_{fh}} \qquad (2.2.2)$$

$(\sigma_1 - \sigma_3)_{fh}$ is then given for any soil by

$$(\sigma_1 - \sigma_3)_{fh} = \frac{2}{R_f(1-\sin\phi)}\,(c\cos\phi + \sigma_{3f}\sin\phi) \qquad (2.2.3)$$

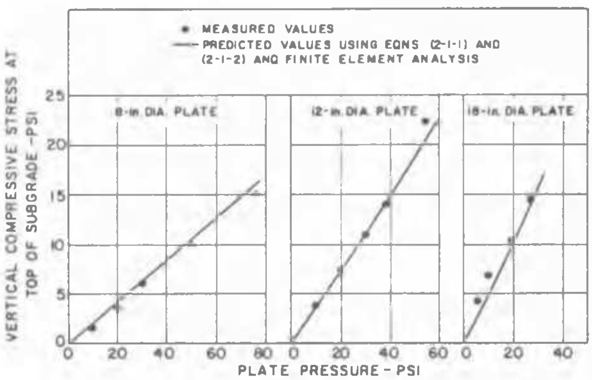

Fig. 4 - Predicted and Measured Vertical Compressive Stresses in Clay Subgrade Underlying Cement-Stabilized Silty Clay (Wang, 1968).

where c and ϕ are the unit cohesion and angle of internal friction, respectively, and σ_{3f} is the minor principal stress at failure. Then through manipulation of equation (2.2.1), Duncan and Chang derived the following expression for tangent modulus at any value of principal stress difference

$$E_t = E_i \left[1 - \frac{(\sigma_1 - \sigma_3)}{(\sigma_1 - \sigma_3)_{fh}}\right]^2 \qquad (2.2.4)$$

where E_i is the initial tangent modulus.

A number of previous studies have shown that E_i can be represented quite well by a relationship of the form

$$E_i = K(\sigma_3)^n \qquad (2.2.5)$$

where K is a constant, σ_3 is the minor principal stress, and n ranges from about 0.4 for gravels and rockfills to 1.0 for saturated clay and intact rock. Kulhawy (1969) has tabulated ranges of K and n for different soil types.

2.2.2 Comparison Between Predicted and Actual Footing Behavior. The linear, elastic, axisymmetric finite element program was modified to incorporate the non-linear representation of modulus given by equation (2.2.4). In addition provision was made to incorporate failure conditions within the analysis.

Full scale footing load tests were made on square footings 5 ft by 5 ft and 1.5 ft by 1.5 ft in plan founded on the soil conditions shown in Fig. 5.

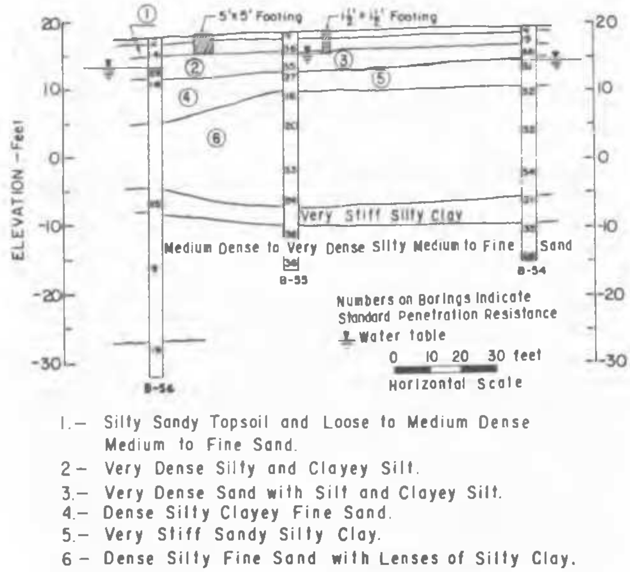

Fig. 5 - Soil Conditions at Footing Load Test Site

It may be seen that the footings were located above several layers of different characteristics, that the deposits were variable, and both cohesionless and cohesive layers were encountered. Since no undisturbed strength values were available it was necessary to estimate the strength and modulus parameters for the different strata. Values assigned, listed in Table II, were selected using the summary ranges developed by Kulhawy (1969).

The finite element analyses were made assuming equivalent circular footings and incremental loading. Pressure was increased in increments of 500 psf. After computation of stresses and displacements for all elements in any increment, new modulus values were computed using the new stresses, and the process was repeated using the next pressure increment.

TABLE II

SOIL PROPERTIERS USED IN FOOTING LOAD-SETTLEMENT ANALYSES

Stratum Number (Fig. 5)	Ø' degrees	c' psf	K psf	n	Rf	Poisson's Ratio	Effective Unit Weight-pcf
1	30	0	20,000	055	0.85	0.45	110.0
2	35	500	25,000	0.65	085	0.45	72.5
3	30	1000	25,000	0.65	0.85	0.45	72.5
4	35	500	25,000	0.60	0.85	0.45	67.5
5	35	500	25,000	0.60	0.85	0.45	67.5
6	35	0	20,000	0.60	0.85	0.45	44.0

Fig. 6 shows a comparison between predicted and measured load-settlement behavior for the two footings. It may be seen that the agreement is quite good, particularly for the 5 by 5 ft footing. In view of the soil property assumptions that were needed and the very small settlements that were observed, it is considered that the results are quite acceptable.

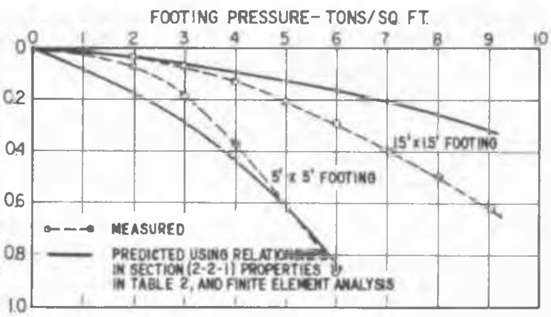

Fig. 6 - Predicted and Measured Settlements of Footings on Sandy Soils.

2.3 Foundation Heave Due to Excavation and Excavation Stability

An excavation 200 ft deep for the Buena Vista Pumping Plant near Bakersfield, California, required removal of over 5 million cubic yards of earth. A photograph of the excavation when the 160 ft depth had been reached is shown in Fig. 7. While the usual methods of slope stability analysis indicated that the excavation would be stable when carried to the 200 ft depth, rebound markers had been sheared off and heave of the excavation bottom had developed causing concern over the stability of the project.

The problem was studied by Chang and Duncan (1969), who analyzed the ground movements and stability using the finite element method and the empirical constitutive relationships presented in Section 2.2.1.

Fig. 7 - Buena Vista Pumping Plant Excavation (State of California, Department of Water Resources)

A generalized soil profile across the excavation is shown in Fig. 8. Strength parameters and hyperbolic stress-strain parameters were determined for the different soils from the results of laboratory tests on undisturbed samples. It was found that one set of property values could be used for the sandy soils in zones I, III, and V (Fig. 8), and another set for the clays in zones II and IV. Values so determined are listed in Table III.

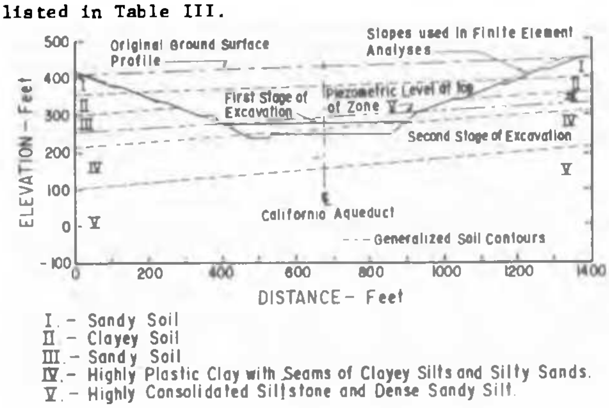

Fig. 8 - Generalized Soil Profile at the Site of the Excavation for the Buena Vista Pumping Plant (Chang and Duncan, 1969).

Two extreme cases regarding drainage conditions were used for analyses. "Undrained" implied that construction proceeded so rapidly that the clays in zones II and IV did not drain during the construction period (although the sandy soils were assumed fully drained). Undrained stress-strain properties were used for the clays in the analysis. "Drained" analyses assumed complete drainage, and drained stress-strain properties were used for both sandy and clayey soils.

Rebound of the bottom of the excavation as a function of excavation depth is shown in Fig. 9. It may be seen that the observed total rebound of 2.43 ft. was within about 8% of the value predicted using the "slow" analysis. Similar computations for other points within the excavation gave comparable results.

TABLE III

VALUES OF PARAMETERS USED IN HYPERBOLIC STRESS-STRAIN FORMULATIONS FOR BUENA VISTA SOILS

	Sandy Soils in Zones I, III, and V	Clayey Soils in Zones II and IV
c' (kg/cm^2)	0.26	1.0
ϕ' (degrees)	38.7	35
R_f	0.74	0.83 (assumed)
Loading-Undrained:		
K (psf)	—	178
n	--	1.0
Loading-Drained:		
K (psf)	16,500	4.0
n	0.59	1.18
Unloading-Undrained		
K (psf)	—	220
n	—	1.0
Unloading-Drained		
K (psf)	48,000	14.4
n	0.59	1.18
Poisson's Ratio:		
Undrained	--	0.49 (assumed)
Drained	0.2	0.3

Contours of percentage mobilized strength for an excavation depth of 160 ft. were determined using the same procedures, with the results shown in Fig. 10. It may be seen that two local zones of failure were predicted on the side slopes, as well as a thin failed zone along the bottom. The rebound markers which were sheared off were located near the base of the excavation where the finite element solution indicated local failure would occur. Since the percentage of strength mobilized decreased rapidly away from these zones no overall instability was indicated and excavation was carried to the 200 ft. design depth without difficulty.

2.4 Conclusion

These three examples indicate that reasonable correlations between predicted and observed stress-deformation behavior are possible using empirical constitutive relationships for the soil and computational methods which allow for spatial variations in properties, such as the finite element method. None of the constitutive relationships used in these studies satisfy the criteria established for an ideal relationship; nonetheless, if used with care and for loading conditions reasonably representative of those used for determination of the necessary soil constants, they can provide very acceptable solutions to complex problems.

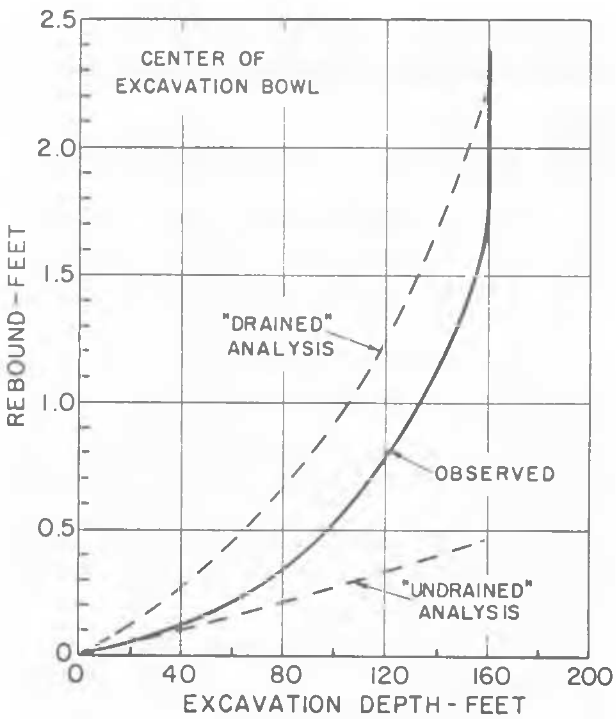

Fig. 9 - Variations of Calculated and Observed Rebounds with Excavation Depth (Chang and Duncan, 1969).

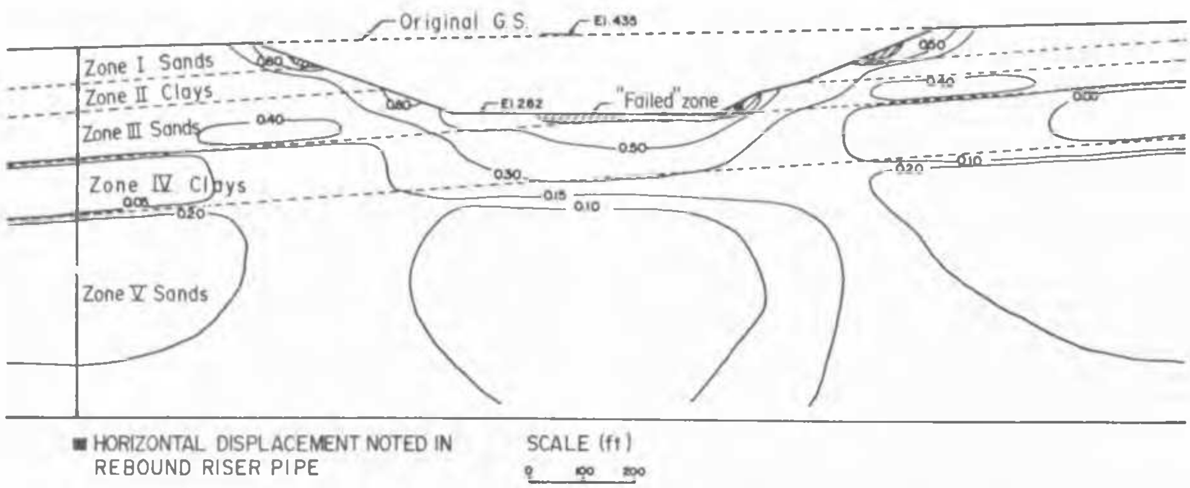

Fig. 10 - Contours of Mobilized Strength $((\sigma_1 - \sigma_3)/(\sigma_1 - \sigma_3)_f)$ after Excavation (Chang and Duncan, 1969)

3. PROPERTY INTERRELATIONSHIPS IN SENSITIVE CLAYS

Scott and Ko refer to a recent study by Houston and Mitchell (1969) which indicated that the properties of sensitive clays fit a pattern which can be predicted using concepts concerning the influence of effective stress and void ratio on volume change tendencies during shear. Liquidity index was used as a normalized water content, and a generalized relationship between sensitivity, effective stress, and liquidity index was derived that appears applicable to a range of normally consolidated clays. This relationship is shown in Fig. 11 as a sensitive contour pattern.

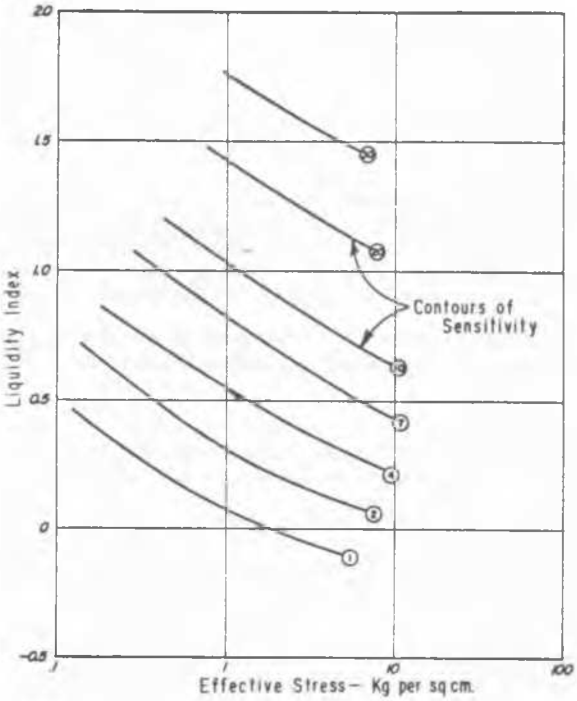

Fig. 11 - General Relationship between Sensitivity, Liquidity Index, and Effective Stress.

The relationship shown in Fig. 11 may be used to:

1. Estimate sensitivity when undisturbed samples are not available.

2. Estimate changes in sensitivity that will accompany a change in effective stress and/or liquidity index.

3. Extrapolate a small amount of sensitivity data into a larger, more useful pattern.

4. CONSOLIDATION AND SWELLING

4.1 Validity of Darcy's Law

A number of investigators have suggested that under some conditions at least, Darcy's law may not be valid in fine-grained soils (Hansbo, 1960; Miller and Low, 1963; Swartzendruber, 1962; Florin, 1951; Mitchell and Younger, 1967). The possible consequences of a threshold gradient and non-linearity between hydraulic flow velocity and hydraulic gradient at low gradients on the consolidation of clays has been considered in several papers; e.g. Hansbo (1960), Florin (1951), Roza and Kotov (1958); Mitchell and Younger (1967).

Fig. 12 shows the relationship between hydraulic gradient and time factor according to the Terzaghi theory for various depths in a clay layer during consolidation. This figure shows that hydraulic gradients during consolidation of clay layers in the field rarely exceed one or two. On the other hand, in laboratory tests the gradients may be several hundred. If significant deviations from Darcy's law do exist, and in particular if there is a threshold gradient for flow, it is surprising that consolidation in the field bears any relationship to predictions according to theory. The fact that it does suggests that actual discrepancies with Darcy's law may be minor.

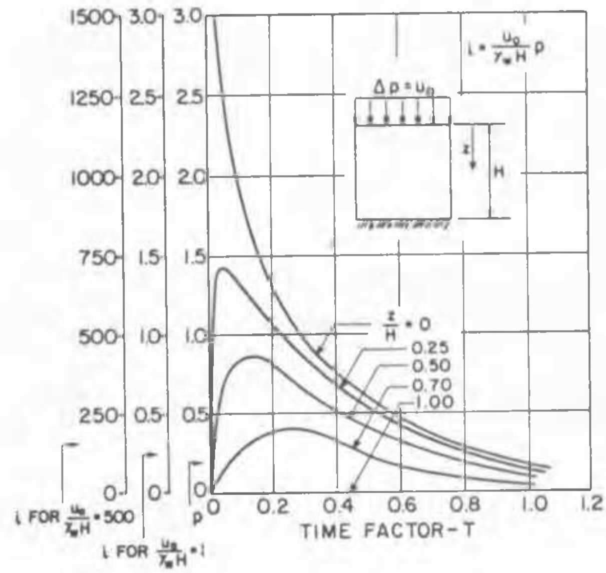

Fig. 12 - Hydraulic Gradients during Consolidation According to Terzaghi Theory.

Olsen (1965) has reviewed the evidence in support of deviations from Darcy's law in saturated inorganic clays and has shown that much of it could be challenged on experimental grounds. He subsequently (Olsen, 1966) presented data to show that flow rate was linearly related to gradient in the low gradient range for kaolinite. Similar results for other clays were reported by Gray (1966), and Miller *et al* (1969) have shown the absence of threshold gradients in a clay-water system which had been thought (Miller and Low, 1963) previously to exhibit large threshold effects.

Thus available evidence from recent carefully conducted experiments supports the validity of Darcy's law in intact saturated clays. It does appear, however, as noted by Scott and Ko in their review of the work of Olsen (1969), that since flow of water through clays may be induced by electrical and chemical gradients as well as hydraulic gradients, Darcy's Law alone may not be a sufficient basis for analysis of water movement in all cases.

4.2 Coupled Flows in Saturated Clays

It has been found that hydraulic flows may be induced through saturated clays under the action of several different types of gradient; e.g., hydraulic, electrical, chemical, and thermal. Such flows are termed "coupled" flows. Advantage of electrically induced hydraulic flow has been taken in the application of electro-osmosis to soil stabilization for many years. Relatively less is known about the practical importance of flows (and therefore consolidation) caused by chemical and thermal gradients. The work of Gray (1969), however, suggests that thermally induced flows may be of relatively minor importance in saturated clays.

On the other hand, water flow induced by chemical concentration differences across a clay layer may be significant, particularly under conditions where the hydraulic permeability is very low; e.g. in highly plastic clays at the low void ratios corresponding to very high consolidation pressures.

Treatment of flows of one type in response to gradients of another type are most conveniently handled within the framework of irreversible thermodynamics. If all flows are linearly related to the driving forces, as appears to be the case for saturated clays, then the following equation is applicable,

$$J_i = L_{ij} X_j \tag{4.2.1}$$

where J_i = flow of type i

L_{ij} = coefficient for flow of type i due to gradient of type j

X_j = gradient of type j.

Olsen (1969) has adapted equation (4.2.1) to the special case of one-dimensional hydraulic flow under hydraulic, electrical, and chemical gradients according to

$$q_H = -k_H\left(\frac{A}{\ell}\right)\Delta H + k_c\left(\frac{A}{\ell}\right)\log\frac{C_1}{C_2} - k_E\left(\frac{A}{\ell}\right)\Delta E \tag{4.2.2}$$

where k_H, k_C, k_E = coefficients of hydraulic flow due to hydraulic, chemical, and electrical gradients, respectively

A = cross sectional area for flow

ℓ = thickness of clay layer

C_1,C_2 = electrolyte concentration on opposite sides of the clay layer

ΔE = electrical potential difference between opposite sides of the clay layer.

The ratios k_C/k_H and k_E/k_H provide a measure of the relative importance of chemically and electrically induced hydraulic flows. The fact that k_E/k_H is significant in fine-grained soils has been well established, and Esrig (1968) has shown that the negative pore pressure, u, that can be induced at any point during electro-osmotic consolidation is given by

$$u = -\frac{k_E}{k_H}\gamma_w V \tag{4.2.3}$$

where V = the voltage at the point

γ_w = unit weight of water.

Only limited data are available to indicate the possible importance of chemically induced flow and consolidation in clays. One set of such data from Olsen (1969) is shown in Fig. 13 for tests on kaolinite. It may be seen that for consolidation pressures greater than about 10 atmospheres, the hydraulic flow rate under a 10-fold difference in concentration across the clay layer becomes significant relative to the flow rate induced by a unit hydraulic gradient. Since kaolinite is a very inactive clay mineral, it is likely that chemically coupled flow may be of even greater importance in more active clays.

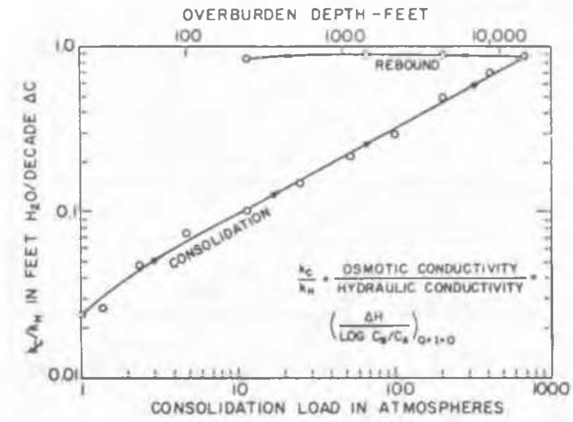

Fig. 13 - Ratio of Chemical-Osmotic to Hydraulic Conductivity Coefficients for Kaolinite (Olsen, 1969).

Hydraulic flows and possible consolidation of clay layers due to chemical osmotic effects may become of importance in soil mechanics in connection with problems of sea water intrusion of aquifers in contact with clay layers, underground storage, and waste disposal. Analysis of consolidation in such situations will require extension of classical consolidation theory to take account of flows induced by other than hydraulic gradients.

ACKNOWLEDGEMENTS

The assistance of Professor J. M. Duncan, who carried out the modification of the axisymmetric finite element computer program for use in the footing settlement study is appreciated. In addition Professor Duncan kindly furnished the material for the Buena Vista Pumping Plant case history.

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Chairman J. G. ZEITLEN

Thank you Dr. Mitchell for your interesting discussion. Dr. Mogami, will you please let us have your comments?

Panelist T. MOGAMI (Japan)

1. Various Models.

When we study the properties of the material, we have several ways of approach. Replacement of the material by simpler model is a powerful one. Granular material is composed of particles, hence a model composed of spheres is attractive. Even if we adopt an assembly of equal spheres as a model, the mechanics on this model is not so easy, so that some regularly packed equ-

al spheres are widely used. Though the difficult mechanics on such regular model could be solved, it would be much harder to generalize the results to actual material which is composed of irregularly packed irregularly shaped particles.

On the other hand, the continuum models are also familiar to researchers. When the stress distribution in a ground is studied, the Boussinesq's or modified Boussinesq's theory based on elastic model is used.

Theory of plasticity offers also a powerful model. Researches on mechanical behaviour of clay or granular material which have successfully been promoted by Cambridge school concern with this model.

Whatever model one may adopt, he would aim at

(i) clarifying the behaviour of the material by replacing it by a model of which properties we are familiar,

(ii) finding out some new postulate, on which one could build up a new theory,

(iii) finding out some law useful for engineering purpose between quantities obtainable by some measurement.

The first and second of aims are of rather academical character, whereas the third is much practical. The third would be most important, because the engineering science is to supply materials to the engineering, however we have to polish up our theory on any model so as not to contain contradictions in itself. It is because we have an anticipation that incomplete theory would not bring fruitful result.

2. A Model which the Author Proposed.

The author himself has developed for years a model, which belongs to the probabilistic model. When the author wanted to develop a theory of mechanics of granular material in similar way to the classical statistical mechanics, the author felt that his way was entirely in the midst of dense fog. On the one hand, he is not so familiar with the theory, on the other hand, as far as he knew classical statistical mechanics deals with conservative system. He noticed that the mechanical behaviour of the material is governed by its void ratio, but the void ratio is an average quantity. He noticed also that two samples can have the same void ratio even when the distribution of voids in sample differs from each other.

At the first step, the author assumed the material composed of particles of equal volume and calculated by combinatory analysis the number (Z) of configurations of particles for given void ratio and given deviation of void ratio from its mean defined by

$$\left.\begin{aligned} s &= \frac{1}{n}\sum(e_i-\bar{e})^2 \\ \bar{e} &= \frac{1}{n}\sum e_i \end{aligned}\right\} \quad (1)$$

, where e_i is the i-th void ratio when the total range of void ratios is divided into n subinterval of equal width, $\bar{e}$ is the mean void ratio, the $\bar{e}$ is verified equal to the void ratio e.

Later, the author could show that even when the material is composed of particles of various sizes, the number of configurations of particles for given void ratio and given deviation of void ratio can be expressed in similar form to that for equal particles. By experiment on an assemblage of equal spheres, he could confirm that the frequency curve of voids in the material is sufficiently represented by e and s.

After getting the expression of the number of configurations in terms of e and s, he had to proceed assuming that the entropy of the material is proportional to the logarithm of the number of configurations as in the classical statistical mechanics. He thought that this assumption is questionable, because as stated before, the classical statistical mechanics is constructed for conservative system and our material is not conservative system. Still he thought that if he could have some relationship between quantities which can be measured by direct observation and the relationship could be shown valid, the validity of the assumption would be confirmed at least approximately. Fortunately he could get some relationships between void ratio and the angle of internal friction which were shown valid by experimental data.

Gudehus told his question about the author's assumption in his private letter to the author and in his paper. His question was the same as that of the author, however the author relied upon the fact that his relationships were confirmed by experimental data.

3. Refinement of the Author's Theory

When the author could know the thermodynamics based on the theory of information, he felt if he could have a little hope of refining his theory.

As explained above, some finite numbers of configurations correspond to given e and s.

These configurations are designated as A_1, A_2, ..., A_z and each of them is called the "state".

The configuration of particles in the material is one of A_1, A_2, ..., A_z, however, we cannot know in which state the material is. When the material is deformed the state of the material changes. The number of possible states also changes as the void ratio and its deviation change.

Notwithstanding we cannot know in which state the material is, we can assume that a probability; $P(A_i)$, $i = 1,2, \ldots, z$; is attributed to each state. The values of such probabilities cannot be known in general case. The degree of uncertainty about the knowledge concerning the material, in another word knowledge about the state in which the material is, is expressed, following the theory of information, as

$$U = -K_1 \sum P(A_i) \log P(A_i) \qquad (2)$$

, where K_1 is a constant.

This function U plays similar role to the entropy in thermodynamics. hence this is called the entropy also in the theory of information.

By observation of an assemblage of equal spheres, it was recognized that some structure is developed in a material when the failure stage approaches. In actual material direct observation of its interior part cannot be made. However, formation of slip surface in the sample is observed on its outer surface. The fact that slip surface is formed means the development of some structure.

The structure means that constituting particles of the material cannot move freely and particles are gathered into groups. Each group of particles moves as one body.

In another word, all of the mathematically possible configurations are not necessary realizable, hence the physically possible configurations, realizable states, are a part of all mathematically possible states. Therefore, for example, we have

$$\left.\begin{array}{l} P(A_1) \neq 0,\ P(A_2) \neq 0,\ \ldots, P(A_m) \neq 0, \\ P(A_{m+1}) = P(A_{m+2}) = \ldots P(A_z) = 0 \end{array}\right\} (3)$$

as the shearing deformation proceeds and approaches the failure stage, the number m in (3) would decrease.

If at a stage of deformation far from failure hence the distribution of particles ie almost random, each particle is fixed with each other with a kind of paste, and this configuration is called as A_1, after this stage configuration of particles cannot change and we have

$$\left.\begin{array}{l} P(A_1) = 1 \\ P(A_2) = P(A_3) = \cdots\cdots = P(A_z) = 0 \end{array}\right\} (4)$$

Hence, the condition (3) is satisfied, but there exists no structure in the material.

