1/24

On the Movement of Groundwater through Anisotropic Soil

Etude sur l'écoulement dans les terrains anisotropes

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SUMMARY

In most research on seepage carried out in the past, the soil has been considered as a homogeneous and isotropic material. This means that the permeability in both directions, horizontally and vertically, is the same. However, the composition of natural soil shows that in nearly all cases the permeability in the horizontal direction is greater than that in the vertical direction. This phenomenon is known as anisotropy. Because of the difficulties in the analytical treatment of anisotropic soil problems a study was made at the Technische Hochschule of Stuttgart using an electrical analogue. In this paper it will be shown that the results obtained by assuming material to be isotropic are not correct and in many cases favour smaller dimensions. This may reduce the safety of structures. In the present paper the design of the analogue model and its application will be described.

SOMMAIRE

Jusqu'à présent la plupart des études sur l'écoulement ont été établies en général sous l'hypothèse d'un terrain homogène et isotrope, c'est-à-dire d'une perméabilité constante dans toutes les directions. Or on a trouvé dans presque tous les terrains naturels des coefficients de perméabilité différents pour la direction verticale et horizontale (anisotropie du terrain). Etant donné que l'étude théorique de ces problèmes se montre difficile et de longue durée, on a utilisé à la Versuchsanstalt für Grundbau und Wasserbau der Technischen Hochschule, Stuttgart, une méthode d'analogie électrique, qui permet d'introduire les différents coefficients de perméabilité et d'étudier leur influence. Les résultats obtenus montrent que l'hypothèse d'un terrain isotrope fournit en général des valeurs trop favorables pour le dimensionnement d'un ouvrage. L'exposé suivant décrit le système de montage et l'application de la méthode.

IN MOST STUDIES OF SEEPAGE PROBLEMS made in the past, the assumption is made that the soil under consideration is a homogeneous isotropic mass; i.e., a mass that exhibits essentially the same permeability properties in every direction. However, this is generally not the case, because soil by formation, form of grains, and consolidation has different coefficients of permeability for the vertical and horizontal directions. In other words natural soil is anisotropic. According to the experiments of Casagrande (1935) the quotient k_x/k_z ranges between 2 and 10.

An analytical treatment of isotropic seepage areas is possible by application of the potential theorem and Laplace's differential equation. In case of two-dimensional flow this gives

$$\partial^2\Phi/\partial x^2 + \partial^2\Phi/\partial z^2 = 0 \qquad (1)$$

where $\Phi(x, z)$ is the potential function. Any point on one of the equipotential lines has the same value. This is also the case for the stream function $\Psi = \Psi\ (x, z)$ which satisfies Laplace's equation too. Applying the Cauchy-Riemann differential equations it can be proved that the flow lines and equipotential lines form a grid of curves wherein all intersections are at right angles. Therefore Laplace's differential equation represents an interchangeable network of orthogonals. Thus we are able to obtain a picture of the groundwater flow in an isotropic mass by integration of the Laplace equation.

However, in order to do this one has to know and be able to analyse the boundary conditions. In many cases the mathematical analysis of these conditions is very difficult and time-consuming. For this reason model tests are quite common, using filters with coloured probes, flow tank, Hele-Shaw model, or electrical analogue.

ANISOTROPIC SOIL

In the case of anisotropic soil, Laplace's equation of the form (1) is not applicable any more. A differential equation has to be substituted which takes into consideration that the coefficient of permeability for the horizontal (k_x) and the vertical (k_z) is different. This gives

$$k_x\partial^2\varphi/\partial x^2 + k_z\partial^2\varphi/\partial z^2 = 0. \qquad (2)$$

Since in this equation the stream lines and the equipotential lines no longer represent an interchangeable orthogonal network, a solution of this problem is very difficult. At the Versuchsanstalt für Grundbau und Wasserbau, Technische Hochschule, Stuttgart, a method is used which has been proved to be of value.

THE TEST EQUIPMENT

The method was originally developed by Pavlovsky (1933) and is known in the literature as the electrical analogue in which the correspondence between the steady-state flow of water through porous media (Darcy's law) and the flow of electric current in a conductor (Ohm's law) has been established.