The "structure" should be considered in close connection with the deformation. When the material is not deformable, the arrangement pattern of particles has no concern. The fact that the material has structure during deformation means that the configulation of particles does not change so much during deformation. Hence we can say that the number of physically possible, realizabele states in material having structure is not so large and the state of the material travels around among the possible states one after another. In another word, the probabilities of all physically possible states do not differ so much with each other when the stage of deformation of material approaches failure and structure is formed in the material.

Hence the mathematical formulation of the condition that the material is in the failure stage and structure is formed in the material is

$$\left.\begin{array}{l} P(A_1) = P(A_2) = \cdots\cdots = P(A_m) \\ P(A_{m+1}) = P(A_{m+2}) = \cdots\cdots = P(A_z) = 0 \end{array}\right\} (5)$$

By the above explanation, it is clear that in failure stage, the scattering of voids in the material around their mean is not so large.

In the previous paper, the author assumed the scattering of void ratios around their mean, expressed by a letter n, be very small compared with the total number N of particles. Therefore, the number of states calculated in the previous paper by combinatory analysis can be considered as equal to m in the expression (5)

When the condition (5) is satisfied, the entropy defined by the expression (2) is reduced to

$$U = -K_2 \log P(A_i), \quad i \leq m \qquad (6)$$

, which has the same form adopted in the previous paper.

Chairman J. G. ZEITLEN

Thank you very much Dr. Mogami. I would like to ask professor Biarez to present us with his prepared discussion.

Panelist J. BIAREZ (France)

RESUME:

L' auteur suggère quelques definitions pour les lois rhéologiques et les limites de lois. Pour la plasticité parfaite, il lui semble bien établi qu'il n'y a pas de variation de volume à tenseur de contrainte constant, donc que le potentiel plastique standard ne peut être accepté. D'autres hypothèses, à preciser semblent préférables, comme la similitude du tenseur accroissement de déformation et du tenseur déviateur de contrainte ou la dérivation d'une fonction constante $G(\sigma_{ij})$ de section hexagonale régulière differente de la fonction statique $F(\sigma_{ij})$.

Depuis quelques annés, les recherches en Mécanique des Sols se rapprochent de la Mecanique des Milieux Continus classique mais l'on peut se demander si l'étude détaillée des lois générales tensorielles peut aider l'ingénieur qui "calcule" toujours ses fondations à partir du SPT. Il serait souhaitable que d'autres recherches, plus difficiles, essaient d'améliorer la pratique actuelle:

- Mesures en chaque point du sol interessé par la construction. Il est préférable de connaître sommairement les propriétes de chaque zone du terrain (par exemple d'une couche d' un centimètre d'argile), plutôt que de parfaitement connaître les propriétés de quelques décimètres cubes qui se prêteraient à une loi mathématiquement simple. D'importants progrès sont à faire dans les prélèvements continue et la connaissance des appareils de mesure in situ.

- Mesure des propriétés des sols difficiles: enrochements, graviers in situ, vases, sols fissurés, fragilité et naissance de fissuration, relation eau-air-solids.

Si des travaux doivent être orientés dans ce sens, à court terme, il faut tenir compte de la mutation apportée en mécanique par les ordinateurs. Dans quelques années, ceux-ci seront assez importants pour calculer un problème à trois dimensions avec une loi rhéologique satisfaisante depuis les petites jusqu'aux grandes déformations, avec conditions limites et hétérogénéités quelconques. Il est nécessaire de préparer cette période par des recherches permettant d'expliciter les propriétés des sols sous une forme acceptable, ce qui se fait, mais aussi de préparer les techniques de mesure pour le terrain intéressé, ce qui reste à faire.

Les propriétés mécaniques du sol font intervenir les lois rhéologiques de chaque phase (solide, liquide, gaz) et les lois inter - phases (solubité...). Les théories mathématiques actuelles (comme la plasticité) sont trop élémentaires pour satisfaire le constructeur, car elles ne coîncident en général qu'avec des parties de loi du solide seul, ignorant le rôle de l'eau par exemple, qui constitute le grande particularité du sol.

Si l'on se contente de la phase solide, la loi est une relation entre des chemins repérés en fonction du temps dans les espaces des contraintes et déformations. Seules les experiences où ces champs sont homogènes permettent la mesure de ces lois. D'importants progrès ont été faits grâce aux appareils triaxiaux avec lubrification des extrêmités, mais ceux-ci sont limités aux chemins sans rotation des directions principales par rapport au matériau (axes rhéologiques). Les deformations avec rotation sont étudiées par ROSCOE et nous avons proposé récemment un appareil pour de très grandes déformations de ce type (Réf. 1-2). Pour obtenir un chemin probable, il faut faire un calcul préalable approximatif élastique plastique par exemple, puis faire des expériences avec des chemins voisins, et recalculer avec cette nouvelle loi, par elements finis par exemple (Réf. 1-2-8) (Fig. 1-2). Mais l'on sera longtemps encore incapable de reproduire ces déformations experimentalement d'une manière correcte sur le sol représentant l'état in situ, sauf les cas simples où la rotation est négligible.

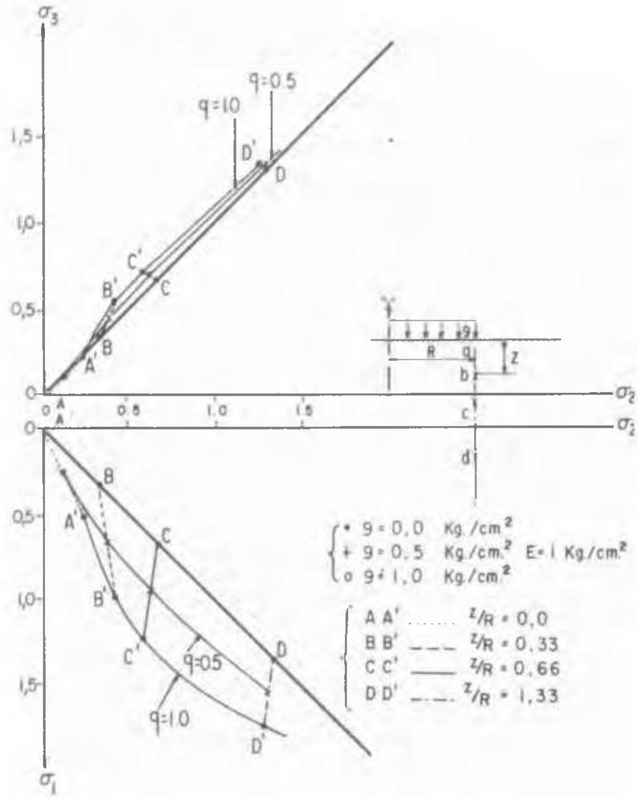


Fig. 1 - Chemin dans l'espace des contraintes

plaque circulaire souple chargée uniformt (élasticité). points situés sous le bord de la plaque, à la profondeur z.
materiau pesant (γ = 1 t/m^3; Ko = 1)

Pour faciliter la discussion, bornons-nous à favoriser un vocabulaire comun pour l'experimentation et le calculateur. L'essai selon le chemin triaxial habituel conduit à une loi que l'on sépare souvent en limites. On peut distinguer les parties suivantes par exemple:

1. Elasticité.

a) Au sens strict, il s'agit d'une relation biunivoque linéaire ou non contrainte-deformation, donc pour phénomènes réversibles (avec une précision donnée). Cette loi n'est valable qu'à l'intérieur de la limite élastique qui peut s'exprimer sous forme $f(\sigma_{ij})=0$. C'est une surface dans l'espace des contraintes.

b) En pratique, "la" courbe contrainte-deformation présente souvent un "coude" assez net (la loi de variation de pente change). Ce coude sépare un domaine peu réversible d'un domaine qui l' est beaucoup plus, et limite

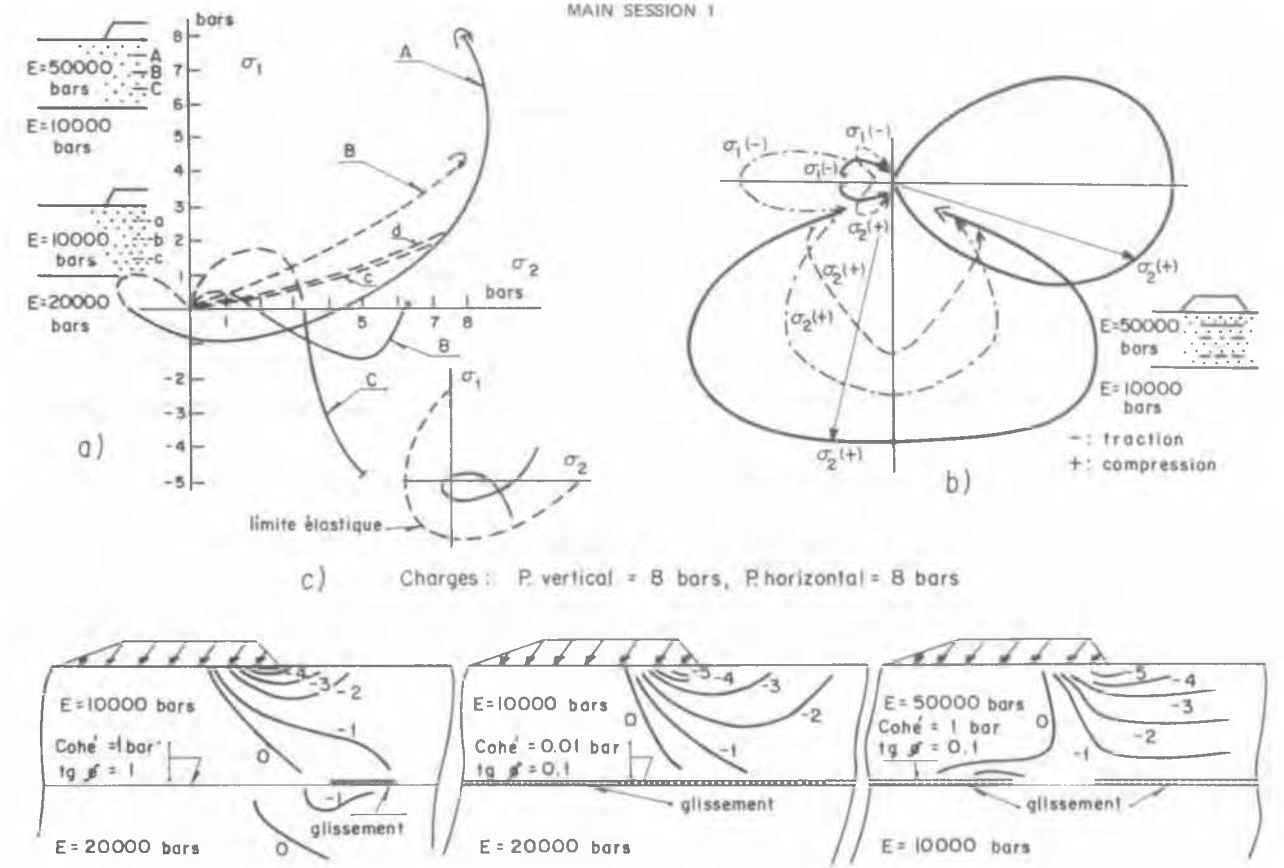


Fig. 2. a) Trajet dans l'espace des contraintes. principales pour différentes cotes A, B, C, a, b, c, lorsque la charge se déplace.

b) Contraintes principales: rotation, intensité (charge mobile).

c) Isobars de la contrainte moyenne en traction.

la validité <u>pratique</u> des calculs d'élasticité pour un <u>premier chargement</u>. Nous suggérons d'appeler <u>le lieu du "coude"</u>: surface limite "elastique" ou surface de surconsolidation et de discuter la validité des calculs d' écrouissage en ce lieu. Cette expressionsemble plus précise que surface d'écoulement. Le coude et la différence de réversibilité seront d' autant plus mets que le chemin utilisé sera proche du chemin de surconsolidation si elle est anisotrope.

2. <u>Surface maximale dans l'espace des contraintes ou courbe intrinsèque maximale.</u>

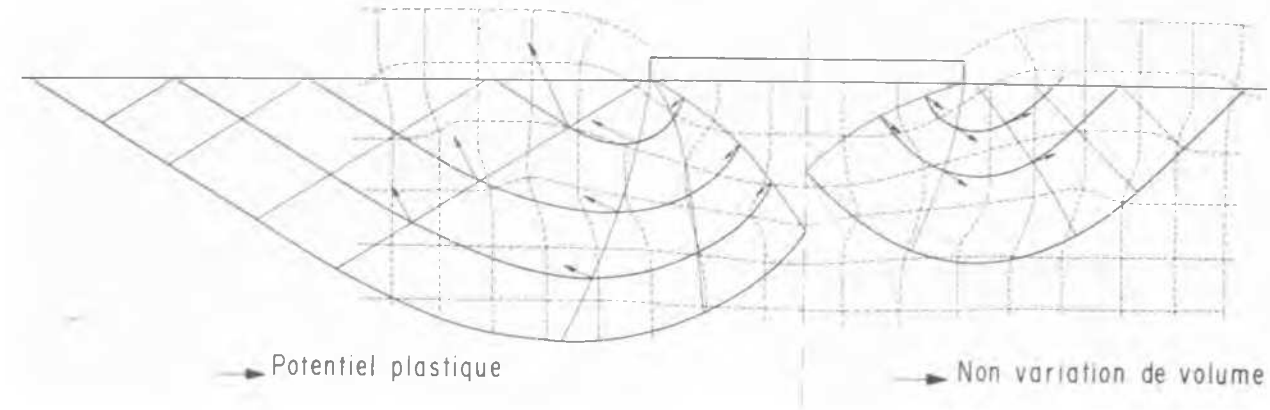
Si l'on trace l'ensemble des cercles de MOHR (effectif...) pour un triaxial a pression latérale donnée (totale), il existe une enveloppe que nous appelons courbe intrinsèque maximale; ceci correspond a l'enveloppe des chamins dans l'espace des contraintes, au-dela de laquelle il n'existe aucun point (pour une vitesse donnés de déformation...) On peut tracer tout autre courbe ou surface selon le critere choisi, par exemple maximun de $\sigma 1/ \sigma 3$ etc... m ais il faudrait en montrer l'intéret, sauf pour les sols ou c=0 ou ø = 0.

3. <u>Plasticité parfaite.</u>

A partir d'une certaine déformation dans le triaxial. le tenseur de contrainte reste constant, d' où une loi en contraintes F(ij) que l' on peut appeler <u>surface de plasticité parfaite</u>. Elle peut être par exemple conique de section hexagonale et correspondre a la loi de COULOMB. Cette loi n'est applicable qu'à partir de déformations assez grandes, correspondant a ce palier contrairement a ce que peut laisser supposer l'expression "rigide plastique". La déformation n'est pas quelconque d'où une loi <u>cinématique</u> complétant la précédente.

a) A notre connaissance, si le tenseur de contrainte est constant (Réf. 3-4...), il n'y a pas de <u>variation de volume</u>. Le vecteur déformation <u>est donc orthogonal</u> à la trissectrice et non a la surface F(σ ij). Cette non variation de volume ne peut être constatée que si l'on effectue d'assez grandes déformations homogènes d'où le grand intérêt du perfectionnement des appareils dans ce domaine.

Si l'on se déplace sur la surface de plas-



3. Potentiel Plastique. Non variation de volume.

ticité parfaite, il sera parfois nécessaire de modifier cette loi en tenant compte de l' augmentation densité critique avec la partie isotrope du tenseur de contrainte (Réf. 4-7 ...) Ceci est facile si le chemin dans l' espace des contraintes croit d' une maniere simple.

b) En premiere approximation il est peut-etre possible d'admettre l'identité des directions principales des tenseurs de contraintes et accroissement de déformation: à deux dimensions, ces deux conditions cinématiques sont suffisantes, un calcul de force portante de fondation a montré que les trajectoires ainsi obtenues étaient nettement en meilleur accord avec l'expérience que le calcul avec le potentiel plastique (Réf. 5-6) (Fig. 3). Toutefois, ces directions sont peut etre différentes dans les cas ou la rotation des directions principales du tenseur de contrainte est importante (dans les sols).

c) A trois dimensions, on ajoute une condition supplémentaire qui ne semble pas encore parfaitement définie. On pourrait supposer que l'accroissement de déformation est semblable au déviateur de contrainte. Une autre hypothèse consiste à supposer que le vecteur accroissement de déformation est orthogonal à un cylindre G (σ ij) parallèle à la trissectrice etvde section hexagonale reguliere. On peut encore modifier cette loi si l'on veut tenir compte de l'augmentation de densité avec la pression moyenne. Divers calculs on été faits avec ces hypothèses (Réf. 9-10).

Par ailleurs, il serait souhaitable d'utiliser des monts différents pour les domaines de contraintes et déformations. Par exemple, ne pas appeler ligne de rupture ou de glissement une ligne où la loi de COULOMB est satisfaite ($\tau = c + \sigma \text{ tg } \emptyset$), mais bicaractéristique statique; car c'est une condition en contraintes et les mots rupture ou surtout glissement ont un sens flou ou plutôt cinématique. L'expression ligne de glissement désigne la discontinuité cinématique (que l'on peut parafois observer). Ceci est d'ailleurs a distinguer des bicaractéristiques cinématiques. Parfaois, ces lignes sont différentes.

De même, dans l'espace des contraintes, la surface maximale ne devrait pas s'appeler surface de rupture car une discontinuité cinématique ne lui est pas nécessairement lies.

En dernier lieu, il est bon de distinguer les lois relatives a des déformations continues et les lois pour déformation discontinue. Par exemple, si l'on impose une déformation continue homogène, c'est-a-dire qu'un carré se transforme en rectangle par exemple, le palier de plasticité parfaite peut être différent de la déformation qui permet ou impose une surface de discontinuité. Nous avons montré à deux dimensions (Fig. 4) (Réf. 1-8) que cette différence était négligeable pour des particules circulaires, èt tres importante pour des particules hexagonales; il en est probablement de même pour des argiles à particules plates. Il nous semble nécessaire de préciser si la loi de plasticité parfaite correspond à une déformation continue ou avec une surface de discontinuité.

4. Ecrouissage.

Nous appelons écrouissage une modification due à une déformation irréversible. Il nous semble indispensable de séparer les parties isotropes et déviatoires de cette déformation. La déformation isotrope modifie toute la loi, en particulier "la surface maximale" dans l' espace des contraintes; par contre, nous pensons qu'elle ne modifie pas la surface de plasticité parfaite en déformation continue. Celle-ci nous paraît indépendante de l'his-toire des déformations irréversibles et donc de la fabrication de l'échantillon, au moins en première approximation (Réf. 2-8).

La déformation déviatoire, bien connue dans les métaux, modifie la limite élastique mais en général d'une manière anisotrope (généralisation de l'effet BAUSCHINGER-Réf. 8); sa représentation complète doit donc être envisagée dans un espace a six dimensions et non dans $\sigma 1$ $\sigma 2$ $\sigma 3$. Par ailleurs, pour une contrainte moyenne donnée, l'échantillon se

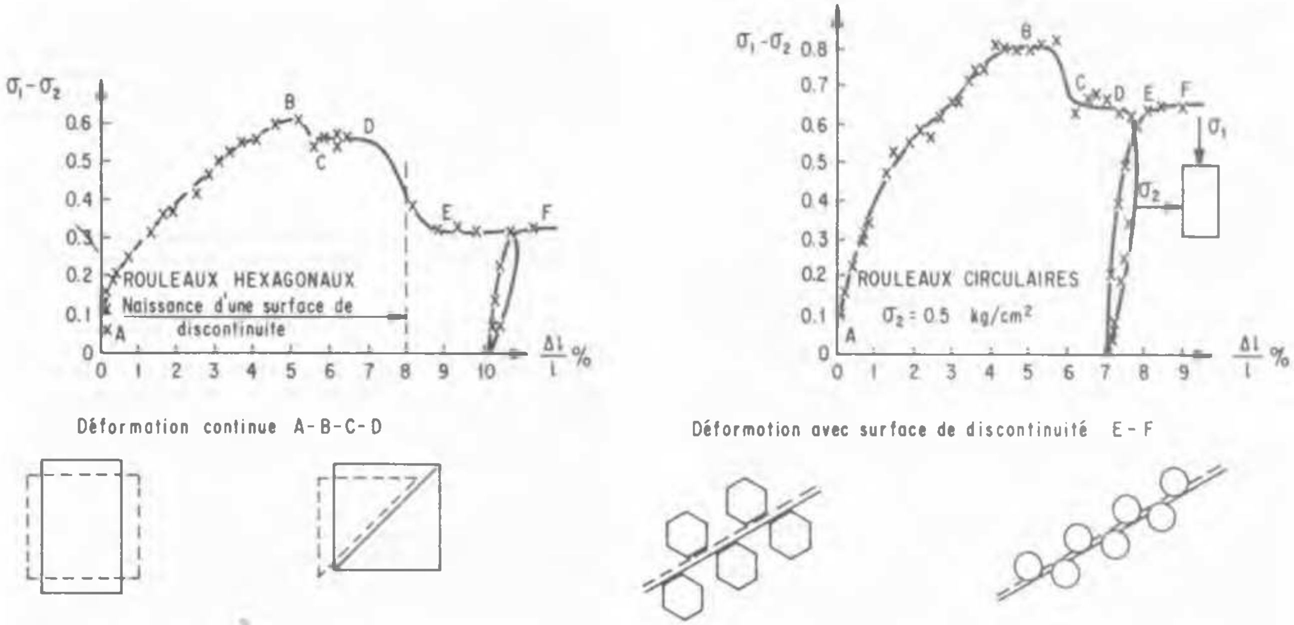


FIG. 4 **CISAILLEMENT BIAXIAL SUR ROULEAUX**

tasse ou se dilate selon la valeur de sa densité initiale par rapport a sa densité critique. Il ne nous parait pas possible, ici encore, d'utiliser le potentiel plastique standard, et meme le potentiel plastique généralisé. Pour le cas d'une augmentation continue des contraintes, il semble que la simulation approximative du chemin dans l'espace des contraintes donne une loi utilisable pour la methode des éléments finis, comme l' ont fait de nombreux auteurs.

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Chairman J. G. ZEITLEN

Thank you Dr. Biarez. Dr. Gibson, please.

Panelist R. E. GIBSON (England)

1. The Reporters for this Session deserve our thanks and congratulations for having carried out their difficult task with imagination and presenting their material in such a way as to stimulate interest. They have referred, of course, to recent work on stress-deformation and strength characteristics of soils published in the engineering and geotechnical literature. But in addition they have performed the valuable function of drawing our attention to pertinent references culled from the wide field of general continuum and granular mechanics. Many of these appear in comparatively unfamiliar journals but report advances which will, no doubt, influence future developments in our discipline.

The experienced soils engineer when faced with a problem may draw upon theory, from his own and other's case records and upon intuition and judgment. But this does not and must not prevent him, on occasion, from treating the first with apparent contempt and the second with suspicion. This can often prove puzzling to the young engineer, but is dictated by the nature and special features of each case. In other words: his method of procedure is appropriate to the circumstances.

I wish to limit my remarks in this Panel Discussion to two specific investigations in the area of mechanics with which I have been concerned. These exemplify the role which I believe is appropriate to mechanics in the practice of soil and foundation engineering. It happens also that they serve to contrast the advantages and dangers of using either analytical or numerical techniques, each to the exclusion of the other, in solving problems where a choice between these approaches is open to us.

2. Before turning to these I shall mention, briefly, circumstances where the application of sophisticated mechanics at the design stage would, in some sense, be inappropriate for one of the following reasons:

a) *Entirely misleading conclusions may follow.*

The physically relevant factors have either not been identified, or have not been - and perhaps cannot be - taken account of in the analysis.

b) *It is unnecessary.*

Simpler procedures are entirely adequate and, being more flexible, may lead to sounder conclusions.

c) *The reliability of the predictions may be illusory.*

An adequate theory can be rendered impotent in practice by our inability to measure and assign numerical values to the parameters. This lack of knowledge may be peculiar to a particular project, or it may reflect difficulties of a more fundamental nature. If the forecast is sensitive to changes in the values of these parameters, the uncertainties will be correspondingly magnified.

The illustrations I have chosen are naive and well-known, but I make no apology for that.

2.1 A good deal of attention has been given to the problem of determining the magnitude and distribution of vertical stress beneath a loaded area. But a knowledge of this quantity per se is very rarely required. The design of a lining for a tunnel beneath a pre-existing surface load, for example, is hardly furthered by this information. Even if the solution to the problem in mechanics takes account of the presence of a cylindrical cavity in the medium, of a lining around this cavity and of the non-linear stress-strain behaviour of the soil, its worth would scarcely be enhanced. In reality any of the following factors may be relevant either in the short or long term:

(a) the depth of the tunnel;

(b) the type of lining and its mechanical behaviour;

(c) the stresses induced by the erection procedure;

(d) the nature of the contact between lining and soil;

(e) the type and properties of grout if this is used; and the grouting pressure;

(f) the time-dependent creep and swelling characteristics of the soil, and the time-dependent distribution of pore water pressure; and this catalogue could easily be extended.

The majority of problems of this kind are not susceptible to overall analysis, although the influence of isolated factors may be examined.

2.2 In the application of one-dimensional consolidation theory a knowledge of the vertical stresses through the clay stratum is needed, both to estimate the initial pore water pressures developed and the primary settlement likely to occur. If the layer is fairly thin this can be estimated with sufficient exactness by taking an appropriate 'angle of spread' of the load. A more 'exact' treatment would be both unjustified and unnecessary. Of course, if the layer were thick a complete investigation might be required.

2.3 Finally, it is sometimes advocated that the design of foundations on sands and gravels should be based on ultimate bearing capacity theory. Many expressions relating this quantity to the angle of shearing resistance are available in the literature, some of alarming complexity, but all based on simplifying assumptions.

But how is the angle of shearing resistance to be determined? By what process is a factor of safety or load factor selected which will ensure that the settlements are acceptable? In fact, used in this way, the equation merely replaces one unknown by two.

Both theories I have referred to have, of course, their rightful place and function.

I hope it is clear that it is not mechanics or its application as such that I have questioned, only its unnecessary or inappropriate use.

3. I present now two cases where solutions were developed to answer specific questions. The answers were found, but in both cases more emerged: in the first a lesson in tactics, in the second a result of some generality.

3.1 The first concerns the progress of three-dimensional consolidation of a clay layer of thickness h and infinite lateral extent, loaded on its surface by a pressure p distributed over a circular area of radius R.

This reduces, effectively to a standard one-dimensional problem when (R/h) is large. The Reporters have drawn attention to a solution available for a Biot half-space: this corresponds to small (R/h).

In many cases of practical importance, the conditions for the validity of these limiting solutions are not even approximately satisfied and to meet this need the layer problem was examined. The tedious but straightforward analysis was completed some years ago and led to this unpromising expression for the surface settlement S:

$$\frac{S(r,t)}{R} = \frac{p(1+\nu)}{E}\int_0^\infty \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} K(r,\lambda)\, F(s,\lambda) \times \exp(-\lambda^2+s)T\, ds\, d\lambda \qquad (1)$$

where

$$K(r,\lambda) = \frac{J_o(\lambda r/_h)J_1(\lambda R/_h)\tanh\lambda}{\lambda(1+\lambda\,\mathrm{cosech}\lambda\,\mathrm{sech}\lambda)},$$

$$F(s,\lambda) = \frac{1}{s+L(\lambda)-M(\lambda)s^{\frac{1}{2}}\tanh s^{\frac{1}{2}}},$$

$$M(\lambda) = \frac{(1-2\nu)\,\lambda\coth\lambda}{(1-\nu)(1+\lambda\,\mathrm{cosech}\lambda\,\mathrm{sech}\lambda)},$$

$$L(\lambda) = \lambda M(\lambda)\tanh\lambda - \lambda^2.$$

The moving singularities made a hand calculation impossibly lengthy and as we had no access to a large computer at that time, work ceased. I then realized that this was in no real sense a solution at all: the problem, essentially one of numerical analysis, had merely been recast in a different form.

Some years later Professor Robert L. Schiffman, of the University of Illinois at Chicago Circle, rescued the algebra and undertook with Dr. Pu the formidable task of numerical evaluation. It took some weeks to evolve a suitable strategy and by the end a lot of computer time. The paper by Gibson et al (1968) records the results but the real lessons were not reported there. They were:

1. Despite very simple geometry, loading and highly idealized constitutive relations, algebra of such complexity results that to obtain numerical answers a comparatively large amount of time and effort must be expended on numerical analysis and programming.

2. A comparable investment would be required for each variation of the basic problem.

3. A numerical attack from the outset might be rewarding. This approach could possess a generality lacking in our procedure.

Progress along these lines by us and others has been referred to in general terms by Professor Scott.

3.2 That analysis still has its place is illustrated by my final example.

In connection with studies of the settlement behaviour of structures on over-consolidated clays, Dr. Noel Simons, then of Imperial College, asked me to extend the classical solution for a flexible circular load on an incompressible elastic half-space, to take account of a linear increase with depth (z) of Young's modulus:

$$E(z) = E(o) + 3mz \qquad (2)$$

Solutions for particular values of Poisson's ratio and depth variations of E are available in rather inaccessible journals but nothing quite meeting our needs could be discovered. A numerical approach was indicated but, because the incompressibility condition can cause trouble, I chose to use analysis. By pure chance it was found possible to obtain expressions for the displacements and stresses (Gibson, 1967, 1968; Gibson et al, 1969). In particular the surface settlement, under uniform pressure p, is:

$$S(r) = \frac{pR}{4m}\int_0^\infty \frac{J_o(r\xi)\,J_1(R\xi)\,d\xi}{\xi\beta\Lambda\,(\xi\beta)}, \qquad (3)$$

where Λ is defined by

$$\Lambda(\lambda) = \lambda Ei(-2\lambda)\exp(2\lambda) + 1 + 1/2\lambda$$

and

$$\beta = E(o)/3m.$$

This nearly made us revert immediately to a finite element program.

However, it reduced correctly to the classical solution for constant $E(m=o)$. At the other limit $E(o)=o$, a wholly unexpected result emerged from equation (3):

$$S(r) = \frac{pR}{2m}\int_o^\infty J_o(r\xi)\, J_1(R\xi)\, d\xi, \qquad (4)$$
$$= p/2m \qquad (r < R)$$
$$= o \qquad (r > R).$$

This meant that the flexible load settled uniformly by an amount p/2m while no surface settlement whatever occurred outside the loaded area. The result can readily be generalized, using superposition, to show that for arbitrary loading over any area, the settlement at a point of the surface is proportional to the <u>local</u> pressure:

$$s = \frac{p}{2m}$$

This non-homogeneous elastic solid responded exactly like Winkler's layer of springs, while the term 2m can be identified as the familiar coefficient of subgrade reaction k_s.

Incidentally, in this limiting case, it was also shown that the stress components are identical with those in the *homogeneous* elastic medium.

If a numerical procedure had been employed from the outset it is difficult, for the following reasons, to imagine how these results could have been established, *unless they had already been anticipated intuitively.*

(1) The results are valid only for a half-space: a numerical attack would have, of necessity, been concerned with a layer of limited depth.

(2) Serious numerical difficulties would, almost certainly, have occurred due to the presence of a singularity around the edge of the loaded area, which the analysis revealed.

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Chairman J. G. ZEITLEN

Thank you very much Dr. Gibson. Dr. Kenney will you continue?

Panelist C. KENNEY (Canada)

Professors Scott and Ko must be congratulated and thanked for preparing the report on Stress-Deformation and Strength Characteristics. Their coverage of the research developments in this field is so comprehensive and so complete that after reading it several times I found myself asking the question - "is there anything that can be added?" Also, when studying the material content of the report that Professor Scott magnificently organized and presented, I experienced the frightening realization (as I expect many others did) that I had read only a few of the references sited in the report and was unable to understand the basics of many of the recent theoretical developments. As a university professor it is not always easy to admit not being able to understand the basics of one's technical field. The problem is not necessarily lack of mental ability (although this might be my problem) but rather a problem of not having sufficient time to put towards the study of new developments. For the practising engineer the problem is probably an order of magnitude greater than that for the university professor and it comes as no great surprise to any of us that the gap existing between theory and practice appears to be widening at an ever-increasing rate. If we care to look back to the Proceedings of the 1st International Conference it can be seen that many papers contained both theoretical developments and their applications to engineering practice. It is a relatively rare occurrence that this is found in new literature today This general problem is not confined to soil mechanics and foundation engineering but is prevalent in all branches of engineering.

If for no other reason but to be provocative I will say that much of the blame for this situation must be taken by the university communities. There are ever-increasing pressures building up in the academic world for professors to do research and to publish their results. In itself this is not an unhealthy situation because the advancement of knowledge is a prerequisite to all material and human advancements, but the key is that the research should be of value and of relevance. We would all agree that there is being done today perhaps the highest-quality research of all times, but we would also quickly agree that this forms a disturbingly small proportion of the total research being published. Again to be provocative with my academic colleagues, there are many of us at universities that are not scientists and who have little hope of making significant scientific contributions to our field. Rather, we are primarily engineers or scientific engineers, and therefore we are best qualified to work at the interface between science and engineering practice, the interface that we realize is being sadly neglected today. Is it not possible for more university personnel to become involved in research problems associated with engineering practice? And it can also be asked - cannot more consulting companies take some of their difficult problems to the universities for the purpose of applying co-operative approaches of research and engineering experience to the problems? I might simply add to my academic colleagues that the potential rewards in working (and publishing) at the interface between science and engineering practice can be much greater to many of us than working in science or pseudo-science when measured with respect to challenge, stimulation, interest and professional recognition. This is not meant to down-play science but rather to indicate that today the important point on the critical path of engineering progress is the effective application of theory to engineering practice, and that we at universities are best equipped and qualified to attack this critical area.

The four major Sessions following Session 1 deal with various areas of engineering practice and deal with problems centred on stress-deformation and strength characteristics of soils, the subject of Session 1. Perhaps the panel could spend a little time to discuss to what degree has the research activity contained in the Report for Session 1 helped us (or will help us) (i) to understand basically and (ii) to predict the movements of earth structures and foundations. It is probable that, coming out of such a discussion, a way will be indicated by which the gap between research developments and engineering practice can be narrowed.

Chairman J. G. ZEITLEN

Thank you very much Dr. Kenney, and I wish to again thank all the members of the panel. We will have a ten minutes break. Thank you again.

RECESS

Chairman J. G. ZEITLEN

I would like to call on professor Murayama of Kyoto University. Will he come up, please?

Panelist S. MURAYAMA (Japan)

1. Theoretical Considerations

In the author's paper "Stress-strain-time Behavior of Soils Subjected to Deviatoric Stress" presented to this Conference, the flow behavior of clay skeleton under the triaxial compression test has been expressed by the following equations (the following notations are identical with those in the paper):

$$\left.\begin{array}{ll} \gamma = A.W.z & W.Z = P \\ \text{where } z = \sigma_d/(\sigma_m' + \sigma_b), & \sigma_d = \sigma_1 - \sigma_3 \end{array}\right\} \quad (1)$$

If the flowing proceeds under the state of retarded elasticity, it has been given as

$$\left.\begin{array}{ll} z < z_{el} & , \ W\text{: constant} \\ A = A_e \cdot \psi(t) & , \ A_e\text{: constant} \end{array}\right\} \quad (2)$$

According to the paper, the time function $\psi(t)$ can be calculated by simulating the flow behavior with that of the generalized Voigt model whose individual retardation time is τ ($\tau = \eta_i/G_i$). If the retardation time spectrum $M(\ln \tau)$ is expressed by a box-type distribution from τ_i to τ_m, $\psi(t)$ is given by

$$\left.\begin{array}{ll} \psi(t) = a + b \ln t & \text{for } \tau_i < t < \tau_m \\ \psi(t) = 1 & \text{at } t \to \infty \end{array}\right\} (3)$$

where a and b are constants at a constant temperature. At the end of flowing after a long period of stress application, the mobilizing probability P reaches a finite value, which is denoted by P_∞. Hence,

$$P_{t\to\infty} = P_\infty \quad (4)$$

Since the increase in the probability of the actual mobilizing particle P is caused by the displacement of the particle which is related to the displacement factor A, the following relation may be proposed.

$$\frac{1}{A}\frac{dA}{dt} = \frac{1}{P}\frac{dP}{dt} \quad (5)$$

Solving the above equation and substituting Eqs. (2), (3), and (4) for the condition of $\tau_i < t < \tau_m$, we get

$$\left.\begin{array}{l} A = A_e\,(a + b \ln t) \\ P = P_\infty(a + b \ln t) \end{array}\right\} \quad (6)$$

Therefore, γ in Eq. (1) becomes

$$\gamma = A_e P_\infty (a + b \ln t)^2 \quad (7)$$

From Equations (1), (6), and (7), the following relation is obtained.

$$\left.\begin{array}{l} \ln\left(\frac{d\gamma}{dt}/\sigma_d\right) = B - \ln(\sigma_m' + \sigma_b) - \ln t \\ B = \ln(2A_e bW)\text{: (const)} \end{array}\right\} (8)$$

This is a relation between the strain rate and time t as far as the deviatoric stress σ_d is constant. If the mean effective stress σ_m' is constant, the strain rate or the creep compliance decreases with log t. As σ_m' is time dependent, however, the rate of strain does not simply decrease with log t.

When the clay sample is subjected to deviatoric stress, a porewater pressure u is usually generated. In the case of the normally consolidated clay whose preconsolidation stress is σ_{mo}, σ_m is given as follows:

$$\left.\begin{array}{l} \sigma_m' = \sigma_{mo} + \Delta\sigma_m' \\ \Delta\sigma_m' = \sigma_d/3 - u \end{array}\right\} \quad (9)$$

As the internal confining stress σ_b is affect ed by the nearness among particles due to the consolidation, σ_b should vary with the preconsolidation stress σ_{mo}. If a certain value of σ_b is adopted as a standard and is denoted as σ_{bo}, σ_b of any clay can be expressed as follows:

$$\sigma_b = \sigma_{bo} + \Delta\sigma_b \tag{10}$$

Hence,

$$\left.\begin{aligned} &\sigma'_m + \sigma_b = \left\{1 + a_s(\Delta\sigma'_m + \Delta\sigma_b)\right\}(\sigma_{mo}\,\sigma_{bo}) \\ &\text{where} \quad a_s = 1/(\sigma_{mo} + \sigma_{bo}) \end{aligned}\right\} \tag{11}$$

$$\left.\begin{aligned} &\text{In Eq. (11), if } -1 < {}_s(\sigma_{mo}+\sigma_{bo}) \le 1 \\ &\text{then, } \log(\sigma'_m + \sigma_b) \doteqdot a_s(\Delta\sigma'_m + \Delta\sigma_b) + \log(\sigma_{mo} + \sigma_{bo}) \end{aligned}\right\} \tag{12}$$

Moreover, if σ_b of the normally consolidated clay can be assumed to be proportional to the consolidation stress, $\Delta\sigma_b$ is expressed as

$$\left.\begin{aligned} &\Delta\sigma_b = \beta\cdot\Delta\sigma_{mo} \\ &\text{or} \\ &a_s\cdot\Delta\sigma_b = a_s\cdot\beta\cdot\Delta\sigma_{mo} = a_c\Delta\sigma_{mo}\ \ a_s\beta = a_c \end{aligned}\right\} \tag{13}$$

where β : the coefficient of proportion, $\Delta\sigma_{mo}$ the difference between the consolidation stress of a clay sample and the standardized consolidation stress.

Substituting Eqs. (11) and (13) into Eq. (8), we get

$$\ln\left(\frac{d\gamma}{dt}/\sigma_d\right) = B - \ln(\sigma_{mo}\ \sigma_{bo}) - \ln t - a_s\cdot\Delta\sigma_m - a_c\cdot\Delta\sigma_{mo} \tag{14}$$

Besides Eq. (14), we take the following, another equation excluding the last two terms in the right hand side of Eq. (14):

$$\ln\left(\frac{d\gamma}{dt}/\sigma_d\right) = B - \ln(\sigma_{mo}+\sigma_b) - \ln t \tag{15}$$

Since Eq. (15) is represented by a straight line inclined 45° to the axis on a logarithmic paper, Eq. (15) can be represented by shifting the relationship of Eq. (14) by $(a_s\cdot\Delta\sigma_m + a_c\cdot\Delta\sigma_b)$ along log t-axis horizontally.

2. Experimental Results

Triaxial compressive flow tests were performed on normally consolidated clay samples at a constant temperature under undrained condition (Murayama and Morisawa, 1969). The porewater pressure during flow was observed. Testing conditions were as follows: (a) The various deviatoric stresses were applied to the samples of the same pre-consolidation stress of $\sigma_{mo} = \sigma_c = 2.0$ kg/cm^2, (b) The same deviatoric stress of $\sigma_d = 4.0$ kg/cm^2 to the samples of the various pre-consolidation stresses, (c) The various deviatoric stresses to the samples of the various pre-consolidation stresses.

The relationship of $d\gamma/dt/\sigma_d$ - t is identical with the relationship of the creep compliance J(log τ) and the retardation time τ in the generalized Voigt model. Such relationship obtained under the testing condition (a) is shown in Fig. 1-a. When the plots in Fig. 1-a are shifted by $a_s\cdot\Delta\sigma'_m$ ($\Delta\sigma'_m = \sigma'_m\ \sigma_c$, $a_s = 0.96$) along log t-axis or log τ -axis, the shifted relationship obtained is shown in Fig. 1-b. It may be noticed in Fig. 1-b that all plots lie along a straight line inclined 45° to the horizontal axis. Similarly, Fig. 2-a is the creep compliance - τ relationship obtained under the testing condition (c). The new plots obtained by shifting the points horizontally in Fig. 2-a by $(a_s.\Delta\sigma_m + a_c.\Delta\sigma_{mo})$ (where a_s=0.96, a_c = 0.34) are shown in Fig. 2-b, in which the new plots also lie along the straight line of 45° inclination.

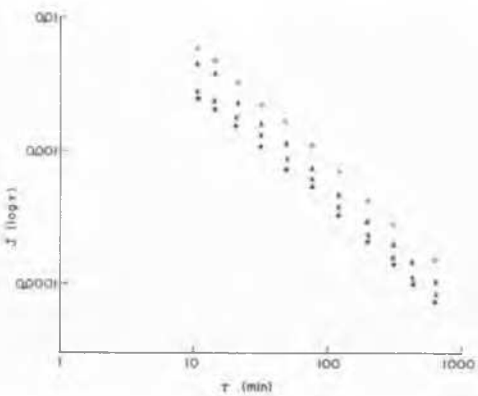


Fig. 1-a. Creep compliance-retardation time relationship obtained by experiments

σ_c : constant = 2.0 kg/cm^2

∘ σ_d = 1.0 kg/cm^2 ∆ σ_d = 0.8 kg/cm^2 x σ_d = 0.6 kg/cm^2 • σ_d = 0.4 kg/cm^2

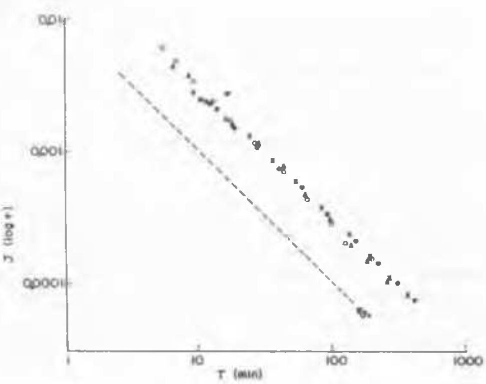


Fig. 1-b. Shifted relationship of Fig. 1-a by $a_s.\Delta\sigma'_m$.

σ_c = 2.0 kg/cm^2. See symbols Fig. 1-a.

From these verifications, the assumptions stated above may be acceptable. It may be concluded from these experiments that the rheological behavior of the skeleton of normally consolidated clay can be simulated by that of the generalized Voigt model of a box-type retardation spectrum.

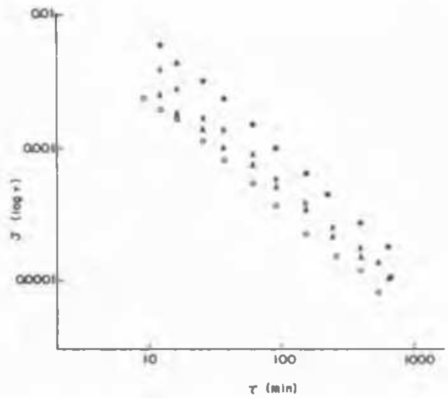


Fig. 2-a. Creep compliance-retardation time relationships obtained by experiments (σ_d , σ_c : variable)

• $\sigma_c = 1.0$ kg/cm^2 , $\sigma_d = 0.4$ kg/cm^2
x $\sigma_c = 2.0$ kg/cm^2 , $\sigma_d = 0.8$ kg/cm^2
Δ $\sigma_c = 3.0$ kg/cm^2 , $\sigma_d = 1.2$ kg/cm^2
o $\sigma_c = 4.0$ kg/cm^2 , $\sigma_d = 1.6$ kg/cm^2

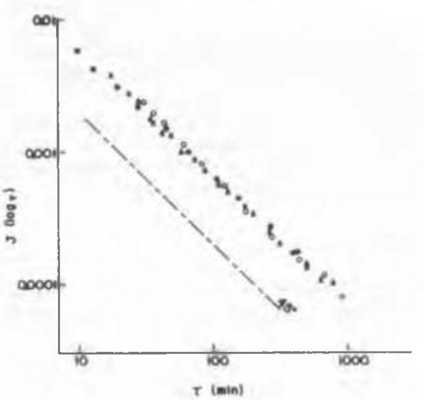


Fig. 2-b. Shifted relationship of Fig. 2-a by ($a_s \cdot \Delta\sigma'_m + a_c \cdot \Delta\sigma_{mo}$). See symbols Fig. 2-a.

REFERENCE

Murayama, S. and M. Morisawa, "On the stress-strain behavior of clays", Proc. of Annual Meeting of Kansai Branch of JSCE, 1969, (in Japanese), pp. III.10.1 - 4.

Chairman J. G. ZEITLEN

Thank you very much Dr. Murayama. We will hear Dr. Adel Saada, Associate Professor of Engineering from Case Western Reserve University. Please, Dr. Saada.

A. S. SAADA (U. S. A.)

Ladies and Gentlemen, my statement and question are related to that part of the State-of-the-Art connected with Anisotropy. The question is addressed to both the General

Reporter and the Members of the Panel:

For a cross anisotropic saturated clay (i.e. with an axis of rotational symmetry), the directions of the principal stress and the principal strains generally do not coincide; different stress-strain curves and failure values are obtained depending on the inclination of the principal stress on the axis of symmetry. This inclination of the stresses will affect the pore pressure responses since this response is the result of the tendency of the material to change volume during strain. All the quantities mentioned depend on and are intimately attached to the arrangement of the particles that gives the clay the property of cross anisotropy. As far as Coulomb's failure criterion is concerned, there are as many strain lines as there are planes and a tangent to a group of Mohr circles is meaningless.

Consequently, how oan a saturated clay be anisotropio with respect to pore pressure development and isotropic with respect to strength parameters (effective or total)?

The answers and comments of the panel are of paramount importance, since they may determine at least one of the directions in which the study of anistropic clays will be pursued by many investigators. Let us limit the question to clays tested under controlled rates of stress, or, if you wish, let us worry about peak points alone; at these points the deformation are small and the anisotropy of the material cannot possibly have been erased.

Chairman J. G. ZEITLEN

Thank you very much for your question; I think we appreciate the briefness as well. We also have a question from Engineer Loof from the Soil Mechanics Laboratory at Delft, in the Netherlands, I am wondering if he could state that also, and we will give Dr. Scott as well as the members of the panel an opportunity to comment on both matters.

W. H. LOOF (Netherlands)

My name is Loof, and I am employed at the Soil Mechanics Laboratory in Delft, Holland. I already have addressed this question in writing to Prof. Scott, but repeating it, I simply want to ask this; in designing a structure, we normally set the limits for settlement or settlement rate, my question is:

Whether it is possible to use a simple linear equation for the stress-strain relationship below these limits?

That is all. Thank you.

Chairman J. G. ZEITLEN

Thank you very much. Dr. Kenney, Dr. Scott, suggests you may wish to answer the first question.

Panelist C. KENNEY (Canada)

In answer to professor Saada he was asking about strength, but he did not specify if it was drained or undrained strength; I will take undrained strength first. The materials you were dealing with, are anisotropic with respect to strength. As regards effective stress, for practical purposes there is no essential difference in strength values measured directionally from evidence, we have now on materials that had intact structures. In other words, with respect to effective stress it does not appear that anisotropy of the shear strength parameters is an important value although it may be measurable value.

A. S. SAADA (U. S. A.)

May I comment?

Chairman J. G. ZEITLEN

Yes, please.

A. S. SAADA (U. S. A.)

I wish to state that, to my knowledge, the only tests that have been conducted on anisotropic clays under combined stresses are those of Saada and Baah published in the 1967 Proceedings of the Third Panamerican Conference on Soil Mechanics and those published by Saada and Bamani in volume one of the Proceedings of this Conference (pp.351). All the tests show that it is impossible for the clay to be anisotropic with respect to strength parameters. More details about this point were also given by Saada in a discussion of the paper by Duncan and Seed quoted in the State-of-the-Art. Thank you.

Chairman J. G. ZEITLEN

Any further comments from the Panel? Doctor Scott, would you care to continue with the question of Dr. Loof?

General Reporter R. F. SCOTT

I think that Mr. Loof has asked a question that is probably in all of your minds. We have the old theories that are linear, and nowadays a great many papers come out discussing this non-linear business, and how much of this is worthwhile, how much of it you should try to read and understand and how much of it is a waste of time because linear is good enough? The answer of course has to be a relatively complicated one, because if you have a particular problem of a footing, or a foundation, or a retaining wall and you wish to calculate the displacements of the soil underneath it, or around it or adjacent to the structure, how well you need to know the deformations and to what level of stresses you need to know them are two of the pertinent points in connection with how nonlinear is the material behaviour. If you only are stressing material anywhere up to let's say 10 or 15 percent of some quote, some failure strength or yield strength, maybe linear is quite good enough. I think that in a variety of real problems it will turn out in many cases that linear is not quite good enough, and I am very lucky to have the best demonstration of that already presented this afternoon, if you can cast your minds back to the second or third last slide presented by Dr. Mitchell today, on the footing loading test that he showed, you may recollect that the curve of load vs. displacement of the footing was extremely curved, and that it would have been very difficult to represent that by any kind of linear behaviour which, of course, put together with the geometry and so on, would have ended up with a linear forced deflection curve; in that particular case, the linear theory would not have predicted within, say, 100% or so, what the actual displacement would have been; however, in the slide he showed after that there is rather a weak non-linear behaviour and possibly in that particular situation one would have been happy with a linear theory and the consequently smaller amount of work required to obtain the answers.

Chairman J. G. ZEITLEN

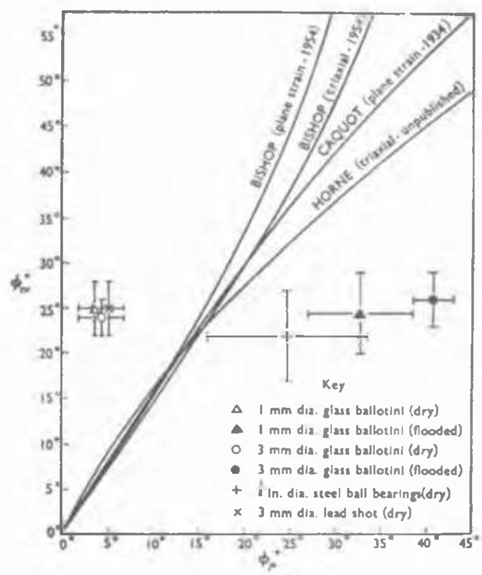
Thank you, Dr. Scott. We will continue with Professor Bishop. Would he be kind enough to come up?

A. W. BISHOP (England)

Professor Scott has referred to the use of granular models in the study of cohesionless materials, and in particular to the theoretical work of Professor M. R. Horne. In Fig. 1.3.1 of Professor Scott's report he produces Horne's theoretical relationship between ϕ'_{cv} and the material coefficient ϕ_{μ}, the coefficient of friction between the individual particles. Experimental data is shown in this figure which appears in general to support this relationship.

I would like to discuss in particular the measurement of the material coefficient

ϕ_μ. Fig. 1 shows the results of tests carried out by my colleague Mr. A. E. Skinner and described in Geotechnique in March 1969. It shows the theoretical relationship obtained by Horne and three earlier theoretical relationships for plane strain and for triaxial compression obtained by more approximate methods. All of these methods ignore rolling of particles. This is an assumption common to Caquot's solution, to Horne's and to my own. In the lower part of the figure are the experimental points from tests on a series of spherical particles of widely ranging coefficients of friction, the lowest being dry ballotini over on the left side, the highest, flooded ballotini under low stresses, on the right side. You will see that there is no significant difference in the values of the parameter ϕ'_{cv} the mass angle of friction, inspite of this wide variation in ϕ_μ.



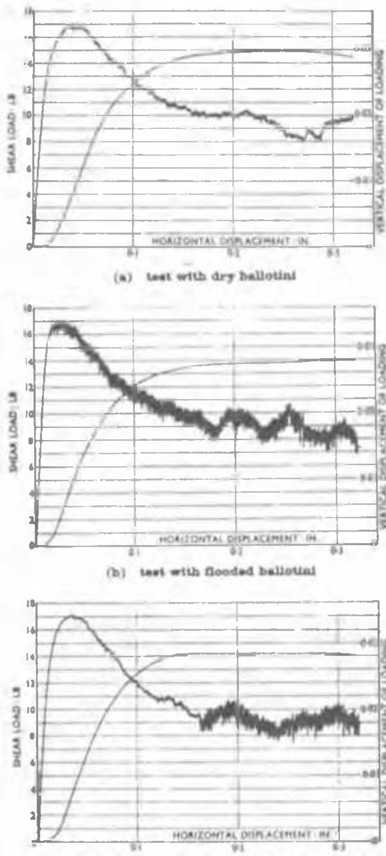
Theoretical and experimental relations between ϕ_μ and ϕ'_{cv}

Fig. 1.

I would like to examine briefly the way in which ϕ_μ has been measured. In tests quoted by Horne, (see Horne, 1969 and Rowe 1969), it was backfigured using the stress-dilatancy relationship (Rowe 1962 and Horne 1965) from drained tests on some samples of the granular material in which dilatancy occurred. In three cases these results have been correlated with tests described by Rowe in which a half shear box of granular material has been slid on a prepared plate of similar material. In neither case, therefore, do we have a direct measurement of interparticle friction.

My colleague Mr. Skinner has been measuring directly the coefficient (since the particles are in all cases large enough) of one particle sliding over two, set up in a very small shear device using electrical measurement techniques.

Table 1 shows the range of values of ϕ_μ for the tests given there, extending from very low values for the glass ballotini tested dry to very high values for the glass ballotini tested wet, and relatively high values for steel balls. These are all spherical particles to a close degree of approximation.



(a) test with dry ballotini

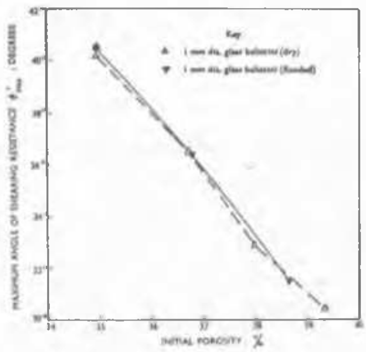
(b) test with flooded ballotini

(c) test started with dry ballotini and flooded after 0·161 in. horizontal displacement

Shear load and vertical displacement of loading platen plotted against horizontal displacement for shear box tests on 1 mm dia. glass ballotini, all at the same initial porosity of 34·8% under a normal load of 20 lb

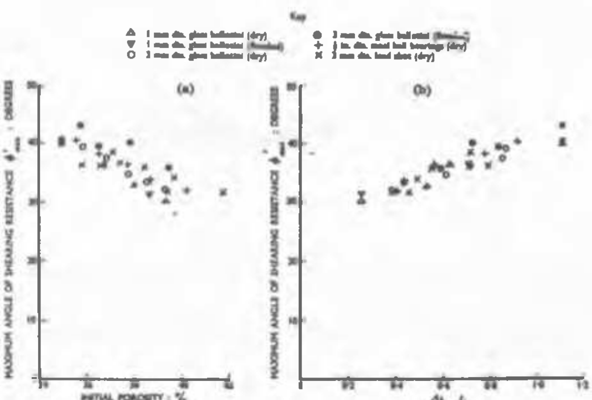
Fig. 2.

Fig. 2 shows three of the shear tests using a shear box (which approximates to plane strain) for dry ballotini (Fig. 2a), flooded ballotini (Fig. 2b), which you will see has a very irregular stress-strain curve at and past the peak; and in Fig. 2c for ballotini which were tested dry in the first half of the test, and then flooded halfway through the test, when this irregular pattern emerges indicating a different mechanism of failure. You will notice that the peak strengths are almost the same in all three cases, each sample being under the same normal load. Fig. 3 shows the results of the strength tests for wet and dry ballotini plotted against initial porosity, and again there is no significant difference in spite of the wide difference in ϕ_μ, between the shear strengths as measured in the shear box test. In Fig. 4b these results, together with those of the tests on steel balls and lead shot, are plotted as the angle of friction against the rate of dilatancy, and although there is some scatter you will see that they can be well represented by a common line. In Fig. 4a the results are plotted on the basis of initial porosity.



ϕ_{max} plotted against per cent initial porosity for dry and flooded shear box tests on 1 mm dia. glass ballotini using a normal stress of 3·58 lb/sq. in.