Considering a natural case of a seepage area, a geometric similar electrical analogue model enables us to simulate the boundary conditions and to find the equipotential lines and the flow lines. However, for tests considering anisotropic soil, the copper foil or conducting papers generally used as a conductive media would not be suitable, since their conductivity for the x and z axes can not be varied. In other words conductivity in both the x and the z direction would be the same. On the other hand, by using a set of single resistors of different capacities for the vertical and horizontal directions

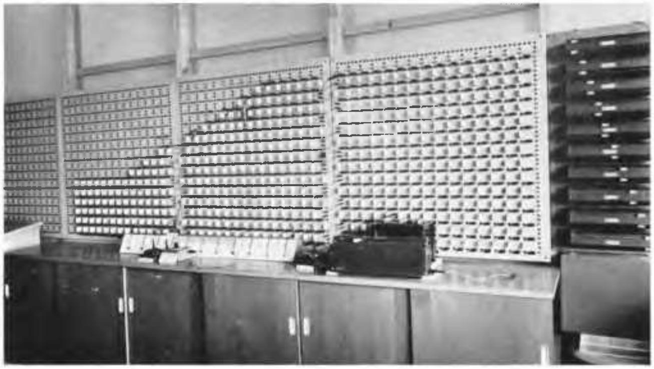

FIG. 1. Equipment of the electrical analogue.

the conductivity could be varied according to the permeability of the natural condition (La Marre and Huard, 1953).

Sheet-metal walls are used to house the sockets for the resistor plugs. Five of the sockets are connected by silver wire, forming a net knot. The resistors are then plugged in, according to the natural geometric shape of the seepage area (Fig. 1). As resistors one can use normal resistors with a capacity ranging from 10, 100, 500, to 1000 Ω having an accuracy of ±0.5 per cent (see Fig. 2). To prevent confusion the resistor plugs should be coloured according to their respective capacities. By putting the necessary resistors in place, the net is completed forming a grid of single meshes. The specific resistance of one mesh can easily be calculated from R by using Kirchhof's laws, where R represents the capacity of a single resistor in ohms.

FIG. 2. Resistors and plugs.

To simulate anisotropic soil, resistors of different capacities have to be used. However attention must be paid to the fact that the coefficient of permeability k in the electrical analogue is in accordance with the ratio of the conducting power $1/\rho$ . . . e.g., in the case of an anisotropic soil with the coefficients of permeability k_x and k_z, the resistors have to be reciprocally proportional to the permeability, that is, $1/k_x$ and $1/k_z$ respectively.

MODEL AND BOUNDARY CONDITIONS

With the grid system and the necessary number of resistors, every variation of seepage area could be set out, if:

1. The geometrical form of the resistor area in the model conforms to the natural seepage area.
2. The resistors correspond to the coefficient of permeability of the natural soil.
3. The boundary conditions are the same in model and nature.

No difficulties should be encountered in obtaining geometric similarity. However to achieve duplication of the boundary conditions a trial and error method might be necessary, especially in cases with free surfaces.

The possible boundary conditions and their analytical treatment may be obtained in the following manner (see Fig. 3).

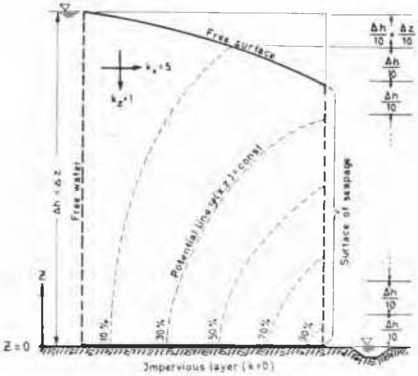

FIG. 3. Possible boundary conditions at a seepage area.

1. *Impermeable layer* (bottom limit). The impermeable layer at the bottom is identical with a special (bottom) flow line.

2. *Free surface* (top limit). Starting with Bernoulli's equation but neglecting the seepage velocity one might write

$$h = p/\gamma + z \qquad (3)$$

and by considering the fact that the atmospheric pressure only acts on the free surface, we have $p = 0$ and can write $h = z$. This means that on the free surface the potential height h corresponds to a certain topographical height z or a certain potential loss Δh corresponds to a certain loss of topographical height Δz, therefore

$$\Delta h = \Delta z. \qquad (4)$$

3. *Free water* (left border of the model). Boundary between flow area and free water with hydrostatic distribution of pressure. Here also is the relationship of $h = p/\gamma + z = \text{const.}$ In such a case the boundary is a potential line, because the piezometric height of all points on this line are the same.

4. *Surface of seepage* (right border of model). Boundary area between solid mass and free atmosphere. As in 2 (free surface) we can write $p = 0$ and

$$\Delta h = +\Delta z. \qquad (4)$$

This means that the arbitrarily chosen number of potential intervals Δh corresponds to the same number of distances Δz of equal length at the surface of seepage.