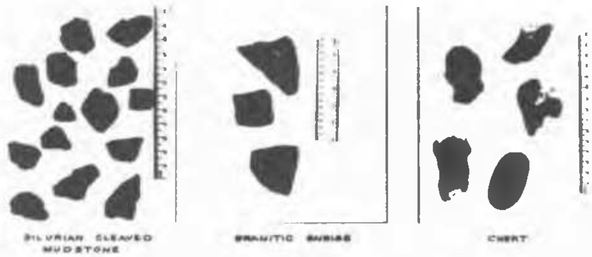
Fig.3.



ϕ_{max} plotted against per cent initial porosity for shear box tests on 1 and 3 mm dia. glass ballotini—dry and flooded, ¼ in. dia. steel ball bearings and 3 mm dia. lead shot
(b) ϕ_{max} plotted against $\Delta hL/h_1\Delta L$ for the same series of tests

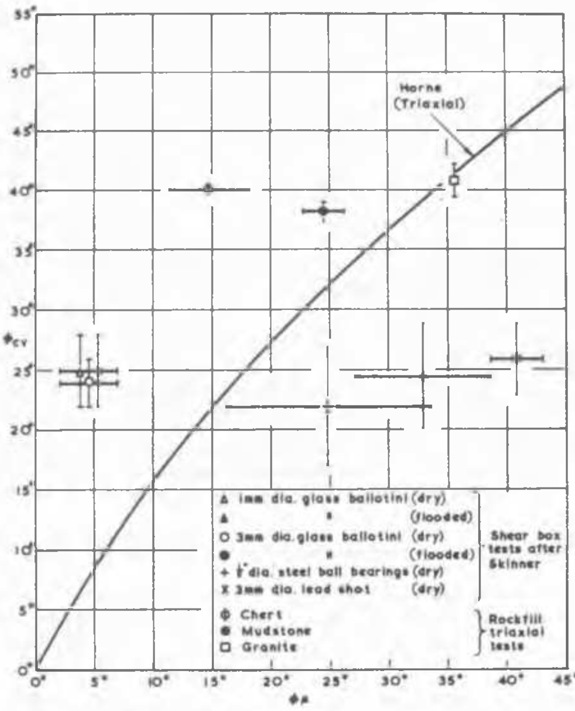
Fig.4.

We have more recently examined the same phenomena with angular particles of two rockfills. Here one can more readily handle representative particles and you do not need a watchmaker's technique in performing the interparticle friction tests. A large particle can be set up in the conventional shear box and slid over another particle. We have tested two rockfills and also chert, which is a gravel sized material of a tuberous shape, Fig. 5. The chert particles, although smooth in surface texture, tend to interlock during shear. In Fig. 6 the results of tests carried out by Tombs (1969) are super-imposed on Fig. 1, the spherical particles forming the lower group at around ϕ'_{cv} of 25°, and the rockfill particles forming another group at a value of ϕ'_{cv} of about 40° or so.



GRAIN SHAPES OF THE MATERIALS TESTED

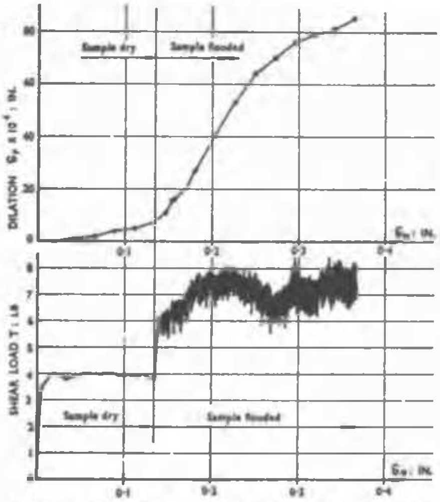
Fig.5.



THE RELATION BETWEEN ϕ_{cv} & ϕ_μ

Fig.6.

These experimental results would appear to me to suggest that Horne's theory is oversimplified and that shape factor plays a very important part in this relationship, and cannot be ignored as is implied by the graph shown by Professor Scott. It may be noted that in Scott's Fig. 1.3.1. the materials range from angular crushed glass particles in the top right hand corner to spherical particles at lower ϕ_{cv} values. Both graphs suggest that the shape factor may play a much larger part than is allowed for in the simple theory.



Shear load ~ horizontal displacement and dilation ~ horizontal displacement curves obtained from a special test with 3 mm dia. glass ballotini in the top half of the shear box, the bottom half being replaced by plate glass, the test being performed partly dry and partly flooded. Normal load 20 lb and initial porosity 37·6%

Fig. 7.

The test result in Fig. 7 suggests one possible explanation of the relatively good correlation between ϕ_{cv} and ϕ_μ in the tests quoted by Horne in contrast to the very different results obtained by Skinner and Tombs. The results are from a shear test in which a half shear box of glass ballotini is slid across a glass plate in a manner similar to that used by Rowe (1962, 1969) in testing several of the materials referred to by Professor Horne. Partway through the test the sample was flooded and you can see that in this case the sliding force increased and stress plot became irregular. However, the sliding force increased only by a factor of approximately 2. Now if you take single particles and slide them under careful control in these circumstances the value of ϕ_μ increases by approximately 5 times (Table 1).

It was suggested by Skinner (1969) that it was the occurrence of rolling which was the principal factor in explaining the small difference in shear force and in the irregular dilatancy curve in tests performed in this manner. Fig. 7 shows that there was almost no dilatancy with the dry particles compared with the flooded particles in this type of shear box test. I gather that Prof. Rowe considers this particular test to be controversial, but it is in any case auxilliary to the main discussion. I would direct your attention principally to the earlier slides that I have shown in which you will see that directly measured values of ϕ_μ show no significant correlation with ϕ_{cv}; but a significant correlation with shape factor is clearly indicated. I feel therefore that this particular field is far more open than has been suggested both in Professor Scott's report and in a good deal of currently published literature.

Table 1

Material	Test condition	Contact load, gm	Coefficient of friction
1 mm dia. glass ballotini	dry	5·7 11·5 23·1 52·0	0·03–0·05 0·05–0·07 0·07–0·09 0·06–0·08
	flooded	5·7 11·5 23·1 52·0	0·50–0·58 0·64–0·72 0·76–0·79 0·78–0·80
3 mm dia. glass ballotini	dry	5·7 11·5 23·1 52·0 109·7	0·03–0·06 0·05–0·07 0·05–0·07 0·07–0·12 0·07–0·08
	flooded	[illegible]7 11·5 23·1 52·0 109·7	0·79–0·89 0·83–0·93 0·87–0·89 0·85–0·86 0·80–0·82
¼ in. dia. steel balls	dry	5·7 11·5 23·1 52·0 109·7	0·29–0·48 0·46–0·62 0·52–0·66 0·63–0·66 0·60–0·62
3 mm dia. lead shot	dry	5·7 11·5 23·1 52·0 109·7	0·07–0·08 0·06–0·08 0·08–0·09 0·10–0·11 0·10–0·12
3 mm dia. glass ball sliding on plate glass	dry	10·0 20·0 40·0 90·0 190·0	0·02–0·07 0·02–0·08 0·06–0·11 0·11–0·13 0·11–0·14
	flooded	10·0 20·0 40·0 90·0 190·0	0·61–0·90 0·85–0·88 0·85–0·88 0·89–0·91 0·80–0·90

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Chairman J. G. ZEITLEN

Thank you very much, Professor Bishop. We greatly appreciate your comments, and also appreciate the conciseness of the remarks. Mr. Mariotti, please?

M. MARIOTTI (Maroc)

Je voudrais appuyer l'impulsion que vient de donner Monsieur Kenney pour une étroite coopération entre les producteurs de méthodes théoriques et les ingénieurs praticiens, et je donnerai de ce point de vue un exemple de problème pour lequel les donnés de la théorie élastique soulèvent des doutes dans son application lorsque l'ingénieur n'examine pas avec attention toutes les données qui lui sont fournies pour contrôler qu'il n'y a pas incompatibilité entre ces donnés et les propriétés des matériaux qu'il utilise.

Ma remarque est faite à propos de l'application des théories élastiques des multicouches au cas de chaussées souples et assises de voies ferrées, application qui a fait l'objet de nombreuses publications dont une dans le cadre de cette session:

Je voudrais signaler en effect que l'ingénieur routier se trouve bien embarrassé lorsqu'il compare les données de la théorie élastique aux possibilités mécaniques des matériaux de la couche inférieure des chaussées souples:

Dans les chaussées souples classiques et à fortiori, dans les assises de voie ferrée, les matériaux de la couche inférieure sont des matériaux graveleux qui peuvent être beaucoup plus rigides que le sol de fondation mais qui sont évidemment dépourvus de résistance à la traction. Or, il apparaît dans les données fournies par la théorie élastique que le plus souvent cette couche est le siege de contraintes de traction qui ne sont pas du tout négligeables et qui sont donc incompatibles avec le comportement réel du matériel.

Les résultats théoriques ne peuvent donc prétendre à traduire fidèlement la distribution réelle des contraintes et des déformations de la chaussée souple. D'ailleurs on peut se demander avec raison si la vie limitée d'une chaussée souple, la fatigue assez mystérieuse qu'elle subit inexorablement, n'est pas attribuable simplement a l'adaptation élasto-plastique de la chaussée à un état de contraintes que certaines de ses parties ne peuvent admettre; certes, il en résulte une autre redistribution des contraintes qui doit sans doute conduire à un accroissement des efforts de cisaillement a l'interface chaussée-sol. Jusqu'ici l'ingénieur routier n'est pas averti de cette distribution et peut-être choisira-t-il pour le dimensionnement et la composition de sa chaussée un critère qui n'est peut-être pas le bon. Nous pensons donc que là se trouve un important exemple à examiner, pour lequel l'analyse théorique est allé trop vite et n'a pas encore tenu compte des limites du comportement élastique des matériaux utilisés; il serait souhaitable que s'établisse un dialogue plus étroit entre le théoricien et l'ingénieur praticien pour aboutir a une distribution plus réaliste des contraintes et déformations dans les chaussées souples et, du point de vue expérimental, pour avoir une connaissance plus profonde des propriétés des matériaux de chaussée pour connaître la forme des ruptures progressives des chaussées souples sous charges répétées.

Peut-être ainsi serait-on amené a changer le choix des critères fondamentaux de dimensionnement des chaussées et peut-être serait-on amené a envisager un choix de matériaux et une composition de la chaussée mieux adaptés aux contraintes réelles.

Chairman J. G. ZEITLEN

Thank you very much. I would like to ask Professor Schiffman from the University of Illinois to give us his remarks.

R. L. SCHIFFMAN (U. S. A.)

Professor Gibsons remarks are, as usual, most cogent and enlightening. His generosity to Dr. Pu and myself, while appreciated, is not an accurate statement of our contributions to the problem discussed. Our contribution was meaningful only in the sense that we performed a small amount of "technology" to implement Gibsons "science".

The "tactics" of problem solving, that Professor Gibson discussed, should be thoroughly considered prior to taking the field. Unfortunately there are no universal rules which can be applied as the tactics vary with geographical location, time and prejudice.

Basically the tactics used in problem solving involve first the point at which a numerical scheme is to be adopted; secondly the type of numerical method which one adopts; and thirdly the manner of computation. It is emphasized that all problems must ultimately provide numerical results. Thus to paraphrase G. B. Shaw we are only 'haggling over the price'. The principal has already been established.

The point within a problem solving process at which we develop a numerical process depends on a variety of parameters. Certainly, availability and access to a computer of sufficient size and speed is a major consideration. However, within a given technological state the tactical decisions are all too often guided by prejudice. We may choose to carry the analysis beyond a reasonable termination in the hope that it will avoid using the computer. On the other hand, we may prematurely jump to the computer in order to avoid the noxious necessity of thinking deeply about the problem. Obviously, both extremities are to be avoided. In general, a proper tactic is one which carries the analytical portions of a problem to a point where the numerical computations are most efficiently performed. That is, they are performed in the least time for a given total effort. In earlier days the comparison between machine and hand computation was so great that we considered any machine computation as a major breakthrough for that problem. Now that machine computations are common place it is no longer sufficient to be using the machine. It must be used properly.

The proper balance between numerical and analytical portions of a problem can vary over the full range of consideration. Clearly, the problem of stress distribution in a semi-infinite solid is one in which as much analysis as possible should be done prior to performing numerical computations. Just as clearly the one-dimensional consolidation of multi-layered clay deposits (Davis and Lee, 1969; Schiffman and Stein, 1969) is one which must be solved by a wholly numerical scheme. In the middle ground are a series of problems typified by Professor Gibson's examples where a practical choice exists concerning the problem solving tactics. The unfortunate fact of life is that our knowledge of numerical techniques for solving three-dimensional consolidation problems is limited. The methods are crude and inefficient. The results are inaccurate and use a substantial amount of time even on the largest and fastest computer available. This would imply, as intended, that for these problems the proper approach, at this time, is to carry the analysis as far as possible before entering the computational phase. When dealing with the semi-infinite solid (Gibson and McNamee, 1963) or the single layer (Gibson, Schiffman and Pu, 1968) it is clear that the balance is on the side of analysis. On the other hand, when dealing with many layers or non-regular boundaries the algebra reaches such proportions of intractability that practicality dictates a wholly numerical solution, even though the method may be poor.

This brings up the second point in the tactics of problem solving; namely the choice of technique. The two principles which should be followed in developing a tactic; assuming a choice exists, are computational efficiency and accuracy. The one-dimensional multi-layer consolidation problem is a good case in point. This problem is treated by a wholly numerical technique (Abbott, 1960; Jordan and Schiffman, 1967; Davis and Lee, 1969; Schiffman and Stein, 1969) by a variety of finite difference procedures. The more recent studies have considered explicit (Jordan and Schiffman, 1967; Davis and Lee, 1969) and implicit methods of the Crank-Nicholson (1947) type (Schiffman and Stein, 1969). A comparison over a wide range of problems points to the comparative disadvantages of the explicit scheme when compared to the Crank-Nicholson approach. The later method is faster by several orders of magnitude and generally permits a greater accuracy. Clearly, the proper tactic in this case in favor of the Crank-Nicholson scheme.

The apparent algebraic simplicity of numerical procedures is often deceptive and can lead to the accumulation of numerical errors and subsequent substantial losses in accuracy. In general, the least accurate part of a total problem is the strongest influence on the overall accuracy of the problem. The numerical analysis of layered one-dimensional consolidation by finite differences requires numerical approximations of the governing equation, the boundary conditions, the initial conditions and the conditions at the layer interface. In general, the error of approximation of the governing equation, the boundary conditions and the initial conditions is of the order of the square of the spatial mesh size (Δz). The choice of a finite difference approximation for the interface conditions should be of the same order of error as the other approximations. If a first order forward or backward difference approximation is used at the layer interface (Davis and Lee, 1969) that segment of the solution has an error whose order is to the first power of (Δz). As shown in Figure 1 (Christian, 1969) this disparity of error is

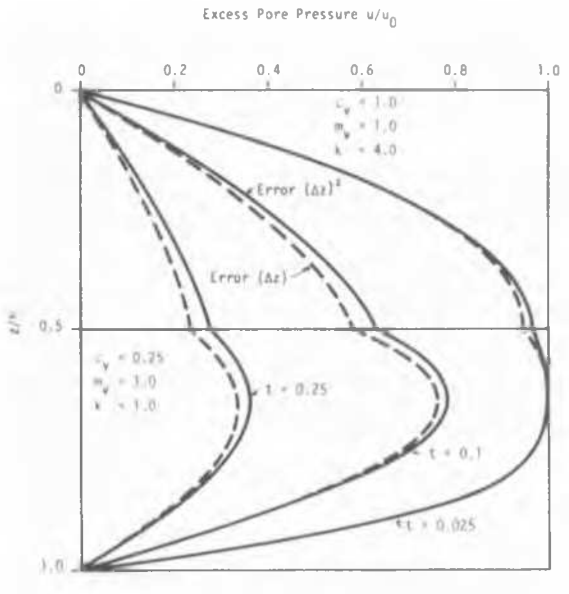


Figure 1

applicable at early times in the vicinity of the layer interface. However, with progressing time the larger interface error propagates throughout the medium until it overshadows the better accuracy of the other entities of the problem. Conversely if central differences are used for approximating the interface conditions (Jordan and Schiffman, 1967; Schiffman and Stein, 1969) a uniform order of error of $(\Delta z)^2$ is maintained. As seen in Figure 1 this maintains the level of accuracy throughout the consolidation process and throughout the medium.

Once a proper numerical scheme is adopted the problem solving tactician must look to the manner in which the computations are to be performed. One very important consideration is the precision of the computing machine. The fact that a computer printout may show five significant figures is often deceptive. A computational scheme may require substantially more significant figures to approximate the answer to a given problem to a lower order of precision. As an example, consider the one-dimensional consolidation of the multi-layered deposit shown in Table 1. This is an overconsolidated glacial till deposit which is free draining at the top and bottom boundaries. The clay is interspersed

TABLE 1

SOIL PROFILE - 6 LAYER SYSTEM

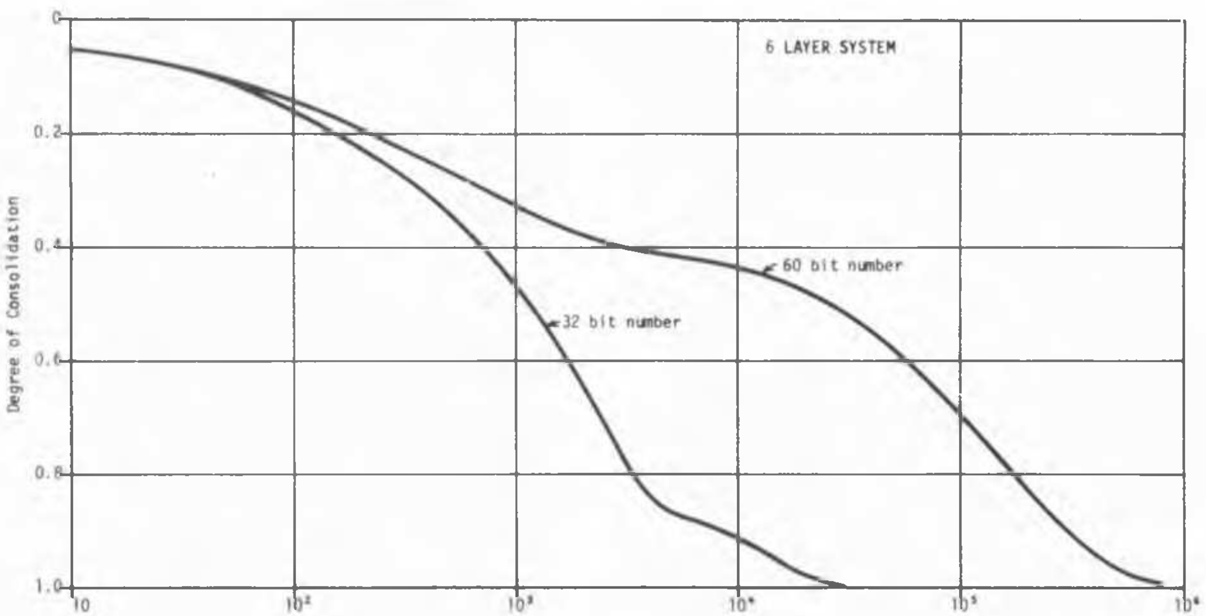
Depth(ft)	Layer No.	H(ft)	c_v(ft^2/day)	m_v(ft^2/kip)	k(ft/day)
0					
	1	2.5	.0411	3.07×10^{-3}	7.89×10^{-6}
2.5					
	2	9.5	.1918	1.95×10^{-3}	2.34×10^{-5}
12					
	3	1.0	1.0×10^{7}	1.0×10^{-6}	6.24×10^{-1}
13					
	4	15.0	.0548	9.74×10^{-4}	3.33×10^{-6}
28					
	5	1.0	1.0×10^{7}	1.0×10^{-6}	6.24×10^{-1}
29					
	6	24.5	.0686	1.95×10^{-3}	8.35×10^{-6}
53.5					

with two thin layers (3 and 5) of sand, which are confined within the loaded area and do not drain laterally. Thus, they do not form drainage surfaces.

The computations for this example were performed on two different computing machines using the same computer program (Schiffman and Stein, 1969). The results are shown in Figure 2. In one case the computer had a basic number (word) size of 32 binary bits (approximately 6 significant figures). In the other case the computer number size was 60 binary bits (approximately 14 significant figures)

It is intuitively obvious that the more correct result is the one carrying the largest number of significant figures. This is borne out by the isochrones shown in Figure 3. This plot is at an early time where the initial excess pore pressure within the sand layers has not started to dissipate. The smaller number size has permitted a significant rounding error. This, coupled with the substantial ratio of soil properties and layer thicknesses has created a fictitious appearance of excess pore pressure dissipation.

Thus, it is important in any calculation that the computer used have a number size which is adequate to the problem being solved.



Time, Days

Figure 2

Excess Pore Pressure, u/u_0

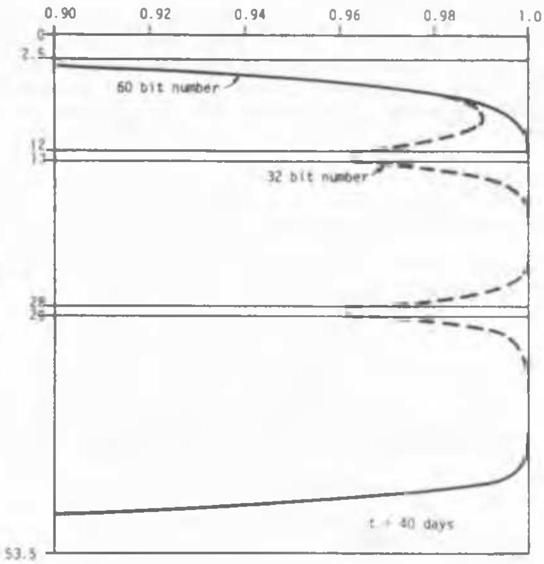


Figure 3

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Chairman J. G. ZEITLEN

Thank you very much, Professor Schiffman. Professor Tsytovich of Moscow, chairman of the National Association of Soil Mechanics of the U.S.S.R. will be giving us a short discussion.

N. A. TSYTOVICH (U. S. S. R.)

Observations of the settlement of structures erected on weak saturated clayey soils show that in most cases the amount of observed settlement differs from the calculated value determined on the basis of the theory of elasticity. In order to establish the actual behavior of foundations erected on silts (weak clays with organic inclusions), investigations were conducted in 1965-1968 to determine the stress at the surface of contact between the foundation and soil, and the variation of the vertical and horizontal pressures along the depth of the soil under the plate.

Since it is extremely difficult to insert a measuring instrument into the soil without impairing its natural structure, we selected lake Siwash for the site of the experiments.

The water in this shallow lake is from 0.5 to 1.0 m deep. To a depth of 5 to 8 m, the lake bottom is made up of homogeneous silts with a very high salt content (up to 20 per cent). The modulus of deformation of these silts ranges from 12 to 28 kg/cm^2, the angle of ineternal friction is 4° to 6°, and the cohesion is 0.1 to 0.25 kg/cm^2.

After being violated, the structural properties of the soil are restored by 90 to 100 per cent after 5 to 7 days as a result of crystallization of the salt between the clay particles. Therefore, after inserting the gauges for stress measurements, the structural bonds around the gauges were completely restored after 5 to 7 days.

Pressure gauges with a hydraulic transducer, similar to those designed by D. S. Baranov, were made to measure the stress at the contact surface and in the depth under the loading plate.

To raise the accuracy of measurement, the thickness of the working membrane on which the strain gauges are glued was reduced to 0.2 mm, and the input membrane was prestressed upon filling the gauge with a liquid. The gauges were carefully waterproofed to enable them to operate for a prolonged period of time in saturated salinated silts.

After being in operation for four (4) months, the gauges were taken out and it was found that their calibration curves exactly coincided with calibrating curves of the gauges plotted before the experiments.

The gauges had a sensitivity of 10 gr/cm^2. A semiconductor strain meter (type ПИД -10M) was employed as the recording instrument. Analytical calculations of the gauge errors, carried out according to the formulae of D. S. Baranov, and Pitti and Sporrow (2) showed that the errors of the gauges when inserted into the silt were within 12 per cent and, upon variation of the contact stresses, within 2 per cent.

Research was conducted on a site that was separated from lake Siwash by an earth dam. To install the loading plates having an area of 10,000 sq.cm., pits 0.8 m deep were dug. At the bottom of the pits, three holes, 80 mm in diameter and 1.8 m deep, were bored.

The holes were bored with a special soil sampler and the removed undisturbed samples of silt were retained and arranged in a definite sequence. Then the gauges were lowered into the holes by means of a special device which enabled the horizontal position of the gauge to be controlled. After this the holes were refilled by the removed samples to a depth of 0.5 m, tamping lightly with a special rammer.

In all, 15 gauges were installed at depths of 0.2, 0.7, 1.2 and 1.7 m under the center of the loading plate and its edges.

In addition, three gauges were installed under the edges and centre of the plate for measuring horizontal pressures inside the silts at a depth of 0.8 m.

Ten days after installing the gauges (time required for restoring the soil structure), a loading plate with an area of 10,000 sq.cm was set on the carefully levelled surface at the bottom of the pit.

Fourteen gauges were installed flush with the underside of the plate at various distances from its centre. This enabled the pressure to be measured under the centre and edges to determine the contact pressures in the metal plate. The gauges were installed in pairs (two each at the same distance from the centre) to exclude any chance errors in measurement.

Load was applied to the plate in steps from 0.05 to 0.10 kg/cm^2 by means of a hydraulic jack which bore against a reinforced concrete loading beam weighing 22 tons.

The plate was loaded until the pressure reached 0.9 - 1.0 kg/cm^2. At this, the settlement of the plate reached 9 to 11 cm.

After applying each step of pressure and after the settlement was stabilized, the pressure of the gauges was measured each hour to determine the variation in stress with time.

The investigations established that the diagram of contact pressures is of parabolic shape, convex side upward. The minimum value of the contact stress under the centre of the plate, at pressures on the plate ranging from 0.2 to 0.6 kg/cm^2, equalled from 0.5 to 0.55 of the average pressure under the plate. The maximum value of the contact pressure under the edges of the plate equalled 2.7 to 2.9 times the average pressure on the plate in the same range of pressures. Upon a further increase in the pressure up to 0.8 and 1.0 kg/cm^2 under the plate, the curvature of the parabolic diagram was reduced, i.e., the pressure was increased under the centre of the plate and was somewhat reduced under the edges (see Fig. 1).

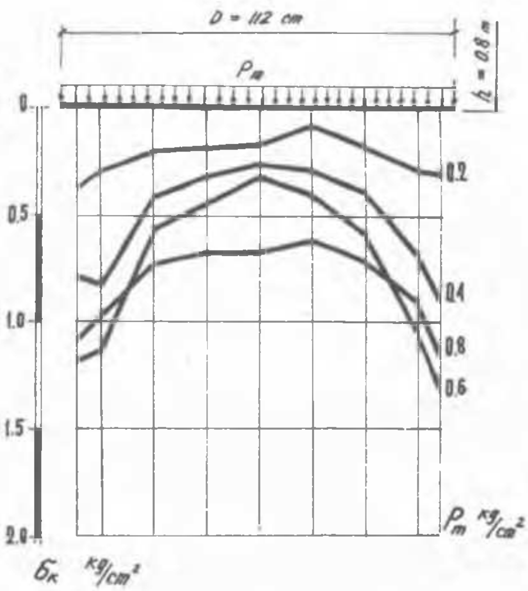

Fig. 1

This phenomenon can apparently be explained by the fact that at pressures of 0.5 to 0.6 kg/cm^2, the formation of a "rigid" wedge (3) was completed. This changes the nature of the contact stresses.

A study of the distribution of stress σ_z along the depth of the base under the plate, indicates that at pressures of 0.1 to 0.15 kg/cm^2 under the plate, at which practically no settlement occurred, the gauges registered pressures that were approximately the same along the depth (up to a depth of 1.8 m) and equal to the average pressure under the plate. With an increase in the load on the plate, the distribution of vertical stresses under the centre and edges of the plate, to a depth equal to one diameter of the plate, conforms sufficiently well with the character of stress distribution obtained from calculations according to the theory of linearly deformed bodies (the deviation was 10 to 12 per cent).

Beginning with a depth equal to one diameter of the plate, the measured vertical stresses are 30 to 40 per cent higher than the values obtained by the theory of linearly deformed bodies. This is witness to the fact that stresses in silts spread to a greater depth than in elastic media, and the "bulb" of equal vertical stresses is stretched out in the direction in which these stresses act.

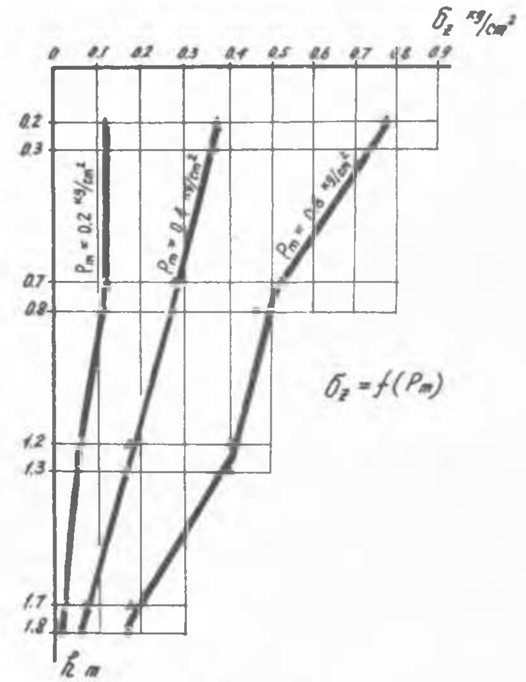

Fig. 2

Fig. 2 shows the results of vertical stress distribution along the depth under the centre of the plate. The curves indicate the average pressure on the plate. Fig. 3 shows the results of vertical stress distribution under the edges of the plate according to the data of experiments No. 4 and No. 5 (the results of investigations in experiments Nos. 1, 2, and 3, were of a similar nature).

Evidently, the depth of the active zone which determines the settlement of the plate is greater for weak saturated soils than is accepted for other kinds of soils.

Data of the experiments show that the method of calculating the settlement for weak saturated clayey soils, proposed by N.A. Tsytovich (4) and M. Yu. Abelev (5), corresponds well with the actual stress distribution under plates on weak clayey soil.

An investigation of the variation of the horizontal pressures with depth indicates that the coefficient of lateral pressure, equal to the ratio of the horizontal pressure increment to the vertical pressure increment, is not constant for silts, and depends essentially on their stressed state.

Thus, at pressures of 0.3 kg/cm^2 on the plate, the coefficient of lateral pressure was equal to 0.6, and at a pressure of 0.9 kg/cm^2 it was equal to 0.95.

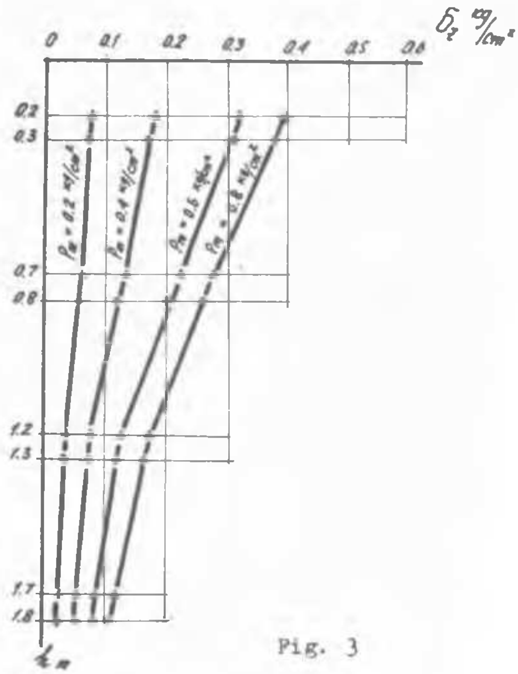

Fig. 3

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Chairman J. G. ZEITLEN

Thank you very much, Professor Tsytovich.

We will now hear Dr. Lo of Canada, Laval University.

K. L. LO (Canada)

The discussion refers to the second subject proposed for discussion: statistical approaches; granular models. In their State of the Art Report on Stress Deformation and Strength Characteristics, the General Reporters have noted that existing theories for shear behavior of real granular materials do not take into account particle breakage. In several engineering problems it is of major interest to know the amount of crushing and the predominant factors that control particle breakage. To obtain a solution for these problems it is therefore necessary to obtain (a) a complete quantitative description of the size distribution of the granular mass before and after action by an imposed system of stresses, and (b) the component of the total energy input which is absorbed in crushing. Because we are directly interested in the ability of tne particles to withstand the contact stresses, it is necessary to use particle statistics and mechanics rather than continum mechanics in these problems, at least as a starting point.

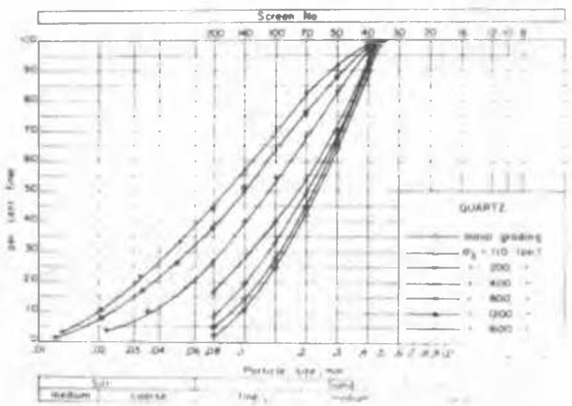

Fig. 1 shows the change in particle size distribution of a quartz sand subjected to drained triaxial tests with "free ends" under various confining pressures up to 1600 lb/sq.in. (112.6kg/sq.cm). Although the diagram gives a qualitative description of particle degradation, it is clear that such a plot is not amenable to mathematical treatment.

Grain size of many granular materials generally follows a logarithmic normal distribution. In other words, the frequency of occurrence of particles having a certain diameter is given by the normal distribution with a logarithm variate. The grain size curves presented in Fig. 1 may therefore be normalized in a socalled log probability plot as shown in Fig. 2.

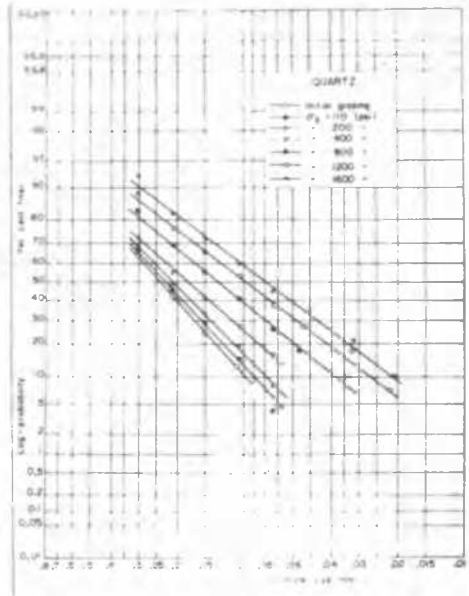

Fig. 2

In this plot, the ordinate is the value of the probability integral and the abscissa is the diameter. Each grain size distribution curve is now rigidly defined by the geometric mean diameter and the standard deviation (Hatch and Choate, 1929). From these quantities any "size properties" such as diameter, surface area and volume of the particles may be calculated.

Without going into mathematical details, the physical model for the computation of crushing energy may be described briefly as follows, in a somewhat oversimplified way. A sample of granular material initially posseses a certain total surface area of component particles. As crushing occurs, additional new surfaces are formed. This process involves the separation of atomic bonds and therefore energy is consumed. Therefore a relationship exists between the change in surface area and the crushing energy absorbed. The crushing energy to produce a unit change in surface area is a characteristic of the material of the particle and may be termed the "characteristic energy". This characteristic energy may be determined by the energy required for the crushing of a single spherical particle. It may be noted that this concept is similar to the classical Griffith theory of fracture. However, the characteristic energy is not the same as the surface energy postulated by Griffith, since the Griffith theory deals with initiation of crack propagation while we are concerned with complete failure of the particle. The two quantities may, however, be related.

The stress-dilatancy theory proposed by Rowe (1962) and elucidated by Horne (1965) may also be extended to include crushing energy, providing an alternative but independent means of estimating the crushing energy.

CELL PRESSURE σ_3' lb/sq. in	GEOMETRIC MEAN DIAMETER d_m mm	SPECIFIC SURFACE AREA S_w in^2/lb	CRUSHING ENERGY (lb-in/sq.in)	
			(a) SOIL MECHANICS	(b) FRACTURE MECHANICS
0	0.238	860	-	-
400	0.184	1280	220	250
800	0.134	1820	610	580
1200	0.102	2530	1250	1050
1600	0.082	3320	2220	1450

TABLE 1. COMPARISON OF CRUSHING ENERGY COMPUTED BY TWO METHODS OF A QUARTZ SAND

The crushing energy of a quartz sand calculated by these two methods are compared in Table 1. It will be seen that the agreement is reasonable, in view of the simplifying assumptions made. It is also interesting to note that when sheared at a cell pressure of 1600 lb/sq.in., the specific surface area increases by about four (4) times the initial value and the geometric mean diameter decreases to approximately one-third (1/3) of its original value.

The relationship between the geometric mean diameter and crushing energy is illustrated in Fig. 3 for three "sands" of (a) aluminum oxide, (b) quartz, and (c) limestone. It will be seen that the geometric mean diameter decreases with increase in crushing energy input. The amount of crushing increases with the decrease in hardness of the material from aluminum oxide to limestone sand, as expected. It is also interesting to note that significant crushing occurs at relatively low pressures for limestone.

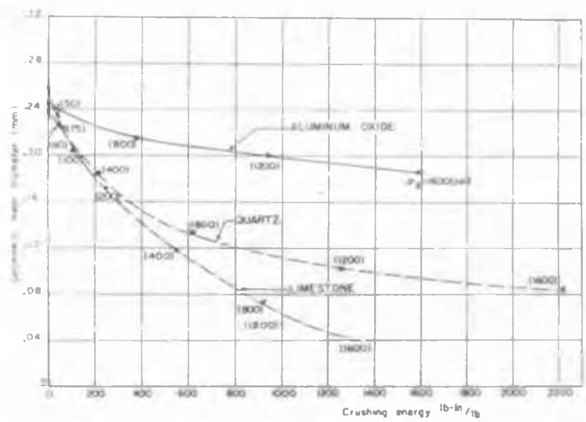

Fig. 3.-

The approach described and the results presented should be of interest in the formulation of a general stress deformation theory of granular materials including particle degradation, in the separation of strength into its physical components for analysis of test results and to several practical problems where particle breakage is an important consideration.

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Chairman J. G. ZEITLEN

Thank you very much Dr. Lo. I have been searching for Professor Roscoe... Here is Professor Roscoe of Cambridge University in England. He intends to discuss measurement of stress-strain behavior under generalized Three-dimensional stress and strain.

K. H. ROSCOE (England)

I was interested that Prof. Scott thought that the computer might help to bridge the gap between the practicing engineer and the research worker. Personally, I am frightened of the misuse that may be made of a computer and of the power that it gives to theoretical experts who invent parameters to describe soil properties which they never attempt to measure and which usually have no real physical significance. The output from a computer is worth nothing more than the parameters that are fed into it. Our capacity to compute has far outstripped our understanding of the stress-strain behaviour of soils and until we develop this understanding we can not feed the right parameters into the computer. I attribute much of the dealy in obtaining a knowledge of the generalized stress strain behaviour of soils to the incredible concentration there has been throughout the years on the so-called "triaxial" test. The conditions of this test are really axi-symmetric and are only relevant to field conditions in very rare cases. It is the duty of all research workers, who have a genuine interest in the correct prediction of the behaviour of real soils, to study the characteristics of such soils under as wide a variety of stress and strain conditions as posible. Many types of test equipment, including for example the axi-symmetric "triaxial" apparatus, suffer from the limitation thatthey compel the principal axes of stress and of strain to coincide at all times within the soil specimen. Even in the new truly triaxial test machines this is usually the case and the data obtained from them will not necessarily be of general application in field problems. Consider for example an element of soil in the neighborhood of a footing. As the load on the footing is increased the axes of stress and of strain within the element will rotate. It is essential that we should augment the knowledge obtained from truly triaxial test equipment with that obtained in types of apparatus in which the principal axes of stress and of strain rotate. I know of only two types of test suitable for this purpose. The first is to apply torsion to a hollow thin cylinder of soil which is subjected to internal and external radial pressures (see for example Saada 1968); this type of test is capable of producing a truly triaxial system of stress with the intermediate principal stress always in the radial direction, but is complicated to carry out and samples are difficult to prepare. The second type of test can be carried out in the simple shear apparatus, and in all the Cambridge models of this apparatus later than the Mk. 4 the directions of the principal axes of stress, stress rate, strain and strain rate can be determined independently at any stage of a test (see Roscoe, Basset and Cole, 1967). The Mark 7 model, designed by M.A. Stroud, with associated X-ray and data logging equipment is shown in Fig. 1. In the simple

Fig. 1. The Mk. 7 simple shear apparatus with X-ray and data logging equipment.

shear apparatus the samples are relatively easy to prepare but the conditions of test are restricted to those of plane strain. Contrary to the remarks of Professor Scott in his State-of-the-Art Report the simple shear apparatus can be used to impose any desired stress path within the restriction that the intermediate principal strain is zero (see Poorooshash and Roscoe 1961). May I also make three brief comments on the paper to this session by Duncan and Dunlop (1969). Firstly their comparison of simple and pure shear is erroneous; the main distinction is that simple shear imposes body rotation. Secondly the shear imposed by one dimensional consolidation while preparing a specimen in the simple shear apparatus has always been incorporated in any work at Cambridge when comparing data from this type of test directly with that from other tests such as the axi-symmetric test (see for example section 13.3 (i) of Roscoe and Burland, 1968). Thirdly the onset of progressive failure has been experimentally observed by X-and γ -ray methods (see Roscoe, 1967). This I would suggest, is more realiable than their computerised finite element analysis which is of no greater value than the soil moduli and the assumed boundary conditions that have been fed into it.

The main restriction that is evident in the types of true triaxial apparatus that have been developed so far has been the extremely limited strains that can be imposed. Fig.2

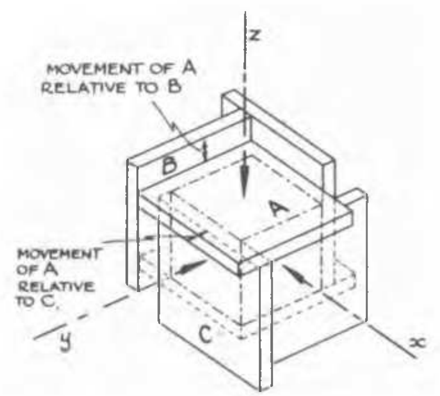

Fig. 2. Platen movement in large strain true triaxial. (After Hambly 1969)

shows the principle of a new true triaxial apparatus that has been made at Cambridge of a design by J.A. Pearce according to suggestions made by E.C. Hambly. The sample is a rectangular prism, each face of which is in contact with a platen (such as A) which is connected to, and can slide, as shown, relative to two neighboring platens (B & C). Any side of the sample can be made to change its length from 13 cm to 7 cm independently of the other two sides under either strain controlled or stress controlled conditions. Each platen is covered with nine square faced load cells capable of measuring the magnitude and position of the normal force and the magnitude and direction of the resultant shear forces on their active faces. The sample container is shown in the centre of its loading frame in Fig. 3.

Fig.3. The true triaxial apparatus, in its loading frame.

In conclusion, I would like to emphasise that the X-ray and lead shot techniques and the load cells we have developed at Cambridge provide a new method studying stress and strain behaviour not only in shear test apparatus but also in the mixed boundary value problems provided by soil model tests. If the model data is to represent to scale the behaviour in the field of the prototype, and if the self-weight of the soil is a significant parameter, the only satisfactory way of achieving this is by centrifugal model testing (see Roscoe 1968). Fig. 4 shows the 3½ m.

Fig. 4. 3½ m. radius centrifuge for testing scaled soil models.

radius centrifuge used by L.J. Endicott in his study of model slopes and Fig. 5 is a picture of a section of the failure plan in one of these slopes taken by N.K. Tovey and A. Balodis in the scanning electron microscope built at Cambridge.

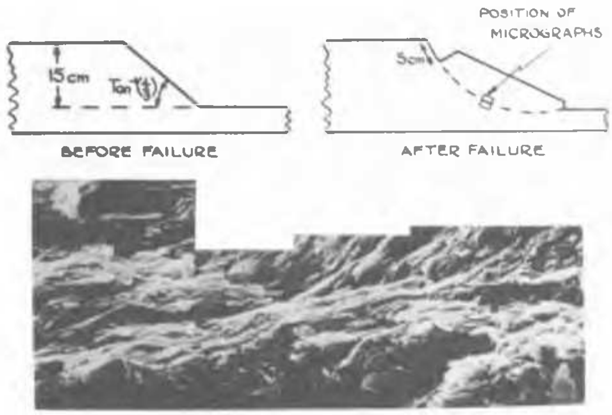

Fig. 5. Portion of failure plane in model slope in centrifuge. Scanning electron micrograph (mag. x 10,000).

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Chairman J. G. ZEITLEN

Thank you very much, Professor. We have a few minutes for Dr. Burland.

J. B. BURLAND (England)

I was interested in the question concerning the relevance in practice of representing soil behaviour by means of non-linear stress-strain relationships. It seems to me that in many instances a more fundamental question is whether any form of elastic or 'deformation' model, however complex, is adequate, or whether a totally different model might not be more realistic and possibly much simpler.

Consider, for example, the fundamental difference between elastic behaviour and, say, work hardening plastic behaviour. For an elastic material under a given initial state of stress the ratio of the components of the strain increments due to a small change in stress is a function only of the stress-increment. If, however, the material is yielding plastically the ratio of the plastic components of the strain increment is primarily a function of the initial stresses (and stress history). Often the stress-increment only enters the picture in determining the magnitude of the strain-increment.

The differences in mode of behaviour just mentioned are profound. Indeed for conditions in which significant rotations of the stress-increment axes occur, as is the case for many soil problems where the initial stresses are due to self-weight, even the most sophisticated elastic or deformation law may prove inadequate if the material is in a state of yield (i.e. if significant grain slip occurs in the case of a soil). In these circumstances incremental stress-strain relations based on the concepts of plasticity offer realism and simplicity. The use of even the simplest types of stress-strain relation which take account of yield and flow can provide very useful insight into the behaviour of soil under certain conditions (see for example Schofield and Wroth (1968), Poorooshasb et al (1967), Roscoe and Burland (1968) and Burland (1969).

In conclusion I would like to emphasise how important it is to gain a physical understanding of the soil behaviour before embarking upon a very refined analysis making use of sophisticated constitutive relations which may nevertheless fail to take account of the fundamental mode of behaviour of the material. The recent development of exciting and powerful methods of analysis should serve as a stimulus for investigating further the fundamental stress-strain properties of soil.

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Chairman J. G. ZEITLEN

Thank you, Dr. Burland.

General Reporter R. F. SCOTT

May I make a comment?

Chairman J. G. ZEITLEN

Yes, please.

General Reporter R. F. SCOTT

Unless I misunderstand, you are saying that non-linear elastic theory cannot account for volume changes at constant hydrostatic stress with shear, would you not? Second order non-linear elasticity theory does account for coupled shearing stresses and volume changes. Second order theory is the next simplest thing to straight forward linear theory. I do not know how simple you want to get.

Chairman J. G. ZEITLEN

I think Dr. Scott has to remember to get in in his subcomment along with the written comment of Dr. Burland. Mr. Bhatia, please.

H. S. BHATIA (Ghana)

I want to confine my remarks to two types of soils formed as a result of tropical weathering of rocks. One of such soils is a micaceous soil which is found in many tropical countries over muscovite-biotite granite type of rocks. The mica content in such soils is very high, generally between 40 and 70 per cent. The problem with such soils when they are compacted is the resistance of mica particles to densification. When such soils are compacted to high densities they show peculiar stress strain relations. The random arrangement of mica particles in the compacted soil mass depends on factors such as the particle size of mica, moisture content, degree of compaction and method of compaction, i.e., static, dynamic, kneading, etc. The arrangement of mica particles in the compacted material greatly affects the stress strain characteristics in such soils. When samples of these soils are subjected to even small strains, mica particles tend to release strain energy, thus undoing to some extent the effect of compaction. The result is that although such soils are compacted to various densities during shear, the mica particles tend to disturb the soil mass in order to attain a stable condition of no stress. This disturbance in the soil mass is reflected in the pore water pressure-strain relations during the test. With increasing strain the mica particles tend to orient along the shear plane, thus reflecting the shear strength as a result only of friction between the mica particles. It is further noticed that although the peak strength values in most of the soils compacted to increasing densities are quite different, yet at strains of about 23 - 24 per cent they register the same stress. With further strain, they tend to give only one curve for samples compacted to different densities. This strength must not be confused with Skempton's residual strength, as strains beyond 23 - 24 per cent produce further fall in stresses though the value of such stresses is the same for samples compacted to different densities. This arrangement of mica particles depends on several factors as pointed out earlier, and is responsible for the disturbance induced in the structure during shear. Now the question arises whether there is any simple method available with us today by which we can determine the arrangement of mica particles in a compacted mass.

We know through experimental work that certain methods of compaction tend to produce more stable arrangements of mica particles than others. If therefore we could find a simple method to determine in a compacted mass the arrangement of mica particles, that might make it very easy for us to adopt methods of compaction which give relatively stable arrangements of mica particles; arrangements that do not cause much disturbance and release of energy during shear. Such methods of compaction will help in making use of higher densities for achieving better strengths.

The second group of soils that I want to refer to is that of lateritic soils, which are formed in the tropics, on all types of rocks. Such soils are deprived of silica and have an accumulation of iron and alumina in them. The environmental conditions in lateritic soil profiles have a pronounced effect on the shear parameters, the stress-strain relations, the pore pressure-time relations, etc. If samples of these soils in the laboratory are air dried before compacting they will give completely different sets of stress-strain-pore pressure relations, as compared to the same soil obtained wet from borrow pits. These soils are usually in a wet condition in the field due to thick vegetation over them, and have never had a chance to dry out. The difference in stress-strain relations in the air-dried samples and in the wet samples is due to the presence of gel type iron-alumina hydroxides in the soil pores of wet samples which tend to dehydrate on exposure. The question therefore rises, if there are any methods available, based on the principle of electric conductivity or otherwise, which could indicate the state of dehydration of the iron-alumina hydroxide in the soil. If we could use a convenient method of checking the state of dehydration of the hydroxide of iron-alumina or, in other words, what we call the degree of laterization, this might make the interpretation of stress-strain-pore water pressure relations much easier. In the absence of such means,the highly sensitive soils of this group may register shear values in the laboratory which are not attainable in the field. This is due to the fact that sometimes such soils never have an opportunity to get even air dried in the field. The standardization of stress-strain relations in such soils is therefore of considerable importance for rational design.

Thank you very much Mr. Bhatia. The suggestion from the Panel is that this is an excellent subject to give to our neighbors at specialty session number seven, "Structural and Physico-Chemical Effects on the Properties of Clay" and six as well, on "Engineering Properties of Lateritic Soils. So, this being the hour, I thank you for your patience and I wish to thank the Panel very much, as well as the General Reporter for his excellent work.

WRITTEN CONTRIBUTIONS

CONTRIBUTIONS ECRITES

E. T. HANRAHAN (Ireland)

The problem of treating soil as a two-phase or three-phase material can be bypassed by dealing with effective stresses or by concentrating on granular materials in which time effects are relatively unimportant. However, such approaches cannot solve the important problem of estimating the magnitude and rate of settlement of a soft soil which is subject to load.

This deformation is time-dependent and is due to the combined effects of the spherical component and the deviatoric component of the stress tensor; both effects operate simultaneously and exert a fundamental and important interaction.

As regards behavior, a two-phase material differs in one very significant respect from a single-phase material. The rate of shear is independent of the dimensions of the specimen, or element, or deposit. The rate of compression, which is determined by hydrodynamic effects, is a variable which is influenced not only by the properties of the material but also by the dimensions of the specimen or deposit.

In this paper, it is shown that to achieve compatibility of strains imposed by a boundary, as in the oedometer test, adjustment of the lateral stress is required.

It is probable that the Terzaghi equation

$$c \frac{\partial u^2}{\partial z^2} = \frac{\partial u}{\partial t}$$

is not of general application to one-dimensional consolidation since the soil property Mv (i.e., the coefficient of unit volume change) is likely to be a variable depending on the lateral stress, which, in turn, is a function of the dimensions of the sample.

Finally, the treatment presented is believed to be applicable to the general condition of three-dimensional strain. However, since most of the tests reported were, in effect, oedometer tests, it is felt that the term "uniaxial consolidation" would be preferable to the term "plane strain" which has been used throughout the paper.

A. J. L. BOLOGNESI (Argentina)

This contribution refers to the author's overall stress-strain result of the paper "The Resistance Concept Applied to Deformation of Soils".

Instead of Janbu's $M = \frac{d\bar\sigma_v}{d\varepsilon_z}$ it is proposed $m_{\varepsilon z} = \frac{d\varepsilon_z}{d\bar\sigma_v}$

which might be called the coefficient of axial strain change. For any path $\Delta\varepsilon_z = m_{\varepsilon z}\ \Delta\bar\sigma_v$

thus generalizing the equation $\Delta\varepsilon_z = m_v\ \Delta\bar\sigma_v$ valid only for drained paths under K_o stress conditions. It is thought that the advantages of using $m_{\varepsilon z}$ instead of M derive from the fact that in $m_{\varepsilon z}, \bar\sigma_v$ diagrams the area between the path and the abscissa is the strain without any limitation whatsoever. The simplest case, first loading drained sand paths under symmetrical stress states, will be used to illustrate the proposition. Stress conditions are defined by $\bar\sigma_v$, effective vertical stress and K, ratio between $\bar\sigma_h$, horizontal or radial effective stress and the effective vertical stress.

In the analysis of practical problems it is possible to relate $d\bar\sigma_h$ and $d\bar\sigma_v$ by means of $d\bar\sigma_h = K_d\ d\bar\sigma_v$ where K_d is constant for differential increments of vertical effective stress. K_d is constant all along the test in constant stress ratio tests; K_d is 0 in constant confining pressure tests. In fig. 1, K = 1 along path AB and $K_d = 0$ along BC. The axial strain increment $\Delta\varepsilon_z$ between $\bar\sigma_{v_{n-1}}$ and $\bar\sigma_{v_n}$ is given by the shaded area.

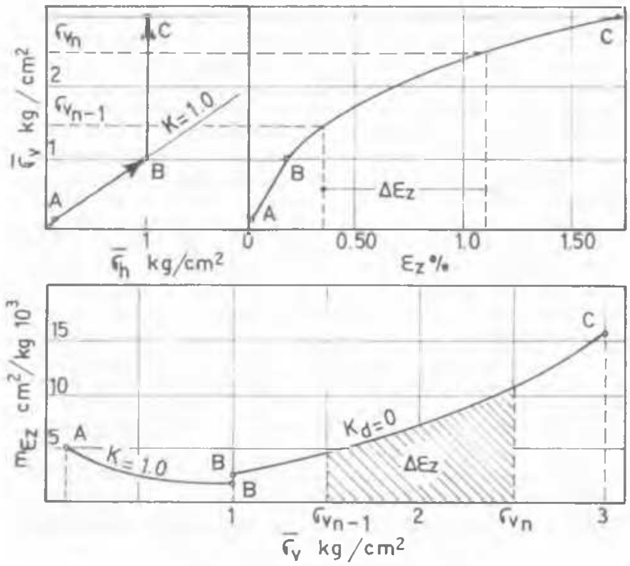

Fig. 1 - Graphycal determination of strains by means of $m_{\varepsilon z}$, $\bar\sigma_v$ diagrams.

$m_{\varepsilon z}$ and consequently Janbu's M are stress and path dependant. Fig. 2 shows values of $m_{\varepsilon z}$ obtained from K_d constant and $K_d = 0$ tests which limits the lower and upper limits of possible results.

Points C and D have the same stress conditions but widely different $m_{\varepsilon z}$ along paths ABC or AC and ABD or AD respectively. CC and DD show the range of possible values. A fine medium sand was used for comparisons. K at C is twice and D is one and a half times K_f for the stress range under consideration.

In opposition to the variations of $m_{\varepsilon z}$ and of Janbu's M for the same stress conditions reached along different paths, it is found that when the factor of safety is larger than about 2, actual stress paths below foundations can be replaced by constant stress ratio paths with no large differences between strains. Actual paths on the $m_{\varepsilon z}$, $\bar\sigma_v$ diagrams show clearly the mechanism of failure and the changes of ratio between horizontal and vertical stresses. Failure will be identified by $m_{\varepsilon z}$ values tending to ∞ or values of Janbu's M tending to 0.

The path followed by a point located at depth Z = 0,75 B below a 20 cm shallow foundation is shown in Fig. 3. The $m_{\varepsilon z}$ values have been obtained from the strains of Eggestad's tests N* 3 published in the Proceedings of the European Conference on Soil Mechanics and Foundation Engineering of 1963. A fluvial sand of the same grading and porosity as used in that test has been employed to determine the required stress path. The initial stress conditions,

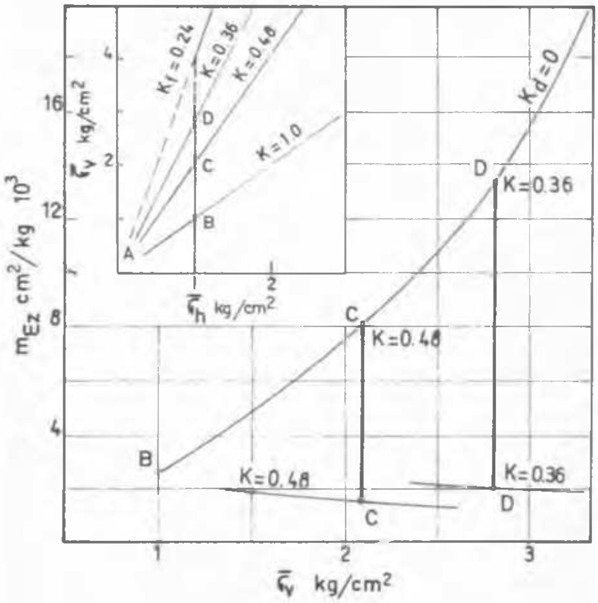

Fig. 2 - Example of range of m_{Ez} variations along different first loading paths for medium dense sand.