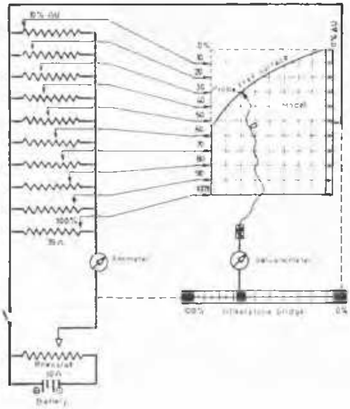

FIG. 4. Potential circuit.

THE ELECTRICAL CIRCUIT

The circuit must be laid out in such a way that the boundary conditions 1 to 4 can be simulated. To accomplish this, a potential circuit is necessary (Fig. 4) to find the free surface (top limit) by the step-by-step method.

In the case of flow problems without a free surface (for instance free standing-sheet piling with flow around it) the electrical circuit becomes much simpler (Fig. 5).

THE MEASURING APPARATUS

In order to find the potential lines a certain current loss ΔU, say 30 per cent, has to be set up on the Wheatstone bridge. With the aid of a probe all points with the potential 30 per cent ΔU can be found, because at all points where the galvanometer indicates zero, the current flow is zero. This in turn means that the potential on the grid point and bridge is the same. Since the potential lines in only a very few cases run exactly through grid points a second 10,000-ohm resistor bridge has been employed (Fig. 6, right) to find the correct point by electrical interpolation. The potential lines are obtained by connection of the single points on a sketch. By reversal of the boundary conditions the flow lines can be obtained in the same manner if necessary.

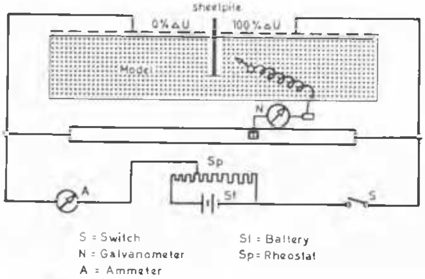

FIG. 5. Circuit for problems without free surface.

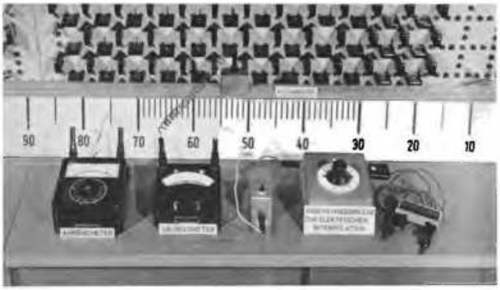

FIG. 6. The measuring apparatus.

EXAMPLES OF APPLICATION

The application of the analogue method in the case of anisotropic soil will be discussed in the following examples. The influence of the different coefficients of permeability $n = k_r/k_z$ will be demonstrated on the seepage area shown on Fig. 7a with a side ratio of 13/18, assuming the bottom to be impermeable.

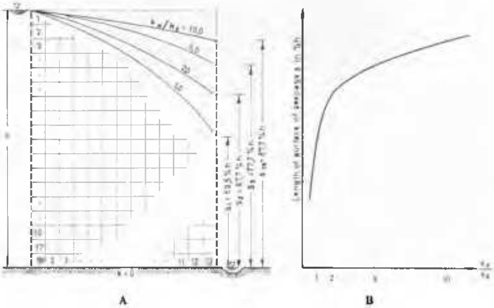

FIG. 7. Influence of ratio k_x/k_z on the length of the surface of seepage.

In case of an isotropic material ($n = 1.0$) the surface of seepage has a length of $s_1 = 50.5$ per cent of h. Considering an anisotropic material of $n = 2.0$ the length is $s_2 = 63.7$ per cent of h, producing a flatter slope of the free surface than in the first case. This inclination further decreases if the ratio n becomes still larger: for example $n = 5.0$, $s_5 = 79.7$ per cent of h and $n = 10.0$, $s_{10} = 87.7$ per cent of h (see also Fig. 7b). That means that the inclination of the free surface is not altered much if n is increased to 100 or 1,000. The same tendency was found in a theoretical study on another seepage problem made by Giesecke (1960).