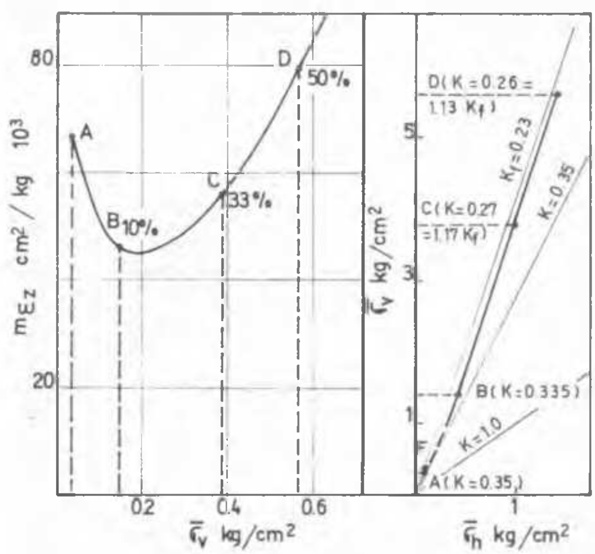

Fig. 3 - Path at depth where maximun strains occur below shallow small circular loaded area when factor of safety is smaller than about 10.

vertical equal to overburden pressure and horizontal corresponding to $K = K_0$, are represented by point A. Points B, C and D indicate the stress conditions when the change in vertical stress on ground surface Δqs is equal to 10, 33 and 50 % of the ultimate pressure Δq_{ult}. AF is the path for Boussinesq stress conditions. For each point Boussinesq's solutions give a single constant ratio between horizontal and vertical stress increments while actually this ratio is variable, starting from larger values and diminishing toward failure. Since K_f, ratio between horizontal and vertical stresses at failure, varies with the stress range, horizontal stresses might be introduced by means of ratios between K and K_f at the chosen vertical stresses. For example, a stress path for a given depth below a foundation might be outlined by stating that a 10 % of failure $K = K_0$ and that at 50 % of failure $K = 1.1\ K_f$. Fig. 3 shows, for the actual path represented there, that at $Z = 0.75$ B, K is about 1.17 K_f when the factor of safety for the vertical load is about 3 and 1.13 K_f when the factor of safety is about 2.

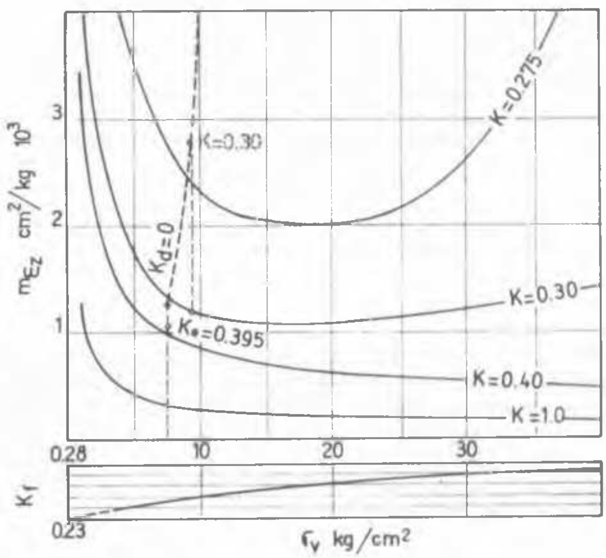

Fig. 4 - Coefficients of axial strain change along constant stress ratio curves for medium dense sand.

m_{Ez}, $\bar{\sigma}_v$ diagrams for preliminary analysis can be prepared to reduce laboratory testing. Fig. 4 shows such a diagram drawn from tests on Parana river fine sand and relative density $D_r = 60$ %. Solid lines are coefficients of axial strain change along constant stress ratio curves. The relationship between K_f and $\bar{\sigma}_v$ is given at the seat of each diagram. Because K_f changes from a minimun to a practical constant value, constant stress ratio curves with K values smaller than this constant will disappear progressively as $\bar{\sigma}_v$ increases. Dotted lines are coefficients of axial strain change along $K_d = 0$ paths, starting from K_o stress conditions.

With due allowance to the fact that m_{Ez} and M are path dependant but profiting from the circumstance that actual paths below foundations follow closely constant stress ratio

paths, it is possible, in many problems, to draw stress paths on m_{ε_z}, $\bar{\sigma}_v$ diagrams and obtain satisfactory strain determinations. In such cases strains can be quickly estimated for any stress range.

Also the effects of overconsolidation can be analyzed simply. In Fig. 5, the path at Z = 0.75 B for preloading of a large diameter pile is ABCP. On second loading is A_2P_2.

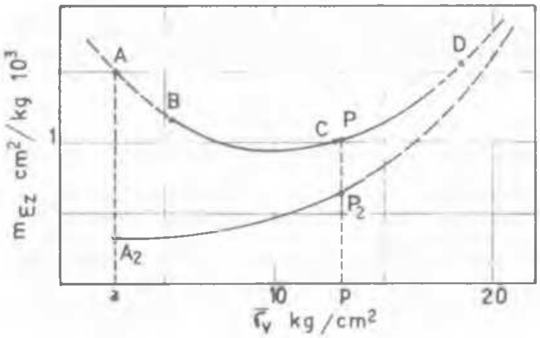

Fig. 5 - Effects of soil precompression below pile.

The strain at such depth changes from the area aAPp for preload induced vertical stress p to a A_2P_2p for reloading after unloading. Preloading effects on settlements can thus be estimated by introducing simplifying assumptions. For example, few tests make it possible to determine relations such as the ratio between aA_2 and pP that can be used for preliminary analysis of many alternatives.

The purpose of this contribution has been to summarize briefly some of the advantages that, on the writer's opinion, can be derived by using:

$$m_{\varepsilon z} = \frac{d\varepsilon_z}{d\bar{\sigma}_v} \quad \text{instead of Janbu's} \quad M = \frac{d\bar{\sigma}_v}{d\varepsilon_z}$$

. For the consolidation test stress conditions, axial strains are equal to volumetric strains and $m_{\varepsilon z} = m_v$.

R. H. FOSTER (England)

Fractionated Kaolin having a particle size range of 1-2µ was dispersed in a suspension having a pH of 8. Lightly over-consolidated (referred to as soft) and heavily over-consolidated (referred to as hard) clays were produced by one dimensional consolidation from respective moisture contents of 150% and 500%, the corresponding pressures were 4.75 kg/cm² and 45 kg/cm² respectively.

Electron micrographs have been produced by P. K. De (1969), using a transmission electron microscope, from resin impregnated thin sections having a thickness of 500 Å. Both materials possess some degree of preferred orientation perpendicular to the direction of consolidation; the hard material (Fig. 1) to a high degree; the soft material (Fig. 2) to a low degree, and the structure of the latter may indeed be regarded as random. The void ratio of the soft material is seen to be considerably higher.

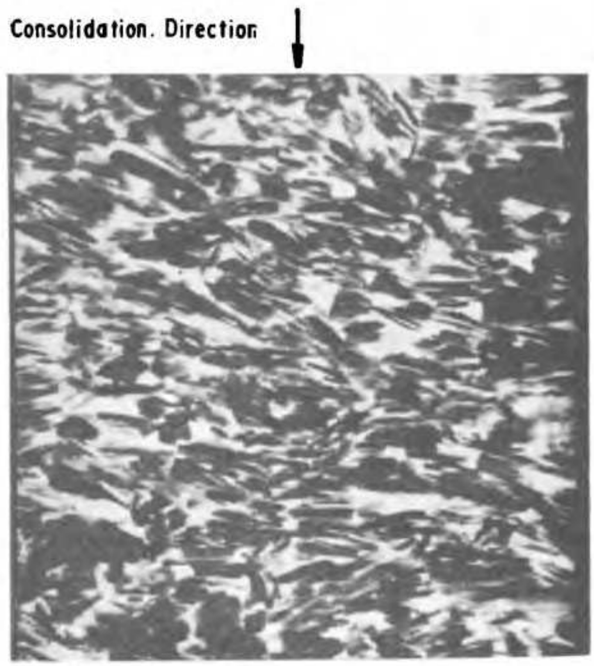

Fig. 1. Electron-Micrograph (6,300) of vertical section through heavily over-consolidated Kaolinite (P. K. De 1969)

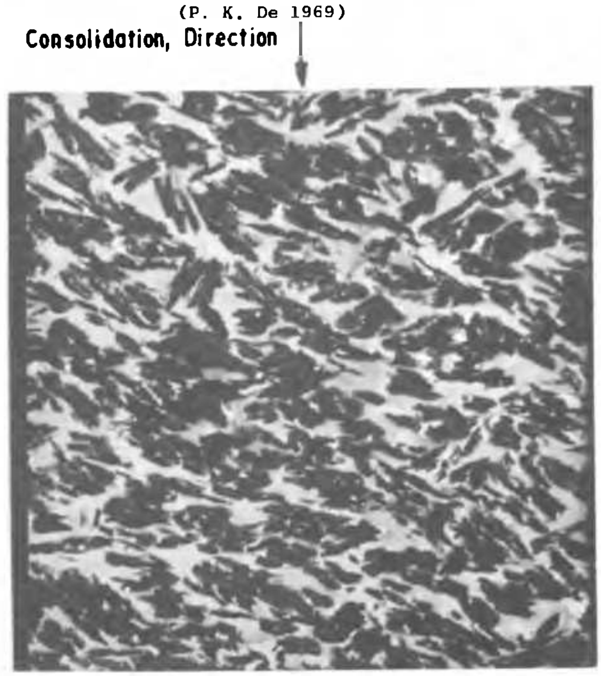

Fig. 2. Electron micrograph (x6,300) of vertical section through lightly over-consolidated Kaolinite (P. K. De 1969)

Fig. 3. Electron-Micrograph (x3,800) of vertical section through lightly over-consolidated Kaolinite after shear failure. (P. K. De 1969).

Both materials were sheared in a direct shear box until straining continued at a constant volume, when the tests were stopped. The hard material exhibited dilatancy at peak stress whilst the soft material compressed. Vertical sections were taken through main shear discontinuities (De 1969) parallel to the direction of shearing.

The soft material (Fig. 3) suffered a reduction in void ratio, its particles became well ordered within the imposed shear structures with a high degree of preferred orientation within each structural zone. The orientation in the main shear discontinuity is in the direction of straining and it is noticed that at the boundary between the main discontinuity and the adjoining kink band structures (Morgenstern and Tchalenko 1967) the relative motion has resulted in dragging of the particles forming the kink band structure which has thus become distorted.

In the hard material (Fig. 4) it is clear that there has been considerable disturbance within the main shear discontinuities; particles appear to have been fractured; boundaries are irregular, suggesting interlocking; and some material has been turned through almost 90°. Nevertheless there is a high degree of orientation in the direction of the discontinuity.

It is suggested that the ordered shear structures found in the soft material are consistent with those which would be expected within clay materials which compress under shear loading, whilst the disturbance found in the shear structures within the hard material are consistent with the behaviour of dilating clays.

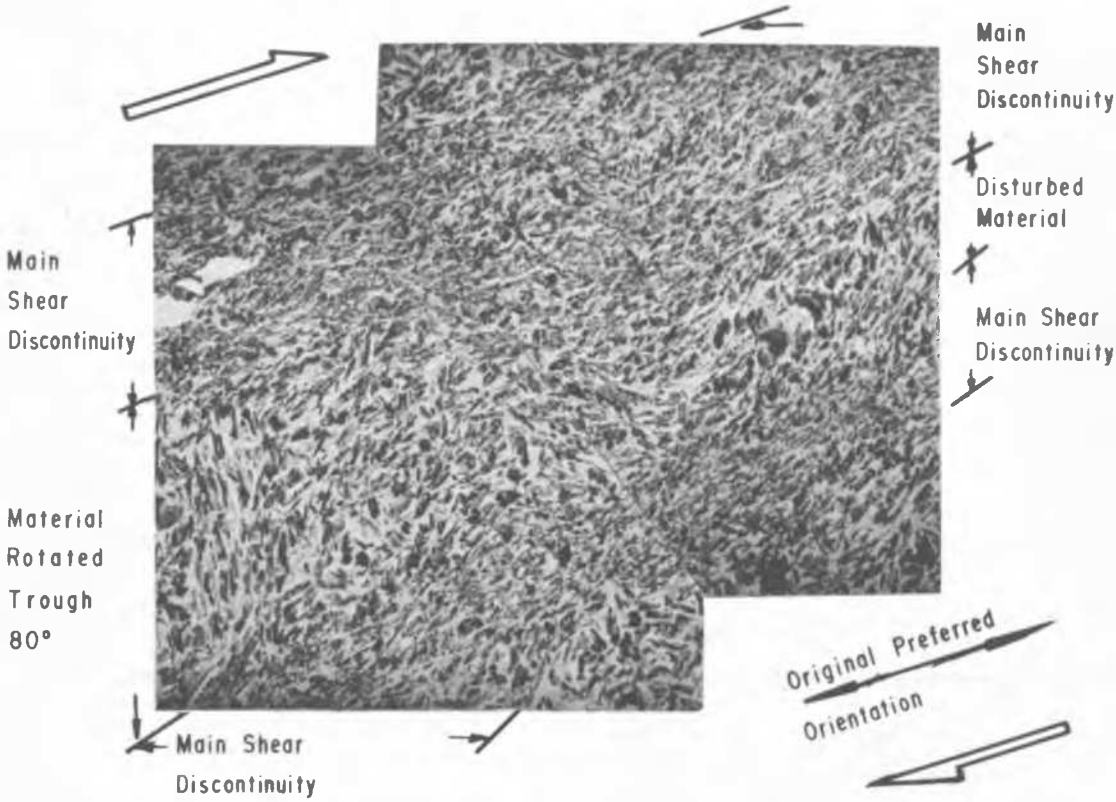

Fig. 4. Electron-Micrograph (x2,600) of vertical section through heavily over-consolidated Kaolinite after shear failure (P. K. De 1969)

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"Kaolin micro-structure after consolidation and direct shear"
Ph.D. thesis, The City University, London.

2. Morgenstern N. R. and J. S. Tchalenko (1967)
"The Optical Determination of Preferred Orientation in Clays and its Application to the study of Microstructure in consolidated Kaolin I and II"
Proc. Royal Society A300 218-250.

N. JANBU (Norway)

My comments are concerned with the interpretation of strain-time curves. As we all know, every deformation test on a soil sample leads to a strain-time curve for each stress level. But even a million ϵ -t curves will be of limited help to us if they are not properly interpreted. With this I mean that the curves should be interpreted so that meaningful soil properties or soil parameters come out of the investigation.

There are just two ways of going about it in my judgment: Namely, either to study the strain rate $\dot{\epsilon}$, or the time resistance R= $\frac{dt}{d\epsilon}$ both as functions of time.

The panel members, the State of the Art paper, and several of the papers in this Session have concentrated on the study of strain rate and its time dependency. And, as we have heard, read, and seen, a number of interesting results have certainly been achieved this way. But at present it may appear very difficult to extract from this study simple engineering parameters, each with a distinct meaning.

The writer would therefore like to offer an alternative way of interpretation, namely, to study the time resistance and its dependency

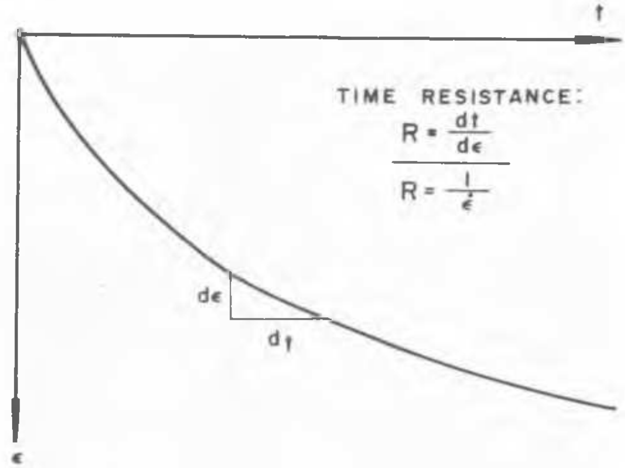


Fig. 1. Definition of Time Resistance

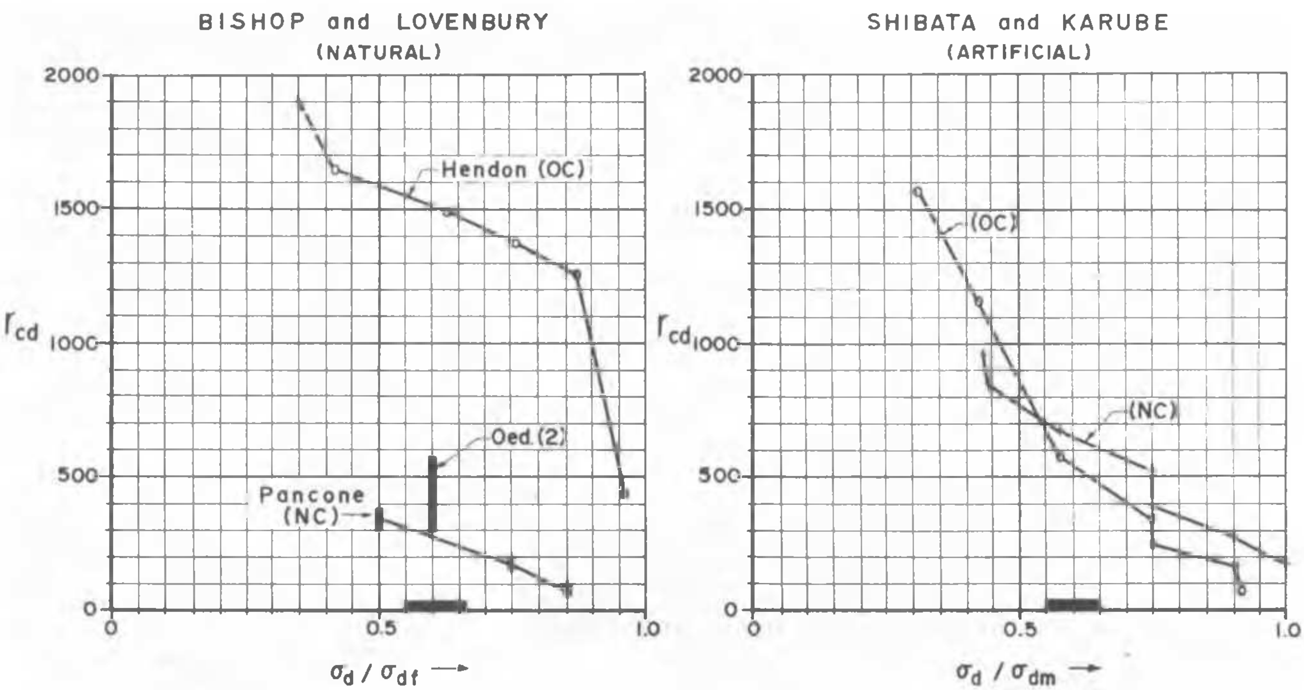


Fig. 2. Time Resistance versus time for different safety factors.

on time, see Fig. 1. The main reason is simplicity, and here is the argument for that: Time resistance has the dimension of time. Therefore, if one plots time resistance versus time, the slope at any point of this curve is a pure number, and over very large time intervals, this number is a constant, which we could call the resistance number = r. Consequently, the long-term time dependency can be characterized by one pure number for each stress level, see Fig. 2.

I have looked into three papers of this Session for the purpose of obtaining resistance numbers to compare with own experience, partly reported in a paper to this Session.

1. First, Bishop and Lovenbury have carried out some very interesting drained creep tests on two undisturbed clays, one overconsolidated and one normally consolidated. From their data it is found that the time resistance for the OC-clay was some 5 or 6 times higher than that for the NC-clay, see Fig. 3. This agrees with my own findings, reported in a paper in Session 1. Moreover, Bishop and Lovenbury's data show that r decreases with decreasing safety factor F, but the decrease is fairly slow until F approaches a low value of say, 1.2 to 1.0.

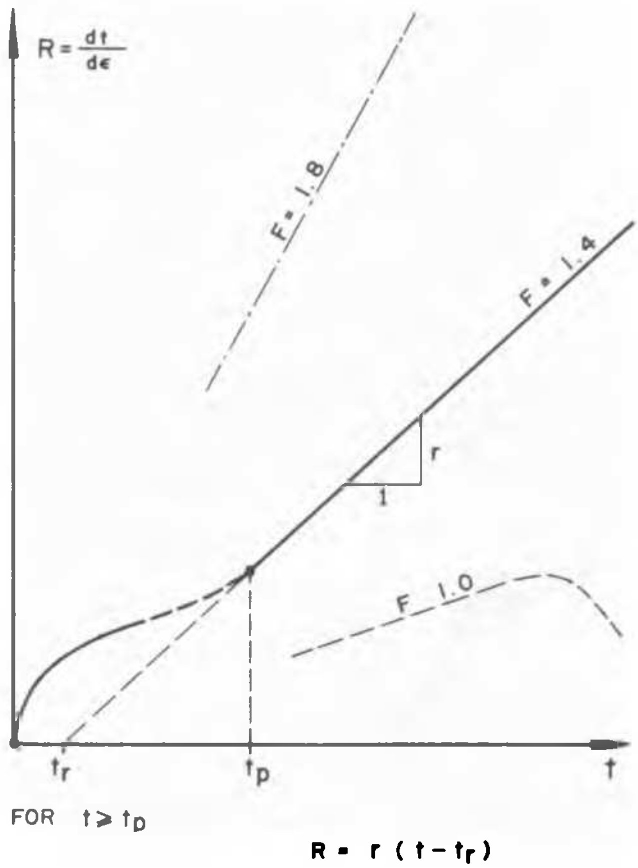


Fig. 3. Resistance Number versus Deviator Stress ratios.

2. From the paper by Shibata and Karube, on drained creep tests, it is also found that r decreases with decreasing F, but now there is no significant difference between the OC- and the NC-clay. This I believe is due to the fact that Shibata and Karube used artificially sedimented clays, where it seems impossible to duplicate the clay structure of undisturbed clays of 10,000 years of age.

3. Finally, I may mention that a study of Singh and Mitchell's paper on undrained creep tests showed that the undrained resistance numbers, over the applicable time ranges, and clay types, were somewhat lower than the corresponding drained creep number, but many of the values are of the same order as for the NC-clays, say 50 to 200.

In conclusion, I strongly recommend that more investigators try to interpret also their long-term deformation tests in terms of the resistance concept. It is my belief that by using the resistance concept, one can contribute to closing the enormous gap between theoretical considerations on the one hand and immediate practical applications on the other.

G. M. LOMIZE (U. S. S. R.)

SYNOPSIS

The paper deals with the experimental study on deformations of a clayey kaolinite three-phase soil of disturbed structure under a complex (unproportionate) loading, carried out with ollow cylindrical specimens subjected to loading by hydrostatic pressure, axial compression and torque (acting on the end surfaces of the specimen).

The expriments have shown that unproportionate loading accompanied by rotation of the principal stress axes considerably affects the regularities of deformations, resulting in misalignment of the axes of the principal stresses and principal strains. In this case, the similarity between stress and strain conditions disappears i.e., the conditions for the use of relationships between stresses and strains of the Hookean type are not met.

Unproportionate loading accompanied by rotation of the principal stress axes (complex loading) is the most general and, at the same time, the most prevailing case of work of an element in the soil medium.

There are few reported studies showing the effect of rotation of the principal stress axes on the stress-strain condition of different types of continuous media (A.A. Iliushin, 1948) including soils (B.B. Broms, A.O. Casbarian, 1965).

The program of the experiments described below has been designed to solve the following problems: to verify the coincidence of the axes of stress and strain tensors and the similarity of stress and strain conditions, and to study the basic regularities of deformations.

The work has been carried out at the Department of Soil Mechanics, Bases and Foundations headed by Prof. N. A. Tsytovich at the Moscow Civil Engineering Institute.

Three-dimensional stress conditions necessary to the performance of the program and rotation of the principal stress axes have been obtained by loading hollow cylindrical specimens by means of application of axial force, hydrostatic pressure (the pressure values were different inside and outside the specimen) and torque acting on the end surfaces of the specimen.

The experiments have been carried out with a specially designed apparatus making it possible to obtain, without substantial distortions, the predetermined stress condition and to measure three linear and one angular deformation. The height of the specimen was 80 mm, the inside and outside diameters were 35 and 60 mm respectively.

The tested kaolinite loam of disturbed structure had liquid limit of 30 per cent, plasticity index of 10 per cent, void ratio of 0.76, water content by weight of 12.3 per cent and saturation factor of 0.44. The initial condition of density and water content of soil was achieved by compaction of previously humidified powder. The experiments have been performed in drained conditions under stepwise loading with stabilization of deformations after each step.

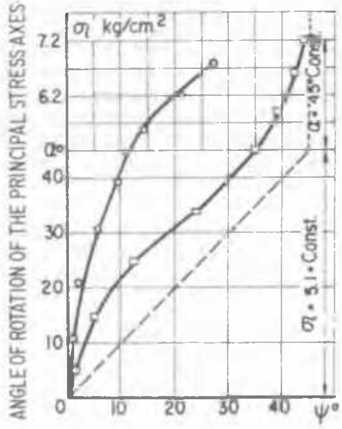
The experiments of the study have been carried out with a constant value of the first invariant of stress tensor which was equal to 15 kg/cm^2. In tests 1 and 2 proportionate loading was applied with a constant value of the parameter of stress condition type $\mu = \frac{2\sigma_2-\sigma_1-\sigma_3}{\sigma_1-\sigma_3}$ which was equal to -1 and 0, respectively.

In test 3 the proportionate loading with μ=0 was applied up to the stress intensity of σ_i=5.10 kg/cm^2. Then the gradual rotation of the principal stress axes at the angle of 45° (μ=0=Const.) was performed. Test 4 was conducted under the proportionate loading and with the value of μ=-1 up to the value of σ_i=5.10 kg/cm^2.

Then the change in the type of stress conditions was made (the transition from μ=-1 to μ=0) under simultaneous rotation of the principal stress axes at 45°.

After rotation of the stress axes, tests 3 and 4 were completed under the proportionate loading with μ=0.

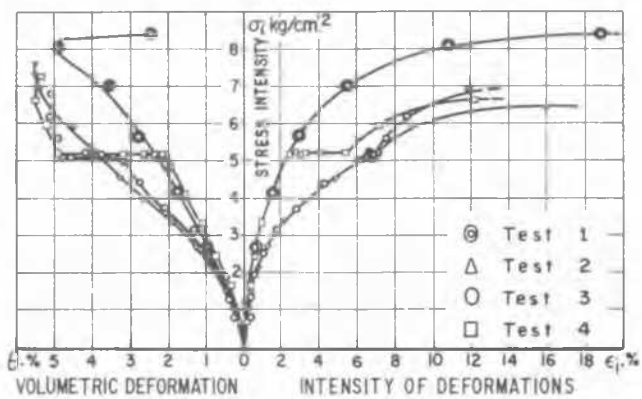
The chart in Fig. 1 shows the variation of angle of rotation of the principal strain axes ψ under rotation of the principal stress axes. The condition of coincidence of the axes is indicated by the dotted line. It is evident from the chart that rotation of the principal stress axes causes no change in position of the principal strain axes, but the angles of rotation differ from each other and the axes do not coincide (with α=45°, ψ varies from 12 to 35°). The relative position of curves $\sigma_i-\epsilon_i$ and $\sigma-\theta$ shown in Fig. 2 for unproportionate loading (curves 1 and 2, with μ=-1 and 0 respectively) reveals the effect of stress condition type and corresponds to data reported earlier (G.M. Lomize, A.L. Kryjanovsky, 1965). Rotation of the principal stress axes with μ=0=Const., resulted in an appreciable increase in volumetric deformation (up to 40 per cent) and caused no substantial growth of deformation of form change. The change in type of stress condition (the transition from μ=-1 to μ=0) with simultaneous rotation of the principal stress axes resulted in a twofold increase in θ and ϵ_i values.



ANGLE OF ROTATION OF THE PRINCIPAL STRAIN AXES

Fig. 1. Variation of angle of rotation of the principal strain axes ψ under rotation of the principal stress axes at:

(1) Unproportionate loading with μ=0;
(2) Unproportionate loading with a change in stress condition type.



VOLUMETRIC DEFORMATION INTENSITY OF DEFORMATIONS

Fig. 2. Variation of volumetric deformation θ and intensity of deformation ϵ_i under the loading:

Test (1): μ =-1; Test (2): μ =0; Test (3): μ =0, with subsequent rotation of the principal stress axes; Test (4): μ =-1, with subsequent rotation of the principal stress axes and transition to μ =0.

In Soil Mechanics, the relationships of plasticity deformation theory are used (Y.K. Zaretsky, 1967), the matrix of the strain tensor (e_{ij}) being expressed in form of a matrix polynomial of the stress tensor (σ_{ij}):

$$\left[e_{ij}\right] = f(\left[\sigma_{ij}\right]) = \phi_0 \left[1 \right] + \phi_1 \left[\sigma_{ij}\right] + \phi_2 \left[\sigma_{ij}\right] \quad (1)$$

Generally adopted assumption about the proportionality between stress and strain deviators results in elimination of the third term on the right side of Eq. (1), the equation of stress-strain relation having in this case a form similar to the generalized Hooke law.

The necessary condition making it possible to write down Eq. (1) is the coincidence of the axes of stress and strain tensors. Therefore, misalignment of the axes of the principal stresses and strains revealed in the tests does not permit use of Eq.(1) for the cases of unproportionate loading which differ considerably from simple (proportionate) loading (I.K. Ivastchenko, M.N. Zakharov, 1969). It is to be noted that the tests performed have shown the disappearance of similarity between stress and strain conditions under unproportionate loading and rotation of the principal stress axes. The physical basis of failure to satisfy Eq. (1) and to maintain the similarity principle is the appearance of anisotropy in soil properties during the loading as a consequence of particularities of structure of soil dispersive medium. As is known, the coefficients ϕ_0, ϕ_1, ϕ_2 in equation (1) are functions of invariants of stress and strain tensors. The experimental results considered above are not contrary to the known requirement necessitating to describe unproportionate loading by three relationships between three invariants of stress and strain (I.I. Goldenblat, 1965). However, the experiments have shown that these relationships were not sufficient under rotation of the principal stress axes. In particular, rotation of the axes while maintaining constant the values of three invariants of stress tensor causes no change in invariants of strains.

The experiments performed have shown that:

- the axes of the principal strains and stresses were in a wide misalignment (12° - 35°) under rotation of the principal stress axes at the angle of 45°;
- rotation of the principal stress axes caused a change in invariant of strains, the volumetric deformation having been particularly increased (by 40 per cent);
- it is necessary to take into account rotation of the axes in relationships between stresses and strains in soil.

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SOMMAIRE

Les études effectuées pour la définition de la résistance au cisaillement des sols argileux non consolidés, ont montré que: a) la possibilité d'utiliser la méthode de "pression interstitielle" (K. Terzaghi) est limitée aux sols sable argileux et limoneux; b) la méthode "compactité - teneur en eau" (N. Maslov) est d'un caractère plus général et possède un nombre des avantages dans l'application pratique. Le rapport décrit le rôle du gradient initial dans la consolidation des sols argileux du point de vue réduction de l'affaissement de l'ouvrage et compactege des terrains dans les zones d'épaisseur limitée. En même temps, il s'agit de la possibilité de consolider les sols argileux sous des charges de courte durée mais appliquées fréquemment. En conclusion de nos travaux on peut considérer comme déterminé que le poids propre de la couche de sables saturés, lors de la perte de la stabilité sous des charges dynamiques, est d'une importance limitée dans le temps.

Comme le montre la pratique, il est d'une importance particulière que lors de la définition des caractéristiques de calcul de la résistance au cisaillement du sol, on ne tient pas compte de ce que la résistance totale au cisaillement du sol argileux équivalente à la charge appliquée, peut exiger parfois plusieurs années pour être atteinte, à moins qu'elle ne soit atteinte du tout (voir

plus bas). De ce fait, la résistance au cisaillement du sol non consolidé complètement s'avère parfois brusquement réduit (2 à 3 fois) par rapport au sol complètement consolidé.

Les circonstances citées ci-dessus ont conduit à mettre au point des mèthodes spéciales pour apprécier la résistance au cisaillement des sols argileux non consolidés. Les méthodes principales dans ce sens sont les méthodes de "pression interstitielle" et "compacité-teneur en eau" proposées respectivement par les professeurs K. Terzaghi (Autriche) et N. Maslov (URSS, 1949).

Selon la théorie de pression interstitielle, la résistance au cisaillement du sol varie dans le temps seulement avec l'augmentation des forces de frottement dans les sols sous charge, due à la diminution, dans le temps, de la pression interstitielle. Cette circonstance a trouvé son expression dans l'équation connue de Terzaghi:

$$\tau_{ft} = (\sigma - U_t) \cdot tg\,\varphi' + c' \quad (I)$$

La différence de principe entre la méthode "compacité-teneur en eau" et la méthode de "pression interstitielle" consite en ce que la première tient compte de l'influence des variations de la compacité-teneur en eau du sol, au cours de la consolidation de celui-ci, sur les valeurs de l'angle de frottement et de la cohésion. Donc l'équation de la résistance au cisaillement prend la forme suivante:

$$\tau_{fw} = \sigma \cdot tg\,\varphi_w + c_w \quad (2)$$

La méthode de "compacité - teneur en eau" est exposée plus détaillement dans les publications du V^{me} Congrès de la Mecanique des sols et fondations (rapport I/4I). Il convient de noter que la méthode de compacité - teneur en eau du sol, vu sa simplicité et sa conviction d'engineering est reconnue dans notre pays et essayée lors des études et de la construction d'un nombre d'ouvrages importants. Mais restait imprécis le problème sur la relation des resultats de la résistance au cisaillement du sol non consolidé pour les deux méthodes précitées. Pour étudier ce probléme on a effectué largement une vérification expérimentale des hypothèses de base et des conclusions des méthodes de "pression interstitielle" et de compacité - teneur en eau" du sol. Pour les études expérimentales on a utilise quelques types d'argile dont la teneur en particules argileuses ($<$ 0,005 mm) variait de 4 à 74%, et l'indice de plasticité (PI) - de 7 à 65%. L'angle de frottement effectif (φ') des sols étudiés variait de 6 à 27°. Les essais ont été effectués sur un appareil de cisaillement et un appareil à compression triaxiale. Lors des essais, la pression interstitielle était mésurée. Les résistances au cisaillement obtenues dans les essais ont été comparées à celles calculées par les mét-

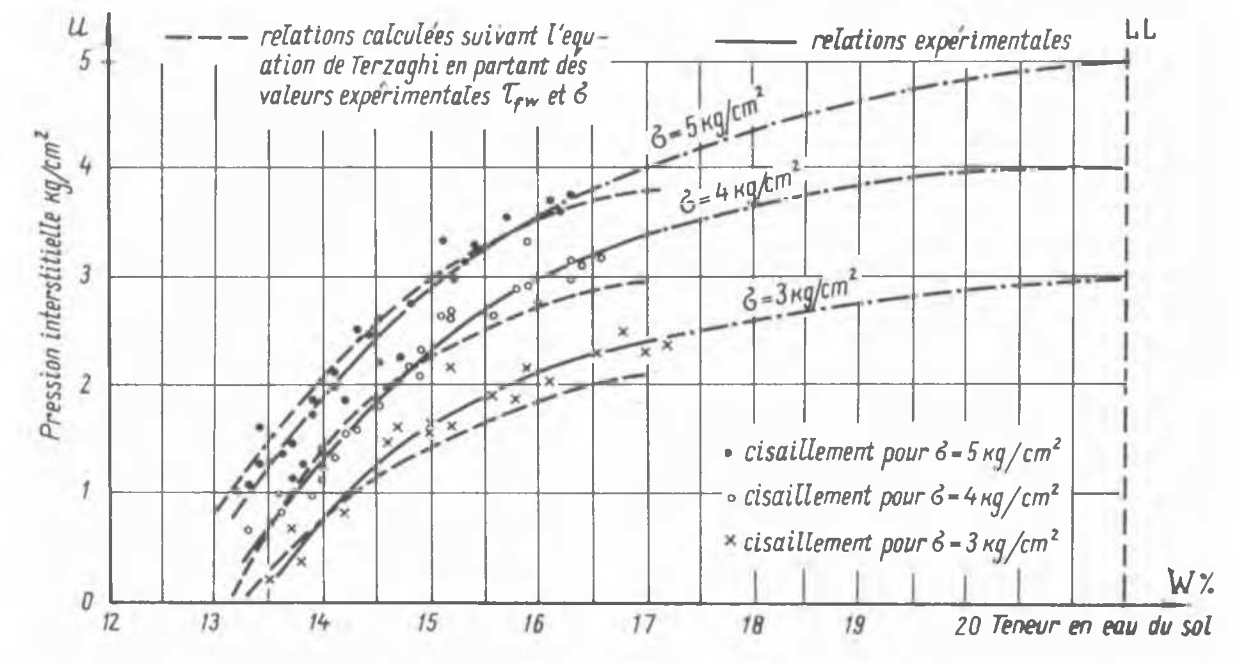

Fig. I. Variation de la pression interstitielle en fonction de la teneur en eau du sol, et de la charge normale au cisaillement. Sable argileux du diluvium.

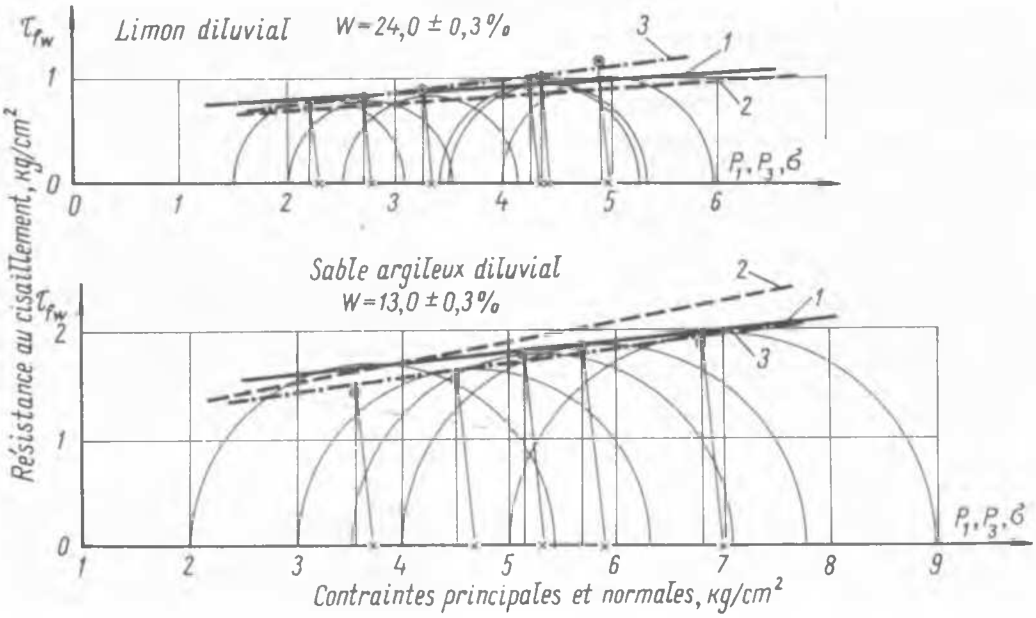

Fig. 2. Comparaison des résistances au cisaillement obtenues à l'appareil à compression triaxiale (I) et calculées par les méthodes "compacité-teneur en eau" (2) et "pression interstitielle" (3).

hodes de "pression interstitielle" (équation I) et de "compacité - teneur en eau" (équation 2).

Les valeurs des pressions interstitielles, relevées lors du cisaillement, ont été mises à la base pour la construction des courbes de relation U = f (W; σ). Ces relations pour une des catégories du sol essayé sont données, à titre d'exemple, sur la fig. I. Sur cette même courbe sont portées les relations obtenues par voie de calcul la pression interstitielle pour chaque essai, d'après la relation de Terzaghi (I), à partie des valeurs expérimentales de la résistance au cisaillement (τ_{fw}) et de la charge normale (σ) ainsi que des valeurs φ' et C' pour le sol considéré.

La bonne coincidence des courbes expérimentales et calculées U = f (W; σ), analoques à celle de la fig. I, a eu lieu pour les sables argileux et les limons essayés. Pour les sols à grande teneur en argile (> 50%), l'écart entre les valeurs de pression interstitielle obtenues par les essais et calculées par l'équation (I), s'est avéré beaucoup plus considérable.

Les résultats d'essais des sols, à la compression triaxiale, sont réunis sur les courbes dont une est représentée sur la fig. 2.

Chaque courbe expérimentale $\tau_{fw}=f(W)$ construite à partir des résultats d'écrasement en un systeme forme C' un nombre d'échantillons et pour une teneur en eau considérée du sol, a été comparée avec des courbes analogues calculées par les méthodes de "compacité - teneur en eau" et de "pression interstitielle". Le calcul de la résistance au cisaillement mayennant l'équation (2) a été effectué par la substitution, pour chaque essai, de la valeur de la tension normale (σ), ainsi que de l'angle de frottement (φ_w) et de la cohésion (C_w) correspodant à la teneur en eau qui nous intéresse. Les dernières valeurs ont été empruntées aux courbes $\varphi_w = f(w)$ et $C_w = f(w)$ construites pour chaque variété du sol. Pour les calculs de la résistance au cisaillement suivant l'équation (I), on a fait substituer les valeurs de la tension normale (σ) et de la pression interstitielle (u) mesurée lors de l'essai, ainsi que les valeurs de φ' et C' correspondants au sol.

De plus, en partant de la valeur expérimentale de τ_{fw} on a calculé, pour chaque essai, moyennant l'équation (I) - la pression interstitielle qui devrait avoir lieu dans le cas où la relation de Terzaghi est juste. Ces relations de calcul (u) ont été comparées également avec les valeurs d'essai de la pression interstitielle (U_{exp}).

De nombreux expériments et comparaisons effectués, les conclusions suivantes ont été faits:

I. Pour les sables argileux et les limons, les résistances au cisaillement à un stade quelconque de consolidation, définies par les méthodes de "pression interstitielle" et de "compacité - teneur en eau" du sol; sont assez comparables. Cette circonstance montre la possibilité de définir par les deux méthodes la résistance au cisaillement dans l'état non comolidé, pour ces catégories du sol.

2; Pour les sols d'une grande teneur en argile (PI > 30%), la méthode de "pression interstitielle" devient moins applicable: l'écart entre les résistances au cisaillement récelles et les résistances calculées à parti de la relation de Terzaghi, est surtout sensible (en moyenne, 40 à 50% de la valeur expérimentale), pour les sols nettement argileux, les sols auxquels justement est destinée cette méthode.

L'écart entre les valeurs expérimentales et les valeurs calculées selon la relation de Terzaghi, croissant au fur et à mesure qu'augmente la teneur en argile, est dû, selon nous, à ce que cette relation est, dans certaine mesure, de forme et ne traduit pas toute la complexité de relations entre les particules argileuses et entre celles-ci et l'eau. A la base de l'équation de Terzaghi est mise la conception des tensions effectives ($\sigma' = \sigma - U$), selon laquelle la résistance au cisaillement du sol ne varie dans le temps qu'au fur et à mesure de l'augmentation des forces de frottement due aux variations de la pression interstitielle. Etant donnée que pour les sols purement argileux, la résistance au cisaillement est définie, en premier lieu, par la résistance visquese des enveloppes colloidales entourant les particules et empêchant le contact direct entre celles-ci, l'applicabilité de la conception de Terzaghi est douteuse. Cet état de choses devient plus évident à mesure que le pourcentage de particules argileuses augmente.

3. La méthode qui tient compte de la "compacite - teneur en eau" permet de fairenune appéciation plus objective des conditions de travail du sol. Pour les calculs de la résistance au cisaill ment moyennant la relation (2), sont utilisées les caractéristiques de cisaillement φ_w et C_w qu'en définit à partir des courbes du type $\tau_{fw} = f\ (w, \sigma)$ construites à la base des valeurs moyennes d'un grand nombre de points expérimentaux. C'est pourquoi les caractéristiques φ_w et C_w traduisent non seulement le changement de rôles des forces de frottement ($\sigma \cdot tg\ \varphi_w$) et de cohésion ($C_w$) en fonction de la compacité - teneur en eau, mais aussi les variations de ces forces en fonction de la composition du sol argileux. C'est ainsi que pour les sols à grande teneur en argile dont la résistance est définie, principalement, par les forces de cohésion, l'angle de frottement φ_w, pour toutes les valeurs de la compacité - teneur en eau, est égal à zèro. Probablement cet état est plus proche de la nature des argiles grasses que celui qui résulte de la conception des contraintes effectives de Terzaghi.

Donc, les faits exposés limitent la possibilité d'utiliser largement, dans tous les cas, la méthode de "pression interstitielle". De plus, les études effectuées ont montré que la méthode de "compacité - teneur en eau" est de caractère plus général et possébe un nombre des avantages dans l'application pratique par rapport à la méthode de "pression interstitielle": possibilité d'utiliser des appareils de cisaillement ordinaires, simplicité du contrôle de la résistance au cisaillement in situ (en partant de la compacité et de la teneur en eau des échantillons) en comparaison avec les mesures de la pression intérstitielle au moyen d'un système d'appareils, etc.

On sait pourtant que le compactage des sols argileux de grande épaisseur dure longtemps du fait de la perméabilité extrêmement faible de ceux-ci et de la viscosité souvent considérable. De plus, comme l'ont montré les études des savants soviétiques (N. Pouzirevsky, I93I; S. Roza, I937 à I959), les sols argileux sont pratiquement imperméables dans les cas où les gradients de drainage du sol s'avèrent inférieurs à une certaine valeur connue sous le nom de gradient initial (J_0). Dans ces conditions, les sols argileux ne se laissent pas pratiquement consolider sous charge. L'influence possible du gradient initial sur le compactage des sols se manifeste surtout dans le désaccord observé souvent entre le caractère de départition de la compacité - teneur en eau et la charge naturelle en profondeur de la couche.

Notons que les valeurs du gradient initial (J_0) pouvent, dans certains cas, atteindre 5 à I0 et plus. Ce dernier fait devient d'une signification particulière du point de vue de l'appreciation de la résistance et de la stabilité des ouvrages fonfés sur les sols argileux, étant donné que l'échappement de l'eau des interstices du sol de fondation peut avoir lieu non sur toute la zone "contrainte" mais seulement sur de petits troncons adjacents directement aux couches drainantes (fig. 3).

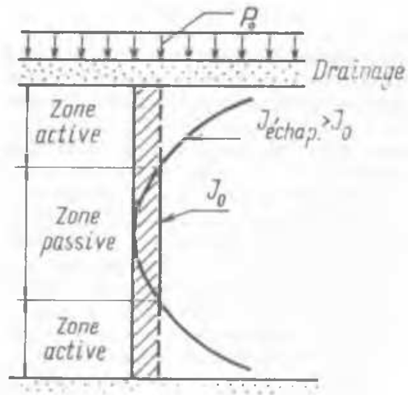

Fig. 3. La consolidation ne se fait qu'aux zones où le gradient d'échappement d'eau I éch. dépasse le gradient initial J_0

La teneur en eau et la résistance du sol dans les zones plus éloignées ("zones passives") peuvent rester, dans ce cas, ratiquement invariable pendant longtemps, et l'effet désiré de consolidation du sol de fondation par le poids propre de l'ouvrage en construction, ne peut être par conséquent obtenu pleinement dans les délais acceptables pour la pratique.

En même temps, la considération pratique du gradient initial constitue un problème très dificile qui consiste à prévoir la pression interstitielle et à apprécier préalablement la valeur du gradient initial (J_0). Pourtant les méthodes théoriques de pronostic de la pression interstitielle sont élaborées d'une manière insuffisante, et a définition du gradient initial par la perméabilité fait intervenir un nombre de daoteurs influençant le résultat. Tout cela crée des difficultés qui excluent nombre de facteurs influençant le résultat.

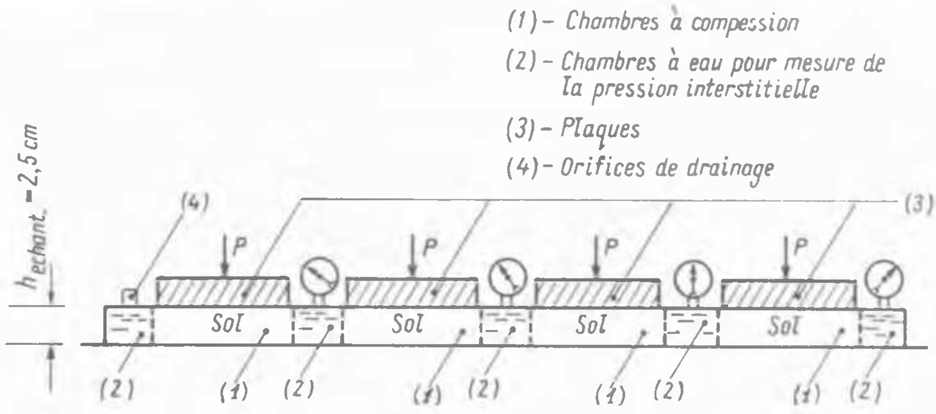

Fig. 4. Appareil à compression KMD-3

pratiquement la possibilité de tenir compté des gradients initiaux ($\mathcal{J}_o$) lors de la conception de l'ouvrage.

Dans le but d'élaborer une méthode directe à tenir compte du gradient initial ($\mathcal{J}_o$) dans les phénomènes de consolidation des sols argileux dans les fondations d'ouvrages, un appareil à compression KMD-3 *) (fig. 4) a été mis an point par E. Dobrov, licencié en technique. L'appareil KMD-3 permet de reproduire les conditions de consolidation du sol argileux remanié ou non remanié, d'épaisseur considérable de la couche à consolider (au-delà de I m), ce qui est obtenu par un nombre de chambres à compression raccordées en série, et ne faisant hydrauliquement qu'une seule - (I).

La charge sur le sol est transmis par l'intermédiaire de plaques rigides - (3), à travers de membranes finesen caoutchouc (ép. 0,I à 0,2 mm) qui ferment hermetiquement les chambres à compression - (I). L'échappement de l'eau du sol sa fait uniquement à travers l'orifice - (4).

L'emploi d'une ligne de filtration plus longue (jusqu'à IOO cm) que celle des appareils à compression classiques (2 à 3 cm) permet d'effectuer la consolidation des sols argileux dans les conditions de faibles gradients de l'eau d'échappement (I éch.) jusqu'aux valeurs ayant lieu dans les conditions réelles et comparables aux valeurs de gradients initiaux ($\mathcal{J}_o$). De plus, l'appareil KMD-3 permet de différencier la valeur de déformations de compactage le long de la voie de filtration dans la couche que l'on compacte.

Suivant les relations expérimentales obtenues (fig. 5) qui donnent le caractère de distribution de la déformation de compactage (molule de tassement - e_p) et de la teneur en eau (W) du sol, suivant la ligne de filtration, la compressibilité des sols diminue de plus en plus.

Le module de tassement (e_p) caractérise la valeur de déformation résiduelle de, compactage et est défini par la formule:

$$e_p = \frac{\Delta h}{h_{in}} \cdot 1000 \text{ mm/m} \qquad (3)$$

ou:

Δh - déformation absolue de l'échantillon mesurée à partir du début de l'essai, en mm;

h_{in} - Hauteur initiale de l'échantillon, en mm.

Les résultats de l'essai décrit confirment la situation mentionnée précédement où les processus d'échappement de l'eau interstitielle se localisent par les gradients initiaux ($\mathcal{J}_o$) dans les limites d'une zone assez restereinte, située près de la surface drainante. En utilisant le caractère linéaire des relations $e_p = f(H_f)$ et $W = f(H_f)$ (fig. 5) on peut définir graphiquemment, par l'extrapolation de celles-ci, la valeur de la zone active c'est-à-dire, la zone où le sol, dans les conditions envisagées, est encore compressible sous charge. Cependant, l'épaisseur de la zone active peur, dans certaines conditions, s'avérer plus considérable que la hauteur totale (H_f) des échantillons essayés du sol consolider dans l'appareil a compression KMD-3. Il en résulte un problème de la durée de consolidation du sol dans les limites de toute la zone active, possible dans les conditions envisagées de drainage et de chargement.

Pour la solution de ce problème on utilise la relation expérimentale du type $\lg T_W = f(H_f)$ qui représente la ici de variation du temps de consolidation T_W jusqu'à une teneur en eau (W) constante suivant la ligne de filtration (H_f).

Donc, le gradient initial peut être pratiquement pris en considération par la voie des études expérimentales des sols argileux à l'appareil KMD-3 qui permet de déterminer les caractéristiques de compression de ceux-ci et d'apprécier la grandeur possible de la zone active et le temps de sa consolidation.

Comme le montre la pratique de construction, l'augmentation de la résistance du sol en diminuant sa teneur en eau (W) peut être provequée non seulement sous l'effet de charges statiques, mais aussi sous l'effet de charges intermittantes (par exemple, fondations des piles de ponts). La mise en évidence de la nature des processus qui ont lieu dans le sol est d'actualité du point de vue de la possibi-

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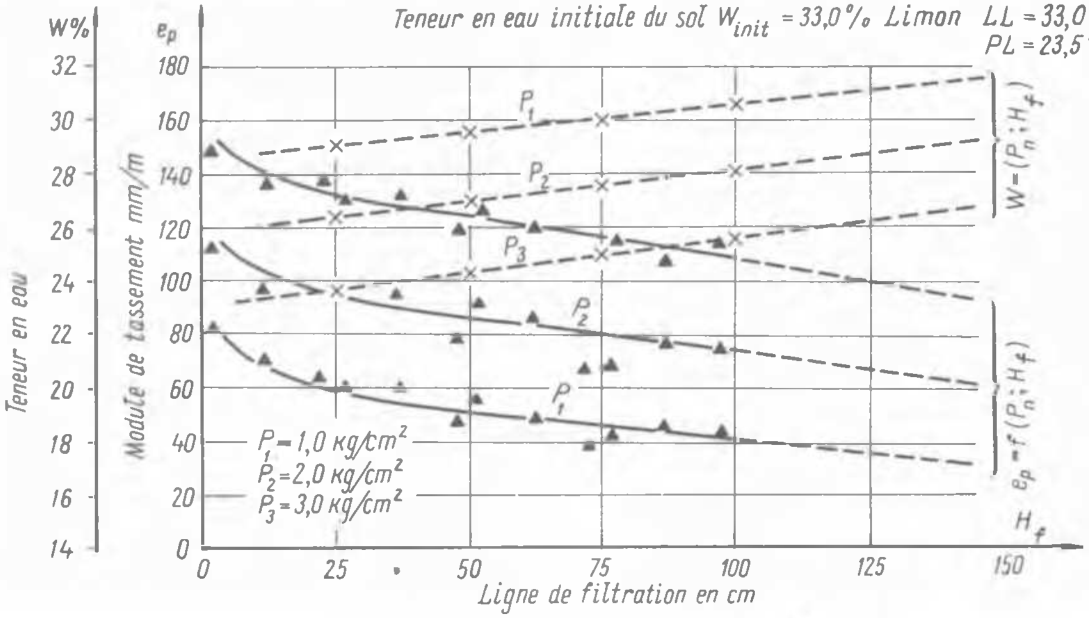

Fig. 5. La déformabilité du sol diminue avec éloignement de la surface drainante, à la suite de l'apparition du gradient initial ($\mathcal{J}_o$).

lité d'exploiter les ouvrages sans accidents, dans les nouvelles conditions de leur fonctionnement (notamment, augmentation de la charge sur les structures de pont due à l'accroissement du trafic). La nécessité de tenir compte du facteur mentionné plus haut s'impose également lors du rentorcement des structures de pont en les adaptant à ces nouvelles conditions de charge. De plus, l'influence de charges réiterées sur les caractèristiques de résistance est loin d'être étudiée, de même qu'il n'y a pas de méthodes accessibles à définir ces caractéristiques.

Dans le but d'établir la nature des phénomènes qui ont lieu dans les sols cohérents sous une charge intermittente une installation a été conçue, permettant d'appliquer sur le sol une charge à différente fréquence et à de différente durée d'action. Les essais ont été effectués dans les conditions qui excluaient toute possibilité d'expansion latérale du sol. Les essais ont été réalisés pour les sols à structure remantée et de différente granulométrie. La déformation résiduelle accumulée, pour une période donnée, sous une charge intermittente, a été caractéristé par un module de tassement e_p (voir formule 3).

On a étudie l'influence de la compacité initiale du sol, de la charge de courte durée, du nombre et de la fréquence de ses applications sur la déformation résiduelle de consolidation. Des échantillons au coefficient de saturation G = 0, 69 à I, 0 ont éte essayés. La fréquence d'application de la charge ν = 15 et 30 appl/mn. La durée d'un cycle de chargement était de 0,2 à I sec. A la suite de nombreaux essais experimentaux nous avons constaté que suivant l'état initial du sol, la deformation de celui-ci est de caractere élaste-résiduel, et au nombre considérable d'applications de la charge, l'accumulation de la déformation résiduelle s'affaiblit vu l'angmentation toujours croissante de la compacité (et, par conséquent, de la résistance) du sol. En outre, il s'est révété que la valeur de la déformation résiduelle de consolidation sous charges cycliques dépendait: de sa compacité initiale, de la charge appliquée, de la fréquence d'application, de la durée du cycle de chargement, ainsi que du nombre de cycles (Fig. 6), l'influence de la fréquence d'application de la charge étant d'autant plus grande que le sol contient plus de grains de sable.