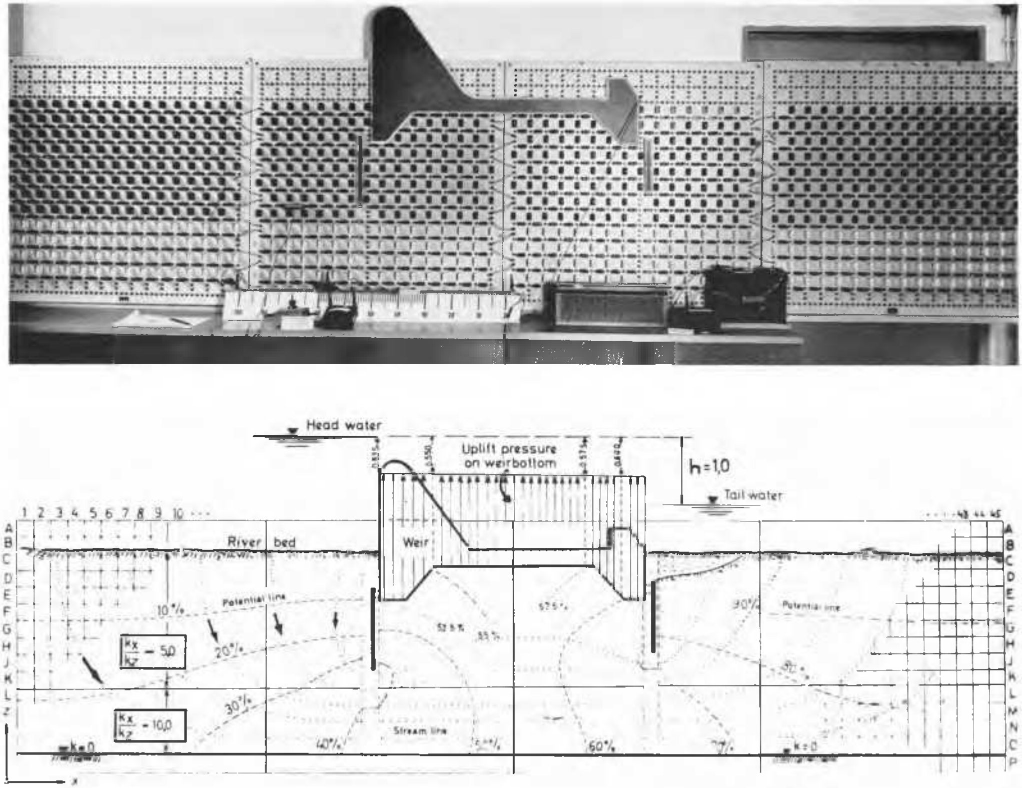

FIG. 8. Flow and potential lines in anisotropic material under a sill.

The foundations of sills present another problem in hydraulic structures, which can be investigated with the electrical analogue. In such cases the following factors are of interest: (1) uplift pressure on the foundation, (2) safety against hydraulically caused ground failure, (3) loss of water through underflow, and (4) seepage velocity at certain points of the structure. These factors can be obtained rather accurately in the above-described analogue model because the anisotropy of the soil can be reproduced. Fig. 8 demonstrates such a case. In the geometrically similar model the top layer has a ratio $n = k_x/k_z = 5.0$ (horizontal resistors 100 Ω, vertical resistors 500 Ω). The bottom layer has a ratio $n = k_x/k_z = 10.0$ (resistors 100 and 1,000 Ω). The distribution of the potential lines and the flow lines is shown on Fig. 8 (bottom). The results can be analyzed in accordance with different known methods (Davidenkoff, 1955; Zweck and Davidenkoff, 1957). As soon as the potential and flow lines are located, the questions concerning the safety and underflow of the structure can be answered.

The method could also be applied to problems of stability of bank slopes protected by impermeable asphaltic revetments (Marotz, 1964). In this paper, the author points out that anisotropic material creates unfavourable conditions when compared with isotropic material.

CONCLUSIONS

In conclusion it can be said that, with the aid of an electrical analogue, almost all of the soil conditions occurring in nature can be reproduced independently from the ratio of permeability in the horizontal and vertical directions. The choice of the resistors used in the analogue depends on the coefficients of permeability in the *x* and *z* directions. They must be obtained from a soil laboratory test. Using the distribution of the potential lines achieved through the model tests, conclusive statements about the hydraulic conditions and the safety of the structure can be drawn. It should be emphasized here, that without considering the fact of anisotropic layers in the soil the factor of safety could be less than assumed.

The analogue method described above can also be used to investigate problems of agricultural irrigation, river training, coastal protection works and questions of ground water flow. At the Institute of Soil Mechanics and Hydraulics at Stuttgart, further studies are under way to investigate problems of unsteady flow by using the electrical analogue. This would open up a new field of application, to supply the planning engineer with reliable data thus contributing to the safety of hydraulic structures.

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