La déformation de consolidation des sols renfermant de l'air, lors du chargement intermittent, est due en partie à la compression de l'air dans les intertices du sol, et en partie à l'échappement de l'eau interstitielle. Les bulles d'air dans le sol comprimées sous une charge de courte durée sont pareilles à des foyers d'accumulation de la tension résiduelle. C'est pourquoi, après le déchargement de ces sols, l'eau continue à s'échapper sans que la déformation ait lieu.

Donc, tout en appliquent sur le sol une charge intermittante (dans les conditions de l'essai de compression) il s'avère possible de prévoir les déformations comme résultat du changement du régimé d'exploitation de l'ouvrage. S'il est nécessaire, on peut représenter, dans le temps, la vitesse d'accumulation du tassement prévu du sol en l'adaptant à la compacité existante du sol, compte tenu du passage des engins plus lourds.

Le problème du rôle du poids propre des sables saturés dans la stabilité dynamique (sismique) de ceux-ci est d'une grande importance pour l'appréciation de la stabilité des talus de grands ouvrages en terre construits dans des régions sismiques et mesurant

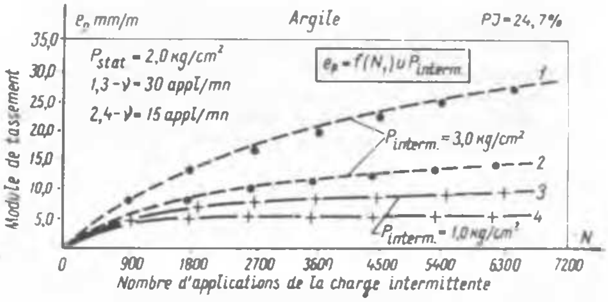

Fig. 6. Représentation du compactage de l'argile comme variation du module de tassement en fonction du nombre d'applications, de la frequence d'applications et de la charge intermittente. La durée d'une seule application de la charge t = I,0 sec. Coefficient d'imbibition du sol G = I,0; densite seche I,59 g/cm3; structure remaniée.

plusieurs dizaines et centaines de mètres de hauteur.

Selon la"théorie de perméabilité dans la stabilité dynamique des sables saturés" (N.N. Maslov, 1953) mise au point il y a un temps en Union Soviétique, la perte de la stabilité des sables saturés, sous l'effet d'un séisme, est provoquée par l'affaiblissement de la résistance au cisaillement de ceux-ci dans les conditions dynamiques. Cet affaiblissement est dû, dans les conditions considérées à l'apparition éventuelle dans les sables de la sous-pression sous l'effet de la charge dynamique (h_z). A cette condition, la résistence au cisaillement des sables non consolidés, sous la charge dynamique, est définie par la rélation:

$$S_{dyn} = / P_{st} - \Delta_e \cdot h_z / \operatorname{tg} \varphi_n \qquad (4)$$

où P_{st} - contrainte normale dans les conditions statiques;

Δ_e - poids spécifique de l'eau;

$\operatorname{tg} \varphi_n$ - angle de frottement interne du sol pour un pourcentage de vides " n ".

La charge dynamique h_z qui apparait dans le sol sableux saturé lors de la consolidation dynamique des sables dans ces conditions, est définie par la relation:

$$h_z = \frac{\nu_n}{K_p} H \cdot Z - \frac{Z^2}{2} \qquad (5)$$

où ν_n - coefficient de consolidation dynamique, égal numériquement à $\frac{dn}{dt}$ et qui témoigne de la vitesse de consolidation (par la diminution des vides " n ") des sables dans le temps sous l'effet dynamique.

K_p - coefficient de perméabilité du sable.

H - épaisseur de la couche sableuse seconée.

t - temps.

Notons que le coefficient de consolidation dynamique ν_n dépend des propriétés du sable, de sa compacité, de la grosseur de grains de même que du caractère et de l'intensité de l'effet dynamique. En même temps, de nombreux essais effectués ont montré que lors de l'application de la charge dynamique de nature sismique (intensité uniforme sur toute l'épaisseur de la couche), le coefficient de consolidation dynamique ν_n est une valeur constante sur l'épaisseur de la couche (ν_{nz} = const) et peut être admis constant en fonction du temps (ν_{nt} = const) pendant un laps de temps qui nous intéresse.

Cependant cette position n'est juste qu'à la condition: $\alpha_{th} > \alpha_{cr}$

où α_{th} - valeur théorique de l'accélération du mouvement oscillatoire.

α_{cr} - accélération critique du mouvement oscillatoire capable de mettre les grains de sable en état d'excitation dynamique.

En même temps, l'accélération critique est, dans le cas général, fonction de la contrainte normale P_n. Il y est tout à fait évident que la charge du poids propre de l'ouvrage, de l'enrochement caractérisé par un coefficient de perméabilité K_p élevé, de même que du poids propre des sables secs recouvrant les sables saturés, peut être considéré comme facteur qui fait augmenter la valeur de α_{cr} et, par conséquent, le degré de la stabilité de la couche de sable saturé sous-jacente.

En même temps, le problème du rôle du poids propre des sables se trouvant au dessous du niveau de l'eau restait non résolu jusqu'à présent.

Pour resoudre ce problème, les auteurs du présent rapport ont réalisé des études correspondants, ayant pour but:

a/ apprécier les conditions de développement dans le temps de la sous-pression dans les sables suivant leur profondeur, en vue de définir l'épaisseur de la zone active passant à l'état d'excitation dynamique;

b/ déterminer la vitesse (v) du développement de la sous-pression dans les sables en fonction de facteurs qui l'influencent;

c/ verifier les formules de relations en vue de pronostiquer le rôle du poids propre et d'en tenir compte dans le comportement des sables saturés sous l'effet sismique.

Pour réaliser ces études il a fallu créer une installation permettant d'effectuer des essais sur échantillons de divers sables de l'épaisseur considérable à consolider.

La particularité caractéristique du dispositif de vibration utilisé dans les essais est la verticalité des vibrations. Cela a permis d'effectuer les essais, à l'accélération donnée, en augmentant l'amplitude des oscillations, de fréquence constante et voisine de celle de la couche saturée (IO à

25 Hz). Cette installation a permis de varier l'amplitude des oscillations de 0,0I mm à I,0 mm, pour la fréquence constante, et a donné la possibilité de varier, dans des conditions convenables, la fréquence de 5 à 30 Hz. Dans ce cas, l'accélération du mouvement oscillatoire a varié de 0,00I g à 0,3 g. Pendant les essais, des dispositifs et un appareillage moderne, tels que: capteurs de vibration, capteurs de pression, oscillographes, etc ont été utilisés.

Dans les études expérimentales on a utilisé des sables de différente grosseur: grossier avec prédominance de grains de 0,5 à 2 mm - 79%, moyen - de 0,25 à 0,5mm - 50 à 70%, et fin - inférieur à 0,25 mm - 57%. Regardons la fig. 7.

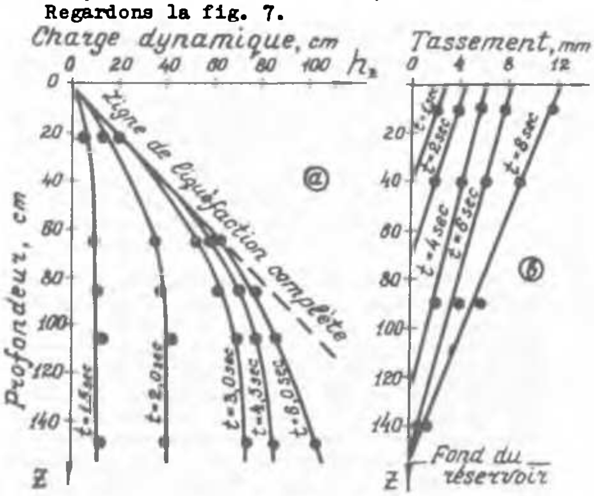

Fig. 7. Variations dans le temps des charges dynamiques (h_z) et du tassement du sable suivant la profondeur. Sable - à grains moyens.

Ici sont données les courbes de relation du type $h_z = f(H,t)$ et $\eta = f(H,t)$ comme exemple de nombreuses courbes construites d'après les mesures de pression dans l'eau effectuées à l'aide de capteurs de pression et de repères spéciaux. La figure 7-a met en évidence un phénomène commun à tous les sols étudiés ne se différant que quantitativement. La charge dynamique ne pourrait atteindre sa valeur maximum possible, à un niveau et à une intensité de secousse donnés, qu'au bout d'un certain temps. Au moment où l'accélération dépasse la valeur α_{cr} , la charge dynamique est limitée, en profondeur, à un certain horizon. Au fur et à mesure de l'augmentation de la charge dans le temps, cet horizon limitant la zone morte (fig. 7-a) s'approfondit de plus en plus. Ce caractère du phénomène a été observé dans tous les essais, et plus nettement dans le cas où le sable était assez consolidé et l'intensité de secousse était assez élevée. Ce phénomène de l'augmentation en profondeur de la zone active Z a été désigné, dans notre pratique, sous le nom de "imprégnation du sol par la dynamique". Le même phénomène a pu être nettement observé également par les tassements des repères disposés à profondeurs différentes du sol (fig. 7-b). Il est évident que le phénomène "imprégnation par la dynamique" est en relation avec l'augmentation de la charge dynamique h_z et est accompagné de la sous-pression provoquée par celle-ci. Le gradient de la charge dynamique, au niveau Z, est défini comme suit:

$$\mathcal{J}_z = \frac{dh_z}{dz} = \frac{\nu_n}{K_p}(H - Z) \qquad (6)$$

La variation de $\mathcal{J}_z$ en fonction de Z est de caractère linéaire. A la surface de la couche de sable, pour Z = 0, le gradient $\mathcal{J}_z$ atteint son maximum. Cela veut dire que les conditions de travail des sables de la couche de surface sont les plus difficiles. Compte tenu de cette circonstance, nous avons:

$$\mathcal{J}_{max} = \frac{\nu_n}{K_p} \cdot H \qquad (7)$$

Au contact de la couche imperméable, le gradient hydrodynamique $\mathcal{J}_z = 0$, ce qui ressort de la relation (6), pour Z = H. Comme on sait, dans les conditions dynamiques, le sable se trouve complètement en suspension pour $\mathcal{J}_z = 1$. De là nous pouvons définir l'horizon critique Z_{cr} à partir duquel et jusqu'à la surface, le sable se trouvera complètement en suspension:

$$Z_{cr} = H - \frac{K_p}{\nu_n} \qquad (8)$$

Admettons que K_p = const et ν_n = const
Désignons par "m" la valeur K_p/ν_n de la relation (8) et nous aurons:

$$Z_{cr} = H - m \qquad (9)$$

Il s'ensuit que, toutes les conditions égales, soit m - const, la profondeur Z_{cr} dépendra de l'épaisseur H de la couche de sable passant lors des secousses, à l'état d'excitation dynamique. Selon nos études, la couche de sable se met en état excité dès la surface en formant dans la couche H deux zones d'épaisseur variable dans le temps t: L -"zone active" et M - "zone morte". La variation de ces valeurs dans le temps est décrite par les conditions suivantes:

Pour	t_0 ...	L_0	... $M = H$
	t_I ...	L_I	$M = H - L_I$
	t_2 ...	L_2	... $M = H - L_2$
	t_n ...	L_n	... $M = 0$

Il est évident que dans notre relation (8) l'épaisseur H correspondra à l'épaisseur de la zone active, soit $L_I, L_2, \dots L_n$
Alors,

$$Z_{cri} = Li - m \qquad (I0)$$

Dans ce cas-là, il y a intérêt à mettre en évidence les facteurs qui puissent influencer la vitesse de développement dans le temps de l'épaisseur de la zone active L_i, car cette épaisseur, dans le cas général, sera définie, dans le temps t (durée de secousse) par la relation

$$L_i = V/f(t)/\ t \qquad (II)$$

où V - vitesse de développement de l'épaisseur de la zone active.

Pour déterminer les valeurs V, nous avons obtenu la relation suivante:

$$V_{zt} = \nu_n (L_t - Z) \qquad (I2)$$

La fig. 8 donne la courbe de relation $V = f(\nu_n)$ construite d'après les résultats d'essais. La valeur ν_n a été déterminée par une méthode connue (6) en s'adaptant à une intensité de secousses et à un état des sables donnés. La variation de la vitesse "d'imprégnation" (V) en fonction de l'épaisseur des sables passant à l'état d'excitation dynamique se laisse voir sur la fig. 9.

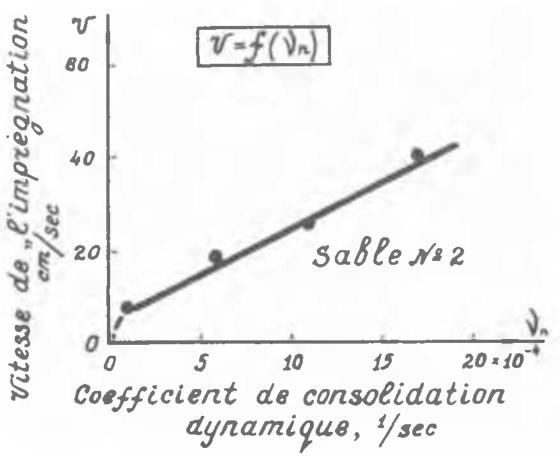

Fig. 8. Courbe de la vitesse de l'enfoncement de la zone active dans le massif sableux (vitesse de "l'imprégnation") en fonction de la valeur du coefficient de consolidation dynamique ν_n.

La cause principale de ces phénomènes est l'augmentation du débit d'eau de drainage, s'écoulant au cours de la consolidation, et son influence grandissnat avec l'augmentation de l'épaisseur de la couche de sables saturée passant à l'état d'excitation dynamique.

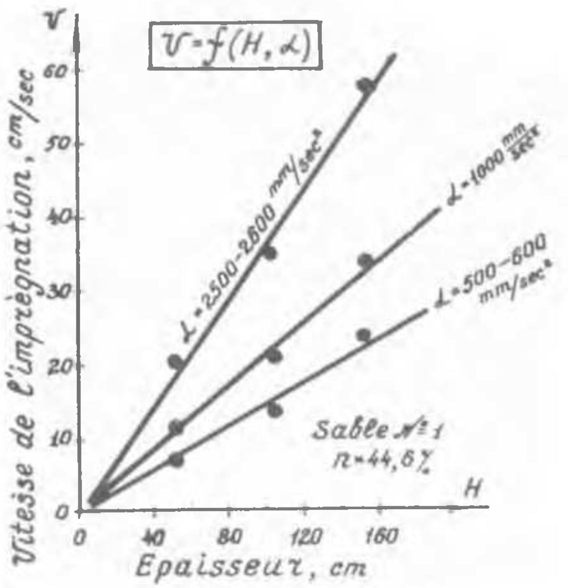

Fig. 9. Relation entre la vitesse de l'expansion en prefondeur de la zone active (vitesse de "l'imprénation") et l'épaisseur totale de la couche sableuse.

Il est évident que la vitesse V_z, à un moment de temps t, sera une valeur variable en profondeur. Dans les conditions considérées du problème, il est intéressant de déterminer la vitesse de drainage, au gradient maximum J_a intervenant pour Z = 0.

Pour ce cas, d'après la relation (I2), nous aurons:

$$V_{tmax} = \nu_n \cdot L_t \qquad (I3)$$

Comme il a été dit plus haut, la vitesse V_{tmax} varie également dans le temps en s'adaptant à la variation, dans ces conditions, de l'épaisseur de la zone active L_t de 0 (pour t = 0) à H (pour t = max). D'où en s'adaptant à la dernière condition:

$$V_{max.max} = \nu_n \cdot H \qquad (I4)$$

Il est évident qu'au moment initial d'un séisme sollicitant le sol, apparaissent des phénomènes assez compliqués dus à la consolidation du sol et à la pression de l'eau drainante sur les grains provoquée par un régime non équilibré. Cette position conduit, sans doute, à accélérer la mise en suspension de grains de sable, et par conséquent, la vitesse d'extension en profondeur de la zone active dans les sables doit augmenter. Donc, la relation (I4) doit être écrit sous forme:

$$V_{tmax} = \xi \cdot \nu_n \cdot H \qquad (I5)$$

où ξ - coefficient tenant compte du régime non équilibré et défini par voie expérimentale.

A cette condition, la valeur L de la zone active, en forme générale, est définie de la relation:

$$L = \xi \cdot V_n \cdot H \cdot t_{seism} \qquad (I6)$$

Ainsi, en conclusion de nos travaux on peut considérer comme déterminé que le poids propre de la couche de sables saturés, lors de la perte de la stabilité sous des charges dynamiques, est d'une importance limitée dans le temps.

Ce phénomène, comme nous l'avons dit, est dûà ce que la charge dynamique d'une certaine intensité appliquée sur le sol fait entrainer toujours de nouveaux horizons (fig.7) dans la zone active, en raison de l'accroissement, dans le temps, de la sous-pression hydrodynamique dans le sol. Selon les études effectuées, on peut admettre, sans prendre de mesures de protection, une perte de la stabilité des sables saturés à des profondeurs de dizaines de mètres, avec une durée suffisante du séisme.

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MASLOV N.N. I958. Problem of high dam density in conditions of seismic activity R. II5, Question N 22, Sixieme Congres des Grands Barrages, New-York.

D. M. MILOVIC (Canada)

I should like to refer to the paper of V.G. Berezantzev et al: "On the Strength of some Soils", Vol I, pp. 11-19.

The authors have shown the variation of cohesion, angle of internal friction and coefficient of lateral pressure of loess soils in function of the water content variation (Fig. 9, eq. 2').

However, on the basis of numerous laboratory shear tests carried out on undisturbed loess samples cut from blocks, and on the basis of field load tests (Clevenger, 1956; Bolognesi and Moretto, 1957; Milovic, 1961; Holtz and Hilf, 1961) it was established that the initial dry density was one of the most important parameters which governed the change in mechanical properties of loess when wetted.

Figs. 1-3 present the variation of some loess properties.

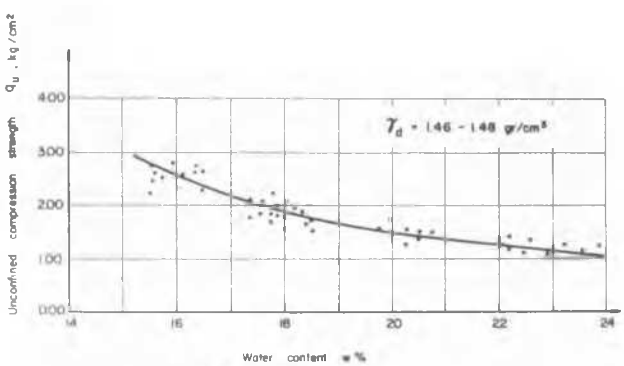

Fig. 1.- Correlation between the unconfined compressive strength q_u and the water content w of the undisturbed samples with the same initial dry density γ_d.

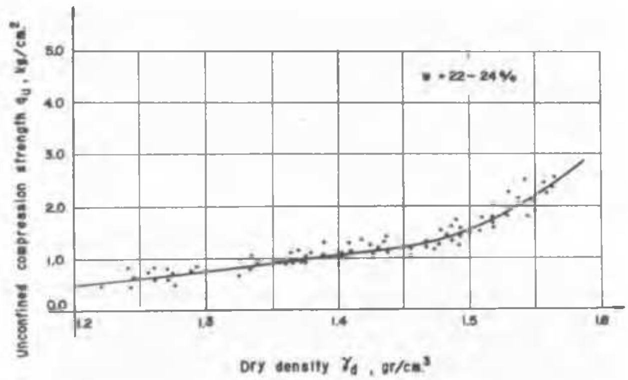

Fig. 2.- Correlation between the strength q_u and the dry density γ_d of the undisturbed samples with the same initial water content w.

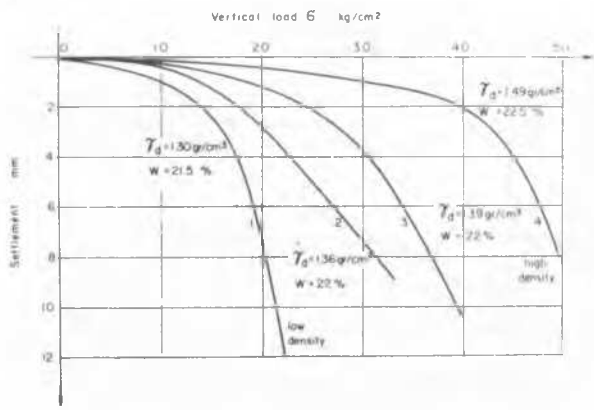

Fig. 3.- Results of the field load tests carried out with 0.7 by 0.7 m load plates. The four curves correspond to four different values of γ_d and each curve represents the average of 3 tests on loess with the same water content.

In other words, it is useful and possible to establish the correlation presented in Fig.9 for each value of γ_d.

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D. M. MILOVIC

Cette discussion se rapporte à la communication "Displacements and Inclinations of Rigid Footings Resting on a Limited Elastic Layer on Uniform Thickness", présentée par I. Sovinc, Vol. I, pp. 385-389.

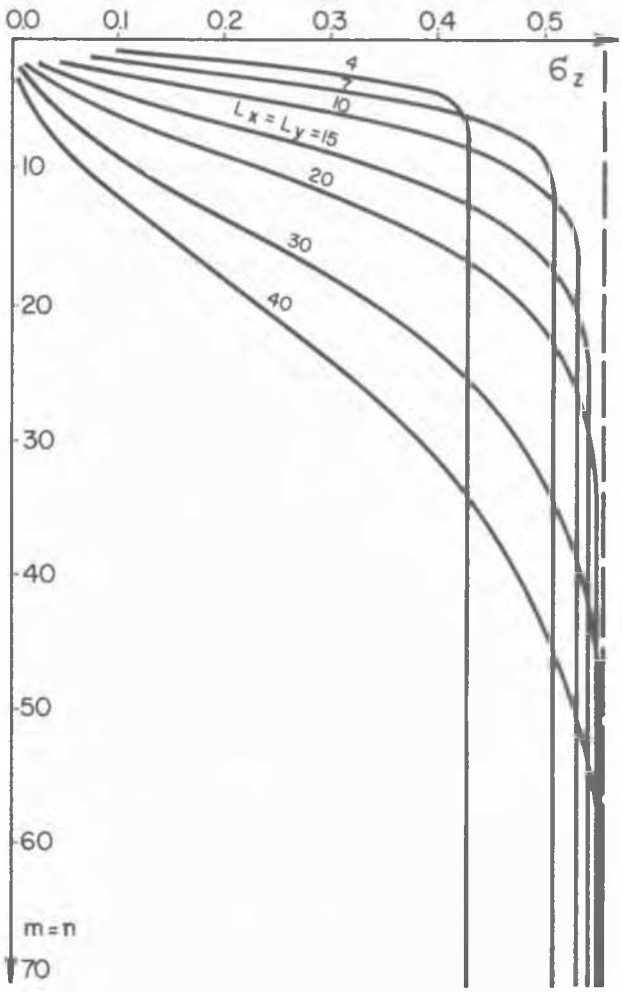

Fig. 1. Rapport nombre d'harmoniques - dimensions de la couche

Dans cette communication le professeur Sovinc traite entre autres, le problème de la détermination des déplacements verticaux d'une fondation rectangulaire et rigide, qui repose sur une couche élastique et isotrope, limitée par une base rigide. Cette étude est faite à la base des doubles séries de Fourier. Cependant l'auteur de cette communication n'a pas donné les dimensions L_x et L_y de la couche compressible, ni le nombre d'harmoniques utilisé dans le calcul des coefficients β, ce qui est très important dans le jugement de la précision des résultats.

J'ai aussi étudié les contraintes et déplacements dans une couche élastique isotrope, puis anisotrope, d'épaisseur limitée à partir des doubles séries de Fourier et je voudrais donner quelques remarques sur l'utilisation de cette méthode.

Fig. 1 illustre certains de mes résultats. Par exemple, si L = 4, dans ce cas 6 harmoniques seulement donnent des résultats qui sont stables mais pas exacts. Les valeurs calculées restent inchangées même pour le nombre d'harmoniques égal à 100.

Par contre, si L est égal à 20, dans ce cas le nombre minimum d'harmoniques qui donne des résultats stables et exacts est de l'ordre de 40.

A la base d'une étude systématique les constatations suivantes ont été faites: d'une part, des résultats précis ne sont obtenus qu'à partir d'une valeur minimale des dimensions L_x et L_y de la couche compressible et, d'autre part, seul le rapport nombre d'harmoniques - dimensions de la couche fait varier la précision des résultats.

Y. NISHIDA (Japan)

Prof. A. Croce, Prof. R. Jappelli, Prof. A. Pellegrino, Dr. C. Viggiani find, in their paper "Compressibility and Strength of Stiff Intact Clays", a linear relation between the compression index, C_c, and in-situ void ratio, e_o. The writer would call their attentions to that the same relationship was already presented by him through some theoretical considerations with many practical data. The compression index seems to be a function of the void ratio where it is measured. Their opinions, that the controlling factor of the compression index is the in-situ void ratio rather than the liquid limit, wL, is entirely agreed with the writer. (Ref. Y. Nishida: A Brief Note on Compression Index of Soil, Proc. Amer. Soc. Civil Eng., Vol. 82, SM3, July 1956). Now the writer studied the compression index by basing on the theory of double difusse layers in colloid chemistry, and he has theoretically obtained the relationship of $C_c = (0.6 \sim 0.8)e$ for fine grain clays as montmolite, which seems to agree with experimental data.

S. OKUSA (Japan)

A soil with weak planes, of cohesion c and internal friction ϕ, fails along the weak planes, before a new fracture cuts the weak planes in the intact soil of cohesion c_o and internal friction ϕ_o, if the angle (α) between the planes and the maximum compressive principal stress lies in the following range in the two dimensional case,

$$\frac{\sin\phi}{\sin(2\alpha+\phi)} \leqq \sin\phi_o \frac{\sigma_m + c_o \cot\phi_o}{\sigma_m + c\cot\phi}$$

where $\sigma_m = (\sigma_1 + \sigma_2)/2$ is the mean normal stress. The range of (α) is independent of σ_m in one of following cases: (a) – $\phi = \phi_o = 0$, (b) $c = c_o = 0$. The condition of failure along the planes is well known as follows,

$$\sigma_1\left[\sin(2\alpha+\phi) - \sin\phi\right] - \sigma_2\left[\sin(2\alpha+\phi)+\sin\phi\right] \geqq 2c\cos\phi,$$

or

$$\sigma_1 \cdot \cos(\alpha+\phi)\sin\alpha - \sigma_2\sin(\alpha+\phi)\cos\alpha \geqq 2c\cos\phi$$

In the case of a three dimensional stress system, failure along the parallel weak planes takes place before the soil itself fails, if direction cosines of the normal to the planes with respect to the three principal axes lie in a certain range.

This range depends on c, c_o, ϕ, ϕ_o and the three principal stresses.

The intermediate principal stress, in the three dimensional case, plays a very important role in the condition of failure along weak planes, and determining the above mentioned range of the direction cosines.

U. SMOLTCZYK (Germany)

Since engineers are generally not very much familiar with continuum theory there seems to open a wide gap between the General Reporter's conclusions and the necessity of every-day engineering practice. Apart from this still much doubt remains about the aid given by continuum mechanics to soil mechanics for two reasons: at first, from a economic point of view it is almost impossible to get such a sufficient knowledge of a natural state of stress as is needed for a 3-dimensional tensor approach of stress-strain analysis, if one would not just go back to that rough and often erratic assumption of an undisturbed general "at rest" state of stress. Second, even when using a simple model of a stable material in a homogenous undisturbed zero state of stress, probably no single-valued constitutive equations for deviator stress increments will exist. The ratio of stress increment and strain increment is different whether the increment is positive or negative, whether principal stress directions change during the process or not and whether the zero state of stress is a result of earlier lateral compression-just to recall some of the more essential factors. Therefore, I am not yet convinced that these fascinating refinements of theoretical mechanics will yield to more reliable results at a certain engineering problem than the rough conventional engineering approaches do, even when excluding such frustrating factors as time or money from our considerations.

As already mentioned by the General Reporter there should as well be research on the engineering side of the problem. My own approach (1) to three-dimensional arching stresses in a soil body which has been exemplified by a paper to session 4 (2) is a kind of generalized earth pressure theory where free parameters of a stress increment tensor are varied such as to maximize or minimize earth pressure at a given point. The soil is considered to remain a continuous body.

The resulting state of stress is in agreement to the three conditions of equilibrium and two independant failure conditions of of the generalized Mohr-Coulomb type. Up to that point, deformation equations are not yet used. They have, of course, to be taken into account as in a conventional settlement analysis when deformations are to be computed for a given stress variation.

References:

(1) Smoltczyk, Stress Computation in Soil Media, Proc.ASCE 93,SM2 (1967),101-124

(2) Smoltczyk,U., Earth Pressure Reduction in Front of a Tunnel Shield, 7.ICSMFE session 4, 473-481.